**Q. Code: 684862**

**Reg. No.** | | | | | | | | | | | | | | |

## B.E / B.TECH. DEGREE EXAMINATION, MAY 2023

First Semester

## MA16151 MATHEMATICS – I

*(Common to all Branches Except MR)*

(Regulation 2016)

**Time: 3 Hours** **Maximum : 100 Marks**

Answer **ALL** questions

**PART A - (10 X 2 = 20 marks)**

**1.** If the sum of two Eigen values and trace of 3×3 matrix A are equal, find the value of |A|.

**2.** State the nature of quadratic form $2xy + 2yz + 2zx$.

**3.** Discuss the convergence of the series $2 - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \ldots$

**4.** Name any two test used in series to test the convergence.

**5.** Find the radius of curvature of the curve $y = 4\sin x - \sin 2x$ at $x = \frac{\pi}{2}$.

**6.** Find the envelope of the family of straight lines $y = mx + \frac{1}{m}$, m being the parameter.

**7.** If $u = (x-y)^4 + (y-z)^4 + (z-x)^4$, then find the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

**8.** If $u = x + y,\ y = uv$ find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$.

**9.** Evaluate $\int_2^4 \int_1^2 \frac{dxdy}{xy}$.

**10.** Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2 yz\, dxdydz$.

**PART B - (5 X16 = 80 marks)**

**11. (a) (i)** Find the Eigen values and Eigen vectors of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. **( 8 )**

**(ii)** Verify Cayley –Hamilton Theorem for the matrix A= $\begin{pmatrix} 13 & -3 & 5 \\ 0 & 4 & 0 \\ -15 & 9 & -7 \end{pmatrix}$ and hence find $A^{-1}$. **( 8 )**

**(OR)**

**(b)** Reduce the quadratic form $x^2 + 3y^2 + 3z^2 - 2yz$ to its canonical using an orthogonal transformation. **(16)**

**12. (a) (i)** Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!\, 2^n}{n^n}$. **( 8 )**

**(ii)** Test for convergence the series $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5.} + \ldots\infty$ **( 8 )**

**(OR)**

**(b)** **(i)** Discuss the convergence of the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p > 0$. **( 8 )**

**(ii)** Test whether the series is conditionally convergent or not. **( 8 )**

$$\frac{1}{2^3} - \frac{1+2}{3^3} + \frac{1+2+3}{4^3} - \frac{1+2+3+4}{5^3} + \ldots\infty$$

**13.** **(a)** **(i)** Find the points on the parabola $y^2 = 4x$ at which the radius of the curvature is $4\sqrt{2}$. **( 8 )**

**(ii)** Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ subject to the condition $a + b = 1$ where a and b are parameters. **( 8 )**

**(OR)**

**(b)** **(i)** Find the equation of the circle of curvature of the curve at the point P on the curve $y = e^x$ where the curve crosses the y – axis. **(10)**

**(ii)** If the centre of curvature of curve is $\left(\frac{c}{a}\cos^3 t, \frac{c}{a}\sin^3 t\right)$, find the evolute of the curve. **( 6 )**

**14.** **(a)** **(i)** If $z$ is a function of $x$ and $y$ and $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. **( 8 )**

**(ii)** Examine $f(x, y) = x^3 + y^3 - 3axy$ for maximum and minimum values. **( 8 )**

**(OR)**

**(b)** **(i)** Expand $f(x, y) = \sin(xy)$ in powers of $(x-1)$ and $(y - \frac{\pi}{2})$ up to second degree terms. **( 8 )**

**(ii)** Find the shortest from the origin to the curve $x^2 + 8xy + 7y^2 = 225$ using the methods of Lagrange's multipliers. **( 8 )**

**15.** **(a)** **(i)** Change the order the integration in $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ and hence evaluate. **( 8 )**

**(ii)** Find the area of the cardioid $r = a(1-\cos\theta)$ by using double integration. **( 8 )**

**(OR)**

**(b)** **(i)** Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}} dydx$ by changing into polar coordinates. **( 8 )**

**(ii)** Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. **( 8 )**