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## B.E. / B.TECH. DEGREE EXAMINATIONS, MAY 2023

First Semester

## MA22152 - APPLIED MATHEMATICS I FOR MARINE ENGINEERS

## (Regulation 2022)

## TIME: 3 HOURS

MAX. MARKS: 100

## COURSE

OUTCOMES
CO 1

- Apply the basic concepts of analytical geometry in

CO 2 Apply the basic notion of calculus to engineering problems and to tackle different geometries.
CO 3 Perform calculus for more than one variable and its applications in engineering problems.
CO 4 Perform integration to compute arc lengths, volumes of revolution and surface areas of
CO 5 Apply integ
CO 5 Apply integration to compute multiple integrals, area, moment of inertia, integrals in polar coordinates, in addition to change of order.

## PART- A ( $20 \times 2=40$ Marks) <br> (Answer all Questions)

|  |  | CO | $\underset{\text { LEVEL }}{\text { RBT }}$ |
| :---: | :---: | :---: | :---: |
| 1. | Find the centre and radius of the sphere $x^{2}+y^{2}+z^{2}-2 x-4 y-6 z-2=0$ | 1 | 2 |
| 2. | Find the equation of the cone with vertex at the origin and passing through the curve $x^{2}+y^{2}=9, z=3$. | 1 | 2 |

3. Find the equation of the tangent plane to the sphere $x^{2}+y^{2}+z^{2}-x+5 y-3 z=0$ at $\mathbf{1}$ $(0,0,3)$.
4. Find the equation of the sphere on the join of $(1,-1,-1)$ and $(-1,2,3)$ as diameter. $\mathbf{1}$
5. Find the Taylor's series expansion of $\mathrm{f}(\mathrm{x})=\cos \mathrm{x}$ about $x=\frac{\pi}{2} \quad \mathbf{2}$
6. If $f(x)=x^{3}-3 x+2$, find $f^{\prime}(3)$.
7. Differentiate $8 x^{8}+2 \tan x+e^{-3 x} \quad \mathbf{2}$
8. Find the $\mathrm{n}^{\text {th }}$ derivative of $\sin (5 x+3) \quad \mathbf{2}$
9. If $u=\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$, find $x \frac{\partial u}{\partial x}+y \frac{\partial u}{\partial y}+z \frac{\partial u}{\partial z}$.
10. Find $\frac{d y}{d x}$ when $x^{2}+y^{2}=2 a x y$.
11. Find the first order partial derivatives of $u=e^{2 x} y^{2} z^{3}$. 3
12. If $x^{y}+y^{x}=c$ find $\frac{d y}{d x}$. 3
13. Find the average value of $f(x)=x^{3}$ in the interval $[1,2] \quad 4$
14. An electric current $i$ is given by the expression $i=L \cos \theta$ where L is constant. Find RMS
15. Find the centre and radius of the sphere $x^{2}+y^{2}+z^{2}-2 x-4 y-6 z-2=0$

2 $x^{2}+y^{2}=9, z=3$.
18. Evaluate $\int_{0}^{2} \int_{0}^{3} e^{3 x} d y d x$
19. Sketch the region of integration for $\int_{0}^{\infty} \int_{0}^{\infty} f(x, y) d y d x$
20. Evaluate $\int_{0}^{a} \int_{0}^{b} \int_{0}^{c}(x y z) d z d y d x$

## PART- B (5 x $10=50$ Marks)

21. (a) Find the equation of the right circular cone whose vertex is at the origin,
(10) 13 whose axis is the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$, and which has semi-vertical angle of $30^{\circ}$. (OR)
(b) Find the equation of the sphere through the circle
(10) 13
$x^{2}+y^{2}+z^{2}+2 x+3 y+6=0 ; x-2 y+4 z-9=0$ and the centre of the sphere
$x^{2}+y^{2}+z^{2}-2 x+4 y-6 z+5=0$
22. (a) If $y=\sin (\sin x)$ then prove that $\frac{d^{2} y}{d x^{2}}+\tan x \frac{d y}{d x}+y \cos ^{2} x=0$
(10) 23
(OR)
(b) If $y=e^{a \sin ^{-1} x}$ then prove that

$$
\left(1-x^{2}\right) y_{n+2}-2(n+1) x y_{n+1}-\left(n^{2}+a^{2}\right) y_{n}=0
$$

23. (a) If $z$ is a function of $x$ and $y$ and $u$ and $v$ are other two variables, such that
(10) 3 $u=l x+m y, v=l y-m x$. Show that
$\frac{\partial^{2} z}{\partial x^{2}}+\frac{\partial^{2} z}{\partial y^{2}}=\left(l^{2}+m^{2}\right)\left(\frac{\partial^{2} z}{\partial u^{2}}+\frac{\partial^{2} z}{\partial v^{2}}\right)$.
(OR)
(b) Find the extreme values of the function
$f(x, y)=x^{3}+y^{3}-3 x-12 y+20$
24. (a) Find the first and second moment of area under $y=1+x+x^{2}$ from $x=0$ to
(10) 4 $x=2$ about $y$-axis.
(OR)
(b) Find the centroid of the region bounded by $y=x^{2}$ and $y=\sqrt{x}$.
(10) 43
25. (a) Find the area of the circle $\boldsymbol{x}^{2}+\boldsymbol{y}^{2}=\boldsymbol{a}^{2}$
(10) 5
(OR)
(b) Evaluate $\int_{0}^{1} \int_{0}^{\sqrt{1-x^{2}}} \int_{0}^{\sqrt{1-x^{2}-y^{2}}} \frac{1}{\sqrt{1-x^{2}-y^{2}-z^{2}}} d z d y d x$
(10) 5

## PART- C ( $1 \times 10=10$ Marks)

(Q.No. 26 is compulsory)
26. Find the maximum value of $x^{m} y^{n} z^{p}$ subject to the condition $x+y+z=a$.

| Marks | CO | RBT <br> LEVEL |
| :---: | :---: | :---: |
| $(10)$ | 3 | 3 |

