Reg. No. | | | | | | | | | | | | | |

B. E / B. TECH.DEGREE EXAMINATIONS, MAY 2023

Fourth Semester

MA18455-PROBABILITY AND STATISTICS

(Biotechnology)

(Regulation2018 & 2018A)

(Statistical Tables are permitted)

TIME: 3 HOURS **MAX.MARKS: 100**

CO1 Gain knowledge in one dimensional random variables and standard distribution.
CO2 Gain knowledge in two dimensional random variables and functions of random variables.
CO3 Gain knowledge in managerial problems especially in quality control problems.
CO4 Exposure to statistical methods designed to contribute to the process of making scientific judgments in the face of uncertainty and variation
CO5 Know the concepts of statistical Quality control theory and their applications on real time problems

PART- A (10x2=20 Marks)

(Answer all Questions)

		CO	RBT LEVEL
1	A discrete random variable X has MGF $M_X(t)=\left(\frac{1}{4}+\frac{3}{4}e^t\right)^5$. Find E(X).	1	2
2	A continuous random variable X has the probability density function $f(x)=k(1+x), 2<x<5$. Find k.	1	2
3	Find the value of k if $f(x,y)=k(1-x)(1-y), 0<x<1, 0<y<1$ is to be a joint density function.	2	2
4	If X has variance 21 and Y has variance 7, and X, Y are uncorrelated, find $Var(X+3Y-15)$.	2	2
5	Two independent samples of observations were collected. For the first sample of 60 elements, the mean was 86 and the standard deviation 6. The second sample of 75 elements had a mean of 82 and a standard deviation of 9. Compute the estimated standard error of the difference between the two means.	3	2
6	Find the value of χ^2 for the following data:	3	2

	Smokers	Nonsmokers
Literates	83	57
Illiterates	45	68

7	Analysis of Variance is a statistical method of comparing the ------- of several populations. (a)standard deviations (b) variance (c) means (d) proportions	4	2
8	To determine whether the test statistic of ANOVA is statistically significant, it can be compared to a critical value. What two pieces of information are needed to determine the critical value? (a) sample size, number of groups (b) mean, sample standard deviation (c)expected frequency, obtained frequency (d) MSE, MSC	4	2
9	Find the lower and upper control limits for $\overline{X}$ and R chart when each sample is of size 4 and $\overline{\overline{X}}=10.8$ and $\overline{R}=0.46$.	5	2
10	If the average fraction defective of a large sample of products is 0.1537, calculate the control limits. (Given that sub-group size is 2,000).	5	2

PART- B (5x 14=70 Marks)

			Marks	CO	RBT LEVEL
11(a)	(i)	A discrete random variable X has the following probability distribution:	(7)	1	3

x	-2	-1	0	1	2	3
$p(x)$	0.1	k	0.2	2k	0.3	3k

Find (i) the value of k (*ii*) $P(X<2)$ (*iii*) $P(-2<X<2)$ and (iv) $P(X>1)$.

	(ii)	The probability of an infinite discrete distribution is given by $P(X=x)=\frac{1}{2^x}$, x = 1,2,…. Find the MGF, mean and variance of the distribution.	(7)	1		3

(OR)

11(b)	(i)	In a certain city, the daily consumption of electric power in millions of kilowatt-hours can be treated as a random variable following Gamma distribution with parameters $\lambda=\frac{1}{2}$ and $\alpha=3$. If the power plant of this city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be adequate on any given day?	(8)	1		3
	(ii)	Suppose that the number of miles that a car can run before its battery wears out is exponentially distributed with an average value of 10,000 miles. If a person desires to take a 5000-mile trip, what is the probability that he/she will be able to complete the trip without having to replace the car battery?	(6)	1		3
12(a)	(i)	Given the joint pdf of (X,Y) as $f(x,y)=\begin{cases}8xy; 0<x<y<1\\0 \quad ,otherwise\end{cases}$. Find the marginal and conditional probability density functions of X and Y. Are X and Y are independent?	(7)	2		3
	(ii)	Let X and Y be two discrete random variables with joint p.m.f $P(X=x,Y=y)=\begin{cases}\frac{x+y}{21}, x=1,2,3, y=1,2\\0 \quad \text{otherwise}\end{cases}$ Find the correlation coefficient between them.	(7)	2		3

(OR)

12(b) **(i)** If the joint pdf of (X,Y) is given by $f_{XY}(x,y) = e^{-(x+y)}$; $x \geq 0$, $y \geq 0$, find the pdf of $U = \frac{X+Y}{2}$. **(7)** **2** **3**

(ii) The life time of a certain brand of an electric light may be considered as a random variable with mean 1200h and standard deviation 250h. Find the probability, using, CLT, that the average lifetime of 60 lights exceeds 1250h. **(7)** **2** **3**

13(a) **(i)** Before an increase in excise duty on tea, 800 people out of a sample of 1000 were consumers of tea. After an increase in excise duty, 800 people were consumers of tea in a sample of 1200 persons. Determine whether there is a significant decrease in the consumption of tea after an increase in excise duty. Test at 5% level of significance. **(7)** **3** **3**

(ii) Test the significance of the difference between the means of the samples drawn from 2 normal populations with the same standard deviation from the following data: **(7)** **3** **3**

Sample	Size	Mean	Standard deviation
Sample I	100	61	4
Sample II	200	63	6

Test at 1% level of significance.

OR

13(b) **(i)** Tests made on the breaking strength of 10 pieces of a metal wire gave the following results: 578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 kg. Test if the mean breaking strength of the wire can be assumed as 577 kg. **(7)** **3** **3**

(ii) The following table gives the number of aircraft accidents that occurred during the various days of a week. Test whether the accidents are uniformly distributed over the week: **(7)** **3** **3**

Day	Mon	Tue	Wed	Thu	Fri	Sat
No. of Accidents	15	19	13	12	16	15

14(a) Three varieties A,B, C of a crop are tested in a RBD with 4 replications. The plot yields in units are as follows: **(14)** **4** **3**

A6	C5	A8	B9
C8	A4	B6	C9
B7	B6	C10	A6

Test whether the yields differ significantly with respect to (a) blocks (b) treatments

OR

14(b) Analyse the following results of a Latin Square Design **(14)** **4** **3**

	1	2	3	4
1	A(12)	D(20)	C(16)	B(10)
2	D(18)	A(14)	B(11)	C(14)
3	B(12)	C(15)	D(19)	A(13)
4	C(16)	B(11)	A(15)	D(10)

15(a) Given below are the values of sample mean $\overline{X}$ and sample range R for 10 samples, each of size 5. Draw the appropriate mean and range charts and comment on the state of control of the process. **(14)** **5** **3**

Sample No.	1	2	3	4	5	6
Mean	43	49	37	44	45	37
Range	5	6	5	7	7	4

Sample No.	7	8	9	10
Mean	51	46	43	47
Range	8	6	4	6

OR

15(b) **(i)** In a factory producing spark plugs, the number of defectives found in the inspection of 15 lots of 100 each is given below: **(7)** **5** **3**

Sample No.	1	2	3	4	5	6	7	8
No. of defective	5	10	12	8	6	4	6	3
Sample No.	9	10	11	12	13	14	15	
No. of defective	4	5	4	7	9	3	4	

Draw the control chart for the number of defectives and comment on the state of control.

(ii) 15 tape – recorders were examined for quality control test. The number of defects in each tap – recorder is recorded below. Draw the appropriate control chart and comment on the state of control. **(7)** **5** **3**

Unit No.	1	2	3	4	5	6	7	8
No. of defects	2	4	3	1	1	2	5	3
Unit No.	9	10	11	12	13	14	15	
No. of defects	6	7	3	1	4	2	1	

PART- C (1x 10=10Marks)

(Q.No.16 is compulsory)

		Marks	CO	RBT LEVEL
16	Two random samples gave the following data:	**(10)**	**3**	**3**

Sample	Size	Mean	Variance
Sample I	8	9.6	1.2
Sample II	11	16.5	2.5

Test whether the two samples have been drawn from the same normal population.

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