Reg. No.

## B. E / B. TECH.DEGREE EXAMINATIONS, MAY 2023

Fourth Semester
MA18456 - QUEUING THEORY AND OPTIMIZATION
(Artificial Intelligence and Data Science)
(Regulation 2018 /Regulation2018A)

## MAX. MARKS: 100

## TIME: 3 HOURS

CO1 Understand the concepts of Linear Programming Problems and its Applications
CO2 Learn the basic concepts of Transportation and Assignment problems
CO3 Understand the fundamental concepts of Queueing systems and its Applications
CO4 Gain more knowledge in analysing queuing models
CO 5 Understand the basic concepts of Optimization and apply the same to problems in Machine Learning

## PART- A(10x2=20Marks) <br> (Answer all Questions)

1. State one Pure birth process with an example.
2. State the Pollaczek-Khinchine formula. $\quad \mathbf{2} \quad \mathbf{1}$
$\begin{array}{lll}\text { 4. What is the basic difference between open and closed Networks? } & \mathbf{2} \quad \mathbf{1}\end{array}$
$\begin{array}{llll}\text { 5. Distinguish between a slack variable and a surplus variable in an LPP. } & \mathbf{3} & \mathbf{2}\end{array}$
3. What is the significance of shadow price in a simplex procedure? $\quad \mathbf{3} \quad \mathbf{2}$
4. How do we address an unbalanced Assignment problem? $4 \quad 2$
5. Can we have degeneracy occur at IBFS Of a TPP? 4
6. What role do constraints play in an optimization problem? $\quad \mathbf{5} \quad \mathbf{2}$
7. How do we confirm a non linear problem to be convex?
8. (a) Customers arrive at a one man barber shop according to a Poisson process with $\begin{array}{ccc}\text { Marks } & \text { (14) } & \mathbf{1}\end{array} \begin{gathered}\text { Co } \\ \text { LBVT } \\ \text { LEVEL }\end{gathered}$ a mean inter arrival of 12 min . Customers spend an average of 10 min . in the barber's chair. a) What is the expected number of customers in the barber shop and in the queue? b) How much time can customer expect to spend in the barber's shop? c) What is the average time customer spends in the queue? d) what is the probability for an empty queue?

## (OR)

(b) A barber shop has two barbers and three chairs for customer. Assume that the customers arrive in Poisson fashion at a rate of 5 per hour and that each barber service customers according to an exponential distribution with mean of 15 minutes. Further if a customer arrives and there are no empty chairs in the shop, he will leave. What is the probability that the shop is empty? What is the expected number of customers in the shop?
12. (a) Derive Pollaczek-Khinchine formula for $M / G / 1$ queue in system.

## (OR)

(b) (i) A patient who goes to a single doctor clinic for a general check-up has to go through 4 phases. The doctor takes on the average 4 minutes for each phase of the check-up and the time taken for each phase is exponentially distributed. If the arrival of the patients at the clinic are approximately Poisson at the average rate of $3 /$ hour, what is the average time spent by a patient (i) in the examination? (ii) waiting in the clinic?
(ii) A car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. Find the average number of cars waiting in the parking lot, if the time for washing and cleaning a car has uniform distribution between 8 and 12 minutes.
13. (a) Apply simplex procedure to solve the following LPP $\operatorname{Max} Z=6 x+4 y+3 z$
s.t $2 x+y+z \leq 2 ; 3 x+4 y-z \leq 12 ; x, y, z \geq 0$
(OR)
(b) Use two phase simplex to solve the following LPP $\operatorname{Max} Z=4 x+y$
s. $t 3 x+y=3 ; 4 x+3 y \geq 6 ; x+2 y \leq 4 ; x, y \geq 0$
14. (a) Vpower company has three electric power plants that supply the electric needs of four cities. The associated supply of each plant and demand of each city is given in the table below.
The cost of sending 1 million kwh of electricity (in lakhs of rupees) from a plant to a city depends on the distance the electricity must travel. Find the optimal solution to the TPP and also find the associated cost.

|  | To |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| From | City1 | City2 | City3 | City4 | Supply (Mkwh) |
| Plant 1 | 8 | 6 | 10 | 9 | 35 |
| Plant 2 | 9 | 12 | 13 | 7 | 50 |
| Plant 3 | 14 | 9 | 16 | 5 | 40 |
| Demand (Mkwh) | 45 | 20 | 30 | 30 |  |

(OR)
(b) Four captain pilots ( $\mathrm{CP} 1, \mathrm{CP} 2, \mathrm{CP} 3, \mathrm{CP} 4$ ) have evaluated four flight officers
(FO1, FO2, FO3, FO4) according to standard criteria in a 1-20 scale, as given below. The company wants to assign one flight officer to one captain. Find out possible crews.

|  | FO1 | FO2 | FO3 | FO4 |
| :---: | :---: | :---: | :---: | :---: |
| CP1 | 2 | 4 | 6 | 10 |
| CP2 | 2 | 12 | 6 | 5 |
| CP3 | 7 | 8 | 3 | 9 |
| CP4 | 14 | 5 | 8 | 7 |

15. (a) (i) A company is planning to construct a rectangular picnic area on the sideroads of a highway with fencing for about 6000 square feet. It needs to construct a rectangular area with fencing in three sides. Find the dimensions which optimize the fence.
(ii) Discuss about Penalty method to solve a non linear programming problem

## (OR)

(b) Form the KKT conditions for the optimization problem below and solve.

## PART-C (1x 10=10Marks) <br> (Q.No. 16 is compulsory)

$$
\begin{array}{ccc}
\text { Marks } & \text { CO } & \begin{array}{c}
\text { RBT } \\
\text { LEVEL }
\end{array} \\
(10) & 4 & 3
\end{array}
$$

16. A Travelling salesman has to visit five cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The travelling cost (in Rs.) of each city from a particular city is given below. Help him get a schedule with minimum cost.

|  |  |  | To City |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
|  |  | A | B | C | D | E |  |
| From | A | $\infty$ | 2 | 5 | 7 | 1 |  |
|  | B | 6 | $\infty$ | 3 | 8 | 2 |  |
|  | C | 8 | 7 | $\infty$ | 4 | 7 |  |
|  | D | 12 | 4 | 6 | $\infty$ | 5 |  |
|  | E | 1 | 2 | 3 | 8 | $\infty$ |  |

