Q. Code: 512793

Reg. No. | | | | | | | | | |

B.E. / B.TECH. DEGREE EXAMINATIONS, DEC 2019

Fourth Semester

MA16454 – PROBABILITY AND RANDOM PROCESSES

(Electronics and Communication Engineering)

(Regulation 2016)

Time: Three Hours **Maximum : 100 Marks**

Answer **ALL** questions

PART A - (10 X 2 = 20 Marks)

		CO	RBT
1.	A continuous random variable X has the probability density function $f(x)=kx^2,\ 0<x<3$. Find k.	**1**	**AP**
2.	Find the mean of a Poisson random variable X which satisfies $P[X=2]=9P[X=4]+90P[X=6]$.	**1**	**AP**
3.	Let X and Y be two discrete random variables with joint pmf $P[X=x,Y=y]=\begin{cases}\dfrac{x+2y}{18}, & x=1,2;\ y=1,2\\ 0 & ,otherwise\end{cases}$. Find the marginal pmf of X.	**2**	**AP**
4.	State any two properties of correlation coefficient.	**2**	**R**
5.	State the four types of stochastic processes.	**3**	**R**
6.	The probability distribution of the process $\{X(t)\}$ is given by $P[X(t)=n]=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,3,..\\ \dfrac{at}{1+at} & n=0\end{cases}$ Find E(x).	**3**	**AP**
7.	Given that the autocorrelation function for a stationary ergodic process with no periodic components is $R_{XX}(\tau)=25+\dfrac{4}{1+6\tau^2}$, find the mean value and variance of the process $\{X(t)\}$.	**4**	**AP**
8.	What is the relation between power spectral density and time auto-correlation function of a random process?	**4**	**R**

9.	If the input X(t) to a LTI system is WSS, then what can you infer about the output?	**5**	**U**
10.	If the input X(t) to a LTI system is Gaussian, then what is the output?	**5**	**U**

PART B - (5 X16 = 80 Marks)

11. (a) (i) Derive the moment generating function, mean and variance of an exponential distribution. **(8)** **1** **AP**

(ii) A discrete random variable X has the following probability distribution: **(8)** **1** **AP**

x	0	1	2	3	4	5	6	7	8
$p(x)$	a	3a	5a	7a	9a	11a	13a	15a	17a

Find the value of 'a' , $P(X<3)$ and $P(0<X<3)$

(OR)

(b) (i) Suppose that a trainee soldier shoots a target in an independent fashion. If the probability that the target is shot on any one shot is 0.7. What is the probability that (i) the target would be hit on the tenth attempt? (ii) it takes him less than 4 shots to hit the target? **(8)** **1** **AP**

(ii) The peak temperature T, as measured in degrees Fahrenheit, on a particular day is the Gaussian (85, 10) random variable. Determine $P(T>100)$ and $P(70\le T\le 100)$ **(8)** **1** **AP**

12. (a) (i) If $f(x,y)=\begin{cases}2-x-y\ , & 0\le x\le 1,\ 0\le y\le 1\\ 0 & ;elsewhere\end{cases}$ is the joint pdf of the random variables X and Y, find the correlation co-efficient of X and Y. **(8)** **2** **AP**

(ii) If the joint pdf of (X,Y) is given by **(8)** **2** **AP**

$f_{XY}(x,y)=e^{-(x+y)}\ ;x\ge 0\ ,\ y\ge 0$, find the pdf of $U=\dfrac{X+Y}{2}$.

(OR)

(b)	(i)	The joint pdf of a two dimensional random variable (X, Y) is given by $f(x, y) = x y^2 + \frac{x^2}{8}, 0 \le x \le 2; 0 \le y \le 1$. Compute (1) $P[X > 1]$ (2) $P\left[Y < \frac{1}{2}\right]$ (3) $P\left[X > 1 \middle/ Y < \frac{1}{2}\right]$	**(8)**	**2**	**AP**
	(ii)	If $X_1, X_2, ..., X_n$ are Poisson variates with parameter $\lambda = 2$, use the central limit theorem to estimate $P(120 \le S_n \le 160)$, where $S_n = X_1, X_2, ..., X_n$ and $n = 75$.	**(8)**	**2**	**AP**
13. (a)	(i)	Verify whether the random process $X(t) = A\cos(\omega t + \theta)$ is wide sense stationary when A and ω are constants and θ is uniformly distributed on the interval $(0, 2\pi)$	**(8)**	**3**	**AP**
	(ii)	The transition probability matrix of a Markov chain $\{X_n\}$, three states 1,2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ Classify the states of the Markov chain.	**(8)**	**3**	**AP**

(OR)

(b)	(i)	Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 6 per minute. Find the probability that during a time interval of 1 minute (a) exactly 4 customers arrive (b) more than 4 customers arrive	**(8)**	**3**	**AP**
	(ii)	Define a semi-random telegraph process and verify whether it is evolutionary.	**(8)**	**3**	**AP**
14. (a)	(i)	Given that a random process $X(t)$ has the autocorrelation function $R_{XX}(\tau) = Ae^{-\alpha\lvert\tau\rvert}\cos\omega\tau, A > 0, \alpha > 0, \omega$ are real constants, find the power spectrum of $X(t)$.	**(8)**	**4**	**AP**
	(ii)	If $X(t)$ and $Y(t)$ are uncorrelated random processes, and if $Z(t) = X(t) + Y(t)$, find $S_{ZZ}(\omega)$ and $S_{XZ}(\omega)$.	**(8)**	**4**	**AP**

(OR)

(b)	(i)	If the cross-power spectrum of two random processes $X(t)$ and $Y(t)$ is given by $S_{XY}(\omega) = a + \frac{ib\omega}{W}, \lvert\omega\rvert < W$ and $S_{XY}(\omega) = 0$, elsewhere, where $W > 0, a, b$ are real constants, determine the cross-correlation function.	**(8)**	**4**	**AP**
	(ii)	The autocorrelation function for a stationary process $X(t)$ is given by $R_{XX}(\tau) = 9 + 2e^{-\lvert\tau\rvert}$. Find the mean value of the random variable $Y = \int_0^2 X(t)dt$ and the variance of $X(t)$.	**(8)**	**4**	**AP**
15. (a)		State and prove Wiener-Khinchine theorm.	**(16)**	**5**	**AP**
		(OR)			
(b)	(i)	A system has an impulse response $h(t) = e^{-bt}U(t)$ where $U(t)$ represents the unit step function. Find the power spectral density of the output $Y(t)$ corresponding to the input $X(t)$.	**(8)**	**5**	**AP**
	(ii)	Find the mean square value of a linear system with input autocorrelation function $R_{XX}(\tau) = e^{-4\lvert\tau\rvert}$ and impulse response $h(t) = 2e^{-7t}$	**(8)**	**5**	**AP**