## Reg. No.

## B.E./ B. TECH.DEGREE EXAMINATIONS, MAY 2023

First Semester

## MA22253 - MATHEMATICS FOR DATA SCIENCE

(Artificial Intelligence and Data Science) (Regulation 2022)

## TIME: 3 HOURS

| COURSE <br> OUTCOMES <br> CO 1 | STATEMENT |
| :--- | :--- |
| CO 2 | Terform operations on various discrete structures such as sets, functions and relations. |
| CO 3 | Identify structures on many levels as an application of the concepts and properties of <br> algebraic structures. |
| CO 4 | Apply the basic notions of groups, rings, fields which will be used to solve related <br> problems. |
| CO 5 | Execute the simplification of Boolean algebraic expression. |

## PART- A(20x2=40Marks)

(Answer all Questions)

1. Find all the partitions of $\{x, y, z\}$ and list them.

## RBT

 LEVEL2. If $A=\{0,1\}, B=\{0,-1\}$ then find $A \times B \& B \times A$. Are they equal? $\mathbf{1} \quad \mathbf{2}$
3. Let $f: R \rightarrow R$ defined by $f(x)=x^{2}+1$. Find $f^{-1}(10)$ and $f^{-1}(-4) \quad \mathbf{1} \quad \mathbf{2}$
4. $\operatorname{Letf}(x)=x+5, g(x)=2 x+3$. Compute $f \circ g \& g \circ f \quad 1 \quad 2$
5. Express the statement "Good food is not cheap" in symbolic form $\quad \mathbf{2} \quad \mathbf{2}$
6. Negate the statement: " John is playing football" in two different forms $\quad \mathbf{2} \quad \mathbf{2}$
$\begin{array}{lllll}\text { 7. Write the converse and contra-positive of the conditional statement: Indian Criket team } & \mathbf{2} & \mathbf{2}\end{array}$ wins whenever match is played in Kolkata
7. Find the truth value of $(x)(P \rightarrow Q(x)) \vee(\exists x) R(x)$ where $\mathrm{P}: 2>1, \mathrm{Q}(\mathrm{x}): \mathrm{x}>3$, 2 $R(x): x>4$, with the universe of discourse is $E=\{2,3,4\}$
8. Find the orders of the elements $(-1)$ and 3 in $\left(\mathrm{R}^{*}, \bullet\right)$ where $\mathrm{R}^{*}=\mathrm{R}-\{0\}$
9. Verify whether $\mathrm{G}=\{1,-1, \mathrm{i},-\mathrm{i}\}$ is a cyclic group
10. Find all right cosets of $\{[0],[3]\}$ in the group $\left(\mathrm{Z}_{6},{ }_{6}\right)$

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2. Check whether the permutation $g=\left(\begin{array}{llll}1 & 2 & 3 & 4 \\ 4 & 3 & 1\end{array}\right)$ is odd or even? $\quad \mathbf{3}$
3. What is the degree of the polynomial $f(x)=\mathbf{6} \boldsymbol{x}^{\mathbf{3}}+\mathbf{5} \boldsymbol{x}^{2}+\mathbf{3 x}-\mathbf{2}$ over $\boldsymbol{Z}_{6} \quad 4$
4. What is the remainder when $\mathrm{f}(\mathrm{x})=\mathrm{x}^{5}+2 \mathrm{x}^{3}+\mathrm{x}^{2}+2 \mathrm{x}+3 \in \mathrm{Z}_{5}[\mathrm{x}]$ is divided by $\boldsymbol{x}-\mathbf{1} \quad 4 \quad 2$
5. Is $f(x)=x^{3}+x+4 \in Z_{2}[x]$ over $Z$ and $C$ irreducible? 4
6. What are the units in the ring $(\mathrm{Q},+, \mathrm{x})$
7. Draw the Hasse diagram for $\{(\mathrm{a}, \mathrm{b}) /$ a divides b$\}$ on the set $\{1,2,3,4,6,8,12\}$
8. Prove that $\left(Z^{+}, \mid\right)$is Poset where $Z^{+}$is a set of positive integers.
9. Apply Demorgan's law for $\quad \mathbf{5} \quad \mathbf{2}$ $\overline{[(x+\bar{y})(\bar{x}+y)]}$
10. Check whether $D_{12}=\{1,2,3,4,6,12\}$ is a finite Boolean algebra.

## PART- B (5x 10=50Marks)

21. (a) Let $R$ be a binary relation on the set of positive integers such that (10) 1 $\mathrm{R}=\left\{(\mathrm{a}, \mathrm{b}) / \mathrm{a}=\mathrm{b}^{2}\right\}$. What are the properties of R ? Is R an equivalence relation? Partial ordering?

## (OR)

(b) If $\mathrm{S}=\{1,2,3,4,5\}$ and if $\mathrm{f}, \mathrm{g}, \mathrm{h}: \mathrm{S} \rightarrow \mathrm{S}$ are given by
(10) 13
$\mathrm{f}=\{(1,2),(2,1),(3,4),(4,5),(5,3)\} . \mathrm{g}=\{(1,3),(2,5),(3,1),(4,2),(5,4)\}$.
$\mathrm{h}=\{(1,2),(2,2),(3,4),(4,3),(5,1)\}$. Verify whether $\mathrm{f} \circ \mathrm{g}=\mathrm{g} \circ \mathrm{f}$ (i)explain why f
and $g$ has inverse but $h$ does not (ii) find $f^{-1} \& g^{-1}$ (iii)prove that
$(f \circ g)^{-1}=g^{-1} \circ \mathrm{f}^{-1} \neq \mathrm{f}^{-1} \circ \mathrm{~g}^{-1}$
22. (a) Establish the relation $\mathrm{p} \rightarrow(\mathrm{q} \rightarrow \mathrm{r}) \Rightarrow(\mathrm{p} \rightarrow \mathrm{q}) \rightarrow(\mathrm{p} \rightarrow \mathrm{r})$
(10) 23

## (OR)

(b) Test the validity of the following: "Sonia is watching TV. If Sonia is (10) $2 \mathbf{3}$ watching a TV, then she is not studying. If she is not studying, then her father will not buy her a scooty. Therefore, Sonia's father will not buy her a scooty"
23. (a) Prove that the non- zero elements of $\mathrm{Z}_{7}$ is a group under multiplication

## (OR)

(b) Determine (i) $\alpha \beta$ (ii) $\alpha^{3}$ (iii) $\beta^{4}$ (iv) $\alpha^{-1}$ and $\beta^{-1}$ (v) $(\alpha \beta)^{-1}, \beta^{-1} \alpha^{-1}$ In a group $S_{5}=\{1,2,3,4,5\}$ where
$\alpha=\left(\begin{array}{lllll}1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5\end{array}\right) \& \beta=\left(\begin{array}{lllll}1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 3 & 4\end{array}\right)$
24. (a) Find $[777]^{-1}$ in the ring $Z_{1009}$

## (OR)

(b) Test the polynomial $\mathrm{x}^{2}+\mathrm{x}+4 \in \mathrm{Z}_{11}[\mathrm{x}]$ for irreducibility over $\mathrm{Z}_{11}$
25. (a) Given $P(S)$, the set of all sub sets of $S=\{a, b, c\}$ and $\subseteq$, the inclusion relation on $\mathrm{S}(\mathrm{i})$ Show that $(P(S), \subseteq)$ is a poset (ii) Draw the Hasse diagram of the poset(iii) Find the minimal and maximal element and a chain of length 3 (OR)
(b) Let $\mathrm{B}=\mathrm{D}_{30}=\{1,2,3,6,10,12,15,30\}$, the divisors of 30 with the divisibility as order. For any $a, b \in B, a+b=\operatorname{lcm}(a, b), a . b=\operatorname{gcd}(a, b), \quad a^{\prime}=\frac{30}{a}$, Verify that $(B,+, \ldots, 1,30)$ is a Boolean Algebra

## PART- C (1x 10=10Marks)

## (Q.No. 26 is compulsory)

Verify whether the compound proposition $(p \vee q) \rightarrow(p \wedge q)$ is a tautology LEVEL 3 or a contradiction. Also find the PDNF and PCNF of the same if it exists.
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