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M.E./M.Tech. Degree Examinations, January 2017
First Semester

COMMUNICATION SYSTEMS

MA16182 – APPLIED MATHEMATICS FOR COMMUNICATION ENGINEERS

(Regulation 2016)

QP Code:996058

Time: Three hours

Maximum : 100 marks

Answer ALL questions

PART A - (10 X 2 = 20 Marks)

1. Define Vector space.
2. Write some applications of Toeplitz matrices.
3. Define slack and surplus variables
4. What is an assignment problem?
5. Write the algorithm for fourth order R.K .method.
6. What are the steps involved in shooting method?
7. If X and Y are independent random variables with variance 2 and 3 respectively. Find the variance of $3X + 4Y$.
8. Distinguish between correlation and regression.
9. What is the average waiting time of a customer in the 3 server infinite capacity Poisson queue if he happens to wait, given that $\lambda = 6$ per hour and $\mu = 4$ per hour.
10. Define Little's formula.

PART B - (5 X16 = 80 Marks)

11. (a) Construct a QR decomposition for the matrix $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (16)

(OR)

(b) Determine the Cholesky decomposition for $A = \begin{bmatrix} 16 & -3 & 5 & -8 \\ -3 & 16 & -5 & -8 \\ 5 & -5 & 24 & 0 \\ -8 & -8 & 0 & 21 \end{bmatrix}$ (16)

12. (a) Find an initial basic feasible solution of the transportation problem. (16)

		Destination				supply
		D ₁	D ₂	D ₃	D ₄	
Source	S ₁	3	2	4	1	20
	S ₂	2	4	5	3	15
	S ₃	3	5	2	6	25
	S ₄	4	3	1	4	40
	Demand	30	20	25	25	100

(OR)

(b) Minimize: $Z = x_1 - 3x_2 + 3x_3$ (16)

Subject to $3x_1 - x_2 + 2x_3 \leq 7$

$2x_1 + 4x_2 \geq -12$

$-4x_1 + 3x_2 + 8x_3 \leq 10, x_1, x_2, x_3 \geq 0$

13. (a) (i) Using shooting method, solve the BVP (8)

$\frac{d^2 y}{dx^2} = y + 1, 0 < x < 1, y(0) = 0, y(1) = e - 1.$

(ii) Given $\frac{dy}{dx} = x^3 + y, y(0) = 2.$ Compute $y(0.2)$ and $y(0.4)$ by Runge-Kutta method of fourth order. (8)

(OR)

(b) (i) Using collocation method, Find the approximation solution of $y'' + y + x = 0$ subject to the boundary condition $y(0) = y(1) = 0.$ Also compare your result with exact value. (8)

(ii) Given $\frac{dy}{dx} = x^2(1 + y), y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979,$ evaluate $y(1.4)$ by Adams-Bashforth method. (8)

14. (a) (i) If X and Y are two random variables having joint probability density (8)

function $f(x, y) = \begin{cases} cx(x - y), & 0 < x < 2, -x < y < x \\ 0 & \text{otherwise} \end{cases}$

Find (i) c, (ii) $f_x(x)$, (iii) $f_y(y)$ and (iv) $f_{y/x}(y/x).$

- (ii) Two regression lines are $4x - 5y + 33 = 0$ and $20x - 9y = 107$. The variance of x is 9. Find (i) The mean values of x and y . (ii) Correlation coefficient between x and y . **(8)**

(OR)

- (b) (i) If two random variables X and Y have the joint probability density **(8)**

$$f(x, y) = \begin{cases} 2 - x - y, & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Show that } \text{Cov}(X, Y) = \frac{-1}{144}$$

- (ii) If X and Y are independent random variables with **(8)**

$$f(x) = e^{-x}; x > 0 \text{ and } f(y) = 3e^{-3y}; y > 0, \text{ find } f_z(z), \text{ if } Z = \frac{X}{Y}.$$

15. (a) (i) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is **(8)**

(i) more than 1 min, (ii) between 1 min and 2 min and (iii) 4 min or less.

- (ii) The transition probability matrix of a Markov chain $\{X_n\}$, $n = 1, 2, 3, \dots$ **(8)**

$$\text{having 3 states 1, 2 and 3 is } P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix} \text{ and the initial}$$

distribution is $P^{(0)} = (0.7, 0.2, 0.1)$.

Find (i) $P\{X_2 = 3\}$ and (ii) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

(OR)

- (b) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour. **(16)**

- a) What fraction of the time all the typists will be busy?
- b) What is the average number of letters waiting to be typed?
- c) What is the average time a letter has to spend for waiting and for being typed?
- d) What is the probability that a letter will take longer than 20 min waiting to be typed and being typed?