Q. Code:903982

Reg. No. | | | | | | | | | | | | | | |

B.E / B.TECH. DEGREE EXAMINATION, MAY 2023

Second Semester

MA16251 MATHEMATICS – II

(Common to all Branches Except MR)

(Regulation 2016)

Time: 3 Hours **Maximum : 100 Marks**

Answer **ALL** questions

PART A - (10 X 2 = 20 marks)

1. Find a unit normal vector to the surface of the sphere $x^2 + y^2 + z^2 = 1$.

2. If $\vec{F} = 5xy\vec{i} + 2y\vec{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve $y = x^2$ between $x = 1$ and $x = 2$.

3. Find the particular integral of $(D-1)^2 y = \sinh 2x$.

4. Reduce $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = x$ into a differential equation with constant coefficients.

5. State the sufficient conditions for the existence of Laplace transform.

6. State initial and final value theorems on Laplace transform.

7. Is $f(z) = z^3$ analytic?

8. Find the image of $|z| = 2$ under the transformation $w = 3z$.

9. Evaluate $\int_C \frac{\cos \pi z}{z-1} dz$ if C is $|z| = 2$.

10. Find the residue of the function $f(z) = \frac{4}{z^3(z-2)}$ at a simple pole.

PART B - (5 X16 = 80 marks)

11. (a) (i) Show that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ is irrotational and hence find its scalar potential. **(8)**

(ii) Verify Green's theorem in a plane for the integral $\int_C [(x-2y)dx + xdy]$ taken around the circle $x^2 + y^2 = a^2$. **(8)**

(OR)

(b) Verify Gauss Divergence theorem for the vector function $\vec{F} = (x^3 - yz)\vec{i} - 2x^2y + 2\vec{k}$ over the cube $x = 0,\ y = 0,\ z = 0,\ x = a,\ y = a,\ z = a$. **(16)**

12. (a) (i) Solve $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x \log x$. **(8)**

(ii) Solve the simultaneous equation $\frac{dx}{dt} - y = t,\ \frac{dy}{dt} + x = t$. **(8)**

(OR)

(b) (i) Solve $(D^2+4D+3)y=e^{-x}\sin x.$ (8)

(ii) Using the method of variation of parameters, solve $\frac{d^2y}{dx^2}+4y=\tan 2x.$ (8)

13. (a) (i) Find $L[t^2e^{-3t}\sin 2t].$ (8)

(ii) Using convolution theorem find inverse Laplace transform of $\frac{2}{(s+1)(s^2+4)}.$ (8)

(OR)

(b) (i) Find the Laplace transform of the square-wave function of period a defined as $f(t)=\begin{cases}1 & when\ 0<t<\frac{a}{2}\\ -1 & when\ \frac{a}{2}<t<a\end{cases}.$ (8)

(ii) Solve the differential equation $\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=e^{-t}$ with $y(0)=1$ and $y'(0)=0,$ using Lapalce transform. (8)

14. (a) (i) If $w=f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f'(z)|=0.$ (8)

(ii) Find the bilinear transformation that maps $z=0, 1, \infty$ into points $w=-5, -1, 3$ respectively. What are the invariant points of transformation? (8)

(OR)

(b) (i) Determine the analytic function $f(z)=u+iv$ given that $3u+2v=y^2-x^2+16xy.$ (8)

(ii) Find the image of the strip $1<x<2$ under the map $w=\frac{1}{z}.$ (8)

15. (a) (i) Using Cauchy's integral formula, evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)}dz$ where C is the circle $|z|=\frac{3}{2}.$ (8)

(ii) Find the Laurent's series expansion of $\frac{1}{z^2-3z+2}$ in the region $1<|z|<2.$ (8)

(OR)

(b) Evaluate $\int_{-\infty}^{\infty}\frac{x^2-x+2}{x^4+10x^2+9}dx$ using Contour integration. (16)

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