Reg. No. | | | | | | | | | | | | | |

B.E / B.TECH. DEGREE EXAMINATION, MAY 2023

Fourth Semester

MA16453 – PROBABILITY AND QUEUEING THEORY

(Common to CSE &INT)

(Regulation 2016)

TIME: 3 HOURS **MAX. MARKS: 100**

PART- A (10 x 2 = 20 Marks)

(Answer all Questions)

1. Let X be a discrete R.V. with probability mass function $P(X=x)=\begin{cases}\frac{x}{10}, x=1,2,3,4\\ 0, otherwise\end{cases}$. $Compute P(X<3)$ and $E\left(\frac{X}{2}\right)$.
2. For a Binomial distribution with mean 6 and variance 2, find the first two terms of the distribution.
3. The joint probability mass function of X and Y is

X\Y	0	1	2
0	0.1	0.04	0.02
1	0.08	0.2	0.06
2	0.06	0.14	0.3

   Check if X and Y are independent.
4. The regression equations of X on Y and Yon X are respectively 5x – y = 22 and 64x – 45y = 24. Find the mean values of X and Y.
5. What is a random process? When do you say a random process is a random variable?
6. Consider the Markov chain consisting of the three states 0, 1, 2 and transition probability matrix $P=\begin{bmatrix}0 & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 0 & \frac{1}{2}\\ 1 & 0 & 0\end{bmatrix}$. Draw the state transition diagram.
7. In an $(M/M/1):(\infty/FIFO)$ queueing model if $\lambda=8/hr$, $\mu=10/hr$, what is the average waiting time in the queue?
8. What are the values of P_0 & P_n for the queueing model $(M/M/1):(K/FIFO)$ when $\lambda=\mu$.
9. Explain M/G/1 model.
10. Write the steady state equations for 2 stage series queues with blocking.

PART- B (5 x 14 = 70 Marks)

Marks

11. (a) **(i)** If the density function of a continuous random variable X is given by **(8)**

$$f(x)=\begin{cases} ax, & 0\le x\le 1\\ a, & 1\le x\le 2\\ 3a-ax, & 2\le x\le 3\\ 0, & elsewhere\end{cases}$$

i)Find the value of '**a**' (ii) Find the c.d.f of X (iii) Find $P(X\le 1.5)$.

(ii) Trains arrive at a station at 15 minutes' interval starting at 4 a.m. If a passenger arrives at a station at a time that is uniformly distributed between 9.00 a.m. and 9.30 a.m., find the probability that he has to wait for the train for (i) less than 6 minutes (ii) more than 10 minutes. **(8)**

(b) **(i)** Derive the MGF, mean and variance of Geometric distribution. **(8)**

(ii) X is a normal variate with **mean = 30** and S.D = 5. Find $\mathbf{P[26\le X\le 40]}$. **(8)**

12. (a) **(i)** The joint pdf of the continuous R.V (X,Y) is given as **(8)**

$f(x,y)=\begin{cases}e^{-(x+y)}, x>0, y>0\\ 0 \quad ,elsewhere\end{cases}$. Find the pdf of the random variable $U=\frac{X}{Y}$.

(ii) The life time of a certain brand of an electric bulb may be considered as a random variable with mean 1200h and standard deviation 250h. Find the probability, using central limit theorem, that the average lifetime of 60 bulbs exceeds 1250 hours. **(8)**

(b) Obtain the equation of the regression lines from the following data. **(16)**

X	3	5	6	8	9	11
Y	2	3	4	6	5	8

13. (a) **(i)** Show that the random process $X(t)=A\cos(w_0t+\theta)$ is wide-sense stationary, if A and w_0 are constants and θ is uniformly distributed random variable in $(0,2\pi)$. **(8)**

(ii) A salesman territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However, if he sells in either city B or city C, the next day he is twice as likely to sell in city A as in the other city. In the long run how often does he sell in each of the cities? **(8)**

(b) **(i)** Let $\{X_n : n \geq 0\}$ be a Markov chain having state space $S = \{1,2,3\}$ and one step TPM **(8)**

$$P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

(i) Draw a transition diagram for this chain.
(ii) Is the chain irreducible? Explain.
(iii) Is the state 3 ergodic? Explain.

(ii) Suppose that the earthquakes occurs in a certain region of California, in accordance with a Poisson Process at a rate of 7 per year. What is the probability of no earthquakes in 1 year? What is the probability that exactly 3 earthquakes in 8 years? **(8)**

14. (a) **(i)** There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, **(8)**
(a) What fraction of the time all the typists will be busy?
(b) What is the average number of letters waiting to be typed?
(c) What is the average time a letter has to spend for waiting and for being typed.
(d) What is the probability that a letter will take longer than 20 min waiting to be typed and being typed? Assume that arrival and service rates follow poisson distribution.

(ii) (ii) Patients arrive at a clinic according to Poisson distribution at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour. **(8)**
(a) Find the effective arrival rate at the clinic.
(b)What is the probability that an arriving patient will not wait?
(c)What is the expected waiting time until a patient is discharged from the clinic?

(b) Derive the steady state probabilities of the number of customers in M/M/I queueing system from the birth death processes and hence deduce that the average measures such as L_s, L_q, W_s and W_q. **(16)**

15. (a) **(i)** Consider a queuing system where arrivals are according to a Poisson distribution with mean 5 per hour. Find the expected waiting time in the system if the service time distribution is (i) a uniform distribution between t = 5 minutes and t = 15 minutes. (ii) Normal distribution with mean 3 minutes and standard deviation 2 minutes. **(8)**

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(ii) A repair facility by a large number of machines has two sequential stations with respective rates one per hour and two per hour. The cumulative failure rate of all the machines is 0.5 per hour. Assuming that the system behavior may be approximated by the two-stage tandem queue, determine the average repair time. **(8)**

(b) Discuss an M/G/1/∞: FCFS queueing system and hence obtain the Pollaczek-Khintchine (P-K) mean value formula. Deduce also the mean number of customers for the M/M/1/∞: FCFS queueing system from the P-K mean value formula. **(16)**

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