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# B.E. / B.TECH. DEGREE EXAMINATION, MAY 2023 <br> Third Semester <br> <br> MA18354 - MATHEMATICS FOR DATA ANALYSIS 

 <br> <br> MA18354 - MATHEMATICS FOR DATA ANALYSIS}
(Artificial Intelligence and Data Science)

## (Regulation 2018 / 2018A)

## TIME: 3 HOURS

## MAX. MARKS: 100

CO 1 The students will be able to apply the concepts of basic principles of Combinatorics and its Applications.
CO 2 The students will be able to understand the basic concepts in Number Theory and its Applications in Data Science.
CO 3 The students will be able to provide the required support to develop regression models which can be used in data analytics
CO 4 The students will be able to understand the fundamental concepts of graph theory.
CO 5 The students will be able to understand the advanced concepts of graph theory and its applications to Computer science.

## PART- A (10 x $2=20$ Marks) <br> (Answer all Questions)

1. State the Pigeonhole principle. 1
2. How many ways are there to select five players from a 10 -member tennis team to make $\quad \mathbf{1} \quad \mathbf{2}$ a trip to a match at another school?
3. Express $(28,12)$ as a linear combination of 28 and $12 . \quad 2 \begin{aligned} & 2\end{aligned}$
4. Find the canonical decomposition of 2520 . $\quad \mathbf{2} \quad \mathbf{2}$
5. If $S_{X X}=132000 ; S_{Y Y}=2.13745 ; S_{X Y}=505.40$, find the residual sum of squares. $\mathbf{3} \mathbf{2}$
6. Write the normal equations for polynomial regression to fit a second degree $\mathbf{3} \quad \mathbf{1}$ polynomial.
7. How many edges are there in a graph with 10 vertices each of degree six? $\quad \mathbf{4}$
8. Determine whether the following degree sequence is graphic. If yes, draw the graph $\quad \mathbf{4} \quad \mathbf{2}$ having the degree sequence.
3, 3, 3, 3, 2
9. Explain a maximum matching with an example.
10. Give an example of an M -augmenting path. $5 \quad 2$

## PART- B (5 x $14=70$ Marks)

## Marks Co $\begin{gathered}\text { RBT } \\ \text { LEVEL }\end{gathered}$

11(a)
(i) In a survey of 270 college students, it is found that 64 like brussels sprouts, 94 like broccoli, 58 like cauliflower, 26 like both brussels sprouts and broccoli, 28 like both brussels sprouts and cauliflower, 22 like both broccoli and cauliflower, and 14 like all three vegetables. How many of the 270 students do not like any of these vegetables?
(ii) Use mathematical induction to prove that $n^{3}-n$ is divisible by 3 whenever $n$ is $\begin{array}{lllll}\mathbf{7} & \mathbf{1} & \mathbf{3}\end{array}$ a positive integer.

## (OR)

11(b) (i) Solve the recurrence relation $a_{n}=8 a_{n-1}+10^{n-1}$ with the initial condition $a_{0}=1 . \quad \mathbf{7} \quad \mathbf{1} \quad 3$
(ii) The English alphabet contains 21 consonants and five vowels. How many strings $\quad 7 \quad 1 \quad 3$ of six lowercase letters of the English alphabet contain?
a) exactly one vowel?
b) exactly two vowels?
c) at least one vowel?
(i) Prove that the gcd of the positive integers $a$ and $b$ is a linear combination of $a \quad \mathbf{7} \quad \mathbf{2} \quad \mathbf{3}$ and $b$.
$\begin{array}{lllllll}\text { (ii) Conjecture a formula for the nth row of the following number pattern and prove } & \mathbf{7} & \mathbf{2} & \mathbf{3}\end{array}$ it.

$$
\begin{aligned}
1 \cdot 9+2 & =11 \\
12 \cdot 9+3 & =111 \\
123 \cdot 9+4 & =1111 \\
1234 \cdot 9+5 & =11111 \\
12345 \cdot 9+6 & =111111 \\
123456 \cdot 9+7 & =1111111
\end{aligned}
$$

(OR)
12(b) (i) Using the Euclidean algorithm, express $(4076,2076)$ as a linear combination of $\mathbf{7} \quad \mathbf{2} \quad \mathbf{3}$ 4076 and 2076.
(ii) Show that $[a, b]=\frac{a b}{(a, b)}$, where $a$ and $b$ are positive integers. $\quad \mathbf{7} \quad \mathbf{2} \quad \mathbf{3}$

14(b) Show that a non-empty connected graph is Eulerian if and only if it has no vertices of $14 \quad 4 \quad 3$ odd degree.

| Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ <br> x | 20 | 22 | 25 | 35 | 18 | 29 | 31 | 16 | 13 | 48 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Tearing <br> strength (g) <br> y | 1600 | 1700 | 2100 | 2500 | 1550 | 2600 | 2550 | 1100 | 1050 | 2650 |

Fit a straight line to the given data by the method of least squares and use it to predict the tearing strength one can expect when the temperature is $29^{\circ} \mathrm{C}$.
Also find the residual sum of squares and construct a $95 \%$ confidence interval for the regression coefficient $\alpha \cdot\left(t_{0.025,8}=2.306\right)$

## (OR)

13(b) (i) The following are data on the ages and incomes of 5 executives working for the
$\begin{array}{lll}7 & 3\end{array}$ same company and the number of years they went to college:

| Age $\mathrm{x}_{1}$ | 37 | 45 | 38 | 42 | 31 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Years college $\mathrm{x}_{2}$ | 4 | 0 | 5 | 2 | 4 |
| Income(dollars) y | 71,200 | 66,800 | 75,000 | 70,300 | 65,400 |

Fit an equation of the form $y=\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}$ to the given data.
(ii) Fit a second degree parabola to the following data:
$\begin{array}{lll}7 & 3\end{array}$

| x | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 1.1 | 1.3 | 1.6 | 2 | 2.7 | 3.4 | 4.1 |

14(a) (i) Determine whether the graphs G and H are isomorphic.


G
H
(ii) Show that every non trivial tree has at least two vertices of degree one.

