Q. Code: 429225

Reg. No.

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B.E. / B.TECH. DEGREE EXAMINATIONS, DEC 2019

Fourth Semester

MA16453 – PROBABILITY AND QUEUEING THEORY

(Common to CS and IT)

(Regulation 2016)

Time: Three Hours **Maximum : 100 Marks**

Answer **ALL** questions

PART A - (10 X 2 = 20 Marks)

		CO	RBT
1.	Find the value of k if a continuous random variable X has a density function given by f(x) = k(1+x) if 2<x<5.	**1**	**U**
2.	Define moment generating function .	**1**	**R**
3.	Find the mean of x and y given two regression lines are 4x-5y+33 =0 and 20x-9y =107 .	**2**	**U**
4.	The joint probability mass function of (X,Y) is given by $P(x,y)=\begin{cases}\frac{x+2y}{18}, & x=1,2;\ y=1,2\\ 0 & ,\text{otherwise}\end{cases}$. Find all marginal probability distributions.	**2**	**U**
5.	Define Regular matrix.	**3**	**R**
6.	State any four types of a Stochastic processes.	**3**	**R**
7.	Define effective arrival rate for an (M/M/1):(k/FIFO)queue.	**4**	**R**
8.	For an (M/M/1): (∞/FIFO)queue find the average queue length if $\lambda = 4/hr$ & $\mu = 6/hr$	**4**	**U**
9.	What is a series queue with blocking?	**5**	**R**
10.	Define a tandem queue.	**5**	**R**

PART B - (5 X16 = 80 Marks)

11. (a) (i) The probability mass function of a discrete random variable X is given in the following table: **(8)** **1** **AP**

X	0	1	2	3	4	5
P(X=x)	k	3k	5k	7k	9k	11k

Find (i)the value of k (ii)P(X < 4) (iii)P(3 < X≤ 5) (iv) P(X ≥ 1)

		(ii)	Find the MGF, mean and variance of a random variable X having p.d.f. $f(x)=\begin{cases} x & ,0\le x\le 1 \\ 2-x, & 1\le x\le 2 \\ 0 & ,\text{elsewhere} \end{cases}$.	**(8)**	**1**	**AP**
			(OR)			
	(b)	(i)	The probability of a student passing a subject is 0.8. What is the probability that he will pass the subject? (i) on his third attempt (ii) before his third attempt?	**(8)**	**1**	**AP**
		(ii)	State and prove the Memory less property of Exponential distribution.	**(8)**	**1**	**AP**
12.	(a)	(i)	Compute the correlation coefficient between X and Y using the following data: <table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Y</td><td>2</td><td>5</td><td>3</td><td>8</td><td>7</td></tr></table>	**(8)**	**2**	**AP**
		(ii)	The joint pdf of a two dimensional random variable (X,Y) is given by $f(x,y)=xy^2+\frac{x^2}{8}, 0\le x\le 2; 0\le y\le 1$. Compute (1) $P[X>1]$ (2) $P\left[Y<\frac{1}{2}\right]$ (3) $P\left[X>1/Y<\frac{1}{2}\right]$	**(8)**	**2**	**AP**
			(OR)			
	(b)	(i)	If the joint pdf of (X,Y) is given by $f_{XY}(x,y)=x+y\ ;\ 0\le x,y\le 1$, find the pdf of $U=XY$.	**(8)**	**2**	**AP**
		(ii)	Given the joint pdf of (X,Y) $f(x,y)=\begin{cases} e^{-(x+y)}, & x>0, y>0 \\ 0 & ,\text{elsewhere} \end{cases}$. Find the marginal densities of X and Y. Are X and Y independent?	**(8)**	**2**	**AP**
13.	(a)	(i)	Verify whether the random process $X(t)=A\cos(\omega t+\theta)$ is wide sense stationary when A and ω are constants and θ is uniformly distributed on the interval $\left(0,\frac{\pi}{2}\right)$.	**(8)**	**3**	**AP**

		(ii)	The transition probability matrix of a Markov chain $\{X_n\}$, n=1,2,3, having 3 states 1,2 and 3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $p^{(0)} = (0.7\ \ 0.2\ \ 0.1)$. Find (i) $P\{X_2 = 3\}$ and (ii) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$	**(8)**	**3**	**AP**
			(OR)			
	(b)	(i)	Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes (a) exactly 4 customers arrive (b) more than 4 customers arrive.	**(8)**	**3**	**AP**
		(ii)	A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train, but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if a 6 appeared. Find (i) the probability that he takes a train on the third day and (ii) the probability that he drives to work in the long run.	**(8)**	**3**	**AP**
14.	(a)	(i)	A petrol pump has 2 pumps. The service times follow the exponential distribution with mean of 4 min. and cars arrive for service is a Poisson process at the rate of 10 cars per hour. (i) Find the Probability that a customer has to wait for service (ii) What is the probability that the pumps remain idle?	**(8)**	**4**	**AP**
		(ii)	Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 10 minutes and the service time is an exponential random variable with mean 8 minutes, (i) Find the average number of customers L_s in the shop (ii) Find the average time a customer spends in the shop W_s (iii) Find the average number of customers in the queue L_q (iv) What is the probability that the server is idle.	**(8)**	**4**	**AP**
			(OR)			
	(b)	(i)	Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 5 trains, find the probability that the yard is empty and the average number	**(8)**	**4**	**AP**

of trains in the system, given that the inter arrival time and service time are following exponential distribution.

(ii) A two person barber shop has 5 chairs to accommodate waiting customers. Customers who arrive when all 5 chairs are full leave without entering the barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 minutes in the barber's chair. Compute $p_0, p_1, p_7, and\ L_q$ **(8)** **4** **AP**

15. (a) Derive Pollaczek-Khintchine formula for the average number of customers in the M/G/1 queueing system. **(16)** **5** **AP**

(OR)

(b) (i) Write short notes on open and closed network. **(8)** **5** **AP**

(ii) A one-man barber shop takes exactly 25 minutes to complete one hair-cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer in the spends in the shop. Also, find the average time a customer must wait for service? **(8)** **5** **AP**