Registration No. | | | | | | | | | |

M.E./M.Tech. Degree Examinations, January 2017

First Semester

COMPUTER SCIENCE ENGINEERING

MA16187 –APPLIED PROBABILITY AND STATISTICS

(Regulation 2016)

(Statistical tables are Permitted)

QP Code: 883807

Time: Three hours **Maximum : 100 marks**

Answer **ALL** questions

PART A - (10 X 2 = 20 Marks)

1. Let X be a random variable with E(X) = 1, and E[X(X-1)] = 4. Find Var X and Var (2-3X).
2. The moment generating function of a random variable X is given by $M_X(t) = e^{3(e^t-1)}$ Find P(X=1).
3. If two random variables X and Y have probability density function $f(x,y) = ke^{-(2x+y)}$ $, for \; x, y > 0,$ evaluate k.
4. The regression lines between two random variables X and Y is given by 3X+Y=10 and 3X+4Y=12. Find the coefficient of correlation between X and Y.
5. Let $X_1, X_2, ..., X_n$ represent a random sample of service time of n customers at a certain facility where the underlying distribution is assumed exponential with parameter λ .Find the moment estimator λ .
6. State the principle of least squares.
7. Define critical region and level of significance.
8. State the uses of χ^2 -distribution.
9. Define random vectors and random matrices.
10. Let $X_{(3x1)}$ be $N_3(\mu, \Sigma)$ with $\Sigma = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Are $X_1 \; and \; X_2$ independent?

PART B - (5 X16 = 80 Marks)

11. (a) (i) In a continuous distribution, the probability density is given by $f(x) = k \times x \times (2-x), for \; 0 < x < 2$. Find k, mean, variance and distribution function. **(8)**

(ii) Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls (ii) at least 1 boy (iii) at most 2 girls and (iv) children of both genders. Assume equal probabilities for boys and girls. **(8)**

(OR)

(b) (i) Describe Gamma distribution, Obtain its MGF. Hence or otherwise compute the first four moments. **(8)**

(ii) The saving bank account of a customer showed an average balance of Rs.150 and a standard deviation of Rs.50. Assuming that the account balance is normally distributed (i) What percentage of account is over Rs.200? (ii) What percentage of account is between Rs.120 and Rs.170? (iii)What percentage of account is less than Rs.75? **(8)**

12. (a) (i) Let X_1 *and* X_2 have the joint probability distribution $f(x_1, x_2)$ *of* X_1 *and* X_2 **(8)**

$X_2 \backslash X_1$	0	1	2
0	0 .1	0.4	0.2
1	0.2	0.2	0

(a) Find $P(X_1 + X_2 > 1)$ (b) Find the probability distribution $f_1(x_1) = P(X_1 = x_1)$ of the individual random variable X_1.

(ii) The joint density function of the random variable (X, Y) is given by **(8)**

$$f(x, y) = \begin{cases} 8xy, & 0 < x < 1,\ 0 < y < x \\ 0\ , & elsewhere \end{cases}$$

Find the (i) Marginal density of Y
(ii) Conditional density of X/Y=y and (iii) P(X<1/2).

(OR)

(b) (i) The joint probability density function of the two dimensional random Variable (X,Y) $f(x, y) = \begin{cases} x + y, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0\ , & otherwise \end{cases}$. **(8)**

Find the correlation coefficient between X and Y.

(ii) If X and Y are independent random variables, with p.d.f **(8)**

$f(x) = e^{-x},\ x \ge 0\ ; f(y) = e^{-y},\ y \ge 0.\ show that\ U = \dfrac{X}{X+Y}\ and\ V = X + Y$

are independent.

13. (a) (i) Let $X_1, X_2, ..., X_n$ be a random sample from a distribution with mean μ and variance σ^2, then prove that the estimator $\hat{\sigma}^2 = s^2 = \dfrac{\sum (X_i - \overline{X})^2}{n-1}$ is unbiased for estimating σ^2. **(8)**

(ii) For random sampling from a normal population, find the maximum likelihood estimators for (a) the population mean, when the population variance is known (b) the population variance, when the population mean is known. **(8)**

(OR)

(b) (i) Fit a parabola of second degree $y = a + bx + cx^2$ for the data **(8)**

x	0	1	2	3	4
y	1	1.8	1.3	2.5	2.3

(ii) Marks obtained by 10 students in Mathematics(x) and statistics(y) are given below **(8)**

x:	60	34	40	50	45	40	22	43	42	64
y:	75	32	33	40	45	33	12	30	34	51

Find the two regression lines. Also find y when x=55

14. (a) (i) A manufacturing firm claims that its brand A product outsells its brand B product by 8%. If it is found that 42 out of sample of 200 persons prefer brand A and 18 out of another sample of 100 persons prefer brand B. Test whether the 8% difference is a valid claim. **(8)**

(ii) Two researchers A and B adopted different techniques while rating the student's level. Can you say that the techniques adopted by them are significant? **(8)**

Researchers	Below average	Average	Above average	Genius	Total
A	40	33	25	2	100
B	86	60	44	10	200
Total	126	93	69	12	300

(OR)

(b) (i) Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables **(8)**

Sample I:	18	13	12	15	12	14	16	14	15
Sample II:	16	19	13	16	18	13	15		

Do the estimates of the population variance differ significantly at 5% level?

(ii) The following table gives the values of protein from Kangeyam cow's milk and buffalo's milk. Examine if these differences are significant. **(8)**

Cow's milk:	1.90	1.95	2.00	2.02	1.85	1.80
Buffalo's milk:	2.12	2.00	2.20	2.45	2.20	2.10

15. (a) (i) Find the covariance matrix for the two random variables X_1 *and* X_2 when their j.p.f is represented in the following table **(8)**

X_1	X_2	
	0	1
-1	.06	.24
0	.14	.16
1	.00	.40

(ii) Let X have covariance matrix $\Sigma = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$ **(8)**

Determine ρ *and* $V^{1/2}$.

(OR)

(b) (i) If X is distributed as $N_5(\mu, \Sigma)$, find the distribution of $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$. **(8)**

(ii) Explain the population principal components in details. **(8)**