Reg. No.

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## B.E. / B.TECH. DEGREE EXAMINATIONS, MAY 2023

## Second Semester

## MA22252 - APPLIED MATHEMATICS II FOR MARINE ENGINEERS

(Marine Engineering)
(Regulation 2022)

## TIME: 3 HOURS

COURSE
OUTCOMES
CO 1

Apply the basic concepts of ordinary differential equations and its applications in marineengineering problems.

CO 2 Apply various techniques in solving differential equations.
CO 3 Solve gradient, divergence and curl of a vector point function and related identities, evaluation of line, surface and volume integrals using Gauss, Stokes and Green's theorems.
CO 4 Recognize fundamental properties of analytic functions and construct simple conformal maps.
CO 5 Apply Laplace transforms to solve differential equation.

## PART- A ( $20 \times 2=40$ Marks $)$

(Answer all Questions)

1. Solve $x^{-4} \frac{d y}{d x}=e^{-3 y} \quad 1$
2. Form the differential equation from the following equation $y=A_{1} \cos 3 x+A_{2} \sin 3 x \quad 1 \quad \mathbf{1}$
3. Find the integrating factor of $\frac{d y}{d x}+\frac{y}{x}=x \quad 1$
4. Find the general solution of the differential equation $\frac{d y}{d x}=\frac{1+y^{2}}{1+x^{2}} \quad \mathbf{1}$
5. Solve $\frac{d^{2} y}{d x^{2}}-5 \frac{d y}{d x}+6 y=0$.

22
6. Find the Particular Integral of $\left(D^{2}-4 D+4\right) y=e^{2 x+3} \quad \mathbf{2}$
7. Solve $\left(D^{2}-4\right) y=2$. $\mathbf{2}$
8. $\quad$ Solve $\left(\mathrm{x}^{2} \mathrm{D}^{2}-\mathrm{x} D+1\right) \mathrm{y}=0 \quad 2$
9. Determine the constant ' $a$ ' such that the vector $\vec{F}=(x+z) \vec{\imath}+(3 x+a y) \vec{\jmath}+$
$(x-5 z) \vec{k}$ has its divergence to be zero.
10. What is the greatest rate of increase of $\phi=x y z^{2}$ at $(1,0,3)$ ? 3
11. Prove that $\vec{F}=x^{2} \vec{\imath}+y^{2} \vec{\jmath}+z^{2} \vec{k}$ is a conservative vector field. 3
12. Find a unit normal to the surface $x^{2} y+2 x z^{2}=8$ at the point $(1,0,2)$. 3
13. Is $f(z)=z^{3}+z$ is analytic? Justify. 4
14. Show that $u=2 x-x^{3}+3 x y^{2}$ is harmonic. 4
15. Show that an analytic function with constant real part is constant. 4
16. Find the invariant points of $f(z)=z^{2} \quad 4$
17. Find $\int_{0}^{\infty} e^{-t} \sin 2 t d t \square 5$
18. Find $\mathrm{L}\left[t^{2} e^{-2 t}\right]$
$5 \quad 2$
19. Find $L^{-1}\left(\frac{s}{(s+4)^{2}}\right)$
20. Verify the final value theorem for $f(t)=3 e^{-2 t}$

## PART- B (5 x $10=50$ Marks)

21. (a) Solve $\frac{d y}{d x}=\left(\frac{y}{x} \sec ^{2} \frac{y}{x}-\tan \frac{y}{x}\right) \cos ^{2} \frac{y}{x}$
(OR)
(b) Find the orthogonal trajectory of the cardioids $r=a(1-\cos \theta)$
Marks CO RBT LEVEL
(10) 13
22. (a) Solve $\left(D^{2}+a^{2}\right) y=\tan a x$ by method of variation of parameter.
(10) 23
(OR)
(b) Solve $(1+x)^{2} \frac{d^{2} y}{d x^{2}}+(1+x) \frac{d y}{d x}+y=4 \cos [\log (1+x)]$.
(10) 23
23. (a) Show that $\vec{F}=\left(6 x y+z^{3}\right) \vec{\imath}+\left(3 x^{2}-z\right) \vec{\jmath}+\left(3 x z^{2}-y\right) \vec{k}$ is irrotational
(10) 3 and find its scalar potential.
(OR)
(b) Verify Green's theorem for $\int_{C}\left(x^{2} d x+x y d y\right)$ where $C$ is the curve in the
(10) 3 $X Y$ plane given by $x=0, y=0, x=a, y=a(a>0)$.
24. (a) Find the bilinear transformation which maps the points $-1,0,1$ in the $z$ plane onto the points $0, i, 3 i$ in the $w$ plane.

## (OR)

(b) Find the analytic function $f(z)=u+i v$ whose real part is

$$
u=e^{x}(x \cos y-y \sin y)
$$

25. (a) Find the inverse Laplace transform of the following functions using convolution theorem $\frac{s}{\left(s^{2}+a^{2}\right)^{2}}$

## (OR)

(b) Solve the following differential equations using Laplace transform
(10) 5 $y^{\prime \prime}-2 y^{\prime}+2 y=0$ given that $y(0)=1, y^{\prime}(0)=1$

## PART- C ( $1 \times 10=10 \mathrm{Marks})$

(Q.No. 26 is compulsory)
26. Solve $\frac{d^{2} y}{d x^{2}}+\frac{1}{x} \frac{d y}{d x}=\frac{12 \log x}{x^{2}}$

| Marks | CO | RBT |
| :---: | :---: | :---: |
| LEVEL |  |  |
| (10) | 2 | 3 |

