Reg. No. | | | | | | | | | |

M.E. / M.TECH. DEGREE EXAMINATIONS, MAY 2019

First Semester

MA16183 – ADVANCED NUMERICAL METHODS

(***Internal Combustion Engineering***)

(Regulation 2016)

Time: Three Hours **Maximum : 100 Marks**

Answer **ALL** questions

PART A - (10 X 2 = 20 Marks)

1. State the condition for convergence of iterative methods for solving a system of linear algebraic equations.
2. Find the dominant eigenvalue of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method.
3. Write Adams-Bash forth predictor-corrector method for solving the initial value problem.
4. Using R.K.method of second order, compute y(0.1) from $\frac{dy}{dx} = y - \frac{2x}{y}$, y(0) = 1.
5. State Lax Wandroff formula.
6. State any two explicit and implicit schemes to solve parabolic PDE.
7. Write the diagonal and standard five point formulae for solving the Laplace equation.
8. Write the finite difference form of Poisson equation $\nabla^2 u = f(x, y)$.
9. Explain Galerkin method?
10. Define Conforming elements.

PART B - (5 X16 = 80 Marks)

11. (a) (i) Using Gauss seidal iteration method solve $\begin{aligned} 27x + 6y - z &= 85 \\ x + y + 54z &= 110 \\ 6x + 15y + 2z &= 72 \end{aligned}$ **(8)**

(ii) Using Newton Raphson method find an approximate root of the equation $3x = \cos x + 1$ correct to three decimal places. **(8)**

(OR)

(b) (i) Determine the Eigen values of the characteristic polynomial of the system $(-1-\lambda)x_1 = 0$; $x_1 + (-2-\lambda)x_2 + 3x_3 = 0$; $2x_2 + (-3-\lambda)x_3 = 0$ by Fadeev Leverrier method. **(8)**

(ii) Solve the system of non-linear equations $x^2 + 3x + y = 5$; $x^2 + 3y^2 = 4$ by general iteration method. Taking (0.5, 0.5) is the initial approximation. Perform up to second iteration. **(8)**

12. (a) (i) Solve for y(0.4), Given $\frac{dy}{dx} = xy + y^2$; y(0)=1, y (0.1) =1.1169, y (0.2) =1.2774, y (0.3) =1.5041 using Adam's method. **(8)**

(ii) Using Runge-kutta method of fourth order find the value y(0.1), given $\frac{dy}{dx} = \frac{1}{x+y} \quad y(0) = 1$. **(8)**

(OR)

(b) Using Shooting method solve the Boundary value problem **(16)**

$y'' = y + 1, \quad 0 < x < 1$

$y(0) = 0, \quad y(1) = e - 1, with \; h = 0.5.$

13. (a) (i) Solve using Bender-Schmidt method $u_{xx} = u_t$ subject to u(0,t)=0, u(5,t)=0 and $u(x,0) = x^2(25 - x^2) \; 0 \le x \le 5$, find the value of u up to t=4 seconds by taking h=1. **(8)**

(ii) Solve the one dimensional heat equation $16u_t = u_{xx}$ subject to u(0,t)=0, u(1,t)=100t and $u(x,0) = 0 \; , 0 \le x \le 1$. Compute u for one time step with h=1/4 using Crank Nicolson's method. **(8)**

(OR)

(b) Solve the wave equation $u_{tt} = 4u_{xx}$ with boundary conditions **(16)**

$u(0,t) = 0, \; u(4,t) = 0, u(x,0) = x(4 - x) \quad 0 \le x \le 4 \quad and \quad u_t(x,0) = 0$

up to 6 time level.

14. (a) Solve $\nabla^2 u = 8x^2 y^2$ over the square mesh with sides $x = -2, x = 2, y = -2, y = 2$ given $u = 0$ on the boundary and mesh length=1 unit. **(16)**

(OR)

(b) Solve $u_{xx} + u_{yy} = 0 \; in \; 0 \le x \le 4, 0 \le y \le 4$ correct to two places of decimals by Liebmann method with boundary conditions: **(16)**

$(i) u(0, y) = 0 \qquad 0 \le y \le 4 \quad (ii) u(4, y) = 8 + 2y \quad 0 \le y \le 4$

$(iii) u(x,0) = \frac{x^2}{2} \qquad 0 \le x \le 4 \quad (iv) u(x,4) = x^2 \qquad 0 \le x \le 4$

15. (a) Solve the boundary value problem **(16)**

$\nabla^2 u = 1 \; on \; |x| \le 1, \; |y| \le 1, \; and \; u = 0 \; on \; the \; boundary \; |x| = 1, |y| = 1$ using Galerkin method and determine the solution values at nodes (0, 0), (1/2, 0) and (1/2, 1/2).

(OR)

(b) Obtain one parameter approximate solution of BVP **(16)**

$\nabla^2 u = x^2 - 1 \; on \; |x| \le 1, \; |y| \le 1/2, \; u = 0$ on the boundary by collocation method.