Ref. No. | | | | | | | | | | | | | |

B.E. / B.TECH. DEGREE EXAMINATION, MAY 2023
First Semester
MA18151 - ENGINEERING MATHEMATICS I
(Regulation 2018 & 2018A)

TIME: 3 HOURS **MAX. MARKS: 100**

CO 1 Develop the use of matrix algebra techniques which is needed for practical applications.
CO 2 Apply the skill to solve statistical problems under correlation and regression and acquire the knowledge for fitting the straight line and parabola.
CO 3 Develop skills to find the curvature, evolute and envelope of curves.
CO 4 Acquire the skills to evaluate the functions of several variables
CO 5 Acquaint the student with mathematical tools needed in evaluating multiple integrals and their usage.

PART- A (10 x 2 = 20 Marks)
(Answer all Questions)

		CO	RBT LEVEL
1	Two eigen values of the matrix $A=\begin{pmatrix}2&2&1\\1&3&1\\1&2&2\end{pmatrix}$ are 1 each. Find the eigen values of A^{-1}.	1	2
2	Write down the quadratic form corresponding to the matrix $\begin{pmatrix}2&1&-2\\1&2&-2\\-2&-2&3\end{pmatrix}$.	1	2
3	Find the mean values of X and Y, for the following regression equations $8X-10Y+66=0;\ 40X-18Y=214$.	2	2
4	What are the normal equations to fit a straight line by the method of least squares?	2	1
5	Find the curvature of the curve $x^2+y^2-6x-4y+9=0$ at any point on it.	3	2
6	Find the envelope of the family of lines $\frac{x}{t}+yt=2c$, t being the parameter.	3	3
7	If $u=\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$.	4	2
8	If $x=u^2-v^2$ and $y=2uv$, find the Jacobian of x and y with respect to u and v.	4	2
9	Evaluate $\int_1^b\int_1^a \frac{dxdy}{xy}$.	5	2

		CO	RBT LEVEL
10	Change the order of integration in $\int_0^1\int_0^y f(x,y)dxdy$.	5	2

PART- B (5 x 14 = 70 Marks)

			Marks	CO	RBT LEVEL
11(a)	(i)	Find the eigenvalues and eigenvectors of the matrix $A=\begin{pmatrix}1&1&3\\1&5&1\\3&1&1\end{pmatrix}$.	(7)	1	3
	(ii)	Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix}1&3&7\\4&2&3\\1&2&1\end{pmatrix}$ and hence find A^{-1}.	(7)	1	3

(OR)

		Marks	CO	RBT LEVEL
11(b)	Reduce the quadratic form $x_1^2+2x_2^2+x_3^2-2x_1x_2+2x_2x_3$ to canonical form by orthogonal transformation. Also find its rank, index, signature and nature.	(14)	1	3
12(a)	Calculate the correlation coefficient for the following data. Also obtain the equations of lines of regression.	(14)	2	3

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

(OR)

12(b) **(i)** Obtain the rank correlation coefficient for the following data: (7) 2 3

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

(ii) Fit a parabola by the method of least squares for the following data: (7) 2 3

x	1	2	3	4	5	6	7	8	9	10
y	1.28	1.53	1.03	0.81	0.74	0.65	0.87	0.81	1.10	1.03

13(a) **(i)** Show that the measure of curvature of the curve $\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1$ at any point (x,y) on it is $\frac{ab}{2(ax+by)^{\frac{3}{2}}}$. (7) 3 3

	(ii)	Find the equation of the circle of curvature of the curve $y^2 = 12x$ at $(3,6)$.	**(7)**	**3**	**3**
		(OR)			
13(b)	**(i)**	Find the evolute of the curve $x = a(\cos t + t\sin t),\ y = a(\sin t - t\cos t)$.	**(7)**	**3**	**3**
	(ii)	Find the envelope of the family of straight lines $\frac{x}{a}+\frac{y}{b}=1$, where the parameters a and b are connected by the relation $a+b=c$.	**(7)**	**3**	**3**
14(a)	**(i)**	If $u = \log(x^3+y^3+z^3-3xyz)$, show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$	**(7)**	**4**	**3**
	(ii)	Find the Taylor's series expansion of $e^x \sin y$ near the point $\left(-1,\frac{\pi}{4}\right)$ up to third degree terms.	**(7)**	**4**	**3**
		(OR)			
14(b)	**(i)**	Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 where $y_1 = \frac{x_2x_3}{x_1},\ y_2 = \frac{x_3x_1}{x_2},\ y_3 = \frac{x_1x_2}{x_3}$.	**(7)**	**4**	**3**
	(ii)	A rectangular box open at the top is to have a capacity of 108 cu.ms. Find the dimensions of the box requiring least material for its construction	**(7)**	**4**	**3**
15(a)	**(i)**	Change the order of integration in $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dydx$ and then evaluate it.	**(7)**	**5**	**3**
	(ii)	Find the smaller of the areas by bounded by $y = 2-x$ and $x^2+y^2=4$.	**(7)**	**5**	**3**
		(OR)			

15(b)	**(i)**	Express $\int_0^a \int_y^a \frac{x^2}{\left(x^2+y^2\right)^{\frac{3}{2}}} dxdy$ in polar coordinates and then evaluate it.	**(7)**	**5**	**3**
	(ii)	Find the volume of the sphere $x^2+y^2+z^2=a^2$ by triple integration.	**(7)**	**5**	**3**

PART- C (1 x 10 = 10 Marks)

(Q.No.16 is compulsory)

		Marks	CO	RBT LEVEL
16	Find the maximum and minimum values of the function $f(x,y) = x^3+3xy^2-15x^2-15y^2+72x$.	**(10)**	**4**	**3**