Q. Code: 580884

Reg. No.

B.E. / B.TECH. DEGREE EXAMINATIONS, DEC 2019

Third Semester

MA18352 – DISCRETE MATHEMATICS

(Common to CS and IT)

(Regulation 2018)

Time: Three Hours **Maximum : 100 Marks**

Answer **ALL** questions

PART A - (10 X 2 = 20 Marks)

		CO	**RBT**
1.	Construct a truth table for the compound proposition $(p \to q) \to (q \to p)$.	**1**	**AP**
2.	Let $E = \{-1, 0, 1, 2\}$ denote the universe of discourse. If $p(x, y) : x + y = 1$, find the truth value of $(\forall x)(\exists y)p(x, y)$.	**1**	**AP**
3.	How many permutations of the letters ABCDEFGH contain the string ABC?	**2**	**AP**
4.	Find the recurrence relation satisfying the equation $y_n = A3^n + B(-4)^n$	**2**	**AP**
5.	Draw a complete bipartite graph of $K_{2,3}$ and $K_{3,3}$.	**3**	**U**
6.	Give an example of a graph which is Hamiltonian but not Eulerian.	**3**	**U**
7.	Give an example of a monoid which is not a group.	**4**	**U**
8.	Find the idempotent elements of $G = \{1, -1, i, -i\}$ under the binary operation multiplication.	**4**	**AP**
9.	Let $X = \{1, 2, 3, 4, 6, 8, 12, 24\}$ and R be a division relation. Draw the Hasse diagram of the Poset $\langle X, R\rangle$.	**5**	**AP**
10.	Is there a Boolean Algebra with five elements? Justify your answer.	**5**	**U**

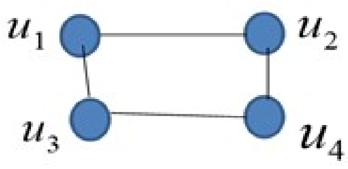
PART B - (5 X16 = 80 Marks)

11.	(a)	(i)	Obtain the principal conjunctive normal form and principal disjunctive normal form of $(P \wedge Q) \vee (\neg P \wedge R)$.	**(8)**	**1**	**AP**
		(ii)	Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.	**(8)**	**1**	**AP**

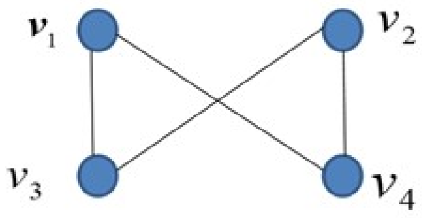
(OR)

	(b)	(i)	Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S), \neg R \vee P$ and Q.	**(8)**	**1**	**AP**
		(ii)	Use rules of inference to obtain the conclusion of the following arguments: "Linda, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."	**(8)**	**1**	**AP**
12.	(a)	(i)	State the principle of strong mathematical induction. Prove that a positive integer greater than 1 is either a prime number or it can be written as product of prime numbers.	**(8)**	**2**	**AP**
		(ii)	Find the number of integers between 1 and 500 both inclusive that are not divisible by any of the integers 2, 3, 5 and 7.	**(8)**	**2**	**AP**
			(OR)			
	(b)	(i)	There are six men and five women in a room. Find the number of ways four persons can be drawn from the room if (1) they can be male or female, (2) two must be men and two women, (3) they must all are of the same gender.	**(8)**	**2**	**AP**
		(ii)	Find the generating function of Fibonacci sequence and hence solve it.	**(8)**	**2**	**AP**
13.	(a)	(i)	Prove that a simple graph with n vertices and k components can have atmost $(n - k)(n - k + 1)/2$ edges	**(8)**	**3**	**AP**
		(ii)	Prove that a connected multigraph has an Euler Tour (Eulerian) if and only if each of its vertices has an even degree.	**(8)**	**3**	**AP**
			(OR)			
	(b)	(i)	Determine whether the following two graphs $G = (V, E)$ and $H = (W, F)$ are isomorphic.	**(8)**	**3**	**AP**

G:



H:



(ii) Explain Konigsberg bridge problem. Represent the problem by means of graph. Does the problem have a solution? **(8)** **3** **AP**

14. (a) (i) State and prove Lagrange's theorem on groups. Is the converse true? **(12)** **4** **AP**

(ii) Prove that the intersection of two subgroups of G is also a subgroup of G. **(4)** **4** **AP**

(OR)

(b) (i) Examine whether $G = \left\{\begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \neq 0, a \in R\right\}$ is an abelian group under matrix multiplication, where R is the set of all real numbers. **(8)** **4** **AP**

(ii) Show that the group $\{Z_n, +_n\}$ is isomorphic to every cyclic group of order n **(8)** **4** **AP**

15. (a) (i) Let S_{30} bet the set of positive divisors of 30. If $\leq$ is the relation of divisibility, prove that $(S_{30}, \leq)$ is a Poset. Draw the Hasse diagram of the Poset. **(8)** **5** **AP**

(ii) Show that every non empty subset of a Lattice has a least upper bound and a greatest lower bound **(8)** **5** **AP**

(OR)

(b) (i) In a distributive Lattice prove that $a * b = a * c$ and $a \oplus b = a \oplus c$ imply $b = c$. **(8)** **5** **AP**

(ii) Show that the De Morgan's laws are valid in a Boolean Algebra. **(8)** **5** **AP**