Q. Code: 405818

M.E. / M.TECH. DEGREE EXAMINATIONS, DEC 2020 (Held during April, 2021)

First Semester

MA18185 Applied Probability and Statistics

(Regulation 2018)

Use of Statistical Tables Permitted

Time: Three hours Maximum : 80 Marks

Answer **ALL** questions

PART A - (8 X 2 = 16 marks)

1. The required condition for a discrete probability mass function is
(a) $p(x)>0, \sum p(x)=1$ (b) $p(x)\leq 0, \sum p(x)=1$ (c) $p(x)\geq 0, \sum p(x)=1$
(d) $p(x)>0, \sum p(x)=0$

2. The following distribution possesses memory-less property
(a) Binomial (b) Poisson (c) Geometric (d) Normal

3. Which of the following statement is true?
(a) Rejecting the null at 5% level of significance implies rejection at 10% level
(b) Rejecting the null at 10% level of significance implies rejection at 5% level
(c) Rejecting the null at 5% level of significance is independent of rejection at 10% level
(d) For all continuous distributions the rejection regions are symmetric around the mean

4. Let $X_1, X_2, .., X_k$ be a random sample of size k from a Binomial distribution with parameters n and p. Which of the following is a maximum likelihood estimate of the parameter p?
(a) $\overline{X}$
(b) $\frac{1}{\overline{X}}$
(c) $\frac{n}{\overline{X}}$
(d) $\frac{\overline{X}}{n}$

5. The joint probability mass function of (X, Y) is given by
$p(x,y)=k(2x+3y), x=0,1,2;\ y=1,2,3$. Determine the value of k.

6. If the equations of the two lines of regression of Y on X and X on Y are respectively, $7x-16y+9=0; 5y-4x-3=0$, calculate $\bar{x}$ and $\bar{y}$.

7. A sample of 100 is chosen from each school and each student is evaluated as pass or fail for a fitness test. Authorities claim that all the students are capable of passing the test. Perform goodness of fit test on the following data:

$City$	O_i	E_i
A	98	100
B	79	100

What is the value of the statistic and does the hypothesis hold?(take critical values for α=0.025)

8. If $X = \begin{pmatrix} 42 & 4 \\ 52 & 5 \\ 48 & 4 \end{pmatrix}$ find $\overline{X}$

PART B - (4 X16 = 64 marks)

09. (a) (i) Suppose that a trainee soldier shoots a target in an independent fashion. If the probability that the target is hit on any one shot is 0.7, **(8)**
(i) What is the probability that the target would be hit in 10^{th} attempt?
(ii) What is the probability that it takes him less than 4 shots?
(iii) What is the probability that it takes him an even number of shots?

(ii) Assume the mean height of soldiers to be 68.22 inches with variance of 10.8 inches squared. How many soldiers in a regiment of 1000 would you expect to be over 6 feet, given that the height of the soldiers is normally distributed. **(8)**

(OR)

(b) (i) A discrete random variable X has the following probability distribution: **(8)**

X=x	0	1	2	3	4	5	6	7	8
P(X=x)	a	3a	5a	7a	9a	11a	13a	15a	17a

Find (i) the value of 'a' (ii) $P(X<3)$, $P(X \geq 3)$, $P(0<X<3)$

(ii) The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$. **(8)**
(a)What is the probability that the repair time exceeds 2 hrs?
(b)What is the conditional probability that a repair takes at least 11 hrs given that its duration exceeds 8 hrs?

10. (a) (i) The joint probability distribution of a pair of random variables (X,Y) is **(8)**

p(x,y)		Y: 0	Y: 1	Y: 2
X	0	0.1	0.04	0.02
X	1	0.08	0.20	0.06
X	2	0.06	0.14	0.30

Find (i) $P(X \leq 1)$ (ii) $P(Y \leq 1)$ (iii) $P(X+Y<2)$
Determine whether X and Y are independent.

(ii) If X and Y are independent random variables with respective probability density functions $e^{-x}, x \geq 0$, $e^{-y}, y \geq 0$, find the density functions of $U = \dfrac{X}{X+Y}$ and $V = X$. Are U and V independent? **(8)**

(OR)

(b) (i) Suppose that the two dimensional random variable (X,Y) has the joint probability density function $f(x,y)=\begin{cases} x+y, & 0<x<1, 0<y<1 \\ 0, & otherwise \end{cases}$ **(8)**

Obtain the correlation coefficient between X and Y

(ii) The joint probability mass function of (X,Y) is given by $p(x,y)=k(2x+3y), x=0,1,2;\ y=1,2,3$. Are X and Y independent. Find all the conditional probability mass functions. **(8)**

11. (a) (i) Water samples of a specified quantity are taken from a river suspected for having been polluted by improper treatment procedure at an upstream sewage disposal plant. Let X denote the number of e-coli organisms found per sample and assume that X is a Poisson random variable with parameter λ. Determine the value of λ that gives the highest probability of observing the sample. Compute λ for a sample of size 4 that yields the data 12,15,16,17. **(8)**

(ii) Let $X_1, X_2, \ldots, X_n$ be a random sample for a uniform distribution in $(\mu-3\sigma, \mu+3\sigma)$. Estimate the parameters μ and σ by the method of moments. **(8)**

(OR)

(b) Fit a straight line and a parabola to the following data by the method of least squares and find the most appropriate fit. Justify your answer. **(16)**

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

12. (a) (i) The mean of 2 large samples 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D 2.5 inches at 1% level of significance? **(8)**

(ii) The following data gives the number of air craft accidents that occurred during various days of the week. Find whether the accidents are uniformly distributed over the week. **(8)**

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents	14	16	8	20	11	9	14

(OR)

(b) (i) In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant at 5% level of significance? **(8)**

(ii) Sandal powder is packed into packets by a machine. A random sample of 12 packets is drawn and their weights are found to be (in kg) 0.49, 0.48, 0.47, 0.48, 0.49, 0.50, 0.51, 0.49, 0.48, 0.50, 0.51 and 0.48. Test if the average weight of the packing can be taken as 0.5 kg at 5% level of significance. **(8)**