Reg. No. | | | | | | | | | | | | | | |

B.E / B.TECH. DEGREE EXAMINATION, MAY 2023

Third Semester

MA18352-DISCRETE MATHEMATICS

(Common to CSE & INT)

(Regulation2018 & 2018A)

TIME: 3 HOURS **MAX.MARKS: 100**

CO1	Acquire the concepts of logic to test the lucidity of a program.
CO2	Describe and apply the counting principles in computer simulations.
CO3	Develop graph theory tools for day – to – day applications.
CO4	Expose the concepts and properties of algebraic structures such as groups, rings and fields.
CO5	Categorize Boolean algebraic structures on numerous levels.

PART- A (10x2=20 Marks)

(Answer all Questions)

		CO	RBT LEVEL
1	Show that the propositions $p \rightarrow q$ and $\neg p \vee q$ are equivalent.	1	2
2	Write the converse, inverse and contrapositive of "If you work hard, then you will be rewarded".	1	2
3	If seven colours are used to paint fifty bicycles, then show that at least eight bicycles must be of the same colour.	2	2
4	Find the recurrence relation for the Fibonacci sequence.	2	2
5	How many edges are there in a graph with 10 vertices, each of degree 5?	3	2
6	For the degree sequence 3,3,3,2,1 either draw a graph if it exists or explain why no graph exists.	3	2
7	Give an example of a semi-group which is not a monoid.	4	2
8	Let $\mathbf{Z}$ be the group of integers with binary operation $*$ defined by $a * b = a + b - 2$ for all $a, b \in \mathbf{Z}$. Find the identity element of the group $(\mathbf{Z}, *)$	4	2
9	Draw the Hasse diagram of $D_{30} = \{1,2,3,5,6,10,15,30\}$.	5	2
10	Let $X = \{1,2,3,4,5,6\}$ and R be the relation defined by $(x, y) \in R$ if and only if $x - y$ is divisible by 3. Find the elements of R.	5	2

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PART- B (5x 14=70 Marks)

			Marks	CO	RBT LEVEL
11(a)	**(i)**	Obtain the principal disjunctive normal form and principal conjunctive normal form of $(p \wedge q) \vee (\neg p \wedge r)$	(7)	1	3
	(ii)	Show that $\neg(p \vee (\neg p \wedge q))$ and $(\neg p \wedge \neg q)$ are logically equivalent.	(7)	1	3
		(OR)			
11(b)	**(i)**	Show that $((p \rightarrow q) \wedge (r \rightarrow q)) \Rightarrow (p \vee r) \rightarrow q$	(7)	1	3
	(ii)	Show that $r \rightarrow s$ can be derived from the premises $p \rightarrow (q \rightarrow s)$, $\neg r \vee p$ and q .	(7)	1	3
12(a)	**(i)**	Using induction principle show that $n^3 + 2n$ is divisible by 3.	(7)	2	3
	(ii)	Find the number of integers between 1 and 250 both inclusive that are divisible by any of 2,3,5,7	(7)	2	3
		(OR)			
12(b)	**(i)**	A total of 1232 students have taken a course in Spanish, 879 have taken a course in French and 114 have taken a course in Russian. Further 103 students have taken courses in both Spanish and French, 23 students have taken courses in both Spanish and Russian, and 14 students have taken courses in both French and Russian. If 2092 students have taken a course in at least one of Spanish, French, Russian, how many students have taken a course in all three languages?	(7)	2	3
	(ii)	Use generating functions to solve the recurrence relation $S(n+1) - 2S(n) = 4n$ with $S(0) = 1, n \geq 0$	(7)	2	3
13(a)	**(i)**	Prove that a simple graph with n vertices and k components can have utmost $\frac{(n-k)(n-k+1)}{2}$ edges.	(7)	3	3
	(ii)	Draw an undirected graph represented by the following adjacency matrix and hence verify the handshaking theorem. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$	(7)	3	3
		(OR)			
13(b)	**(i)**	Determine whether the following graph has an Euler circuit/ Hamilton circuit? If yes, name the circuit.	(7)	3	3

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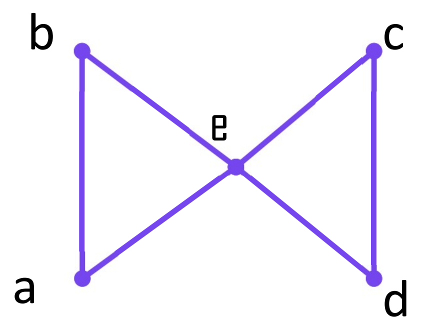

			Marks	CO	RBT LEVEL
	(ii)	Prove that the maximum number of edges in a simple graph is $\frac{n(n-1)}{2}$.	(7)	3	3
14(a)	(i)	Examine whether $G = \left\{\begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \neq 0, a \in \mathbf{R}\right\}$ forms an abelian group under matrix multiplication where $\mathbf{R}$ is the set of all real numbers.	(7)	4	3
	(ii)	Find all the subgroups of $(Z_g, +_g)$	(7)	4	3
		(OR)			
14(b)		State and prove the Lagrange's theorem on groups. Is the converse true? Justify your answer.	(14)	4	3
15(a)	(i)	If S_{42} is the set of all divisors of 42 and D is the relation "divisor of" on S_{42} , prove that $\{S_{42}, D\}$ is a complemented lattice.	(7)	5	3
	(ii)	Establish De Morgan's laws in a Boolean Algebra.	(7)	5	3
		(OR)			
15(b)	(i)	Let $S = \{a, b, c\}$. Show that $(\mathcal{P}(S), \subseteq)$ is a partailly ordered set.	(7)	5	3
	(ii)	Prove that every chain is a distributive lattice.	(7)	5	3

PART- C (1x 10=10 Marks)

(Q.No.16 is compulsory)

		Marks	CO	RBT LEVEL
16	Find the number of distinct permutations that can be formed from all the letters of each word (1) RADAR (2) UNUSUAL (3) MATHEMATICS.	(10)	2	3

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