

Reg. No. 

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**PART- B (5x 14 =70 Marks)**

**B. E / B. TECH.DEGREE EXAMINATIONS, MAY 2023**

Second Semester

**MA18251 – ENGINEERING MATHEMATICS II**

(Common to all branches except MR)

(Regulation 2018 /Regulation2018A)

TIME:3 HOURS

MAX. MARKS: 100

- CO1** Interpret the fundamentals of vector calculus and be fluent in the use of Stokes theorem and Gauss divergence theorem.
- CO2** Express proficiency in handling higher order differential equations
- CO3** Determine the methods to solve differential equations using Laplace transforms and Inverse Laplace transforms.
- CO4** Explain Analytic functions and Categorize transformations.
- CO5** Solve complex integrals using Cauchy integral theorem and Cauchy's residue theorem.

**PART- A(10x2=20Marks)**

(Answer all Questions)

	CO	RBT LEVEL
1. Find a unit normal vector to the surface $x^2 + y^2 - 2z + 3 = 0$ at $(1, 2, -1)$	1	2
2. If $\vec{V} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+\lambda z)\vec{k}$ is solenoidal, find $\lambda$ .	1	2
3. Solve $(D^3 - 4D^2 + 4D) y = 0$	2	2
4. Transform the equation $(2x + 3)^2 y'' - 2(2x + 3) y' + 2y = 6x$ into a differential equation with constant coefficients.	2	2
5. State Initial Value and Final Value Theorem for Laplace transform.	3	1
6. Find $L(\sin^2 2t)$ .	3	2
7. Find the invariant points of the transformation $w = \frac{z-1}{z+1}$ .	4	2
8. Define conformal mapping.	4	1
9. State Cauchy's integral formula.	5	1
10. Evaluate $\int_C \frac{\cos \pi z}{z-1} dz$ if C is $ z =2$ .	5	2

Marks CO RBT LEVEL

<b>11. (a)</b>	(i)	Show that $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is a conservative vector field and find its scalar potential.	(7)	1	3
	(ii)	Find the angle between the normal to the surface $xy^3z^2 = 4$ at the points $(-1, -1, 2)$ and $(4, 1, -1)$ .	(7)	1	3
<b>(OR)</b>					
	(b)	Verify Gauss divergence theorem for $\vec{F} = xy^2\vec{i} + yz^2\vec{j} + zx^2\vec{k}$ over the region bounded by $x = 0, x = 1, y = 0, y = 2, z = 0, z = 3$	(14)	1	3
<b>12. (a)</b>	(i)	Solve $(D^2 - 3D + 2) y = 2\cos(2x+3) + 2e^x$	(7)	2	3
	(ii)	Solve by the method of variation of parameter: $(D^2 + a^2) y = \tan ax$	(7)	2	3
<b>(OR)</b>					
	(b)	(i) Solve the following simultaneous differential equation: $(D+2)x + 3y = 2e^{2t}; 3x + (D+2)y = 0$	(7)	2	3
	(ii)	Solve $[(2x+3)^2 D^2 - 2(2x+3) D - 12] y = 6x$	(7)	2	3
<b>13. (a)</b>	(i)	Find the Laplace transform of the rectangular-wave function $f(t) = \begin{cases} t, & 0 \leq t \leq a \\ 2a-t, & a \leq t \leq 2a \end{cases}$ and $f(t+2a) = f(t)$	(7)	3	3
	(ii)	Find the inverse Laplace transform of $\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$ using convolution theorem.	(7)	3	3
<b>(OR)</b>					
<b>13.(b)</b>	(i)	Solve the differential equation $y'' - 3y' - 4y = 2e^{-t}, y(0) = 1 = y'(0)$ by using Laplace transform.	(7)	3	3
	(ii)	Find the Laplace transform of $\frac{e^{-at} - e^{-bt}}{t}$ .	(7)	3	3
<b>14. (a)</b>	(i)	Find the bilinear transformation that maps the points $z = 0, 1, \infty$ on to $w = i, -1, -i$ .	(7)	4	3
	(ii)	Find the image of the infinite strips $0 < y < \frac{1}{2}$ under the transformation $w = 1/z$ .	(7)	4	3

(OR)

(b) (i) Show that the function  $u(x,y)=3x^2y+2x^2-y^3-2y^2$  is harmonic. (7) 4 3  
Find the conjugate harmonic function.

(ii) Find the analytic function  $f(z) = u+iv$  where  $u - v = e^x (\cos y - \sin y)$ . (7) 4 3

15. (a) (i) Find the Laurent's series expansion of the function (7) 5 3  
 $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$  for  $|z| > 3$ .

(ii) Evaluate  $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)} dz$  where C is  $|z|=4$  using Cauchy's (7) 5 3  
integral formula.

(OR)

(b) (i) Using Cauchy's residue theorem, evaluate  $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z+1)(z+2)} dz$  where (7) 5 3  
C is the circle  $|z|=3$ .

(ii) Using contour integration, evaluate  $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4 \cos \theta} d\theta$  (7) 5 3

**PART- C (1x 10=10Marks)**

(Q.No.16 is compulsory)

	Marks	CO	RBT LEVEL
16. Verify Green's theorem for $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ , where C is the boundary of the region defined by $x = 0, y = 0, x + y = 1$	(10)	1	3

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