Reg. No.

B.E / B.TECH. DEGREE EXAMINATIONS, MAY 2023

Fourth Semester

MA18454 – PROBABILITY AND RANDOM PROCESSES

(Electronics and Communication Engineering)

(Regulation 2018 / Regulation 2018A)

TIME: 3 HOURS **MAX. MARKS: 100**

CO 1 Reproduce and explain the basic concepts such as probability and random variable and identify the distribution.
CO 2 Acquire skills in handling situations involving more than one random variable.
CO 3 Study the characterize phenomena with respect to time in probabilistic manner.
CO 4 Apply the relationship within and between random processes.
CO 5 Apply the response of random inputs to linear time invariant systems.

PART- A (10 x 2 = 20 Marks)

(Answer all Questions)

		CO	RBT LEVEL
1.	If $f(x)=\frac{x^2}{3}, -1<x<2$ is the pdf of the random variable X, then find $P(0<x<1)$.	1	2
2.	Find the expected value of the discrete random variable X with the probability mass function $\boldsymbol{P(X=x)}=\begin{cases}\frac{1}{3} & ;\boldsymbol{x}=0\\ \frac{2}{3} & ;\boldsymbol{x}=2\end{cases}$	1	2
3.	The joint probability density function of bivariate random variable (X , Y) is given by $f(x,y)=\begin{cases}4xy, 0<x<1, 0<y<1\\ 0 \quad , elsewhere\end{cases}$. Find P (X + Y<1)	2	2
4.	The regression equations are 3x + 2y = 26 and 6x + y = 31. Find the means of X and Y.	2	1
5.	Define wide sense stationary process.	3	1
6.	If the customers arrive at a bank according to a Poisson process with a mean rate of 2 per minute, find the probability that, during one-minute interval no customer arrives.	3	2
7.	Find the mean of the stationary process $\{X(t)\}$ whose autocorrelation function $\boldsymbol{R(\tau)=\frac{25\tau^2+36}{6.25\tau^2+4}}$	4	3
8.	State any two properties of the power spectral density function.	4	1
9.	Give an example of a linear system.	5	2
10.	Check whether the system $\boldsymbol{y(t)=tx(t)}$ is time invariant.	5	2

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PART- B (5 x 14 = 70 Marks)

			Marks	CO	RBT LEVEL
11. (a)	**(i)**	A discrete r.v. X has the following probability distribution: (table below) Find (i) the value of 'a' (ii) P(1.5<X<4.5 / X> 2) and (iii) The smallest value of n for which $P(X\le n)>\frac{1}{2}$	**(7)**	1	3
	(ii)	Find the MGF, mean and variance for a Poisson distribution.	**(7)**	1	3
		(OR)			
(b)	**(i)**	A continuous r.v. X has the p.d.f. $f(x)=\frac{K}{(1+x^2)}, -\infty \prec x \prec \infty$. Find k and Distribution function of X.	**(7)**	1	3
	(ii)	If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8. What is the probability that he will finally pass the test (i) on the fourth trial and (ii) in less than four trials?	**(7)**	1	3
12. (a)		The joint probability mass function of (X,Y) is given by $p(x,y)=k(2x+3y)$, $x=0,1,2$; $y=1,2,3$. Find 'k', the Marginal distributions and Conditional probability distributions. Also find the probability distribution of X + Y .	**(14)**	2	3
		(OR)			
(b)		Find the correlation coefficient ρ_{xy} for the following joint density function $f(x,y)=\begin{cases}2-x-y, & 0\le x,y\le 1\\ 0 & \text{elsewhere}\end{cases}$	**(14)**	2	3
13. (a)		Show that the process $X(t)$ whose probability distribution under certain conditions is given by $P\{X(t)=n\}=\begin{cases}\frac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,...\\ \frac{at}{1+at} & , n=0\end{cases}$ is not stationary**.**	**(14)**	3	3
		(OR)			
(b)		If {X(t)}is a Gaussian process with μ(t)=10 and $Cov(t_1,t_2)=16e^{-\lvert t_1-t_2\rvert}$, find the probability that $(i)X(10)\le 8$ *and* $(ii)\lvert X(10)-X(6)\rvert\le 4$	**(14)**	3	3

Table for 11. (a) (i):

Values of X	0	1	2	3	4	5	6	7
P(x)	0	a	2a	2a	3a	a^2	$2a^2$	$7a^2$ + a

			Marks	CO	RBT
14. (a)	(i)	If the power spectral density of a WSS process is given by $S(\omega)=\begin{cases}\frac{b}{a}(a-\lvert\omega\rvert); & \lvert\omega\rvert\le a\\ 0 & ; \lvert\omega\rvert > a\end{cases}$ find its autocorrelation function of the process.	(7)	4	3
	(ii)	If {X(t)} is a WSS process with autocorrelation $R_{XX}(\tau)$ and if $Y(t)=X(t+a)-X(t-a)$. Show that $R_{YY}(\tau)=2R_{XX}(\tau)-R_{XX}(\tau+2a)-R_{XX}(\tau-2a)$.	(7)	4	3
		(OR)			
(b)		Consider two random processes $X(t)=3\cos(\omega t+\theta)$ and $Y(t)=2\cos\left(\omega t+\theta-\frac{\pi}{2}\right)$, where θ is a random variable uniformly distributed in $(\theta,2\pi)$. Prove that $\lvert R_{XY}(\tau)\rvert\le\sqrt{R_{XX}(0)R_{YY}(0)}$.	(14)	4	3
15. (a)		If $\{X(t)\}$ is a WSS process and if $Y(t)=\int_{-\infty}^{\infty}h(u)X(t-u)\,du$ then prove the following: (i) $R_{XY}(\tau)=R_{XX}(\tau)*h(\tau)$ (ii) $R_{YY}(\tau)=R_{XY}(\tau)*h(-\tau)$, where* denotes the convolution.	(14)	5	3
		(OR)			
(b)		If $Y(t)=A\cos(\omega_0 t+\theta)+N(t)$, where A is a constant, θ is a random variable with uniform distribution in $(-\pi,\pi)$ and $N(t)$ is a band-limited Gaussian white noise with a power spectral density $S_{NN}(\omega)=\begin{cases}\frac{N_0}{2}, & \text{for } \lvert\omega-\omega_0\rvert<\omega_B\\ 0, & \text{elsewhere}\end{cases}$. Find the power spectral density of $Y(t)$. Assume that $N(t)$ and θ are independent.	(14)	5	3

PART- C (1 x 10 = 10 Marks)

(Q.No.16 is compulsory)

		Marks	CO	RBT LEVEL
16.	The equations of two regression lines are 3x+12y=19 and 3y+9x=46. Find the mean of X and Y and the correlation coefficient.	(10)	2	3

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