Reg. No. | | | | | | | | | | | | | |

B.E / B.TECH. DEGREE EXAMINATIONS, MAY 2023

Fourth Semester

MA18453 – PROBABILITY AND QUEUEING THEORY

(Common to all CSE and INT)

(Regulation 2018A)

(Statistical tables are permitted)

TIME: 3 HOURS **MAX. MARKS: 100**

CO 1 Describe commonly used univariate discrete and continuous probability distributions by formulating fundamental probability distribution and density functions, as well as functions of random variables.

CO 2 Develop skills in dealing with scenarios involving multiple random variables.

CO 3 Express and characterize phenomenon which evolve with respect to time in a probabilistic manner.

CO 4 Acquire skills in analyzing queueing models.

CO 5 Develop skills in identifying best techniques to solve a specific problem.

PART- A (10 x 2 = 20 Marks)

(Answer all Questions)

		CO	RBT LEVEL
1.	A discrete random variable X has MGF $M_X(t) = \left(\frac{1}{4}+\frac{3}{4}e^t\right)^5$. Find Var(X).	1	2
2.	If X is uniformly distributed over the interval [0,10], compute $P(2 < X < 9)$.	1	2
3.	Find the value of A if the joint pdf of (X, Y) is given by $f(x,y) = Aye^{-x}$ where $x > 0$ *and* $0 < y < 2$.	2	2
4.	If a random variable X has mean 4, variance 9 whereas another random variable Y has mean 2, variance 5 and X, Y are independent, find Var (2X+Y).	2	2
5.	What can you say about the inter-arrival time of a Poisson process with arrival rate?	3	2
6.	State Chapman-Kolmogorov Theorem.	3	1
7.	A TV repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they come in and if the arrival of sets is approximately Poisson, with an average rate of 10 per 8-hour day, what is the repairman's idle time each day?	4	2
8.	What is the probability that a customer has to wait for more than 15 minutes to get his service completed in an (M/M/1): (∞/FIFO) queuing system if λ = 6 per hour and μ = 10 per hour?	4	2

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9.	If the service time of an (M/G/1) queuing model is a constant, how will you find the expected number of customers in the system?	5	2
10.	Define tandem queue and give an example.	5	2

PART- B (5 x 14 = 70 Marks)

Marks | CO | RBT LEVEL

11. (a) (i) A discrete random variable X has the following probability distribution: **(8)** **1** **3**

x	0	1	2	3	4	5	6	7	8
$p(x)$	a	3a	5a	7a	9a	11 a	13 a	15 a	17 a

Find the value of 'a', $P(X<3)$, $P(X \geq 3)$ and $P(0 < X < 3)$.

(ii) The probability of an infinite discrete distribution is given by $P(X = x) = \frac{1}{2^x}$, $x = 1,2, ...$ Find the MGF and mean of the distribution. **(6)** **1** **3**

(OR)

(b) (i) Suppose that the number of miles that a car can run before its battery wears out is exponentially distributed with an average value of 10,000 miles. If a person desires to take a 5000-mile trip, what is the probability that he/she will be able to complete the trip without having to replace the car battery? **(6)** **1** **3**

(ii) The peak temperature T, as measured in degrees Fahrenheit, on a particular day follows Normal Distribution with mean 85 and standard deviation 10. Find $P(T>100)$, $P(T<60)$ *and* $P(70 \leq T \leq 100)$. **(8)** **1** **3**

12. (a) The joint pdf of (X, Y) is given by **(14)** **2** **3**

$$f(x,y) = \begin{cases} k\,xy\,e^{-(x^2+y^2)} & ,x,y \geq 0 \\ 0 & otherwise \end{cases}$$

Find the value of k, both the marginal density functions, both the conditional density functions. Are X and Y independent?

(OR)

(b) Let X and Y be two discrete random variables with joint p.m.f **(14)** **2** **3**

$$P(X = x, Y = y) = \begin{cases} \frac{x+y}{21}, & x = 1,2,3 \text{ \& } y = 1,2 \\ 0, & \text{otherwise} \end{cases}$$

Find the correlation coefficient.

13. (a)	**(i)** Verify whether the random process $\{X(t) = Y\sin(\omega t)\}$ where $Y \sim U[-1,1]$ is WSS or not.	**(7)**	3	3
	(ii) A machine goes out of order whenever a component fails. The failure of this part follows Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If there are five spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks.	**(7)**	3	3

(OR)

(b) **(i)** Find the nature of the states of the Markov chain with the TPM **(6)** 3 3

$$P = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$$

(ii) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train, but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared, find (i) the probability that he takes a train on the third day and (ii) the probability that he drives to work in the long run. **(8)** 3 3

14. (a) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assume that inter-arrival time and service time distribution follows exponential distribution with an average of 36 minutes. Using (M/M/1), calculate the following: **(14)** 4 3

(i) Service rate
(ii) Utilization factor of the service facility
(iii) Expected queue size
(iv) Expected number of units waiting in the system
(v) Expected waiting time in the system
(vi) Expected waiting time in the queue
(vii) Probability that the queue size exceeds 10.

(OR)

(b) A supermarket has 2 girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 min. and if people arrive in Poisson fashion at the rate of 10 per hour, **(14)** 4 3

(i) What is the probability that a customer has to wait for service?
(ii) What is the expected percentage of idle time for each girl?
(iii) If the customer has to wait in the queue, what is the expected length of his waiting time?

15. (a) An automatic car wash facility operates with only one bay. Cars arrive according to Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. Find L_S, L_q, W_S, W_q, if the service time is **(14)** 5 3

(i) normally distributed with mean 12 min and SD 3 min.
(ii) uniformly distributed between 8 and 12 minutes.

(OR)

(b) There are 2 clerks in a bank, one processing house loan applications and the other processing agricultural loan applications. While processing, they get doubts according to an exponential distribution each with a mean of ½**.** To get clarifications, a clerk goes to the Deputy Manager with probability ¾ and to the Senior Manager with probability ¼. After completing the job with D.M., a clerk goes to S.M. with probability 1/3 and returns to his seat otherwise. Completing the job with S.M., a clerk always returns to his seat. If the D.M. clarifies the doubts and advises a clerk according to an exponential distribution with parameter 1 and the S.M. with parameter 3, find **(14)** 5 3

(i) the steady state probabilities $P(n_1, n_2, n_3)$ for all possible values of n_1, n_2, n_3.
(ii) the probability that both the managers are idle.
(iii) the probability that at least one manager is idle.

PART- C (1 x 10 = 10 Marks)

(Q.No.16 is compulsory)

Marks CO RBT LEVEL

16. The joint probability distribution of (X, Y) is given by **(10)** 2 2

X \ Y	1	2	3
1	0.1	0.1	0.2
2	0.2	0.3	0.1

Find the following:

(i) The conditional distribution of X given Y.
(ii) P (X ≤ 2).
(iii) $P(Y \leq 3)$.
(iv) P (X+Y < 4).
(v) Verify whether X and Y are independent or not.

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