Reg. No. $\square$

## B. E / B. TECH.DEGREE EXAMINATIONS, MAY 2023

## Third Semester

## MA18351 - ENGINEERING MATHEMATICS III

## (Common to BIO, CHE, CVE, ECE, EEE \&MEC)

(Regulation 2018 /Regulation2018A)

## MAX. MARKS: 100

## TIME: 3 HOURS

CO1 Express proficiency in handling higher order Partial differential equations.
CO2 Acquire the skill in examining a signal in another domain rather in the original domain by handling Full and Half Range Fourier Series.
CO3 Develop skills in the classification, formulation, solution, and interpretation of PDE models.
CO4 Develops the skill of conversion between time domain to frequency domain using the concept of Fourier Transforms.
CO5 Apply the systematic method for finding the impulse response of LTI systems described by difference equations: partial fraction expansion.

## PART- A (10x2=20Marks) <br> (Answer all Questions)

## Form the partial differential equation by eliminating the arbitrary constants a and b from

$\qquad$ $(x-a)^{2}+(y-b)^{2}=z^{2} \cot ^{2} \alpha$
2. Solve $p+q=p q$
3. Find the root mean square value of $f(x)=x^{2}$ in the interval $(0, \pi)$. $\mathbf{2}$
4. If $\mathrm{f}(\mathrm{x})$ is an odd function defined in $(-l, l)$, what are the values of $a_{0}$ and $a_{n}$ ? $\mathbf{2}$
5. What are the possible solutions of one-dimensional wave equation? $\quad \mathbf{3} \quad \mathbf{1}$
6. The ends A and B of a rod of length 10 cm long have their temperature kept at $20^{\circ} \mathrm{c}$ and $\quad \mathbf{3} \quad \mathbf{2}$ $70^{\circ} \mathrm{c}$. Find the steady state temperature distribution on the rod.
7. Find the Fourier sine transform of $f(x)=1$ in $(0, l)$. $4 \quad \boldsymbol{2}$
8. Define Fourier cosine transform pair. 4
9. Find the Z transform of $(n+1)(n+2)$. $\mathbf{5} \quad \mathbf{2}$
10. Form difference equation from $y_{n}=a+b 3^{n} \quad 1 \begin{array}{lll}5 & 2\end{array}$

## PART- B (5x 14=70Marks)

| Marks | CO | RBT |
| :---: | :---: | :---: |
| LEVEL |  |  |
| $(7)$ | 1 | 3 |
| $(7)$ | 1 | 3 |

(b) (i) Solve $p(1+q)=q z$
(OR)
(ii) Solve $\frac{\partial^{3} z}{\partial x^{3}}-2 \frac{\partial^{3} z}{\partial x^{2} \partial y}=2 e^{2 x}+3 x^{2} y$
(7) 13
(7) 13
12. (a) (i) Find the Fourier series expansion of $f(x)=\mathrm{x}^{2}$ in $-\pi<x<\pi$. Hence
(7) 23 prove $\frac{\pi^{4}}{90}=1+\frac{1}{2^{4}}+\frac{1}{3^{4}}+\ldots$.
(ii) Find the half range Fourier Sine Series of $f(x)=a$ in $(o, l)$. Deduce the sum of $\frac{1}{1^{2}}+\frac{1}{3^{2}}+\frac{1}{5^{2}}+\ldots . \infty$.

## (OR)

(b) (i) Find the Fourier cosine series expansion of $f(x)=x(\pi-x)$ in $0<x<\pi$.
(ii) The table value of the function $y=f(x)$ is given below. Find the Fourier series up to the second harmonic

| x | 0 | $\pi / 3$ | $2 \pi / 3$ | $\pi$ | $4 \pi / 3$ | $5 \pi / 3$ | $2 \pi$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 1.0 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1.0 |

13. (a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in equilibrium position. If it is set vibrating giving each point a velocity $\lambda x(l-x)$, find the displacement of any point on the string at a distance from one end at any time t .

## (OR)

(b) A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered as infinite length. If the temperature along the short edge $y=0$ is given by $u(x, 0)=100 \sin \frac{\pi \mathrm{x}}{8}, 0<x<8$, while the two long edges $x=0$ and $x=10$ as well as the other short edge are kept at $0^{\circ} c$, find the steady state temperature function $u(x, y)$.
14. (a) Find the Fourier transform of $\mathrm{f}(\mathrm{x})$ defined as $f(x)=\left\{\begin{array}{l}a-|x|, \text { for }|x|<a \\ 0 \quad, \text { for }|x|>a>0 .\end{array}\right.$

Hence show that $\int_{0}^{\infty}\left(\frac{\sin t}{t}\right)^{2} d t=\frac{\pi}{2}$ and $\int_{0}^{\infty}\left(\frac{\sin t}{t}\right)^{4} d t=\frac{\pi}{3}$.

## (OR)

(b) (i) Find the Fourier cosine transform of $\mathrm{e}^{-4 \mathrm{x}}$ and hence prove that $\int_{0}^{\infty} \frac{\cos 2 x d x}{x^{2}+16}=\frac{\pi e^{-8}}{8}$
(ii) Using Parseval's identity, prove that $\int_{0}^{\infty} \frac{d x}{\left(a^{2}+x^{2}\right)\left(b^{2}+x^{2}\right)}=\frac{\pi}{2 a b(a+b)}$.
15. (a)
(i) Find the z-transform of $\cos n \theta$.
(ii) Find the Z-transform of $\frac{1}{(n+1)(n+2)}$.

## (OR)

(b) (i) Using partial fractions find the inverse Z-transform of $\frac{2 z^{2}+3 z}{(z+2)(z-4)}$ Solve the difference equation using z-transform
(ii) $y(n+2)+6 y(n+1)+9 y(n)=2^{n}$ given that $\mathrm{y}(0)=0$ and $\mathrm{y}(1)=0$

## PART-C ( $1 \times 10=10 \mathrm{Marks}$ )

(Q.No. 16 is compulsory)

Marks CO RBT Level
16. Find the inverse Z-transforms of $\frac{8 z^{2}}{(2 z-1)(4 z-1)}$ using Convolution theorem (10) $\mathbf{5} \quad \mathbf{3}$
(14) 43
(7) 43
(7) 43
(7) 53
(7) 53
(7) 53
(7) 53

