Q. Code: 705170

Reg. No. | | | | | | | | | |

M.E. / M.TECH. DEGREE EXAMINATIONS, DEC 2019

First Semester

MA18181 – APPLIED MATHEMATICS FOR ENGINEERS

(Common to AL, CU & PD)

(Regulation 2018)

Time: Three Hours **Maximum : 100 Marks**

Answer **ALL** questions

PART A - (10 X 2 = 20 Marks)

No.	Question	CO	RBT
1.	State any two properties of generalized inverse of a matrix.	1	R
2.	Are the vectors $\left\{\begin{pmatrix}1\\1\\2\end{pmatrix},\begin{pmatrix}3\\7\\8\end{pmatrix},\begin{pmatrix}18\\17\\22\end{pmatrix}\right\}$ linearly independent?	1	AP
3.	Define artificial variable in L.P.P.	2	R
4.	Obtain the initial basic feasible solution for the following transportation problem by the North West Corner Rule:	2	AP

		Distribution Centre				
		I	II	III	IV	
Plant	1	10	0	20	11	10
	2	12	7	9	20	25
	3	0	14	16	18	5
		5	15	5	5	

No.	Question	CO	RBT
5.	Write the finite difference equation corresponding to differential equation $y''(x) - y(x) = 0$	3	AP
6.	Using Modified Euler's method, compute $y(0.1)$ from $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1$	3	AP
7.	Let X and Y be two discrete random variable with joint pmf $P[X=x, Y=y] = \begin{cases} \frac{x+2y}{18}, & x=1,2;\ y=1,2 \\ 0 & ,otherwise \end{cases}$. Find the marginal pmf of X.	4	AP
8.	If X_1 has mean 4 and variance 9 while X_2 has mean -2 and variance 5 and the two are independent, find $Var(2X_1 + X_2 - 5)$.	4	AP
9.	What are the basic characteristics of a queuing system ?	5	R
10.	In a self-service system if arrivals are according to a Poisson process at a rate of 50 per hour and the service time is exponential with mean 1 minute, find the expected queue waiting time.	5	AP

PART B - (5 X16 = 80 Marks)

11. (a) Find the characteristic equation, eigen values, eigen vectors and a maximal set of linearly independent eigen vectors for $A = \begin{pmatrix} 5 & 2 & 2 \\ 3 & 6 & 3 \\ 6 & 6 & 9 \end{pmatrix}$ **(16)** **1** **AP**

(OR)

(b) Find the least square solutions of the following system of equations: **(16)** **1** **AP**

$$2x_1 + 2x_2 - 2x_3 = 1$$
$$2x_1 + 2x_2 - 2x_3 = 3$$
$$-2x_1 - 2x_2 + 6x_3 = 2$$

12. (a) **(16)** **2** **AP**

The RM company owns a paint factory that produces both interior and exterior house paints for wholesale distribution. Two basic raw materials A and B are used to manufacture the paints. The maximum availability of A is 6 tonnes a day and that of B is 8 tonnes a day. The daily requirements of the raw materials per tonne of interior and exterior paints are summarized below:

Raw Material	Ton of Raw material per ton of paint	
	Exterior Paint	Interior Paint
Raw Material A	1	2
Raw Material B	2	1

A market survey has established that the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton. The survey also shows that the maximum demand for interior paint is limited to 2 tonnes daily. The wholesale price per ton is \$3000 for exterior paint and \$2000 for interior paint. How much interior and exterior paints should the company produce daily to maximize gross income? Solve using Simplex method.

(OR)

(b) Solve the following assignment problem: **(16)** **2** **AP**

11	17	8	16	20
9	7	12	6	15
13	16	15	12	16
21	24	17	28	26
14	10	12	11	13

13.	(a)	(i)	Use Adam-Bashforth's method to evaluate $y(1.4)$ given $\frac{dy}{dx} = xy, y(1) = 1, y(1.1) = 1.1107, y(1.2) = 1.2461,$ $y(1.3) = 1.412$	**(8)**	**3**	**AP**
		(ii)	Solve $y'' + y + x = 0, y(0) = y(1) = 0$, using Galerkin method	**(8)**	**3**	**AP**
			(OR)			
	(b)	(i)	Using Orthogonal collocation method find the approximate solution of $y'' + y + x = 0, y(0) = y(1) = 0$	**(8)**	**3**	**AP**
		(ii)	Solve the equation $\frac{dy}{dx} = \frac{1}{x+y}, y(0) = 1,$ for $y(0.1)$ and $y(0.2)$ using Runge – Kutta method of fourth order.	**(8)**	**3**	**AP**
14.	(a)	(i)	The joint pdf of a two dimensional random variable (X, Y) is given by $f(x, y) = k(x^3 y + xy^3)$,$0 \le x \le 2; 0 \le y \le 2$. Find the value of k and marginal and conditional density functions. Determine whether X and Y are independent.	**(8)**	**4**	**AP**
		(ii)	If the independent random variables X and Y have the variances 36 and 16 respectively, find the correlation co-efficient between $X + Y$ and $X - Y$.	**(8)**	**4**	**AP**
			(OR)			
	(b)	(i)	If the joint pdf of (X, Y) is given by $f_{XY}(x, y) = x + y$; $0 \le x, y \le 1$, find the pdf of $U = XY$.	**(8)**	**4**	**AP**
		(ii)	Find the correlation co-efficient for the following data:	**(8)**	**4**	**AP**

X	10	14	18	22	26	30
Y	18	12	24	6	30	36

15.	(a)	(i)	Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes (a) Find the average number of persons waiting in the system. (b) What is the probability that a person arriving at the booth will have to wait in the queue? (c) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call?	**(8)**	**5**	**AP**

	(ii)	In a car-wash service facility, cars arrive for service according to a Poisson distribution with mean 5 per hour. The time for washing and cleaning each car follows exponential distribution with mean 10 minutes per car. Find the expected waiting time until a car is washed if (i) there is enough parking space to accommodate all arriving cars (ii) the facility has a total of 5 parking spaces apart from the space for washing and cleaning. How many customers would the facility lose due to the limited parking space of five?	**(8)**	**5**	**AP**
		(OR)			
(b)	(i)	There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, (a) What fraction of the time all the typists will be busy? (b) What is the average number of letters waiting to be typed?	**(6)**	**5**	**AP**
	(ii)	A company uses five robots in the manufacture of its circuit boards. The robots break down periodically, and the company has two repair people to do service when robots fail. When one is fixed, the time until the next breakdown is exponentially distributed with mean of 30 hours. The shop always has enough of a work backlog to ensure that all robots in operating condition will be working. The repair time for each service is thought to be exponentially distributed with a mean of 3 hours. Determine the average number of robots operational at any given time and the expected downtime of a robot that requires repair.	**(10)**	**5**	**AP**