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## B.E. / B.TECH. DEGREE EXAMINATIONS, MAY 2023

## First Semester

## MA22151 - Applied Mathematics-I

(Common to all branches except Marine Engineering) (Regulation 2022)

## TIME: 3 HOURS

## COURSE

statement

## MAX. MARKS: 100

CO 1 Solve eigen value problems in matrices.
CO 2 Apply the basic notion of calculus in Engineering problems and to tackle different 3 geometries.
CO 3 Perform calculus for more than one variable and its applications in Engineering problems. 3
CO 4 Apply definite integrals for design of three-dimensional components. 3
CO 5 Evaluate multiple integrals in cartesian and polar coordinates.

## PART- A (20 x $2=40$ Marks

(Answer all Questions)
CO $\underset{\text { RBT }}{\text { RBEL }}$

1. Find the sum \& product of the eigen values of the matrix $\left(\begin{array}{lll}8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2\end{array}\right)$ using the properties. $\quad \mathbf{1} \quad \mathbf{2}$
2. Find the eigen value of $\left(\begin{array}{ll}2 & 3 \\ 0 & 4\end{array}\right)$ corresponding to the eigen vector $\binom{1}{0}$.
3. The product of two eigen values of the matrix $\left(\begin{array}{ccc}6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3\end{array}\right)$ is 16. Find the third eigen
value.
4. Write down the quadratic form corresponding to the matrix $\left(\begin{array}{ccc}0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2\end{array}\right)$
5. Find the radius of curvature of $y=e^{x}$ at the po int $w$ here it crosses the $y$ axis. $\mathbf{2} \quad \mathbf{2}$
6. Find the curvature of $x^{2}+y^{2}=25$ at $\left.3,-4\right)$. $2 \quad \mathbf{2}$
7. Find the evolute of a curve whose centre of curvature at any point ' $t$ ' is $\mathbf{2} \quad \mathbf{3}$ $\left(3 a t^{2}+2 a,-2 a t^{3}\right)$.
8. Find the envelope of $y=m x+\frac{a}{m}, \mathrm{~m}$ being the parameter.
9. Find the first order and second order partial derivatives of $z=x^{3}+y^{3}-3 a x y$.
10. If $u=e^{x} y z^{2}$ find du.
11. If $u=f(x-y, y-z, z-x)$ find $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$.

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12. If $\mathrm{u}, \mathrm{v}, \mathrm{w}$ are functions of $\mathrm{x}, \mathrm{y}, \mathrm{z}$ and $\frac{\partial(u, v, w)}{\partial(x, y, z)}=4$, find the value of $\frac{\partial(2 u, 2 v, 2 w)}{\partial(x, y, z)}$.
13. Evaluate $\int_{-1}^{1} x^{5} \cos x d x$
14. Evaluate: $\int_{0}^{\frac{\pi}{2}} \cos ^{8} x d x$.
15. Evaluate $\int x \log x d x$
16. Find mean value of the function $f(x)=x^{2}+2$ over the interval $[1,3]$.
17. Evaluate $\int_{2}^{a} \int_{2}^{b} \frac{d x d y}{x y}$
18. Find the value of $\int_{0}^{\pi} \int_{0}^{a \cos \theta} r \sin \theta d r d \theta$
19. Find the area of a circle of radius ' $a$ ' by double integration in polar coordinates.
20. Evaluate $\int_{0}^{1} \int_{0}^{2} \int_{0}^{3} x y z d x d y d z$

## PART- B (5 x 10 = 50 Marks)

21. (a) Find the eigen values and eigen vectors of the matrix $\left(\begin{array}{ccc}2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1\end{array}\right)$.

## (OR)

(b) Diagonalize the matrix $A=\left(\begin{array}{ccc}3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3\end{array}\right)$ by orthogonal transformation.
22. (a) Find the radius of curvature of the curve $\mathrm{x}^{3}+\mathrm{y}^{3}=3 \mathrm{axy}$ at the point $\left(\frac{3 a}{2}, \frac{3 a}{2}\right)$
(OR)
(b) Find the evolute of the curve $x=a(\cos t+t \sin t), y=a(\sin t-t \cos t)$.
23. (a) If $u=\log \left(x^{3}+y^{3}+z^{3}-3 x y z\right)$, show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^{2} u=\frac{-9}{(x+y+z)^{2}}$.

## (OR)

(b) A rectangular box open at top is to have a capacity of 108cu.ms. Find the (10) 3 dimensions of the box requiring least material for its construction.
24. (a) Find the volume of the solid generated by revolving the region between the $y$ - (10) 4
axis and the curve $x=\frac{2}{y}, 1 \leq y \leq 4$ about the $y$-axis. Also draw the region
and solid of revolution.

## (OR)

(b) Find the volume of the solid formed by revolving the region bounded by the graphs of $y=x^{2}+1, y=0, x=0$ and $\mathrm{x}=1$ about y -axis. Also draw the region and solid of revolution.
25. (a) Change the order of integration and hence evaluate $\int_{0}^{a} \int_{y}^{a} \frac{x}{\sqrt{x^{2}+y^{2}}} d x d y$.
(OR)
(b) Find the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0$ $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

## PART- C (1 x $10=10$ Marks)

(Q.No. 26 is compulsory)

Marks CO RBT Level
26. Verify Cayley -Hamilton Theorem for the matrix $\mathrm{A}=\left(\begin{array}{ccc}2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2\end{array}\right)$ and hence find $A^{4}$.
(10) 13

