B.E./B.TECH. Degree Examination, December 2020

Fourth Semester

MA16453-PROBABILITY AND QUEUEING THEORY

(Regulation 2016)

Time: Three hours Maximum: 80 Marks

Answer **ALL** questions

PART A - (8 X 2 = 16 marks)

1. If a continuous random variable X has pdf $f(x)=3x^2,\ 0\le x\le 1$, then $P(0.2\le X\le 0.5)$ is
   (a) 0.117
   (b) 0.217
   (c) 1.117
   (d) 0.107
2. If X has mean 4 and variance 9, while Y has mean -2 and variance 5, and two random variables are independent then $E(XY^2)$ is
   (a) 30
   (b) 36
   (c) 45
   (d) -8
3. Consider a Markov chain with two states and transition probability matrix is
   $$P=\begin{bmatrix}3/4 & 1/4\\ 1/2 & 1/2\end{bmatrix}.$$
   Find the stationary probabilities of a Markov chain.
   (a) $\pi_1=0.25,\ \pi_2=0.75$
   (b) $\pi_1=0.666,\ \pi_2=0.333$
   (c) $\pi_1=0.75,\ \pi_2=0.25$
   (d) $\pi_1=0.5,\ \pi_2=0.5$
4. Customers arrive at a watch repair shop according to Poisson distribution at a rate of one per every 10 minutes and the service time is an exponential random variable with mean 8 minutes. Find the average number of customers in the shop.
   (a) 2
   (b) 3
   (c) 4
   (d) 5

5. The time required to complete a work is an exponentially distributed random variable with $\lambda = 1/2$. What is the probability that time taken to complete the work exceeds 2 hours?

6. The joint probability mass function of two random variable X and Y is given by

$$P_{X,Y}(x,y) = \begin{cases} k(x+2y) & x = 1,2; y = 1,2 \\ 0 & otherwise \end{cases}$$ where k is constant. Find the value of k?

7. A hospital receives an average of 3 emergency calls in a 10-minute interval. What is the probability that there are at the most 3 emergency calls in 10-minute interval?

8. Customers arrive at a one-man barber shop according to Poisson process with a mean inter arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber's chair. If an hour is used as the unit of time, then what is the expected number of customers in the barber shop?

PART B - (4 X16 = 64 marks)

09. (a) (i) If the cumulative distribution function of a random variable X is given **(8)**

by $F(x) = \begin{cases} 1 - \dfrac{4}{x^2}, & x > 2 \\ 0, & x \le 2 \end{cases}$

Find (i) P (X < 3) (ii) P (4 < X < 5) (iii) P (X ≥ 3).

(ii) Suppose that a trainee soldier shoots a target in an independent fashion. **(8)**
The probability that the target is hit on any one shot is 0.7.

(a) What is the probability that the target would be hit on 10^{th} attempt?

(b) What is the probability that it takes him less than 4 shots?

(c) What is the probability that it takes him an even number of shots?

(OR)

(b) (i) It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing at least, exactly, at most 2 defectives in a consignment of 1000 packets using Poisson. **(8)**

(ii) The number of personal computer (pc) sold daily at a computer world is uniformly distributed with a minimum of 2000 pc and a maximum of 5000 pc. **(8)**

(1) Find the probability that daily sales will fall between 2500 and 3000 pc?

(2) What is the probability that the computer world will sell at least 4000 pc?

(3) What is the probability that the computer world will sell exactly 2500 pc?

10. (a) (i) The joint pdf of a two-dimensional random variable (X, Y) is given by **(8)**

$$f(x,y)=\begin{cases} x^2+\frac{xy}{3}: & 0<x<1;\ 0<y<2 \\ 0 \quad : & otherwise \end{cases}.$$

Compute $(i) P\left[X>\frac{1}{2}\right]$ $(ii) P[Y>1]$ $(iii) P[Y<X]$

(ii) If the joint pdf of (X, Y) is given by $f_{XY}(x,y)=e^{-(x+y)}$; $x\geq 0$, $y\geq 0$, **(8)**

Find the pdf of U=(X+Y)/2

(OR)

(b) (i) The joint pdf of random variable (X, Y) is given by **(8)**

$f(x,y)=k\,x\,y\,e^{-\left(x^2+y^2\right)}, x>0, y>0$.

(i) Find the value of k.

(ii) Show that X and Y are independent.

(ii) Find the correlation co-efficient for the following data **(8)**

X	78	89	97	69	59	79	61	61
Y	125	137	156	112	107	136	123	108

11. (a) (i) Show that the random process $X(t)=A\sin(\omega t+\theta)$ is WSS where A and ω are constants and θ is uniformly distributed in $(0,2\pi)$ **(8)**

(ii) A radioactive source emits particles at a rate of 5 per minute according to a Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in a 4-minute period. **(8)**

(OR)

(b) (i) Let $\{X_n : n = 1,2,3..\}$ be a Markov chain on the space S= {1,2,3} with **(8)**

one step TPM $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix}$

(i) Sketch the transition diagram.

(ii) Is the chain irreducible? Explain.

(iii) Is the chain being Ergodic? Explain.

(ii) A man either drives a car or catches a train to get to office each day. He never goes 2 days in a row by train, but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work iff 6 appeared. **(8)**

Find (i) The probability that he takes a train on the third day.

(ii) The probability that he drives to work in the long run.

12. (a) (i) Arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 minutes **(8)**

(a) Find the average number of persons waiting in the system.

(b) What is the probability that a person arriving at the booth will have to wait in the queue?

(c) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call?

(ii) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour. **(8)**

(a) Find the effective arrival rate at the clinic.

(b) What is the probability that an arriving patient does not have to wait?

(c) What is the expected waiting time until a patient is discharged from the clinic?

(OR)

(b) (i) A petrol pump has 4 pumps. The service times follow the exponential distribution with mean of 6 minute and cars arrive for service is a Poisson process at the rate of 10 cars per hour. Find the probability that a customer has to wait for service. What is the probability that the pumps remain idle? **(8)**

(ii) A two-person barber shop has 5 chairs to accommodate waiting customers. Potential customers, who arrive when all 5 chairs are full, leave without entering shop. Customers arrive at average rate of 4 per hour and spend average of 12 minutes in the barber chair. Compute p_0, p_1, p_7. **(8)**