# Engineering Economics and Cost Analysis

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# Syllabus

## UNIT I - INTRODUCTION TO ECONOMICS

Introduction to Economics- Flow in an economy, Law of supply and demand, Concept of Engineering Economics – Engineering efficiency, Economic efficiency, Scope of engineering economics- Element of costs, Marginal cost, Marginal Revenue, Sunk cost, Opportunity cost, Break-even analysis- V ratio, Elementary economic Analysis – Material selection for product Design selection for a product, Process planning.

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## UNIT II - VALUE ENGINEERING

Make or buy decision, Value engineering – Function, aims, Value engineering procedure. Interest formulae and their applications –Time value of money, Single payment compound amount factor, Single payment present worth factor, Equal payment series sinking fund factor, Equal payment series payment Present worth factor- equal payment series capital recovery factor-Uniform gradient series annual equivalent factor, Effective interest rate, Examples in all the methods.

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**UNIT III CASH FLOW**

Methods of comparison of alternatives – present worth method (Revenue dominated cash flow diagram), Future worth method (Revenue dominated cash flow diagram, cost dominated cash flow diagram), Annual equivalent method (Revenue dominated cash flow diagram, cost dominated cash flow diagram), rate of return method, Examples in all the methods.

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UNIT IV - REPLACEMENT AND MAINTENANCE ANALYSIS

Replacement and Maintenance analysis – Types of maintenance, types of replacement problem, determination of economic life of an asset, Replacement of an asset with a new asset – capital recovery with return and concept of challenger and defender, Simple probabilistic model for items which fail completely.

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## UNIT V - DEPRECIATION

Depreciation- Introduction, Straight line method of depreciation, declining balance method of depreciation-Sum of the years digits method of depreciation, sinking fund method of depreciation/ Annuity method of depreciation, service output method of depreciation-Evaluation of public alternatives- introduction, Examples, Inflation adjusted decisions – procedure to adjust inflation, Examples on comparison of alternatives and determination of economic life of asset.

# Contents

# Objective

- The key to success for any productive enterprise is taking the right and result oriented decisions – Objective

- Social problems- unemployment, inflation, poverty, population, taxes, international financial crises have economic roots.

# What is economics about

- Economics is the social science concerned with the analysis of commercial activities and with how goods and services are produced.

- The field of economics studies how the things, people **need** and **want** are made and brought to them.

# Goals of Economics

- A high level of employment
- Price stability
- Efficiency
- An equitable distribution of income
- Growth

# Economics - definition

- “a **social science** concerned chiefly with the way, society chooses to employ its **limited resource**, which have **alternative uses**, to produce goods and services for present and future consumption”.

- Economics is the science that deals with the production and consumption of goods and services and the distribution and rendering of these for human welfare

# Goods

- Economic goods are those goods which possess money value or those for procuring which human effort is required.
- Economic value of a good depends upon the place under consideration.
- Economic goods have the following properties
  - They satisfy human wants or have utility
  - They are scarce or limited in supply.
  - They are marketable i.e have money value and may be exchanged.

# Want vs need

- A *need* is something you *have* to have, something you can't do without.

- A *want* is something you *would like* to have. It is not absolutely necessary, but it would be a good thing to have.

## Functions of an Economic System

- To match supply to the effective demand for goods and services in an efficient manner

- To determine what goods and services are to be produced and in what quantities.

- To distribute scarce resources among the industries producing goods and services

# Functions of an Economic System

- To distribute the products of industry among members of the community.

- To provide for maintenance and expansion of fixed capital investment.

- To fully utilise the resources of society.

# Economic problems

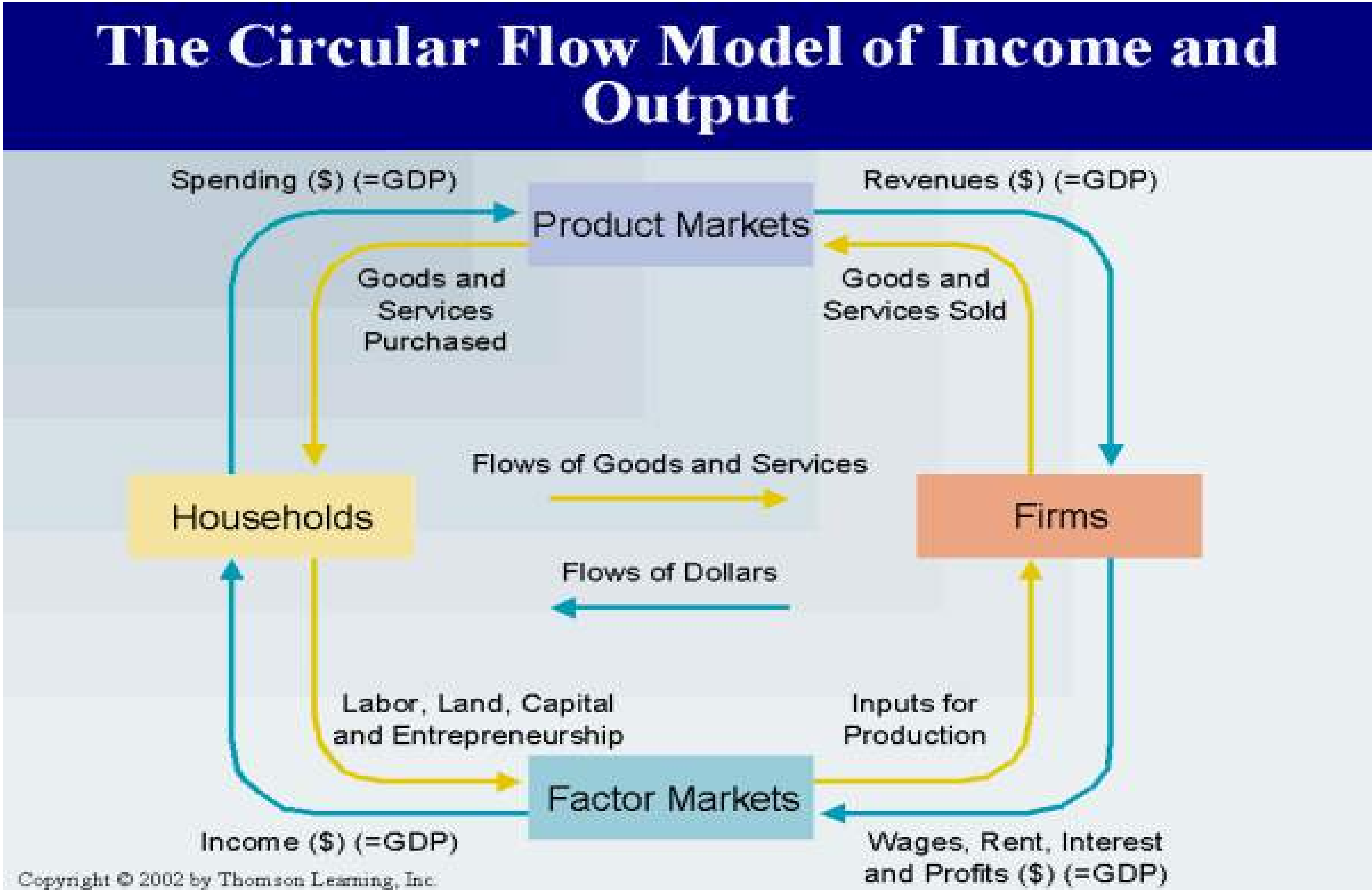
- What goods and services should be produced?
- How should resources be organised for production?
- Who shall get the goods and services?
- How fast shall the economy grow?

# Engineering Economics

- Engineering Economics is a science which deals with the application of economic theory in Engineering Practices.

- It is the study of allocation of resources available to a firm among its activities.

# Flow in an Economy



# Flow in an Economy

- Households and businesses are the two major entities in a simple economy.
- Business organizations use various economic resources like land, labour and capital which are provided by households to produce consumer goods and services which will be used by them.
- Business organizations make payment of money to the households for receiving various resources.
- The households in turn make payment of money to business organizations for receiving consumer goods and services.

# Demand

- Demand is a relation showing the various amounts of a commodity that buyers would be willing and able to purchase at possible alternative prices during a given period of time, all other things remaining the same.

- **The law of demand states that <u>other things remaining the same,</u> when the price of a commodity falls, its quantity demanded rises and when the price of a commodity rises, it quantity demanded falls.**

- In other words, other things remaining the same, **there is an inverse relationship between the price of a commodity and its quantity demanded.**

# Meaning of Demand

- **Individual demand**
  - Demand by one buyer for a commodity is called individual demand.
  - Demand for a commodity by an individual is the quantity of that commodity that the individual is willing to buy at a price over some period of time.
- **Definition include**
  - Quantity of a commodity that a buyer is willing to buy
  - The price of the commodity at which he is willing to buy that quantity, and
  - The time period during which he is willing to buy that quantity at the given price (The time period may be a day, a week, a month, a year or any other period)

# Demand

- Demand for a commodity implies:
  - Desire to acquire it,
  - Willingness to pay for it, and
  - Ability to pay for it.

# Demand

- The demand for mangoes by a consumer is 3kgs per day

- The demand for mangoes by a consumer is 3kgs per day when the price of mangoes is Rs. 10/- kg

- The demand for mangoes is 3 kgs, when the price of mangoes is Rs. 10/- kg

# Market demand

- There are many buyers of a commodity.

- If we add the quantity of the commodity that each of its buyer is willing to buy at a price over a time period, we will get the market demand of the commodity.

- Thus market demand means the total quantity of a commodity that all its buyers are willing to buy at a given price over a time period.

# Want or desire and demand

- Mere want or desire for a commodity by a person is not called his demand for that commodity.
- The want or desire will become a demand if we have the ability to buy it and we are willing to buy it.
- Thus demand is the want or desire for a good backed by the ability and willingness to pay for it.

# True or False

1. My demand for milk is 10 litres
2. My demand for milk is 10 litres per month when the price of milk is Rs. 10 per litre
3. My demand for milk is 10 litres per month whatever may be the price of milk
4. Hari is a rich man and can buy a car, so Hari has a demand for a car.
5. Want for goods means demand for goods
6. Want of a consumer for goods becomes his demand for it when it is backed by ability and willingness of the consumer to pay for it.

# Factors affecting law of demand

- **Price of a commodity**
  - Change in real income or **purchasing power** of the buyer of the commodity
  - Substitution of one commodity for other commodity
- **Other factors which include**
  - Income of the buyer of the commodity
  - Tastes and preferences of the buyer
  - Prices of the related goods

## Change in real income or purchasing power of the buyer of the commodity

- Purchasing power or real income means the quantity of goods and services that one can buy with the given money income.

- An increase in purchasing power means more can be bought with the same money income and a decrease in purchasing power means less can be bought with the same money income.

# Change in real income or purchasing power of the buyer of the commodity

- Example
  - 15 litres of milk per month when the price is Rs. 8per litre
  - Price of the milk falls to Rs. 6 per litre
  - Price of the milk rises from Rs. 8 per litre to Rs. 10 per litre.

## Substitution of one commodity for other commodity

- Two commodities are said to be substitutes of each other when one can be used in place of other.

- Example
  - Kerosene, LPG, electricity
  - Tea and coffee

# Other factors that affect the demand of a commodity

- Income of a buyer **(Normal Vs Inferior Goods)**
- Tastes and preferences of the buyer
- Prices of the related goods
    - Substitute Goods
    - Complementary Goods

# Exceptions to the law of Demand

- Prestige goods
- Giffen goods
- Expectations

# True or False

- The law of demand applies only on essential goods
- The law of demand states that other things remaining the same, the price of a commodity and its quantity demanded are inversely related
- Other things remaining same means the other factors affecting demand do not change.
- The law of demand also applies on goods that have prestige value.
- If the price of a commodity is rising and is expected to continue to rise in future, its quantity demanded will start falling.
- Price of a good is only one of the factors that affects the demand for a good.
- Demand curve slopes downward from left to right.

# True or False

- Other things remaining the same, when the price of a good rises its demand also rises.
- An individual demand schedule shows the quantities demanded of a commodity at different prices.
- When the price of a good rises the purchasing power of its buyer also rises.

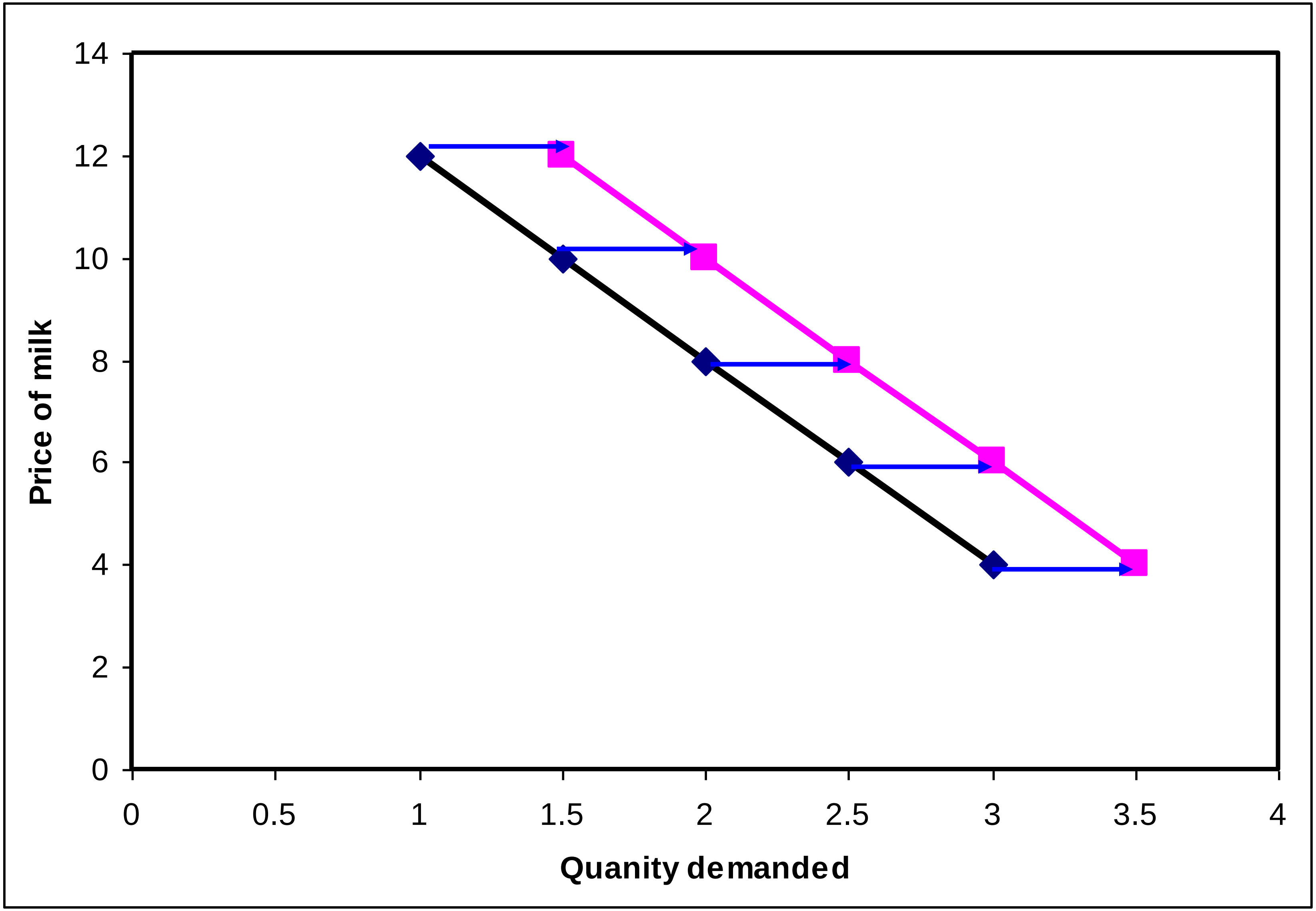
# Fill in the blanks

- The demand for a good is also affected by price of ________ (all goods, related goods)
- In case of a ________good the increase in income of its buyer leads to a fall in its demand. (normal, inferior)
- The demand for a commodity ______________ when the price of its substitute commodity rises. (decreases, increases)
- The demand for a commodity increases if the price of its complementary commodity _____( falls, rises)

# Expansion of Demand and Increase in Demand

| Price of milk per litre (Rs) | Quantity demanded of milk per day (in litres) | Quantity demanded when demand rises |
|---|---|---|
| 12 | 1 | 1.5 |
| 10 | 1.5 | 2.0 |
| 8 | 2 | 2.5 |
| 6 | 2.5 | 3.0 |
| 4 | 3 | 3.5 |

# Expansion of Demand and Increase in Demand



# Expansion of Demand and Increase in Demand

- The expansion of demand results in a downward movement along the demand curve

- Increase in demand results in a rightward shift of the demand curve.

# Contraction of Demand and Decrease in Demand

| Price of milk per litre (Rs) | Quantity demanded of milk per day (in litres) | Quantity demanded when demand rises |
|---|---|---|
| 12 | 1 | 0.5 |
| 10 | 1.5 | 1.0 |
| 8 | 2 | 1.5 |
| 6 | 2.5 | 2.0 |
| 4 | 3 | 2.5 |

# Fill in the blanks

- When demand rises due to fall in price, it is called ______ of demand.
- A rightward shift in demand curve shows ______ in demand.
- A decrease in demand will result in a _____ shift of demand curve
- A upward movement along the same demand curve shows ______ of demand.

# Elasticity of Demand

- The elasticity of demand measures the responsiveness of quantity demanded to a change in any one of the factors by keeping other factors constant.

- **Types**
  - **Price elasticity of demand**
  - **Income elasticity of demand**
  - **Cross Elasticity of demand**

# Price Elasticity of Demand

- Price elasticity of demand is the degree of responsiveness of quantity demanded of a good to a change in its price.

- “The ratio of proportionate change in the quantity demanded of a good caused by a given proportionate change in price”

$$\text{Price Elasticity of Demand} = \frac{\text{Percentage in Quantity Demand}}{\text{Percentage Change in Price}}$$

$$E_d = \frac{\Delta q}{\Delta p} \times \frac{P}{Q}$$

# Income Elasticity of Demand

- Income is an important variable affecting the demand for a good.

- When there is a change in the level of income of a consumer, there is a change in the quantity demanded of a good, other factors remaining the same.

- The degree of change or responsiveness of quantity demanded of a good to a change in the income of a consumer is called income elasticity of demand.

# Income Elasticity of Demand

The ratio of percentage change in the quantity of a good purchased, per unit of time to a percentage change in the income of a consumer".

$$E_y = \frac{\text{Percentage Change in Demand}}{\text{Percentage Change in Income}}$$

# Cross Elasticity of Demand

- The concept of ***cross elasticity of demand*** is used for measuring the responsiveness of quantity demanded of a good to changes in the price of related goods.

"The percentage change in the demand of one good as a result of the percentage change in the price of another good".

$$E_{xy} = \frac{\text{\% Change in Quantity Demanded of Good X}}{\text{\% Change in Price of Good Y}}$$

# Classification of Elasticity of Demand

1. **Price Elasticity of Demand**
   a. Perfectly elastic demand
   b. Perfectly inelastic demand
   c. Relatively elastic demand
   d. Relatively inelastic demand
   e. Unitary elastic demand
2. **Income Elasticity of Demand**
   a. Zero income elasticity of demand
   b. Negative income elasticity of demand
   c. Unitary income elasticity of demand
   d. Income elasticity of demand is greater than one
   e. Income elasticity of demand is less than one
3. **Cross Elasticity of Demand**
   a. Cross elasticity of demand is positive
   b. Cross elasticity of demand is zero
   c. Cross elasticity of demand is negative

# Classification of levels of Price Elasticity

- Perfectly Elastic Demand
- Perfectly Inelastic Demand
- Relatively More Elastic Demand
- Relatively less Inelastic demand
- Unitary Elastic Demand

# Perfectly Elastic Demand

- When there is a small fall in the price of a product, it will result no change in the demand and in a small rise in the prices, it will lead to big contraction of demand, even to zero.

# Perfectly Inelastic Demand

- When there is a big change in the price of certain commodities, there is no change in the demand of those commodities.

- E.g. Most necessity items like salt, rice.

# Relatively More Elastic Demand

- When there is a small change in the price of any commodity, there is a greater change in its demand.

- Commodities fall under comforts and luxury will expand the demand when there is a small change in its price.

- The elasticity of demand is greater than one

# Relatively Less Inelastic Demand

- When there is a large change in the price of any commodity, there is a smaller change in its quantity demand.
- Necessaries of life fall under this category.
- The elasticity of demand is less than one

# Unitary Elastic Demand

- When there is a change in the price of any commodity, there is an equal change in its quantity demanded.

- The elasticity of demand is equal to one.

# Price Elasticity of Demand at a glance

| Numeral value of elasticity | Term for elasticity of demand | Descriptions |
|---|---|---|
| Infinity | Perfectly elastic | Total revenue falls to zero |
| Zero | Perfectly Inelastic | No change in the quantity demanded in response to the change in the price |
| Greater than 1 | Relatively elastic | There is a small change in the price and there is a greater change in its demand |
| Less than 1 | Relatively inelastic | There is a large change in the price and there is a small change in its demand |
| One | Unitary elastic | Percentage change in quantity demanded is equal to the percentage change in its prices |

# Elasticity levels for Income elasticity of demand

- When the income of a person increases, his demand for goods also changes depending upon whether the good is a normal good or an inferior good.

- For normal goods, the value of elasticity is greater than zero but less than one. Goods with an income elasticity of less than 1 are called inferior goods.

- For example, people buy more food as their income rises but the % increase in its demand is less than the % increase in income.

## Elasticity levels for Cross elasticity of demand

(i) **Substitute Goods.** When two goods are substitute of each other, such as coke and Pepsi, an increase in the price of one good will lead to an increase in demand for the other good.

(ii) The numerical value of goods is positive.

# Elasticity levels for Cross elasticity of demand

**Complementary Goods.** In case of complementary goods such as car and petrol, cricket bat and ball, a rise in the price of one good say cricket bat by 7% will bring a fall in the demand for the balls (say by 6%).

The cross elasticity of demand which are complementary to each other is, therefore, 6% / 7% = 0.85 (**negative**).

# Elasticity levels for Cross elasticity of demand

**Unrelated Goods**. The two goods which are unrelated to each other, say apples and pens, if the price of apple rises in the market, it is unlikely to result in a change in quantity demanded of pens.

The elasticity is zero for unrelated goods.

# Importance of elasticity of demand

- Fixing the price of a commodity

- If the demand of the commodity is inelastic, the company may rise the price to maximise the profits.

- If the demand of the commodity is elastic, the company may lower the price to maximise the demand.

# Factors affecting the elasticity of demand

- Nature of the product
  - **Essential goods – inelastic / Comforts - Elastic**
  - **Luxury goods – More Elastic**
- Different use of a commodity / Extent use of the commodity
  - **Variety of uses – Elastic demand / Limited Use – Inelastic demand**
- Range of substitutes
  - **Perfect substitute products – elastic demand**
- Income level
  - **High income – Inelastic / Low income – elastic**
- Proportion of Income spent on commodities
- Purchase frequency of a product – Elastic
- Habit and Fashion
- Deferred Consumption - Elastic

# Law of supply

- The "law of supply" is a fundamental principal of economic theory which states that quantities respond in the same direction as price changes

- In other words, the **law of supply** states that (all other things unchanged) an increase in price results in an increase in quantity supplied.

- This means that producers are willing to offer more products for sale on the market at higher prices by increasing production as a way of increasing profits

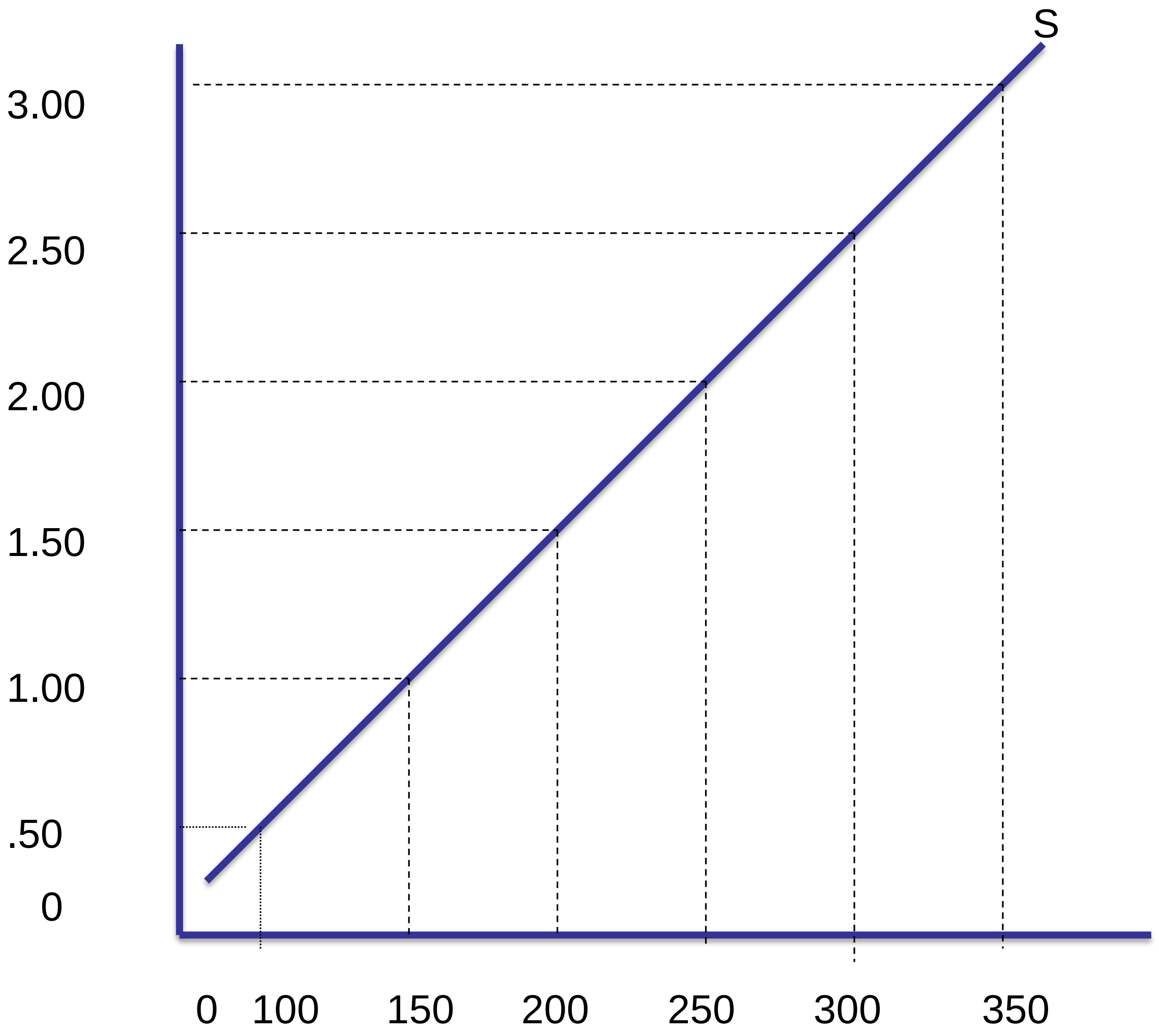
# Law of supply Vs Law of Demand

**Review of the Laws of Supply and Demand**

| The Law of Supply | The Law of Demand |
| --- | --- |
| states that at higher prices, producers are willing to offer more products for sale than at lower prices | states that people will buy more of a product at a lower price than at a higher price, if nothing changes |
| states that the supply increases as prices increase and decreases as prices decrease | states that at a lower price, more people can afford to buy more goods and more of an item more frequently, than they can at a higher price |
| states that those already in business will try to increase productions as a way of increasing profits | states that at lower prices, people tend to buy some goods as a substitute for others more expensive |

# Supply Schedule

| Price Per Slice of Pizza (Rs) | Slices Supplied Per Day (#) |
|---|---|
| .50 | 100 |
| 1.00 | 150 |
| 1.50 | 200 |
| 2.00 | 250 |
| 2.50 | 300 |
| 3.00 | 350 |


S
3.00
2.50
2.00
1.50
1.00
.50
0
0
100
150
200
250
300
350

# Elasticity of Supply

- **Elasticity of Supply**: a measure of the way suppliers respond to a change in price

- **Elastic:** sensitive/responsive to change in price (greater than 1)

- **Inelastic:** not sensitive or not responsive to a change in price (less than 1)

- **Unitary:** Equal change in price to equal change in supply (= to 1)

# Elasticity of Supply

- A supplier's responsiveness to a price change is called ___**Elasticity of Supply**___

  » **(think like a supplier/seller)**

**3 Factors that will determine a product's elasticity**

1. Availability of resources required to make the product
2. Amount of time required to make the product
3. Skill level of the worker needed to make the product

# Elastic Supply

- A product has elastic supply when a price change causes a significant change in the quantity supplied.

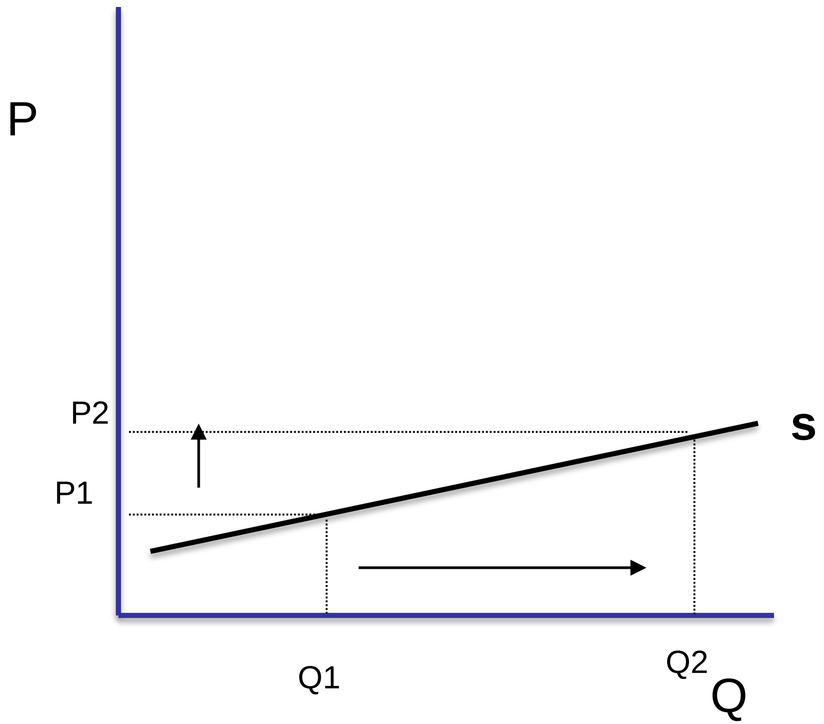
1. Abundance of resources required to make the product

2. Product can be made quickly

3. Low skill level of workers required

# Slope of an Elastic supply curve

- Remember, if the price changes, the quantity supplied changes a lot.  This creates a flatter curve.



# Inelastic Supply

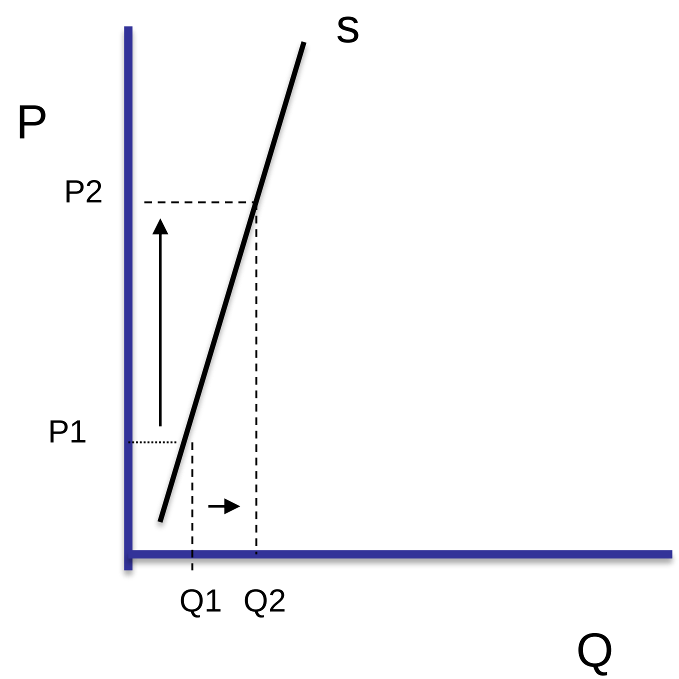
- A price change causes very little change in the quantity supplied.

This happens because…

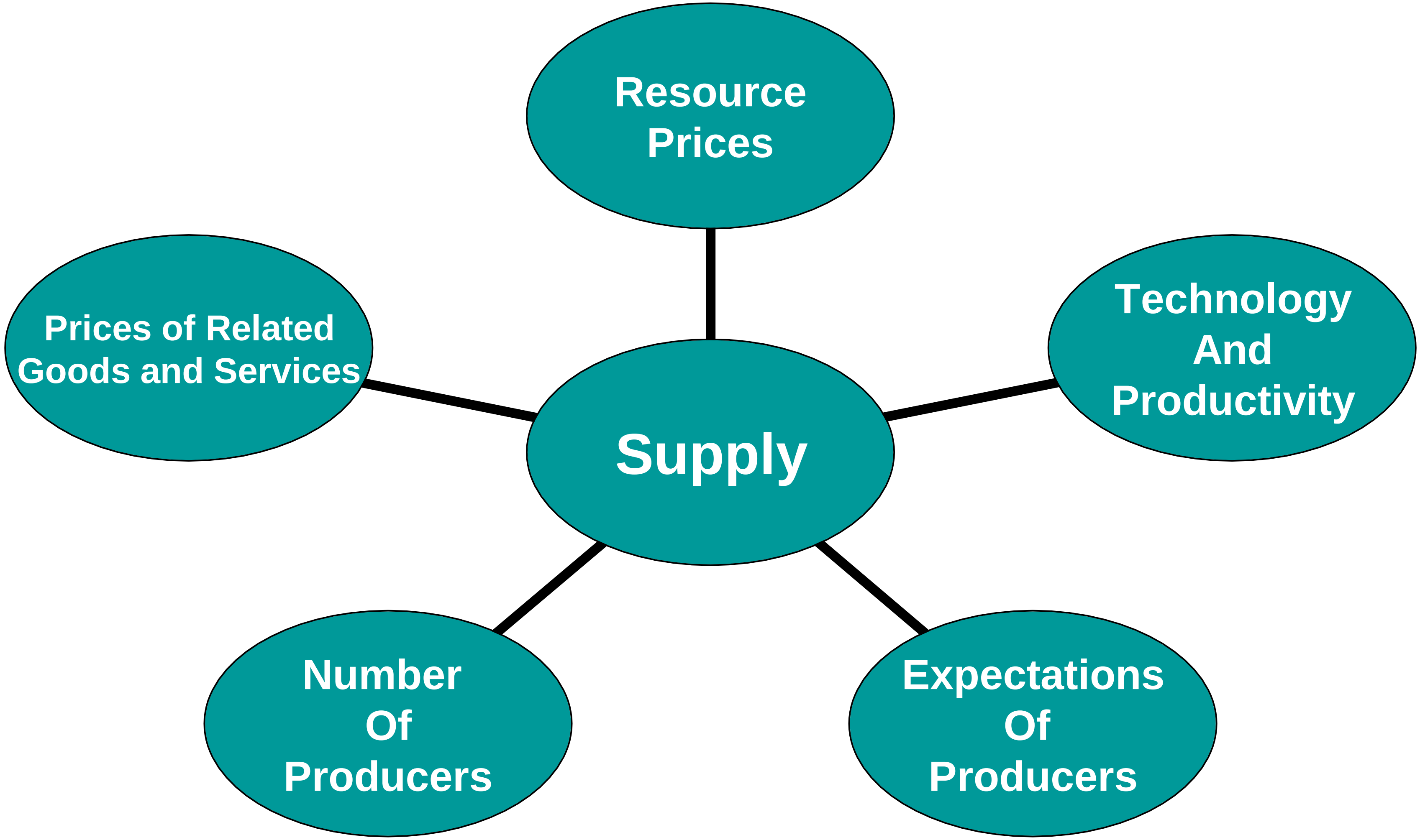
- The product requires scarce resources
- It takes a long time to make
- It requires a high skill level of workers
  - Hand crafted furniture
  - diamonds

# Slope of an inelastic supply curve

- If the market price goes up but the supplier cannot increase production very much, then this creates a steeper curve.



# Factors that Shift Supply



# Price of Inputs (Resource Prices)

- When costs go up, profits go down, so that the incentive to supply also goes down.

# Technology

- Advances in technology reduce the number of inputs needed to produce a given supply of goods.

- Costs go down, profits go up, leading to increased supply.

# Expectations

- If suppliers expect prices to rise in the future, they may store today's supply to reap higher profits later.

# Number of Suppliers

- As more people decide to supply a good the market supply increases (Rightward Shift).

# Price of Related Goods or Services

- The opportunity cost of producing and selling any good is the foregone opportunity to produce another good.

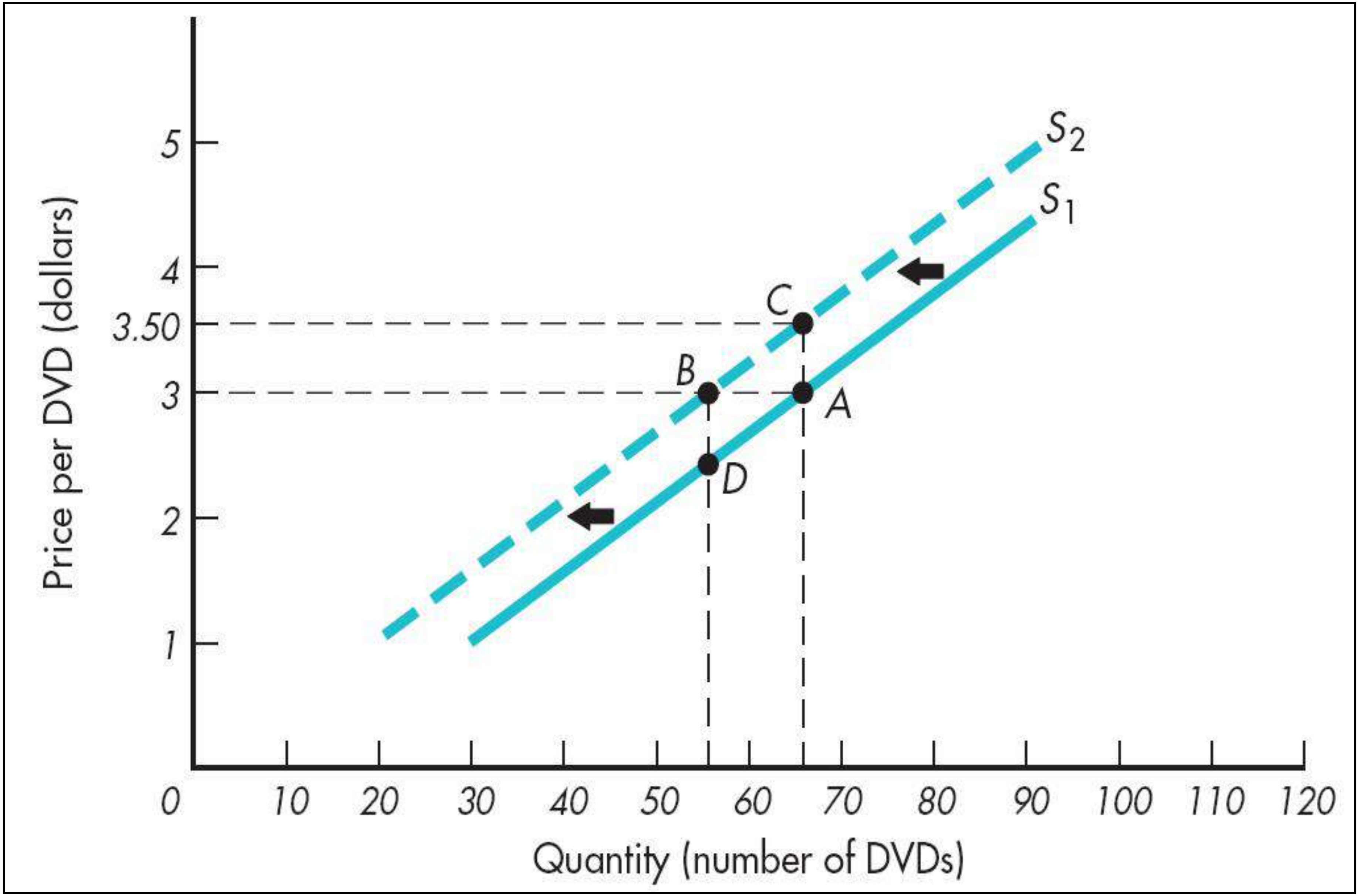
- If the price of alternate good changes then the opportunity cost of producing changes too!

# Taxes and Subsidies

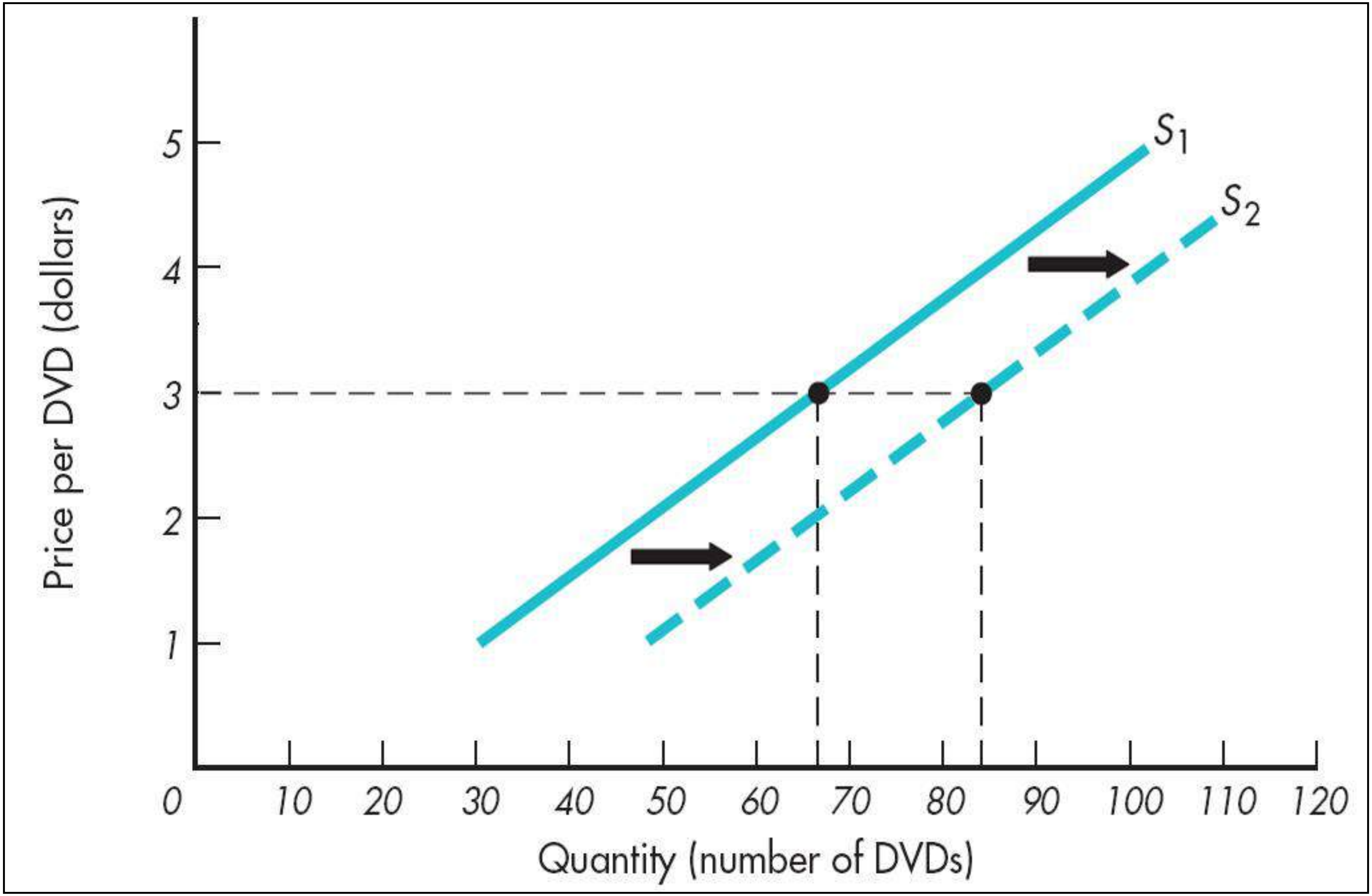
- When taxes go up, costs go up, and profits go down, leading suppliers to reduce output.

- When government subsidies go up, costs go down, and profits go up, leading suppliers to increase output.

# Decrease in Supply

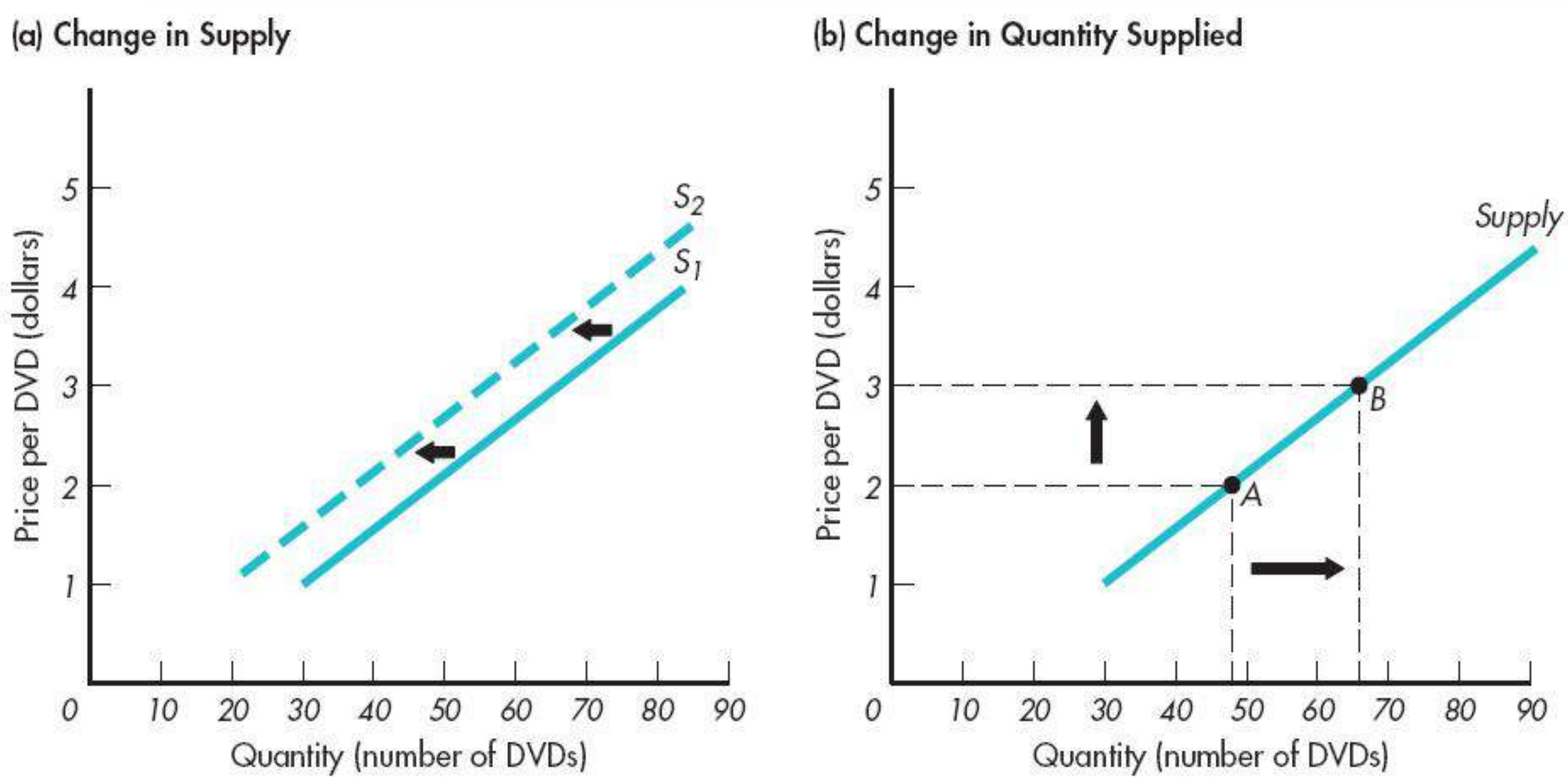


# Increase in Supply



# Change in Supply vs. a Change in the Quantity Supplied

**FIGURE 9** A Change in Supply and a Change in the Quantity Supplied




In Figure 9(a), the quantities that producers are willing and able to offer for sale at every price decrease, causing a leftward shift of the supply curve from $S_1$ to $S_2$. In Figure 9(b), the quantities that producers are willing and able to offer for sale increase, because of an increase in the price of the good, causing a movement along the supply curve from point *A* to point *B*.

# Concept of Engineering Economics

- Engineering is the application of science

- Price has a major role in deciding the demand and supply of a product.

- Hence for a organization, efficient and effective functioning would certainly help it to provide goods/ services at a lower cost which in turn will enable it to fix a lower price for its goods or services. – evolution of Engineering Economics

# Concept of Engineering Economics

- **Engineering Economics** deals with the methods that enable one to take **economic decisions towards minimizing costs and /or maximizing benefits** to business organizations.

# Scope of Engineering Economics

- Interest formulae
- Bases of comparing alternatives
  - Present worth method
  - Future worth method
  - Annual equivalent method
  - Rate of return method
- Replacement analysis
- Depreciation
- Evaluation of public alternatives
- Inflation adjusted investment decisions
- Make or buy decisions
- Inventory control
- Project management
- Value Engineering
- Linear Programming

# Applications of Engineering Economics

- Selection of location and site for a new plant
- Production planning and control
- Selection of an equipment and their replacement analysis
- Selection of a material handling systems
- Determination of plant capacity
- Determination of wage structure of the workers
- It can be applied by a major corporation to analyse plans for a new manufacturing facility or a new research and development centre.

# Advantages of Engineering Economics

- Better decision making on the part of Engineers
- Efficient use of resources result in better output and economic advancement.
- Cost of production can be reduced.
- Alternative courses of action using economic principles may result in reduction of prices of goods and services.
- Elimination of waste can result in application of engineering economics
- More capital will be made available for investment and growth
- Improves the standard of living with the result of better products, more wages and salaries, more output, etc from the firm applying engineering economics

# Types of Efficiency

- **Technical Efficiency**
- **Economic Efficiency**

- **Technical Efficiency** is the ratio of the output to input of a physical system. The physical system may be diesel engine, a machine working in a shop floor, a furnace, etc.

- **Economic efficiency** is the ratio of output to input of a business system or ratio of worth to cost of a business firm.

# Types of Efficiency

- **Worth** is the annual revenue generated by way of operating the business and **cost** is the total annual expenses incurred in carrying out the business.

## Several ways to improve economic efficiency

- Increased output for the same input
- Decreased input for the same output.
- By a proportionate increase in the output which is more than the proportionate increase in the input.
- By a proportionate decrease in the input which is more than the proportionate decrease in the output.
- Through simultaneous increase in the output with decrease in the input.

# Elements of Cost

## Classification of Cost: 1. Variable Cost

**Variable cost varies with the production volume.**

**Classification of Variable Cost:**

| | |
|---|---|
| **1. Direct Material Cost:** | cost of materials that are used to produce the product. |
| **2. Direct Labour Cost:** | amount of wages paid to the labour, who involved in the production activities. |
| **3. Direct Expenses:** | expenses that vary in relation to the production volume, other than the direct material and direct labour costs. |

# Elements of Cost

## Classification of Cost: 2. Overhead Cost

**Overhead cost is fixed, irrespective of production volume.**

**Classification of Overhead Cost:**

**1. Factory Overhead:** indirect expenses incurred in factory right from work-order receipt till goods-despatch.

**2. Administrative Overhead:** all the costs incurred in administering the business.

**3. Selling Overhead:** total expenses incurred in promotional activities & expenses relating to sales force.

**4. Distribution Overhead:** total cost of shipping the items from factory site to customer site.

# Selling Price of a product

| | |
|---|---|
| 1. Prime Cost = | = DMC + DLC + DE |
| 2. Factory cost | = Prime cost + Factory Overhead |
| 3. Production Cost | = Factory cost + Administrative overhead |
| 4. Cost of goods sold | = Production Cost + Opening stock – Closing stock |
| 5. Cost of Sales | = Cost of goods sold + Selling and distribution overhead |
| 6. Selling Price | = Cost of sales + Profit |
| 7. Selling price per unit | = Sales ÷ Quantity sold |

# Marginal Cost

- It is the cost of producing an additional unit of that product.

- Let the cost of producing 20 units of a product be Rs. 10,000.

  Let the cost of producing 21 units of the same product be Rs. 10,045.

  Then the marginal cost of producing the 21st unit is Rs. 45.

# Marginal Revenue

- It is the incremental revenue of selling an additional unit of a product.

- Let, the revenue of selling 20 units of a product be Rs. 15,000.

  Let the revenue of selling 21 units of the same product be Rs. 15,085.

  Then, the marginal revenue of selling the 21st unit is Rs. 85.

# Sunk Cost

- Past cost of an equipment / asset.

- An equipment was purchased for Rs. 1,00,000 about 3 years back.

- If it is considered for replacement, then its present value is not Rs. 1,00,000.

- Instead, its present market value should be taken as the present value of the equipment for further analysis.

- So, the purchase value of the equipment in the past is known as its sunk cost.

- The sunk cost should not be considered for any analysis done from now onwards.

# Opportunity Cost

- A set of alternatives (X &Y) is available for investment.
- Let us assume an alternative (X) is selected and you get return from it.
- If the same money is invested in alternative (Y), it may fetch some more return.
- Since the money is invested in the selected alternative (X), one has to forego the return from the other alternative (Y).

# Opportunity Cost

- The amount that is foregone by not investing in the other alternative (Y) is known as the opportunity cost of the selected alternative (X).

- So the opportunity cost of an alternative is the return that will be foregone by not investing the same money in another alternative.

# Opportunity Cost - Example

- Consider You invested a sum of Rs. 50,000 in shares.
- Let the expected annual return be Rs. 7,500.

- If the same amount is invested in a fixed deposit, a bank will pay a return of 18%. Then, the corresponding total return per year is Rs. 9,000.

- Return from Bank is greater than the return from shares.

- The foregone excess return of Rs. 1,500 by way of not investing in the bank is the opportunity cost of investing in shares.

# Break-even Point

## Break Even Point:

– Total sales revenue equals Total expenses

– Point at which no profit, no loss occurs.

**In Graphical representation,**

the intersection point of Total sales revenue line & Total cost line

# Breakeven Chart

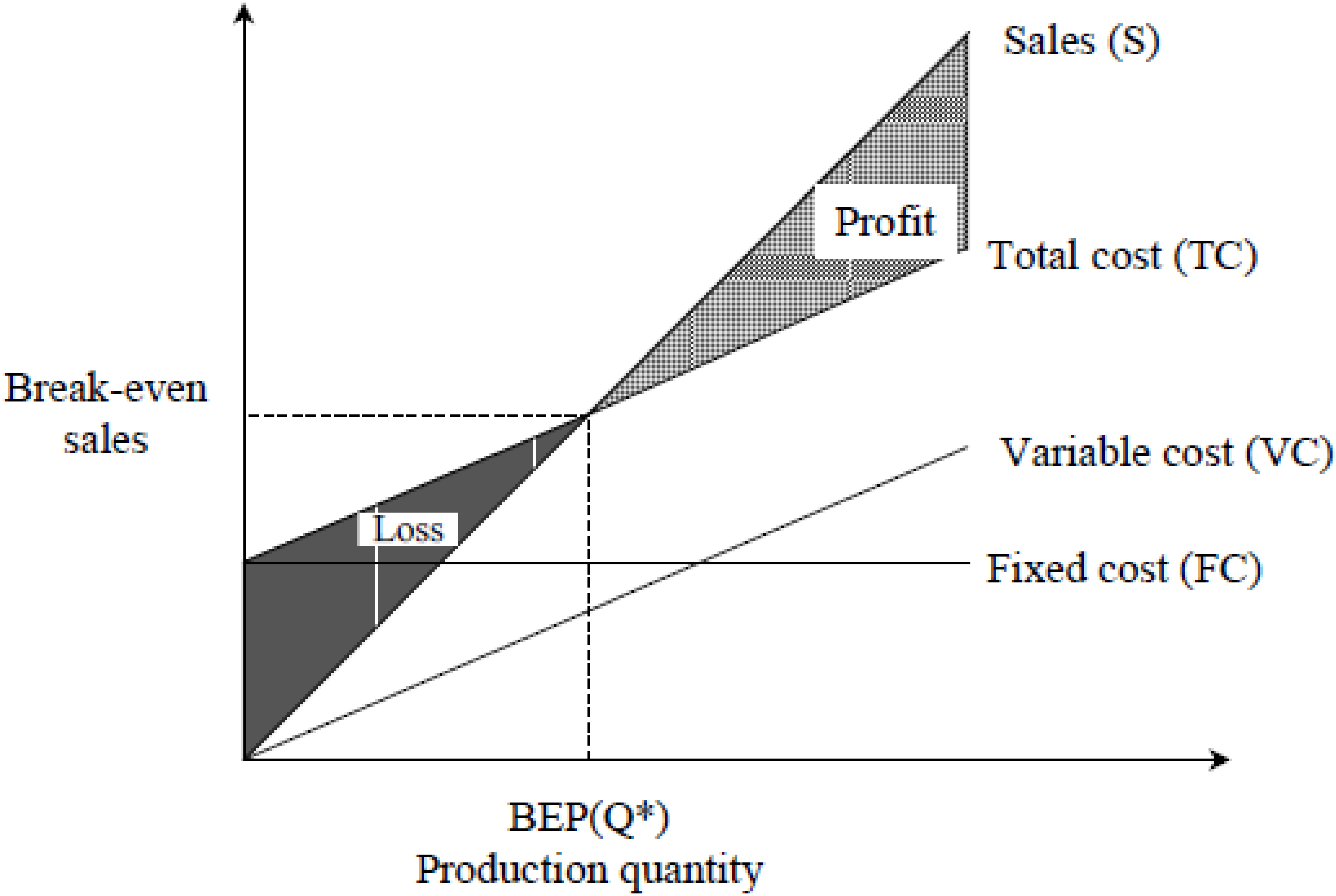



**Fig. 1.3** Break-even chart.

# Break-even Point

- If the production qty < the break-even qty:

  Total expense is more than total revenue.

  Hence, the firm will be making LOSS.

- If the prodn. qty is more than the break-even qty:

  Total revenue is more than total expenses.

  Hence, the firm will be making PROFIT.

# Break-even Analysis

Let s = selling price per unit          v = variable cost per unit

FC = fixed cost per period          Q = volume of production

1. Total sales revenue (S) = s x Q

2. Total cost (TC) = Total variable cost + Fixed Cost

   = (v x Q) + FC

3. Profit = Total Sales Revenue – Total Cost

   = *s x Q – (FC + v x Q)*

4. Break-even quantity (BEQ) = $\dfrac{\text{Fixed cost (FC)}}{\text{Selling price/unit (s)} - \text{Variable cost/unit (v)}}$

5. Break-even sales = BEQ x Selling price/unit

6. Contribution = Sales – Variable costs

7. Margin of Safety = Actual sales – Break-even sales

   = $\dfrac{\text{Profit}}{\text{Contribution}} \times \text{sales}$

# Break Even Analysis: Problem 1

***Alpha Associates has the following details:***

| | |
|---|---|
| Fixed cost | = Rs. 20,00,000 |
| Variable cost per unit | = Rs. 100 |
| Selling price per unit | = Rs. 200 |

Find

(a) The break-even sales quantity,

(b) The break-even sales

(c) If the actual production quantity is 60,000, find

(i) contribution; and

(ii) margin of safety by all methods.

# Break Even Analysis: Solution for Q1

$$\text{(a) Break-even quantity} = \frac{FC}{s - v} = \frac{20{,}000{,}00}{200 - 100}$$

$$= 20{,}00{,}000/100$$

$$= 20{,}000 \text{ units}$$

$$\text{(b) Break-even sales} = \frac{FC}{s - v} \times s = \frac{20{,}000{,}00}{200 - 100} \times \text{Rs.}200$$

$$= 40{,}000{,}000$$

# Break Even Analysis: Solution for Q1

(c) (i) Contribution = Sales – Variable cost

$= (s \times Q) - (v \times Q)$

= (200 x 60,000) – (100 x 60,000)

= 1,20,00,000 – 60,00,000

= Rs. 60,00,000

(c) (ii) Margin of Safety = Sales – Break-even sales

= (200 x 60,000) – (200 x 20,000)

= 1,20,00,000 – 40,00,000

= Rs. 80,00,000

# Contribution

- The **contribution margin** is a concept used with **breakeven point** or in **breakeven analysis.**

- The **contribution margin** is the amount of money a company has to cover its **fixed costs** after it pays all of its **variable expenses.**

- It is also the amount, after covering fixed costs, that contributes to the **net operating profit** or **net operating loss** of the business firm

# Contribution

- **Contribution Margin = Sales Revenue – Variable expenses**

- ***On a per unit basis, contribution is calculated as:***

- **Contribution Margin per unit of sales = Sales Revenue per unit – Variable expenses per unit**

- **Contribution Margin – Fixed Costs = Net Profit or Loss.**

# Contribution - Example

- The break even formula calculates the point at which a company's sales are zero – there is no profit or loss

- Break even in units =

$$\frac{\text{Total Fixed costs}}{\text{Contribution Margin per unit}}$$

- In a company where fixed costs are Rs. 60,000, the price of the product is Rs. 2.00 per unit and variable costs are 80 paise per unit.

# Contribution

- Break even point = 50,000 units
- The contribution margin in this case is Rs. 1.20 per unit.
- The company has to produce and sell 50,000 units of their products in order to cover their total expenses, fixed and variable.
- At this level of sales, they will make no profit but will just break even with a contribution margin towards the fixed costs of Rs. 1.20 per unit sold or Rs. 60,000

# Profit / Volume (P/V) Ratio

- It is the ratio of contribution to sales.

$$P/V\ ratio = \frac{\text{Contribution}}{\text{Sales}} = \frac{(\text{Sales} - \text{Variable costs})}{\text{Sales}}$$

Contribution is the difference between sales revenue and variable cost i.e. (Sales Revenue – Variable Cost).

A high P/V Ratio: indicates that higher profits.

A low P/V ratio: indicates that low profitability.

# Calculating Profit-Volume Ratio

1. look up the price of the product.
2. Calculate the cost involved to produce a single unit of the product.
3. Subtract that number from the initial price number, and this is profit.
4. Divide that profit figure by the price point, and this is profit-volume ratio.
5. Multiply the result by 100 for a percentage.

$$P/V\ ratio = \frac{\text{Contribution}}{\text{Sales}} = \frac{(\text{Sales} - \text{Variable costs})}{\text{Sales}}$$

# Improving Profit-Volume Ratio

By the following ways, an improvement in P/V ratio can be achieved:

1. The selling price increase; but the risk that the volume of sales might be affected.

2. By purchasing the latest machinery, a reduction in the variable cost per unit can be achieved, thereby cutting the production hours.

3. By concentrating on the products by which highest contribution can be achieved.

For doing business analysis, in the hands of management, the P/V ratio is an invaluable tool.

# Advantages of P/V Ratio

1. This ratio determines profitability of a line of product & also overall profitability.

2. This ratio compares the profitability of different lines of products, sales, companies, factories etc.

3. This ratio calculates break-even sales, profit at different levels of output, turnover which may be required for a desired profit or to offset reduction in price or to meet increased expenditure.

# Problem No. 2

Consider the following data of a company for the year 1997:

- Sales = Rs. 1,20,000
- Fixed cost = Rs. 25,000
- Variable cost = Rs. 45,000
- **Find the following:**

    (a) Contribution

    (b) Profit

    (c) BEP

    (d) M.S.

# Solution for Problem No. 2

(a) Contribution = Sales – Variable costs

= Rs. 1,20,000 – Rs. 45,000

= Rs. 75,000

(b) Profit = Contribution – Fixed cost

= Rs. 75,000 – Rs. 25,000

= Rs. 50,000

# Solution for Problem No. 2

(c) BEP

$$P/V \text{ ratio} = \frac{\text{Contribution}}{Sales}$$

$$= \frac{75000}{120000} \text{ x } 100 = 62.50\%$$

$$\text{BEP} = \frac{\text{Fixed } Cost}{\frac{P}{V}\, Ratio}$$

$$= \frac{25000}{62.50} \text{ x } 100 = \text{Rs. } 40{,}000$$

$$\text{(d) M.S.} = \frac{Profit}{\frac{P}{V}\, Ratio} \text{ X } 100$$

$$= \frac{50000}{62.50} \text{ x } 100 = \text{Rs. } 80{,}000$$

# Problem No. 3

Consider the following data of a company for the year 1998:

- Sales = Rs. 80,000
- Fixed cost = Rs. 15,000
- Variable cost = 35,000

- **Find the following:**

  (a) Contribution (b) Profit

  (c) BEP (d) M.S.

# Solution for Problem No. 3

(a) Contribution = Sales – Variable costs

= Rs. 80,000 – Rs. 35,000

= Rs. 45,000

(b) Profit = Contribution – Fixed cost

= Rs. 45,000 – Rs. 15,000

= Rs. 30,000

# Solution for Problem No. 3

(c) BEP

$$P/V \text{ ratio} = \frac{\text{Contribution}}{Sales}$$

$$= \frac{45000}{80000} \text{ x } 100 = 56.25\ \%$$

$$\text{BEP} = \frac{\text{Fixed } Cost}{\frac{P}{V}\,Ratio}$$

$$= \frac{15000}{56.25} \text{ x } 100 = \text{Rs. } 26{,}667$$

$$\text{M.S.} = \frac{Profit}{\frac{P}{V}\,Ratio} \text{ X } 100$$

$$= \frac{30000}{56.25} \text{ x } 100 = \text{Rs. } 53{,}333.33$$

# Problem No. 4

A company produces a single article. About its product, the following cost data has been given:

| | |
|---|---|
| Selling price per unit | Rs. 40 |
| Marginal cost per unit | Rs. 24 |
| Fixed cost per annum | Rs. 1600 |

Calculate:

(a) P/V ratio,

(b) Break-even sales,

(c) Sales to earn a profit of Rs. 200,

(d) Profit at sales of Rs. 12000,

(e) If sales price is reduced by 10%, then a new break-even sales.

# Solution for Problem No. 4

We know that

*Sales – Variable cost = Fixed cost + Profit*

By multiplying & dividing left hand side by Sales,

$$\frac{\textit{Sales x (Sales – Variable Cost)}}{\textit{Sales}} = \textit{Fixed cost + Profit}$$

*i.e*  *Sales x P/V ratio = Contribution*

(a) P/V ratio = Contribution / Sales * 100

= [(40-24)/40] * 100

= 16/40 * 100

= 40%

# Solution for Problem No. 4

(b) *Fixed cost*

*Break even Sales = ------------------*

*P/V ratio*

Sales = 1600/40

= Rs. 4000

(c) Sales to earn a profit of Rs. 200:

*Sales * P/V ratio = Fixed cost + Profit*

Sales * 40% = 1600 + 200

Sales = 1800 / 40%

Sales = Rs. 4500

# Solution for Problem No. 4

(d) Profit at sales of Rs. 12000:

*Sales * P/V ratio = Fixed cost + Profit*

12000 * 40% = 1600 + Profit

Profit = Rs. 3200

(e) New break-even sales, if sales price is reduced by 10%:

New Sales price = Rs. 40 – Rs. 4 = Rs. 36

Marginal cost = Rs. 24

Contribution = Rs. 36 – Rs. 24 = Rs. 12

*P/V ratio = Contribution / Sales*

= (12/36) *100 = 33.33%

*B.E.S * P/V ratio = Fixed Cost (at B.E.P, contribution is equal to fixed cost)*

Or, B.E.S = 1600/33.33%

Or, B.E.S = Rs. 4800

# Elementary Economic Analysis

Introduction: economic decision making involved in day-to-day events.

Example: purchasing of raw materials from a nearby or far-off place.

| Factors affect taking decision | from a nearby place | from far-off place |
|---|---|---|
| Price | more costly | Less cost |
| Transportation cost | Minimum | Very high |
| Availability | not sufficient enough to support the operation throughout the year | Abundant; can support throughout the year |
| Quality | requires pre-processing before it is used in the production process | Does not require pre-processing before it is used in the production process |

# Elementary Economic Analysis

- The procurement of the raw material should be decided in such a way **that the overall cost is minimized.**

# Elementary Economic Analysis

## EXAMPLES FOR SIMPLE ECONOMIC ANALYSIS:

The concept of simple economic analysis is illustrated using suitable examples in the following areas:

- Material selection for a product
- Design selection for a product
- Process planning

# Material selection for a Product

Among various elements of cost,

raw material cost is most significant and it forms a major portion of the total cost of any product.

**Objective :-**

To find a suitable raw material that will bring a reduction in the total cost in any one or combinations of the following ways:

- Cheaper raw material price
- Reduced machining/process time
- Enhanced durability of the product

**Note**

If the new raw material provides any additional benefit, then it should be treated as its welcoming feature.

# Material selection for a Product

- In the design of a jet engine part, the designer has a choice of specifying either an aluminium alloy casting or a steel casting. Either material will provide equal service, but the aluminium casting will weigh 1.2 kg as compared with 1.35 kg for the steel casting.

# Material selection for a Product

- The aluminium can be cast for Rs. 80.00 per kg. and the steel one for Rs. 35.00 per kg. The cost of machining per unit is Rs. 150.00 for aluminium and Rs. 170.00 for steel.

- Every kilogram of excess weight is associated with a penalty of Rs. 1,300 due to increased fuel consumption. Which material should be specified and what is the economic advantage of the selection per unit?

# Material selection for a Product

***(a) <u>Cost of using aluminium metal for the jet engine part:</u>***

- Weight of aluminium casting/unit = 1.2 kg
- Cost of making aluminium casting = Rs. 80.00 per kg
- Cost of machining aluminium casting per unit = Rs. 150.00

# Material selection for a Product

- Total cost of jet engine part made of aluminium/unit = (Cost of making aluminium casting/unit) + (Cost of machining aluminium casting/unit)

  = (80 x 1.2) + 150 = 96 + 150

  = Rs. 246/-

# Material selection for a Product

***<u>(b) Cost of jet engine part made of steel/unit:</u>***

- Weight of steel casting/unit = 1.35 kg
- Cost of making steel casting = Rs. 35.00 per kg
- Cost of machining steel casting per unit = Rs. 170.00
- Penalty of excess weight of steel casting = Rs. 1,300 per kg

# Material selection for a Product

- Total cost of jet engine part made of steel/unit
- = (Cost of making steel casting/unit)

+ (Cost of machining steel casting/unit)

+ (Penalty for excess weight of steel casting)

$= 35 \times 1.35 + 170 + 1{,}300(1.35 - 1.2)$

= Rs. 412.25

# Material selection for a Product: Problem 2

A company manufactures dining tables which mainly consist of a wooden frame and a table top. The different materials used to manufacture the tables and their costs are given in Table:

| Description of item | Quantity | Cost |
|---|---|---|
| Wood for frame and legs | 0.1 $m^3$ | Rs. 12,000/$m^3$ |
| Table top with sunmica finish | 1 | Rs. 3,000 |
| Leg bushes | 4 | Rs. 10/bush |
| Nails | 100 g | Rs. 300/kg |
| Total labour | 15 hr | Rs. 50/hr |

In view of the growing awareness towards deforestation and environmental conservation, the company feels that the use of wood should be minimal. The **wooden top** therefore could be replaced with a **granite top**. This would require additional wood for the frame and legs to take the extra weight of the granite top.

# Material selection for a Product: Problem 2

The materials and labour requirements along with cost details to manufacture a table with granite top are given below:

| Description of item | Quantity | Cost |
| --- | --- | --- |
| Wood for frame and legs | 0.15 $m^3$ | Rs. 12,000/$m^3$ |
| Granite table top | 1.62 $m^2$ | Rs. 800/$m^2$ |
| Leg bushes | 4 | Rs. 25/bush |
| Nails | 50 g | Rs. 300/kg |
| Total labour | 8 hr | Rs. 50/hr |

If the cost of the dining table with a granite top works out to be lesser than that of the table with wooden top, the company is willing to manufacture dining tables with granite tops. Compute the cost of manufacture of the table under each of the alternatives described above and suggest the best alternative. Also, find the economic advantage of the best alternative.

# Material selection for a Product: Problem 2

## (a) Cost of table with wooden top:

| | | |
|---|---|---|
| Cost of wood for frame and legs | = 12,000 x 0.1 | = Rs. 1,200 |
| Cost of wooden top | | = Rs. 3,000 |
| Cost of bushes | = 10 x 4 | = Rs. 40 |
| Cost of nails | = 300 x (100/1,000) | = Rs. 30 |
| Cost of labour | = 50 x 15 | = Rs. 750 |
| Total | | = Rs. 5,020 |

# Material selection for a Product: Problem 2

**(b) Cost of table with granite top:**

| | | |
|---|---|---|
| Cost of wood for frame and legs | = 12,000 x 0.15 | = Rs. 1,800 |
| Cost of granite top | = 800 x 1.62 | = Rs. 1,296 |
| Cost of bushes | = 25 x 4 | = Rs. 100 |
| Cost of nails | = 300 x (50/1,000) | = Rs. 15 |
| Cost of labour | = 50 x 8 | = Rs. 400 |
| Total | | = Rs. 3,611 |

# Material selection for a Product: Problem 2

The cost of a table with **granite top** works out to be **less** than that of a table with a **wooden top**.

Hence, the table with granite top should be selected by the manufacturer.

(c) Economic advantage:

| | |
|---|---|
| Cost of a table with wooden top | = Rs. 5,020 |
| Cost of a table with granite top | = Rs. 3,611 |

| | |
|---|---|
| Economic advantage of table with granite top | = Rs. 1,409 |

# Design selection for a Product

The design modification of a product may result in

reduced raw material requirements,

increased machinability of the materials,

reduced labour, etc.

Design is an important factor.

It decides the cost of the product.

Economic analysis applied to the selection of design for a product.

# Design selection for a Product – Problem 1

Two alternatives are under consideration for a **tapered fastening pin**. Either design will serve the purpose and will involve the same material and manufacturing cost except for the lathe and grinder operations.

1. Design A will require 16 hours of lathe time and 4.5 hours of grinder time per 1,000 units.
2. Design B will require 7 hours of lathe time and 12 hours of grinder time per 1,000 units.
3. The operating cost of the lathe including labour is Rs. 200 per hour.
4. The operating cost of the grinder including labour is Rs. 150 per hour.

Which design should be adopted if 1,00,000 units are required per year and what is the economic advantage of the best alternative?

# Design selection for a Product – Problem 1

Operating cost of lathe including labour = Rs. 200 per hr

Operating cost of grinder including labour = Rs. 150 per hr

**(a) Cost of design A:**

No. of hours of lathe time per 1,000 units = 16 hr

No. of hours of grinder time per 1,000 units = 4.5 hr

Total cost of design A / 1,000 units

= Cost of lathe operation per 1,000 units

+ Cost of grinder operation per 1,000 units

= 16  200 + 4.5 x 150

= Rs. 3,875

Total cost of design A/1,00,000 units = 3,875  x1,00,000/1,000

= Rs. 3,87,500

# Design selection for a Product – Problem 1

**(b) Cost of design B:**

No. of hours of lathe time per 1,000 units = 7 hr
No. of hours of grinder time per 1,000 units = 12 hr

Total cost of design B/1,000 units

= Cost of lathe operation/1,000 units
+ Cost of grinder operation/1,000 units

= 7 x 200 + 12 x150

= Rs. 3,200

Total cost of design B/1,00,000 units = 3,200 x 1,00,000/1,000

= Rs. 3,20,000

# Design selection for a Product – Problem 1

DECISION:

The total cost/1,00,000 units of design B is less than that of design A.

Hence, design B is recommended for making the tapered fastening pin.

Economic advantage of the design B over design A per 1,00,000 units

=

Rs. 3,87,500 – Rs. 3,20,000

= Rs. 67,500.

# Design selection for a process industry

- The chief engineer of refinery operations is not satisfied with the preliminary design for storage tanks to be used as part of a plant expansion programme.

- The engineer who submitted the design was called in and asked to reconsider the overall dimensions in the light of an article in the *Chemical Engineer*, entitled “How to size future process vessels?”

# Design selection for a process industry

- The original design submitted called for 4 tanks 5.2 m in diameter and 7 m in height.
- From a graph of the article, the engineer found that the present ratio of height to diameter of 1.35 is 111% of the minimum cost and that the minimum cost for a tank was when the ratio of height to diameter was 4 : 1.
- The cost for the tank design as originally submitted was estimated to be Rs. 9,00,000.
- What are the optimum tank dimensions if the volume remains the same as for the original design? What total savings may be expected through the redesign?

# Design selection for a process industry

## *(a) Original design*

Number of tanks = 4

Diameter of the tank = 5.2 m

Radius of the tank = 2.6 m

Height of the tank = 7 m

Ratio of height to diameter = 7/5.2 = 1.35

Volume/tank = $(22/7)r^2h = (22/7)(2.6)^2 * 7 = 148.72\ m^3$

# Design selection for a process industry

***(a) New design***

Cost of the old design = 111% of the cost of the new design (optimal design)

Optimal ratio of the height to diameter = 4:1

$h : d = 4 : 1$

$4d = h$

$d = h/4$

$r = h/8$

# Design selection for a process industry

## *(b) New design*

Volume = $(22/7)r^2h$ = 148.72 (since, the volume remains the same)

$(22/7)(h/8)^2h = 148.72$

$$h^3 = \frac{148.72}{22/7} \text{ x } 64$$

$= 3,028.48$

$h = 14.47$ m

$r = h/8 = 14.47/8 = 1.81$ m

# Design selection for a process industry

***(b)New design***

Therefore, Diameter of the new design = 1.81* 2

= 3.62 m

Cost of the new design = 9,00,000 (100/111)

= Rs. 8,10,810.81

Expected savings by the redesign = Rs. 9,00,000 – Rs. 8,10,810.81

= Rs. 89,189.19

# Process Planning /Process Modification

- While planning for a new component, a feasible sequence of operations with the least cost of processing is to be considered.

- The process sequence of a component which has been planned in the past is not static.

- It is always subject to modification with a view to minimize the cost of manufacturing the component.

# Process Planning /Process Modification

- The objective of process planning/process modification is to identify **the most economical sequence of operations to produce a component.**

# Process Planning /Process Modification

**The steps in process planning are as follows:**

- Analyze the part drawing to get an overall picture of what is required
- Make recommendations to or consult with product engineers on product design changes.
- List the basic operations required to produce the part to the drawing or specifications.
- Determine the most practical and economical manufacturing method and the form or tooling required for each operation.
- Devise the best way to combine the operations and put them in sequence.
- Specify the gauging required for the process.

# Example for Process planning

- The process planning engineer of a firm listed the sequences of operations as shown in Table. 1 to produce a component.

| Sequence | Process Sequence |
|---|---|
| 1 | Turning – Milling – Shaping - Drilling |
| 2 | Turning – Milling - Drilling |
| 3 | All operations are performed with CNC machine |

# Example for Process planning

- The details of processing times of the component for various operations and their machine hour rates are summarized in Table 2. Find the most economical sequence of operations to manufacture the component

| *Operation* | *Machine hour rate* (Rs.) | *Process sequence* | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| Turning | 200 | 5 | 5 | - |
| Milling | 400 | 8 | 14 | - |
| Shaping | 350 | 10 | - | - |
| Drilling | 300 | 3 | 3 | - |
| CNC operations | 1000 | - | - | 8 |

# Example for Process planning

## (a) Cost of component using process sequence 1.

The process sequence 1 of the component is as follows

### Turning – Milling – Shaping – Drilling

| Operation No | Operation | Time | | Machine Hour Rate Rs. | Cost Rs. |
|---|---|---|---|---|---|
| | | Min | hr | | |
| 1 | Turning | 5 | 0.083 | 200 | 16.60 |
| 2 | Milling | 8 | 0.133 | 400 | 53.20 |
| 3 | Shaping | 10 | 0.167 | 350 | 58.45 |
| 4 | Drilling | 3 | 0.050 | 300 | 15.00 |
| Total | | | | | 143.25 |

# Example for Process planning

## (b) Cost of component using process sequence 2.

The process sequence 2 of the component is as follows

### Turning – Milling – Drilling

| Operation No | Operation | Time | | Machine Hour Rate Rs. | Cost Rs. |
|---|---|---|---|---|---|
| | | Min | hr | | |
| 1 | Turning | 5 | 0.083 | 200 | 16.60 |
| 2 | Milling | 14 | 0.233 | 400 | 93.20 |
| 3 | Drilling | 3 | 0.050 | 300 | 15.00 |
| Total | | | | | 124.80 |

# Example for Process planning

## (b) Cost of component using process sequence 3.

The process sequence 2 of the component is as follows

**Only CNC operations**

| Operation No | Operation | Time | | Machine Hour Rate Rs. | Cost Rs. |
|---|---|---|---|---|---|
| | | Min | hr | | |
| 1 | CNC Operations | 8 | 0.133 | 1,000 | 133 |

- The Process Sequence 2 has the least cost. Therefore it should be selected for manufacturing the component.

# UNIT II

## Interest Formulae

# Interest Rate

- Interest rate is the time value of money
- It represents the growth of capital per unit period
- The period may be a month, a quarter, semi annual or a year.

# Interest Rate – Types

- Simple Interest
- Compound Interest

# Simple Interest

- Simple interest is defined as a fixed percentage of the principal multiplied by the life the load.

- $I = n \times i \times P$

- $I$ = Total amount of simple interest
- $n$ = Life of the load
- $i$ = interest rate
- $P$ = Principal

# Simple Interest Rate

Fund accumulated at the end of First year

$$= P + iP = P(1+i)$$

Fund accumulated at the end of second year

$$= P + iP + iP = P + 2iP$$

$$= P(1+2i)$$

$III^{ly}$ Fund accumulated at the end if $n^{th}$ year =

$$= P(1+ni)$$

# Compound Interest Rate

When the interest rate is compounded, the total time period is divided into several interest periods.

Interest is credited at the end of each period and is allowed to accumulate from one interest period to the next.

# Compound Interest Rate

During a given interest period, the current interest is determined as the percentage of the total amount owned (i.e the principle plus the previously accumulated interest.

# Compound Interest Rate

For the first period, the interest is determined as

$$I_1 = iP$$

Fund accumulated at the end of First year

$$F_1 = P + I_1 = P+iP = P(1+i)$$

For the second period, the interest is determined as

$$I_2 = iF_1$$

Fund accumulated at the end of second year

$$F_2 = P + I_1 + I_2 = P + iP + i\,P(1+i)$$

$$= P + iP + iP + i^2 P = P(1 + 2i + i^2)$$

$$= P(1 + i)^2$$

III$^{ly}$ Fund accumulated at the end if $n^{th}$ year =

$$= P(1+i)^n$$

# Time value of money

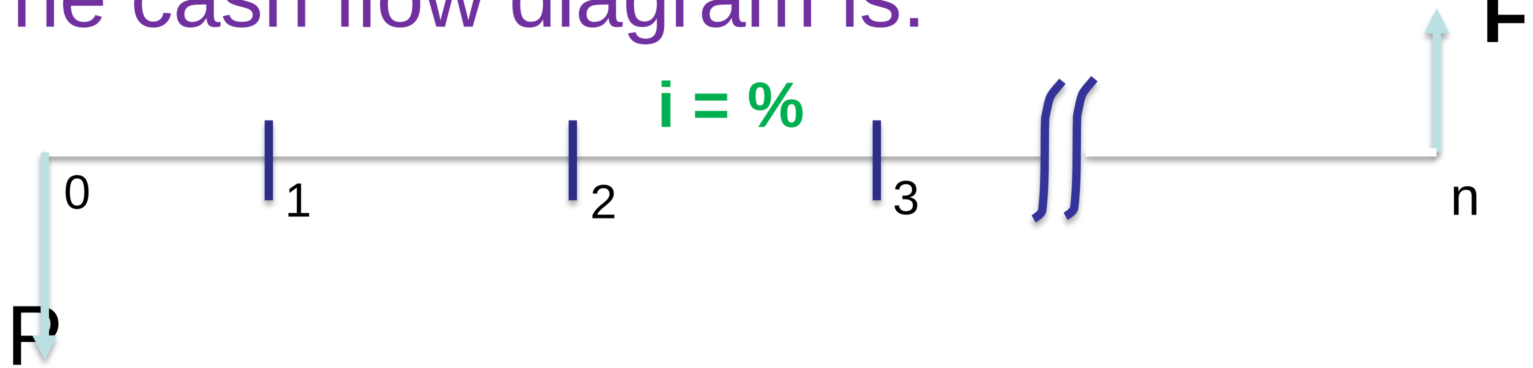
1. Single Payment Compound Amount
2. Single Payment Present Worth Amount
3. Equal Payment Series Compound Amount
4. Equal Payment Series Sinking Fund
5. Equal Payment Series Present Worth Amount
6. Equal Payment Series Capital Recovery Amount
7. Uniform Gradient Series Annual Equivalent Amount

# Single payment compound amount

- Objective is to find the single future sum (F) of the initial payment (P) made at time 0 after 'n' periods at an interest rate 'i' compounded every period.

# Single payment compound amount

- The cash flow diagram is:



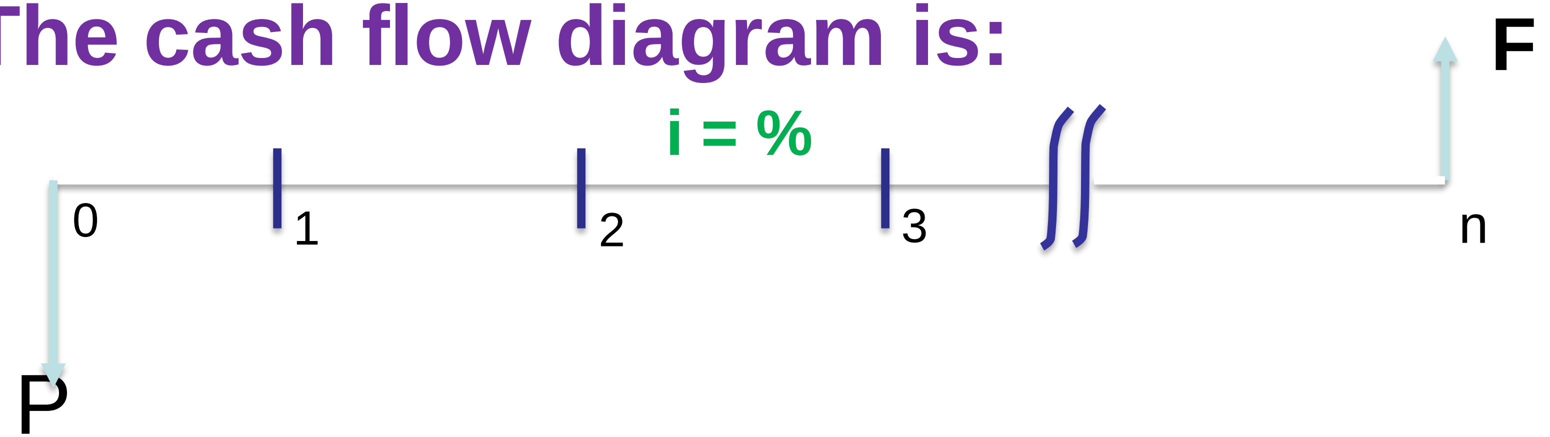

- P is known, F has to be determined
- $F = P(1+i)^n = P(\frac{F}{P}, i, n)$
- Here $\left(\frac{F}{P}, i, n\right)$ is called compound amount factor

# Single Payment Present Worth Amount

- Objective is to find the present worth(P) of a single future sum (F) which will be received after ‘n’ periods at an interest rate of ‘i’ compounded at the end of every interest period

# Single Payment Present Worth

- **The cash flow diagram is:**



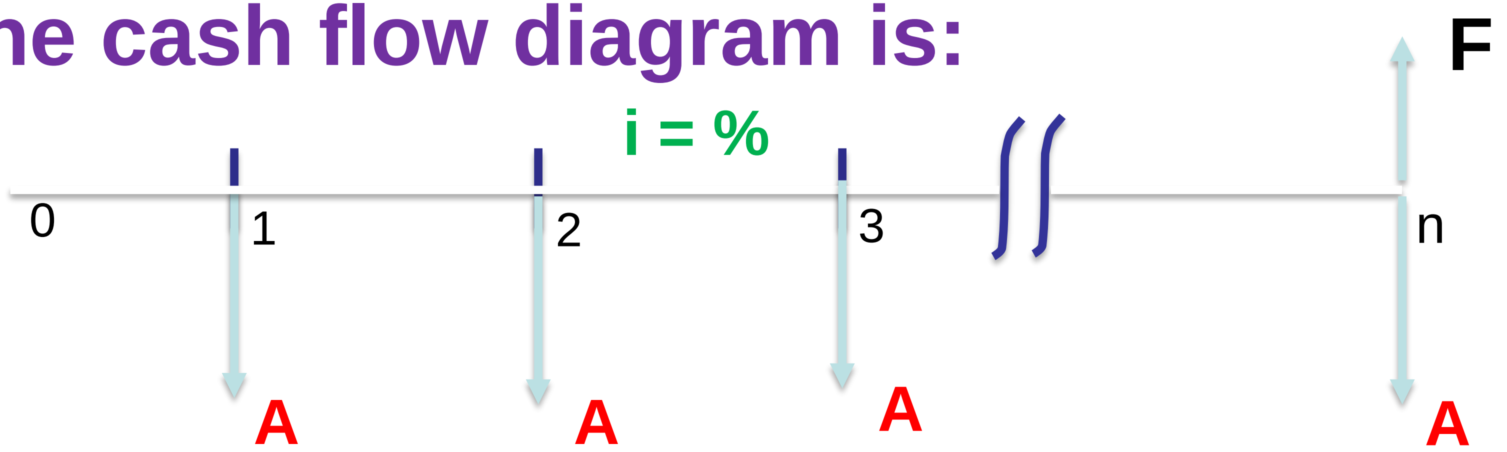

- F is known, P has to be determined
- Present worth P= $\frac{F}{(1+i)^n} = F(\frac{P}{F}, i, n)$
- Here $(\frac{P}{F}, i, n)$ is called single payment present worth factor

# Equal Payment Series Compound Amount

- Objective is to find the future worth of ‘n’ equal payments which are made at the end of every interval period till the end of $n^{th}$ Interest period at an interest rate of ‘i’ compounded at the end of each interest period.

# Equal Payment Series Compound Amount

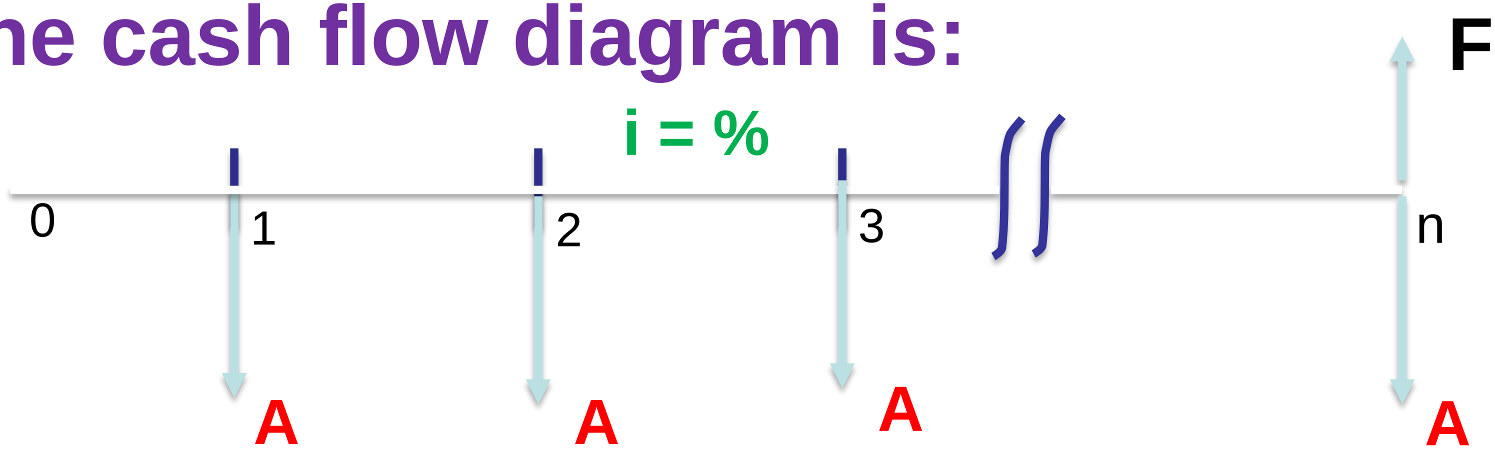
- **The cash flow diagram is:**




- A is known, F has to be determined
- Here $(\frac{F}{A}, i, n)$ is called Equal Payment Series Compound Amount Factor

# Equal Payment Series Compound Amount

- **The cash flow diagram is:**




A = Equal Amount Deposited at the end of each interest period

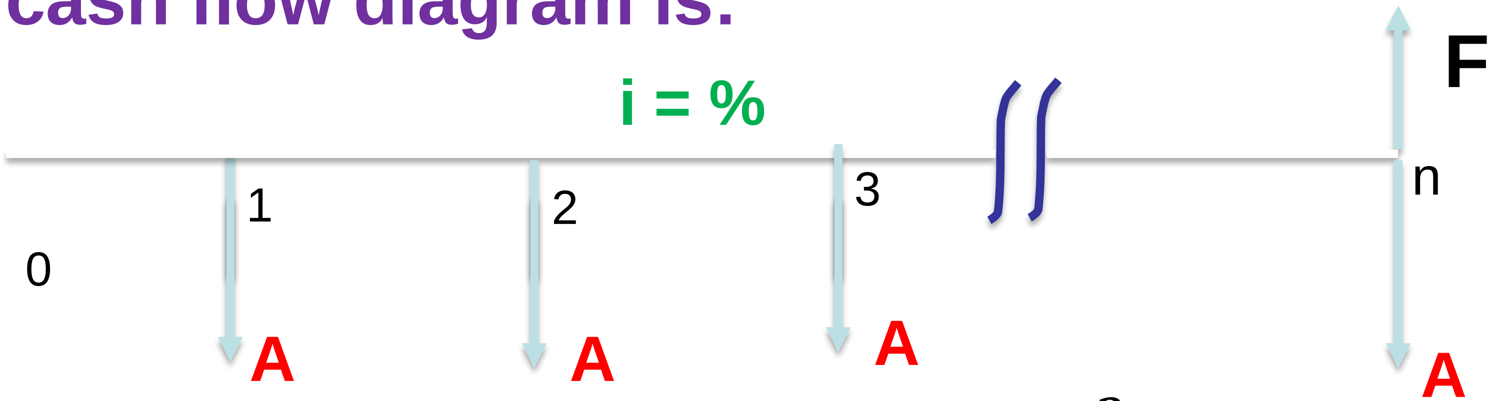
n = Number of interest periods

i = Rate of interest

F = Single Future Amount

# Equal Payment Series Compound Amount

- **The cash flow diagram is:**




Future amount F = A + A(1+i) +A$(1+i)^2$+ ….+A$(1+$

# Equal Payment Series Compound Amount

**Subtract ① from ②,**

$$\frac{F(1+i)}{A} - \frac{F}{A} = (1+i)^n - 1$$

$$\frac{F}{A} + \frac{F}{A} * i - \frac{F}{A} = (1+i)^n - 1$$

$$\frac{F}{A} = \frac{(1+i)^n - 1}{i}$$

$$F = \frac{A * ((1+i)^n - 1)}{i}$$

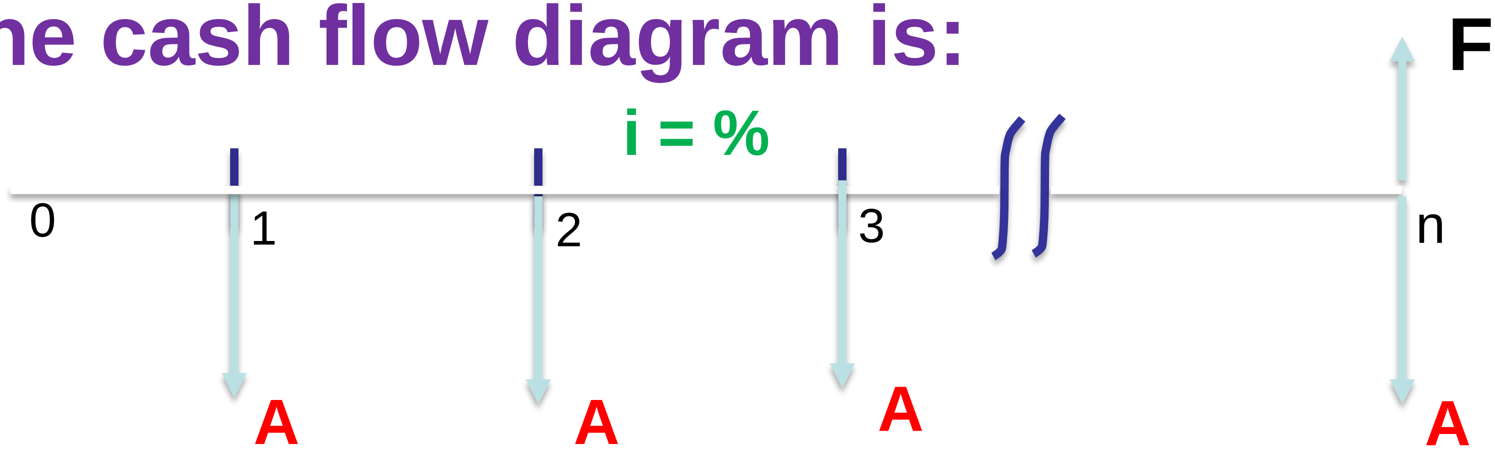
$A(\frac{F}{A}, i, n)$ is termed as equal payment series, compound amount factor

# Equal Payment Series Sinking Fund

- Objective is to find the equivalent amount(A) that should be deposited at the end of every interest period for ‘n’ interest periods to realize a future sum (F) at the end on $n^{th}$ interest period at an interest rate of ‘i’

# Equal Payment Series Sinking Fund

- The cash flow diagram is:



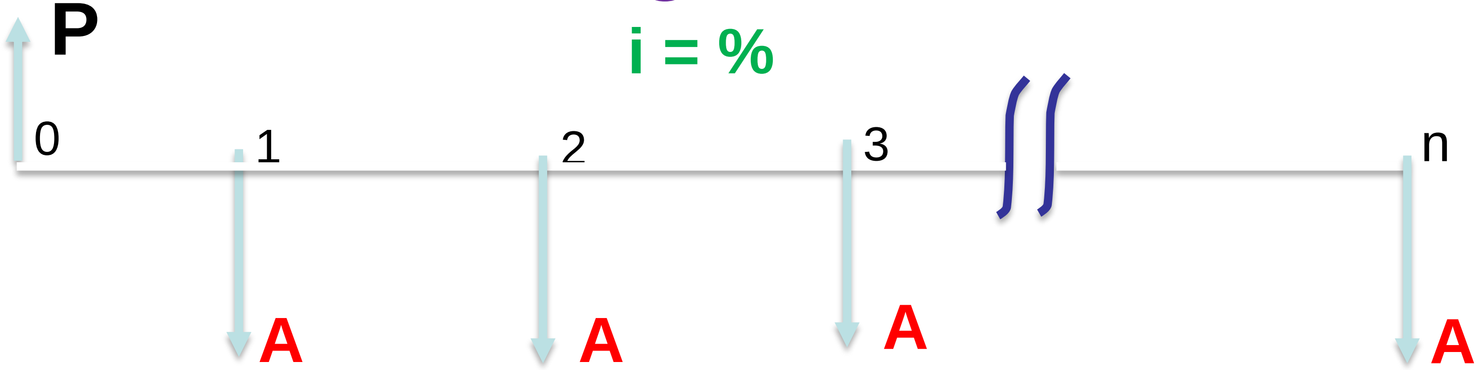

- F is known, A has to be determined
- Here $(\frac{A}{F}, i, n)$ is called Equal Payment Series Sinking Fund Factor
- $A \quad = \quad F \times \frac{i}{(1+i)^n - 1} \quad = \quad F\,(\frac{A}{F}, i, n)$

# Equal Payment Series Present Worth Amount

- Objective is to find the present worth of an equal payment made at the end of every interest period for ‘n’ interest periods at an interest rate of ‘i’ compounded at the end of every interest period.

# Equal Payment Series Present Worth Amount

- **The cash flow diagram is:**



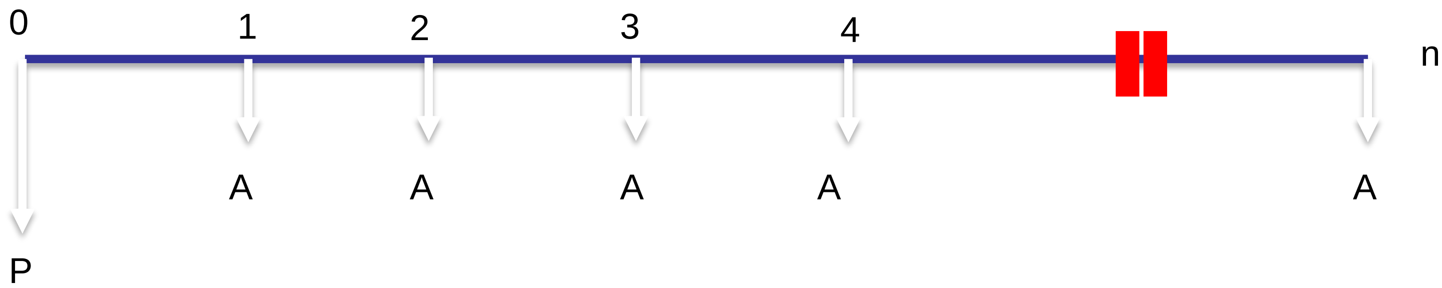

A = Equal Amount Deposited at the end of each interest period

n = Number of interest periods

i = Rate of interest

P = Present Worth

# Equal Payment Series Present Worth Amount




$$P = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \ldots\ldots + \frac{A}{(1+i)^n}$$

$$P = A\left[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \ldots\ldots + \frac{1}{(1+i)^n}\right] - ①$$

Multiply ① by $\frac{1}{(1+i)}$ & subtract ① - ②

# Equal Payment Series Present Worth Amount

$$\frac{P}{(1+i)} = A\left[\frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \frac{1}{(1+i)^4} \ldots\ldots + \frac{1}{(1+i)^{n+1}}\right]$$

$$- \text{②}$$

$$P = A\left[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \ldots\ldots + \frac{1}{(1+i)^n}\right]$$

$$- \text{①}$$

② - ①

$$\frac{P-P-iP}{(1+i)} = A\left[\frac{1}{(1+i)^{n+1}} - \frac{1}{(1+i)}\right]$$

$$-iP = A\left[\frac{1}{(1+i)^n} - 1\right]$$

$$\boxed{P = A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]}$$

# Equal Payment Series Present Worth Amount

- $\frac{P}{A} = \frac{P}{F} \times \frac{F}{A} = \frac{1}{(1+i)^n} \times \frac{(1+i)^n - 1}{i}$

$$= \frac{(1+i)^n - 1}{i\,(1+i)^n}$$

- $P = A \times \frac{(1+i)^n - 1}{i\,(1+i)^n} = A(\frac{P}{A}, i, n)$

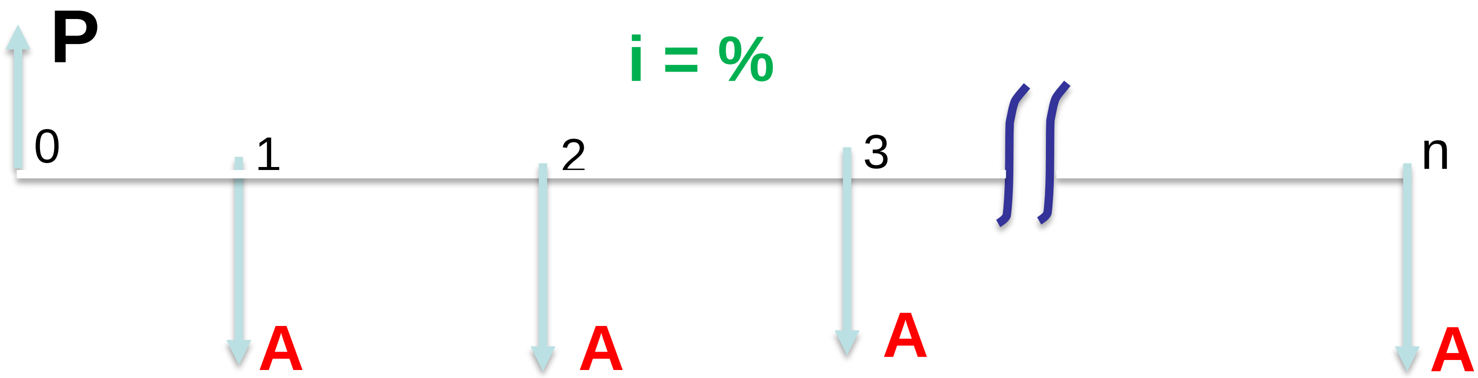
- Here $(\frac{P}{A}, i, n)$ is called equal payment series present worth amount.

# Equal Payment Series Capital Recovery Amount

Objective is to find the annual equivalent amount (A) which is to be recovered at the end of every interest period for ‘n’ interest periods for a loan (P) which is sanctioned now at an interest rate of ‘i’ compounded at the end of every year.

# Equal Payment Series Capital Recovery Amount

- **The cash flow diagram is:**




A = Equal Amount Deposited at the end of each interest period

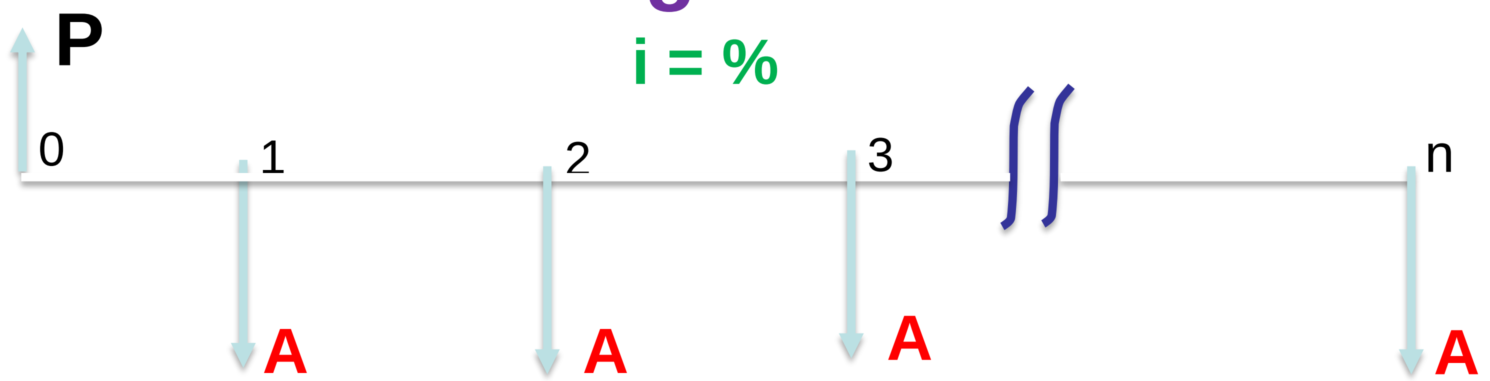
n = Number of interest periods

i = Rate of interest

P = Present Worth / Loan sanctioned at initial period

# Equal Payment Series Capital Recovery Amount

- **The cash flow diagram is:**



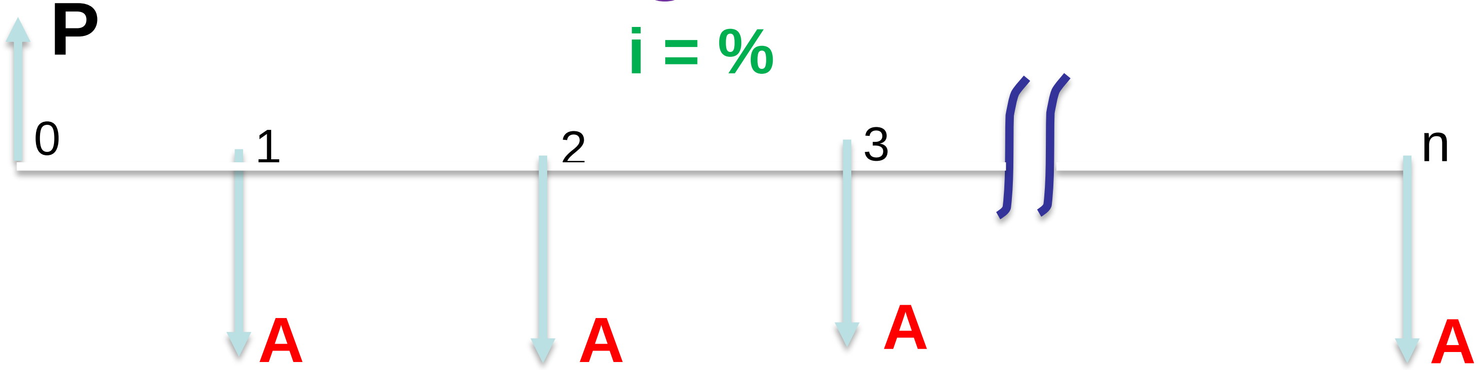

Here P is given and the objective is to determine A

We know that from Equal Payment Series

Present Worth Amount P = A x $\frac{(1+i)^n - 1}{i\,(1+i)^n}$

# Equal Payment Series Capital Recovery Amount

- **The cash flow diagram is:**




$$A = \frac{P[i(1+i)^n]}{(1+i)^n - 1} = P\left(\frac{A}{P}, i, n\right)$$

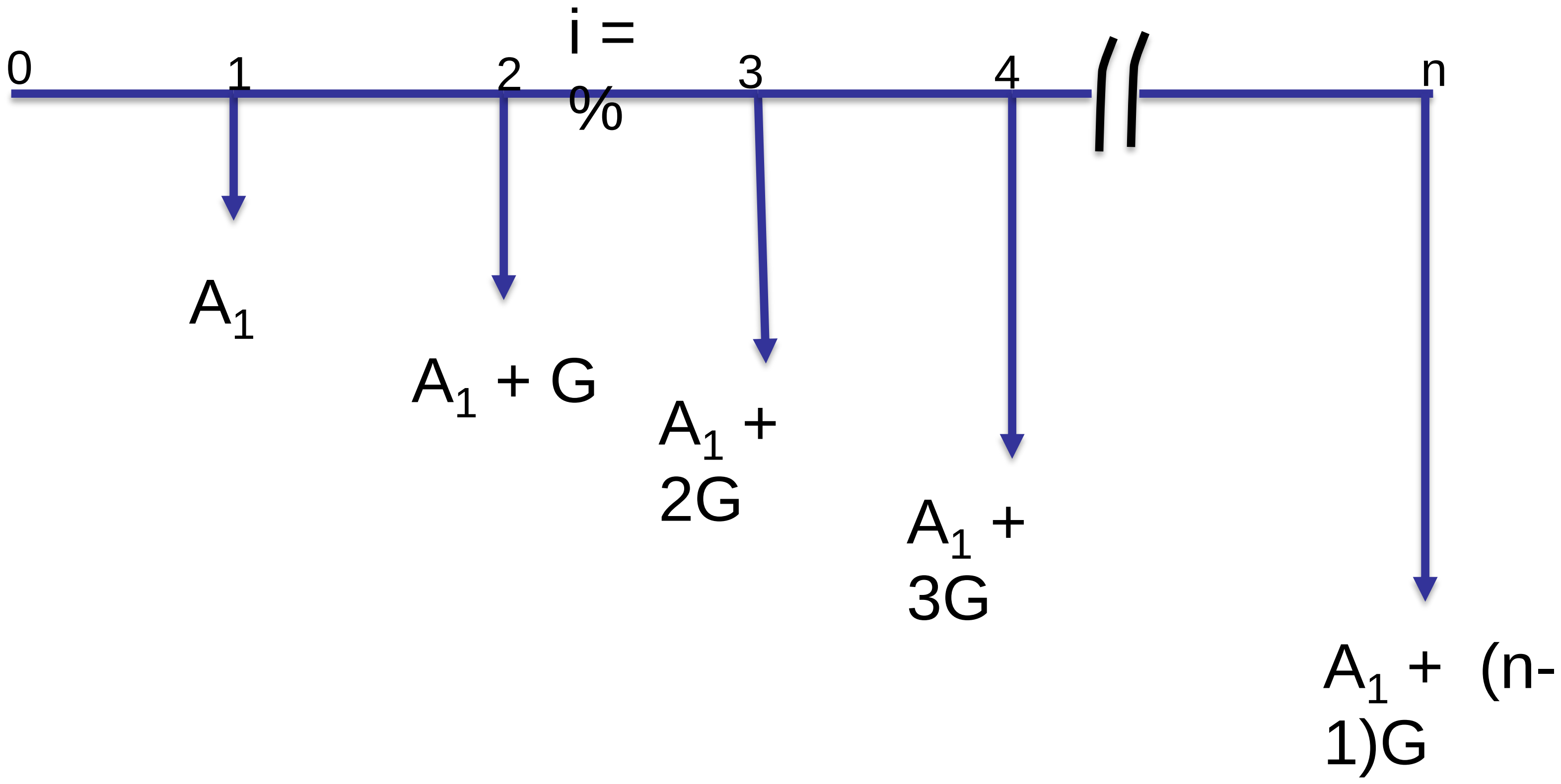
Here $\left(\frac{A}{P}, i, n\right)$ is called equal payment series capital recovery factor.

# Uniform Gradient Series Annual Equivalent Amount

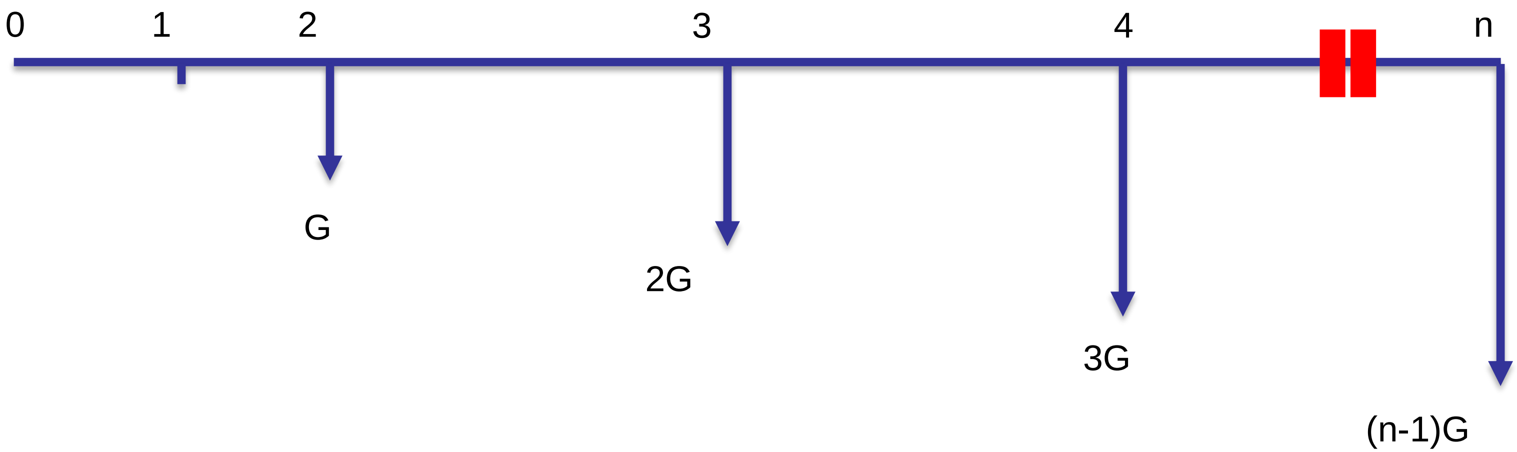
Objective is to find the annual equivalent amount of a series with an amount A at the end of the final year and with an equal increment (G) at the end of each of the following ‘n-1’ years with an interest rate of ‘i’ compounded annually.

# Uniform Gradient Series Annual Equivalent Amount

## Cash Flow Diagram



# Uniform Gradient Series- Present Worth Amount




$$P = G\left(\frac{P}{F}, i, 2\right) + 2G\left(\frac{P}{F}, i, 3\right) + 3G\left(\frac{P}{F}, i, 4\right)$$

$$+ \ldots\ldots\ldots\ldots\ldots$$

$$+ G(n-2)\left(\frac{P}{F}, i, n-1 \; + G(n-1)\left(\frac{P}{F}, i, n\right)\right.$$

$$P = G\left[\frac{1}{(1+i)^2} + \frac{2}{(1+i)^3} + \frac{3}{(1+i)^4} \ldots\ldots + \frac{n-2}{(1+i)^{n-1}} + \frac{n-1}{(1+i)^n}\right] - 1$$

# Uniform Gradient Series- Present Worth Amount

Multiply ① **x** ( i + 1) & subtract ① from ②

$$P(i+1) = G\left[\frac{1}{(1+i)^1} + \frac{2}{(1+i)^2} + \frac{3}{(1+i)^3} + \ldots\ldots\ldots + \frac{n-2}{(1+i)^{n-2}} + \frac{n-1}{(1+i)^{n-1}}\right] \quad \text{- ②}$$

$$P = G\left[\frac{1}{(1+i)^2} + \frac{2}{(1+i)^3} + \frac{3}{(1+i)^4} \ldots\ldots\ldots + \frac{n-2}{(1+i)^{n-1}} + \frac{n-1}{(1+i)^n}\right] \text{-- ①}$$

---

$$iP = G\left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} \ldots\ldots\ldots + \frac{1}{(1+i)^{n-1}} - \frac{n-1}{(1+i)^n}\right]$$

$$iP = G\left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} \ldots\ldots\ldots + \frac{1}{(1+i)^n}\right] - G\left[\frac{n}{(1+i)^n}\right]$$

# Uniform Gradient Series- Present Worth Amount

Refer Equal Payment Series – Present worth Amount

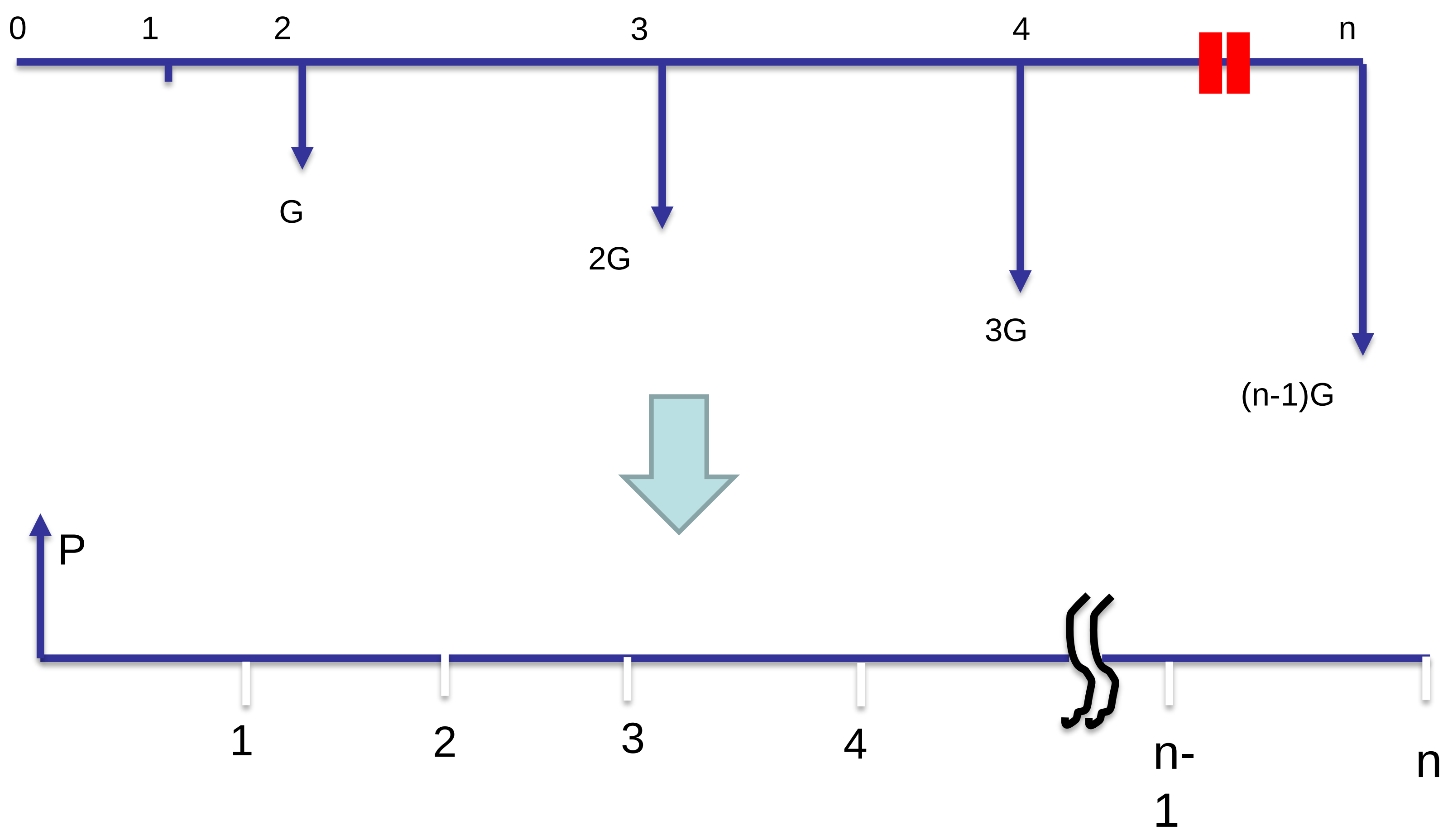
$$iP = G\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] - G\left[\frac{n}{(1+i)^n}\right]$$

$$P = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right] \quad \ldots\ldots\ldots\ldots (3)$$

Eqn. 3 is the general relation to convert an arithmetic gradient 'G' for 'n' years into a present worth at year '0'

# Uniform Gradient Series- Present Worth Amount

The conversion of cash flow diagram



# Uniform Gradient Series- Annual equal Amount

The equivalent Uniform annual series (A value) for an arithmetic gradient 'g' is found by multiplying the present worth in eqn. 3 by the (A/P) factor expression

$$\frac{A_g}{G} = \frac{A}{P} \text{ x } \frac{P}{G}$$

$$\frac{A_g}{G} = \frac{1}{i}\left[\frac{(1+i)^n-1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right] \text{ x } \left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$$

$$Ag = G\left[\frac{(1+i)^n(1+i)^n}{i(1+i)^n\,((1+i)^n-1)} - \frac{(1+i)^n}{i(1+i)^n\,((1+i)^n-1)} - \frac{n(1+i)^n}{(1+i)^n\,((1+i)^n-1)}\right]$$

# Uniform Gradient Series- Annual equal Amount

$$A_g = G \left[ \frac{(1+i)^n}{i((1+i)^n - 1)} - \frac{1}{i((1+i)^n - 1)} - \frac{n}{((1+i)^n - 1)} \right]$$

$$A_g = G \left[ \frac{(1+i)^n - 1}{i((1+i)^n - 1)} - \frac{n}{((1+i)^n - 1)} \right]$$

$$A_g = G \left[ \frac{1}{i} - \frac{n}{(1+i)^n - 1} \right]$$

$$A_g = G \left[ \frac{(1+i)^n - in - 1}{i(1+i)^n - 1} \right]$$

# Uniform Gradient Series- Annual equal Amount

- A = A + $\frac{G(1+i)^n - \text{in} - 1}{i(1+i)^n - 1}$

  = A + $A_G(\frac{A_g}{G}, i, n)$

- Here $(\frac{A_g}{G}, i, n)$ is called uniform gradient series factor.

# Uniform Gradient Series

NOTE:

The total present worth $P_T$ for a gradient series must consider the base and the gradient separately.

- Thus, for cash flow series involving conventional gradient:

1. The base amount is the uniform-series amount ‘A’ that begins in year 1 and extends through year ‘n’. It’s present worth is represented by $P_A$

# Uniform Gradient Series

2. For an increasing gradient, the gradient amount must be added to the uniform series amount. The present worth is $P_G$ .

3. For a decreasing gradient, the gradient amount must be subtracted from the uniform series amount. The present worth is $P_G$ .

The general equations for calculating total present worth $P_T$ of conventional arithmetic gradients are:

1. $P_T = P_A + P_G$

2. $P_T = P_A - P_G$

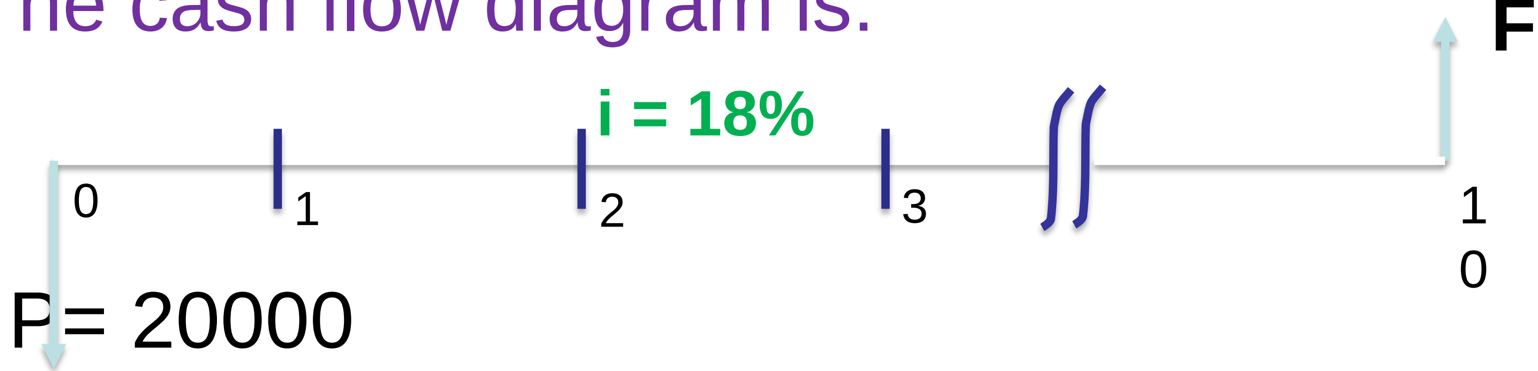
# Problem -1

A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

- **Given Data:**
- P = Rs. 20,000
- i = 18% compounded annually
- n = 10 years

# Solution for problem - 1

- The cash flow diagram is:




$$F = P(1+i)^n = 20000 * \left(1 + \frac{18}{100}\right)^{10}$$

$$= 20000 * 5.234$$

$$= \textbf{Rs. 1,04,680/-}$$

- The maturity value of Rs. 20,000 invested now at 18% compounded yearly is equal to Rs. 1,04,680 after 10 years.

# Problem -2

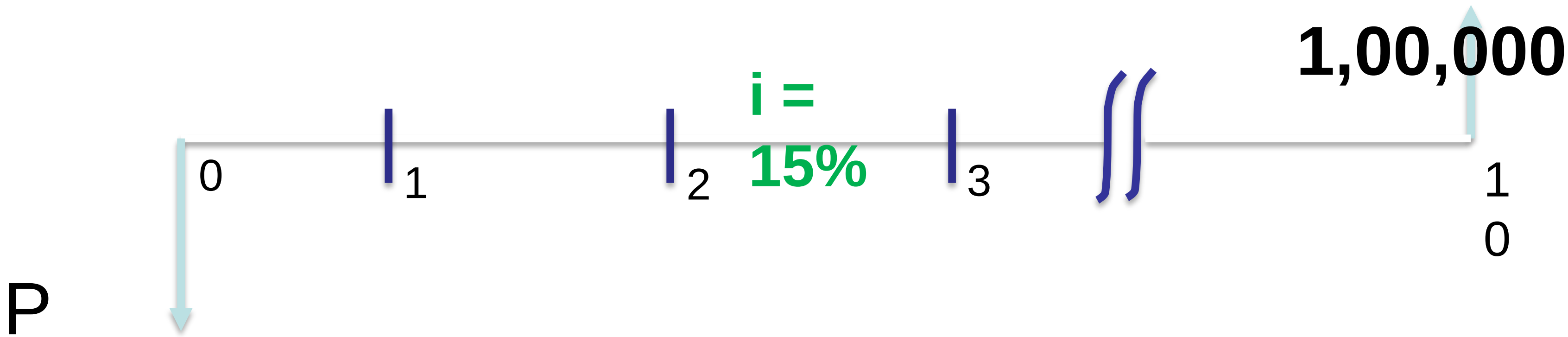
- A person wishes to have a future sum of Rs. 1,00,000 for his son’s education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

- **Given Data:**

- F = Rs. 1,00,000
- i = 15% compounded annually
- n = 10 years

# Solution for problem - 2

- **The cash flow diagram is:**




Present worth P= $\frac{F}{(1+i)^n} = 1{,}00{,}000 \ast \left[\frac{1}{(1+0.15)^{10}}\right]$

= 1,00,000 * 0.2472

= **Rs. 24, 720**

The person has to invest Rs. 24,720 now so that he will get a sum of Rs. 1,00,000 after 10 years at 15% interest rate compounded annually.

# Problem -3

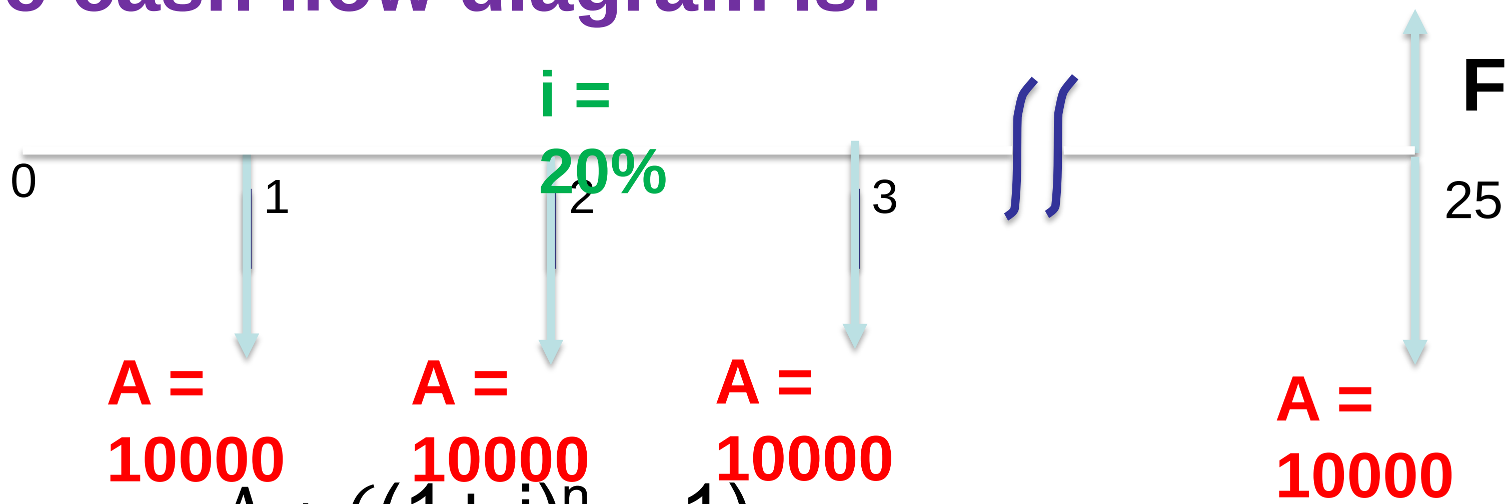
A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

## Given Data

- A = Rs. 10,000
- n = 25 years
- i = 20%
- F = ?

# Solution for Problem - 3

- **The cash flow diagram is:**




$$F = \frac{A * ((1+ i)^n - 1)}{i}$$

$$= \frac{10{,}000 * ((1+0.20)25 - 1)}{0.20}$$

$$= 10,000 * 471.981$$

$$= \text{Rs. } 47, 19, 810/-$$

The future sum of the annual equal payments after 25 years is equal toRs. 47,19,810.

# Problem -4

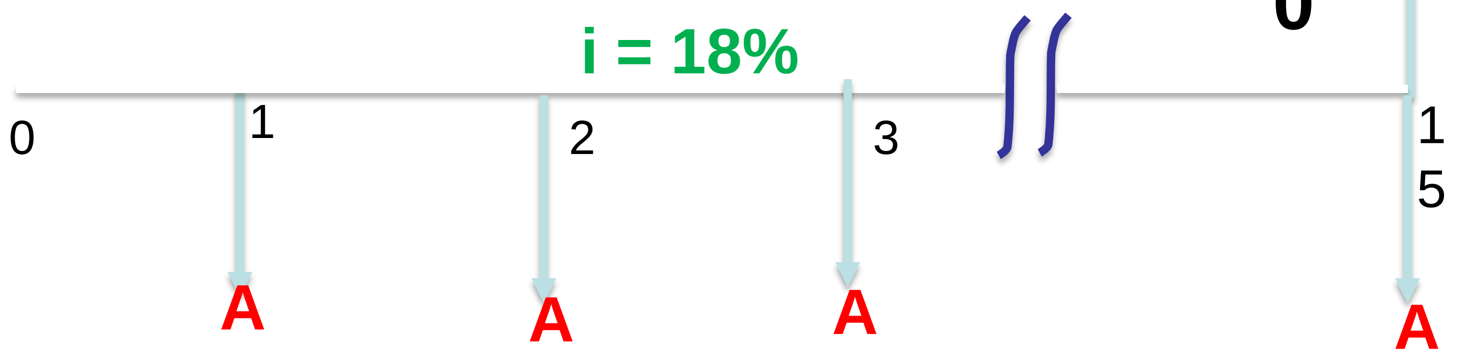
A company has to replace a present facility after 15 years at an outlay of Rs. 5,00,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

## Given Data

- F = Rs. 5,00,000
- n = 15 years
- i = 18%
- A = ?

# Solution for Problem - 4

- **The cash flow diagram is:**




$$A = F \times \frac{i}{(1+i)^n - 1}$$

$$= 5{,}00{,}000 * \left(\frac{0.18}{(1+0.18)^{15} - 1}\right)$$

$$= 5{,}00{,}000 * 0.0164$$

$$= \text{Rs. } 8{,}200/-$$

The annual equal amount which must be deposited for 15 years is Rs. 8,200.

# Problem -5

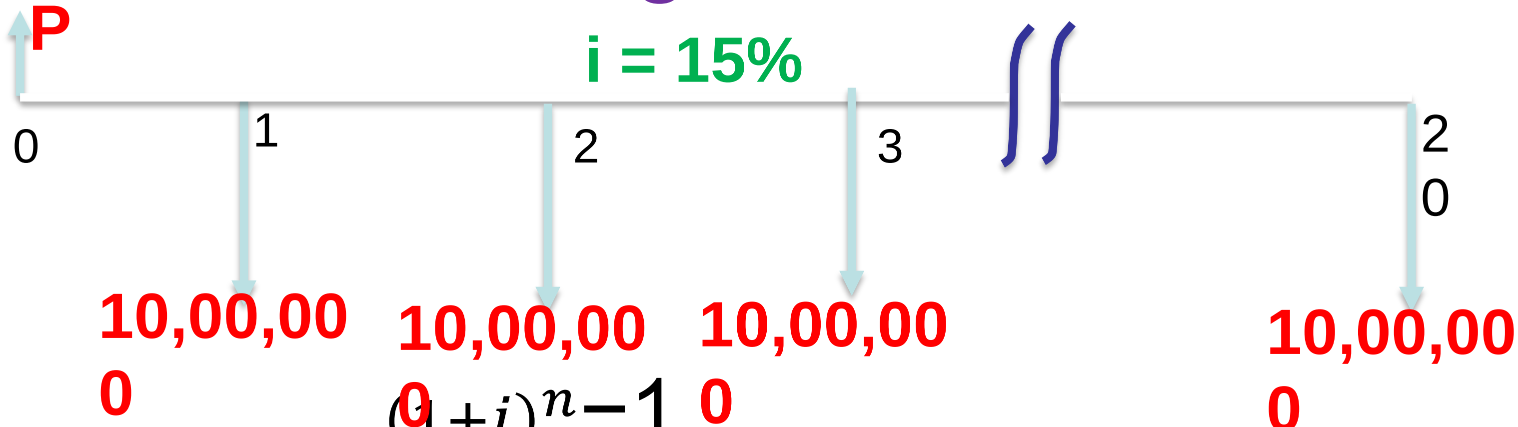
A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

**Given Data**

- A = Rs. 10,00,000
- n = 20 years
- i = 15%
- P = ?

# Solution for Problem - 5

- **The cash flow diagram is:**




$$P = A \times \frac{(1+i)^n - 1}{i\,(1+i)^n}$$

$$= 10{,}00{,}000 * \left(\frac{(1+0.15)^{20} - 1}{0.15 * (1+0.15)^{20}}\right)$$

$$= 10{,}00{,}000 * 6.2593$$

$$= \text{Rs. } 6\ 59,\ 300/-$$

The amount of reserve which must be set-up now is equal to Rs. 62,59,300.

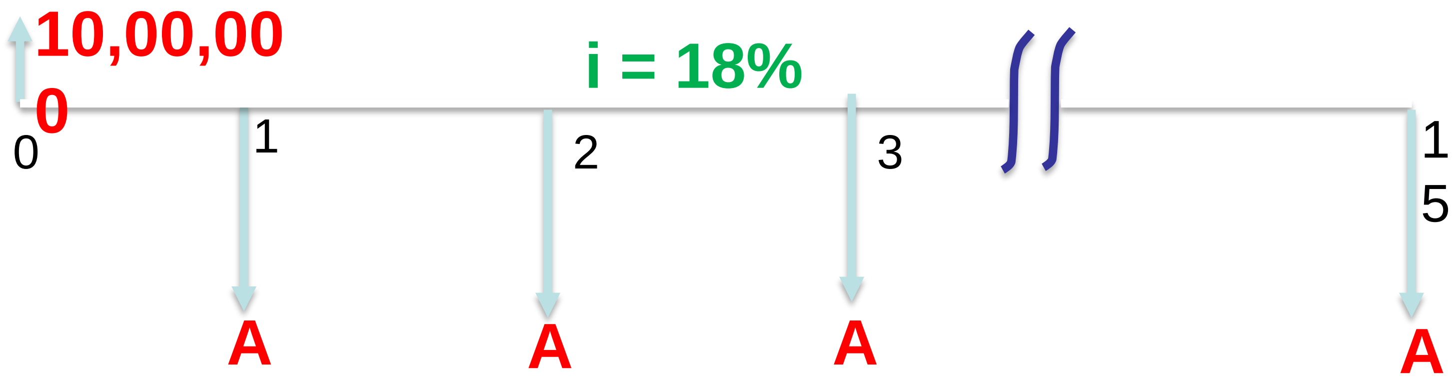
# Problem -6

A bank gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

- **Given Data**
- P = Rs. 10,00,000
- n = 15 years
- i = 18%
- A = ?

# Solution for Problem - 6

- **The cash flow diagram is:**




$$A = \frac{P[i(1+i)^n]}{(1+i)^n - 1}$$

$$= 10{,}00{,}000 * \left(\frac{0.18 * (1+0.18)^{15}}{(1+0.18)^{15} - 1}\right)$$

$$= 10{,}00{,}000 * 0.1964$$

= Rs. 1, 96, 400/-

- The annual equivalent installment to be paid by the company to the bank is Rs. 1,96,400.

# Problem -7

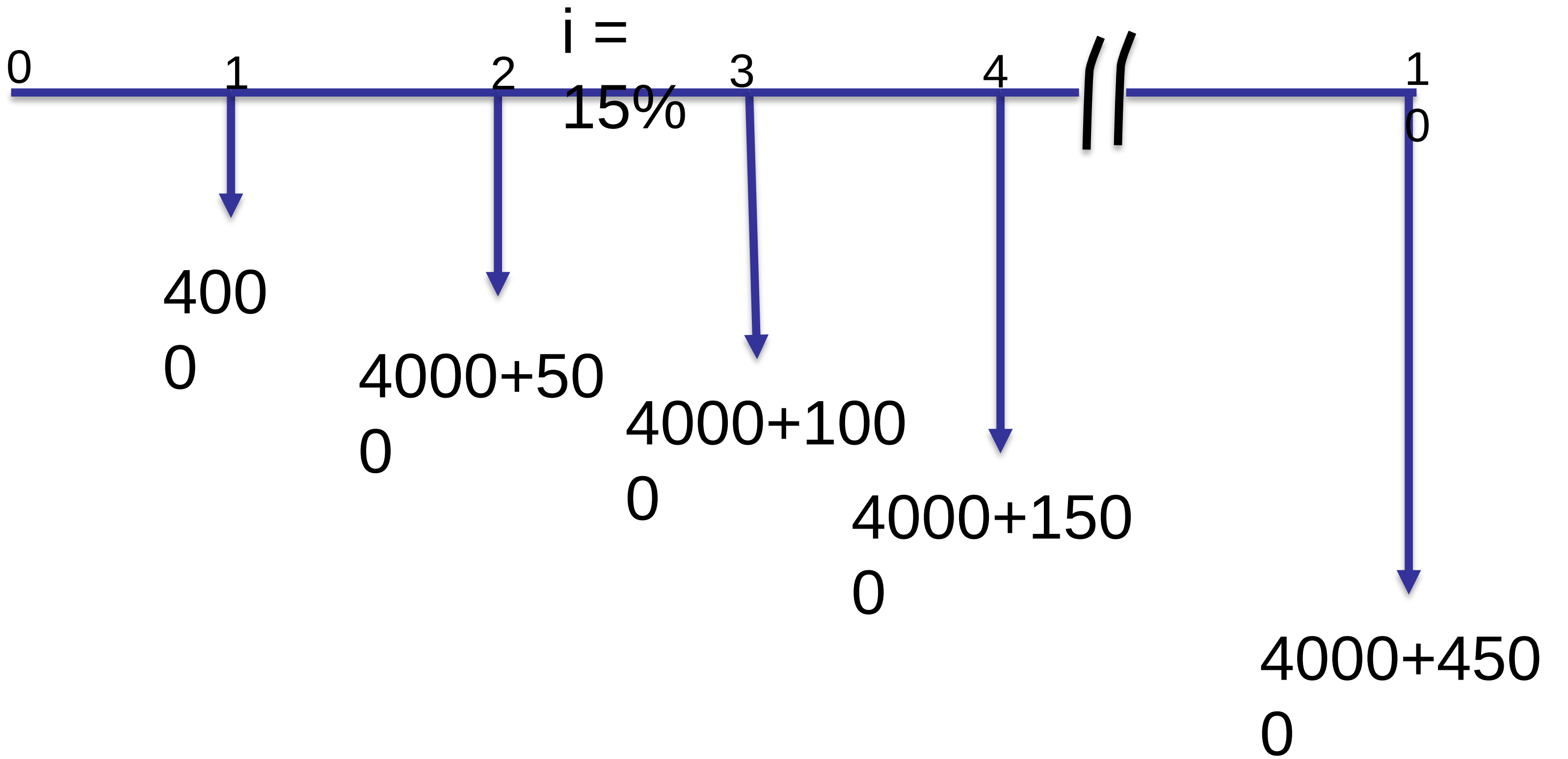
A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs.4,000 at the end of the first year and thereafter he wishes to deposit the amount with an annual increase of Rs.500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10$^{th}$ year of the above series.

**Given Data**

- A = Rs. 4,000 n = 10 years i = 15%
- G = Rs. 500? F= ?

# Solution for Problem - 7

## Cash Flow Diagram



# Solution for Problem - 7

$$A_g = G\left[\frac{(1+i)^n - 1 - in}{i(1+i)^n - i}\right]$$

$$= 500\left[\frac{(1+0.15)^{10} - 1 - 0.15\ X\ 10}{0.15(1+0.15)^{10} - 0.15}\right]$$

$$= 500\left[\frac{1.5456}{0.4568}\right]$$

$$= 500 \text{ x } 3.3835$$

$$A_g = \text{Rs.}1{,}691.75$$

# Solution for Problem - 7

- $A \quad = \quad A_1 + A_G$

  $= \quad 4,000 + 1,691.75$

  $= \quad$ Rs.5,691.75

- This is equivalent to paying an equivalent amount of Rs.5,691.75 at the end of every year for the next 10 years.

# Solution for Problem - 7

- The future worth sum of this revised series at the end of the 10$^{th}$ year is:

$$F = \frac{A((1+i)^n-1)}{i}$$

$$= \frac{5695.75((1+0.15)^{10}-1)}{0.15}$$

$$= \text{Rs.}1,15,563.68$$

- At the end of the 10th year, the compound amount of all his payments will be Rs. 1,15,563.68.

# Problem -8

An engineer is planning for a 15 year retirement. In order to supplement his pension and offset the anticipated effects of inflation, he intends to withdraw Rs.5,000 at the end of the first year and to increase the withdrawal by Rs.1,000 at the end of each successive year. How much money must he have in his savings account at the start of his retirement, if money earns 6% per year, compounded annually.

# Solution for Problem - 8

## Given Data

| | | | | |
|---|---|---|---|---|
| $n$ | = | 15years | | |
| $A_1$ | = | Rs.5,000 | | |
| $G$ | = | Rs.1,000 | | |
| $i$ | = | 6/100 | = | 0.06 |

- To find:

$A$ = ?

$P$ = ?

# Solution for Problem - 8

$$A_g = G\left[\frac{(1+i)^n - 1 - in}{i(1+i)^n - i}\right]$$

$$A_g = 1000\left[\frac{(1+0.06)^{15} - 1 - 0.06 \, X \, 15}{0.06(1+0.06)^{15} - 0.06}\right]$$

$$A_g = 1000\left[\frac{0.4965582}{0.0837935}\right]$$

$A_g$ = Rs.5,926

A = 5,000+ 5,926

$A$ = Rs.10,926

# Solution for Problem - 8

The money that he must have in his savings account at the start of his retirement

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$P = 10926\left[\frac{(1+0.06)^{15} - 1}{0.06(1+0.06)^{15}}\right]$$

$$P = \text{Rs.}1,06,116.03$$

# Problem -9

A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs.8,500 at the end of the first year and there afterwards he wishes to deposit the amount with an annual decrease of Rs.500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the $10^{th}$ year of the above series.

# Solution for Problem - 8

## Given Data:

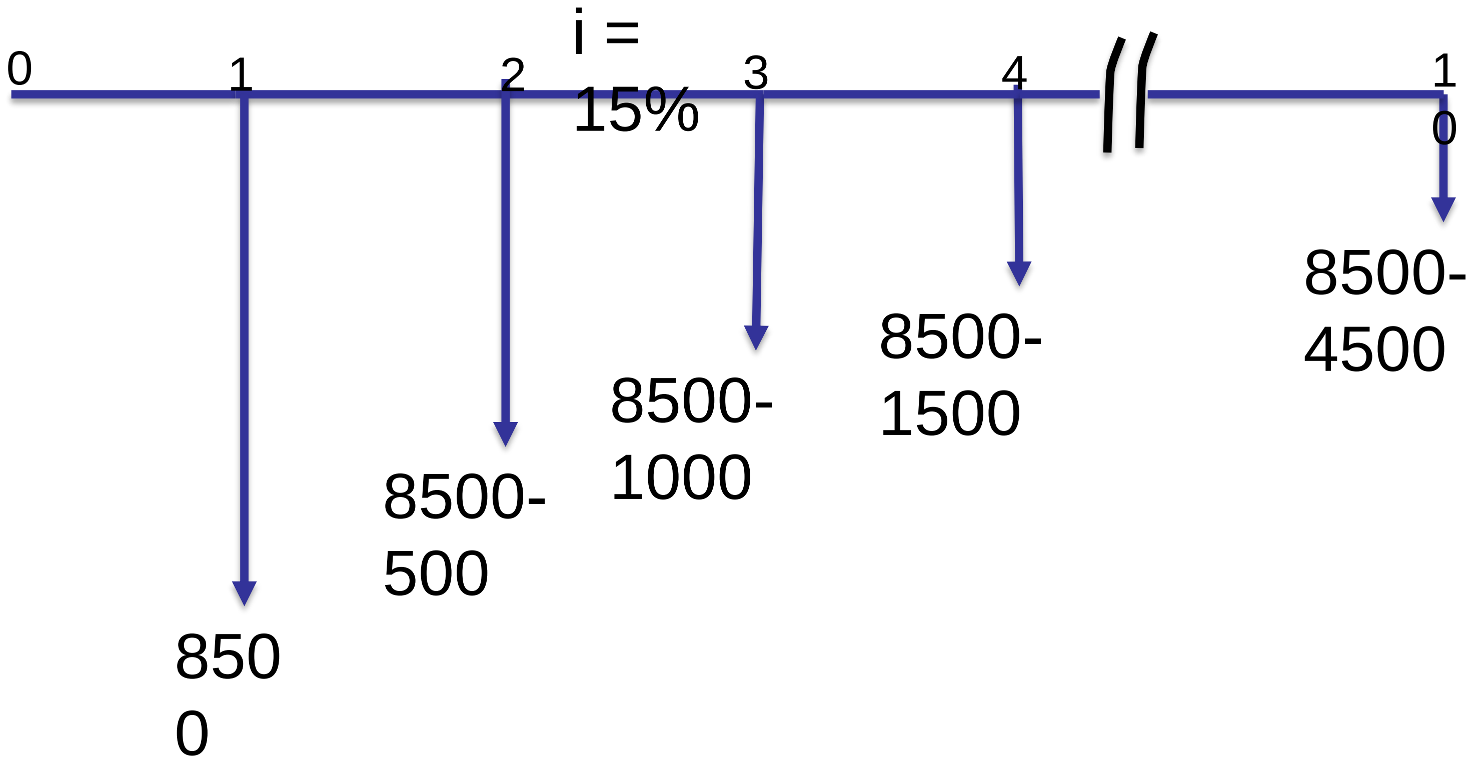
| | | |
|---|---|---|
| A | = | Rs.8,500 |
| G | = | -Rs.500 |
| I | = | 15% |
| n | = | 10 years |

- **To find:**

| | | |
|---|---|---|
| A | = | ? |
| F | = | ? |

# Solution for Problem - 9

## Cash Flow Diagram



# Solution for Problem - 9

$$A_G \quad = \quad G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$A_G \quad = \quad 500\left[\frac{(1+0.15)^{10} - 0.15\ X\ 10\ - 1}{0.15(1+0.15)^{10} - 0.15}\right]$$

$A_G$ = 500 * 3.3832 = Rs.1,691.60

A = 8,500 - 1,691.60

A = Rs.6,808.40

# Solution for Problem - 9

The future worth sum of this revised series at the end of the 10$^{th}$ year is :

$$F \quad = \quad A\left(\frac{(1+i)^n - 1}{i}\right)$$

$$F \quad = \quad 6808.4\left(\frac{(1+0.15)^{10} - 1}{0.15}\right)$$

**F = Rs.1,38, 235.84**

# Effective Interest Rate

- Let ‘i’ be the nominal interest rate compounded annually. But, in practice, the compounding may occur less than a year.

- For example, compounding may be monthly, quarterly or semi-annually.

- Compounding monthly means that the interest is computed at the end of every month

- There are 12 interest periods in a year

# Effective Interest Rate

- If the interest is compounded monthly, then the formula to compute the effective interest rate which is compounded annually is:

$$R \quad = \quad \left(1 + \frac{i}{c}\right)^{N} - 1$$

Where,

i = nominal interest rate

C = The number of interest periods in a year

N = Total number of interest periods in a year

# Problem - 10

A person invests a sum of Rs.5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity amount of the deposit after 10 years.

- **Given Data:**

  P = Rs. 5000

  Nominal Interest rate per year =12%

  Interest rate is compounding quarterly

  Number of years = 10

# Solution for Problem - 10

Number of interest periods per year = 4

Number of interest periods in 10 years=10x4=40

Hence, N=40

Interest rate per quarter= r= 12/4 = 3 %

compounded quarterly

F = P $(1+r)^n$ =5000 $(1+0.03)^{40}$

F = Rs.16,310.19

# Solution for Problem - 10

- **Solution 2:**

Number on interest periods in a year, C = 4

Effective interest rate , R = $\left(1+\frac{i}{c}\right)^{n}-1$

R = $\left(1+\frac{12}{4}\right)^{4}-1$

R =12.55% compounded annually

F = P$(1+R)^{n}$

F = 5000$(1+0.1255)^{10}$

F =Rs.16,308.91

# Problem - 11

How much money must initially be deposited in a savings account paying 5% per year, compounded annually to provide for ten annual withdrawals that start at Rs.6000 and decreases by Rs.500 each year.

- Diagram

# Solution for Problem - 11

$$A_G \quad = \quad G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$A_G \quad = \quad 500\left[\frac{(1+0.05)^{10} - 0.05\ X\ 10\ - 1}{0.05(1+0.05)^{10} - 0.05}\right]$$

$A_G$ = Rs.2,049.50

A = $A_1$ - $A_G$ = 6000 – 2049.5

A = Rs.3950.50

# Solution for Problem - 11

Present worth = P = A $\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

P = $3950.5\left[\frac{(1+0.05)^{10} - 1}{0.05(1+0.05)^{10}}\right]$

P = Rs.30,504.19

# Unit III

## Comparison of Alternatives

## Bases for comparison of Alternatives

- In most of the practical decision environments , executives will be forced to select the best alternative from a set of competing alternatives.

# Bases for Comparing the Worthiness of the Projects.

1. Present Worth Method

2. Future Worth Method

3. Annual Equivalent Method

4. Rate of Return Method

# Present Worth Method of Comparison

- In this method the Cash flows of each alternative will be reduced to time Zero by assuming an interest rate **i.**

- Depending on the type of decision , the best alternative will be selected by comparing the present Worth amounts of the alternatives.

# Present Worth Method of Comparison

The sign of various amounts at different points in time in a cash flow diagram is to be decided based on the type of the decision problem.

- ✓Cash dominated Cash flow diagram
- ✓Revenue/Profit dominated Cash flow diagram

# Present Worth Method of Comparison

In a Cash dominated Cash flow diagram , the cost (out flows) will be assigned with positive sign and the profit , revenue , salvage value (all inflows) etc. will be assigned with negative sign

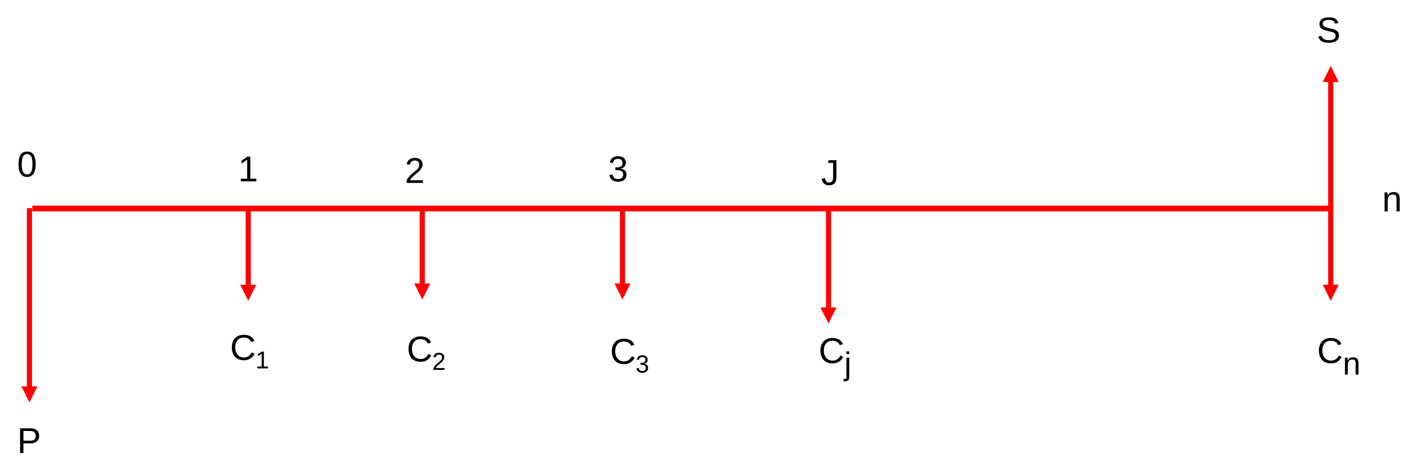
.

# Present Worth Method of Comparison

- In a Revenue/profit dominated Cash flow diagram , the profit , revenue , salvage value(all inflows to an organization) will be assigned with positive sign.

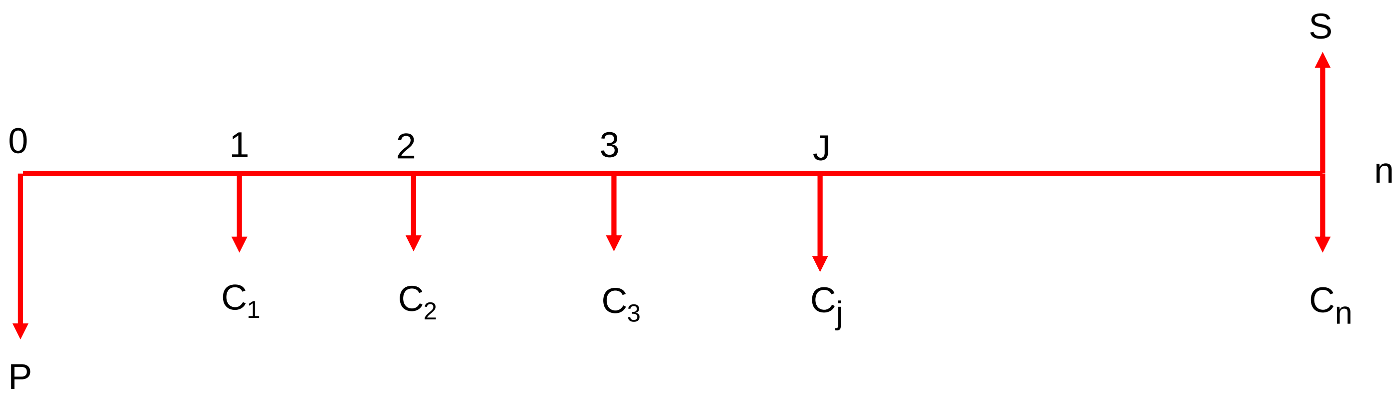
- The costs (outflow) will be assigned with negative sign .

# Cost Dominated Cash flow diagram




- P represents the initial investment.
- $C_j$ the net Cost of operation and maintenance at the end of the $J^{th}$ year.
- S is the salvage Value at the end of the $n^{th}$ year .

# Cost Dominated Cash flow diagram




To calculate the present worth amount of the above Cash flow diagram for a given interest ***i*** , we have

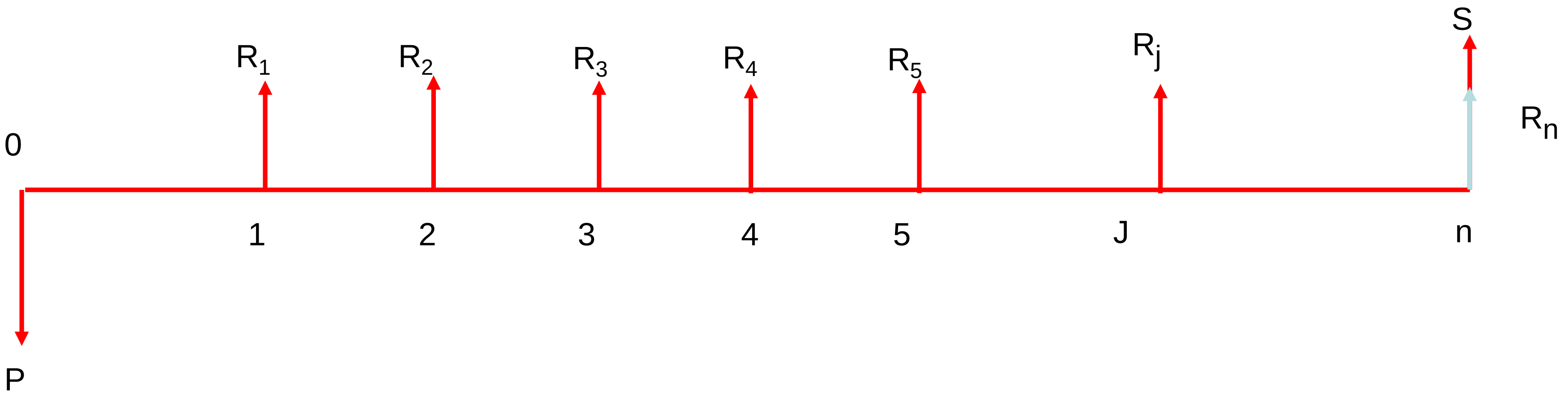
$$\mathbf{PW(i)= P + C_1 \left(\frac{1}{(1+i)}\right) + C_2 \left(\frac{1}{(1+i)^2}\right) + \ldots\ldots + C_j \left(\frac{1}{(1+i)^j}\right) + C_n \left(\frac{1}{(1+i)^n}\right) - S \left(\frac{1}{(1+i)^n}\right)}$$

# Cost Dominated Cash flow diagram

If we have some more alternatives which are to be compared with this alternative , then the Corresponding present worth amount are to be computed and compared .

Finally, the alternative with the minimum present worth amount should be selected as the best alternative.

# Revenue Dominated Cash flow diagram




- P represents the initial investment.
- $R_j$ the net revenue at the end of the $J^{th}$ year.

  The interest rate **i** is compounded annually.
- S is the salvage Value at the end of the $n^{th}$ year .

# Revenue Dominated Cash flow diagram

$$PW(i) = -P + R_1\left(\frac{1}{(1+i)}\right) + R_2\left(\frac{1}{(1+i)^2}\right) + \ldots\ldots + R_n\left(\frac{1}{(1+i)^n}\right) + S\left(\frac{1}{(1+i)^n}\right)$$

Expenditure is assigned a negative sign and revenue a positive sign.

If we have some more alternatives which are to be compared with this alternative , then the Corresponding present worth amount are to be computed and compared .

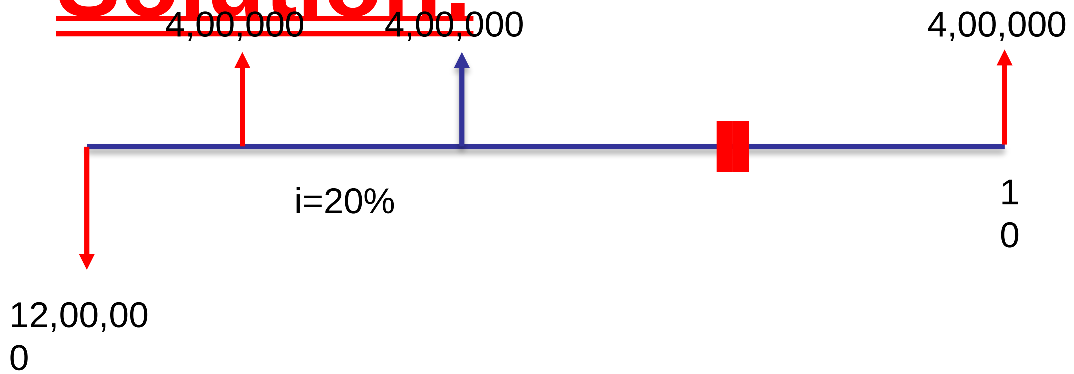
Finally, the alternative with the maximum present worth amount should be selected as the best alternative.

# Problems related to present worth comparison

(1) Alpha industry is planning to expand its production operation .It has identified three different technologies for meeting the goal . The initial outlay and annual revenues with respect to each of the technologies are summarized in table . Suggest the best technology which is to be implemented based on the present worth method of comparison assuming 20% interest rate compounded annually .

| | Initial Outlay (Rs) | Annual Income (Rs) | Life(Years) |
|---|---|---|---|
| Technology 1 | 12,00,000 | 4,00,000 | 10 |
| Technology 2 | 20,00,000 | 6,00,000 | 10 |
| Technology 3 | 18,00,000 | 4,00,000 | 10 |

# Solution:




## Technology-1

Present Worth for this technology is

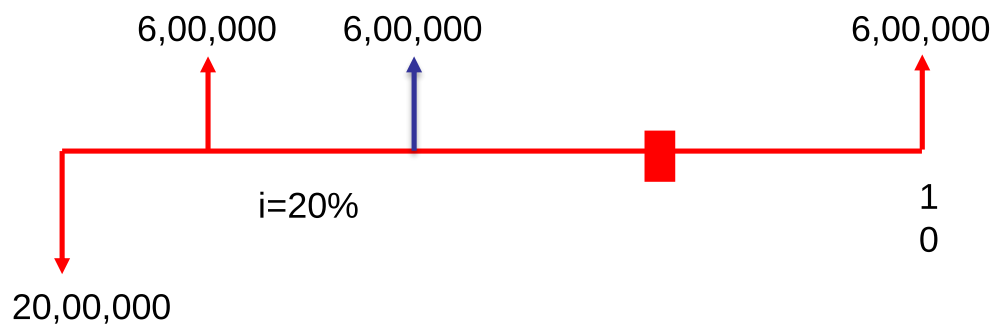
$$PW(20\%)_1 = -12,00,000 + 4,00,000 \text{ X } (\frac{P}{A}, 20\%, 10)$$

$$= -12,00,000 + 4,00,000 \text{ X } (\frac{(1+i)^n - 1}{i(1+i)^n})$$

$$= -12,00,000 + 4,00,000 \text{ X } (\frac{(1+0.2)^{10} - 1}{0.2(1+0.2)^{10}})$$

= Rs 4,76,988/=

# Technology-2




Present Worth for this technology is

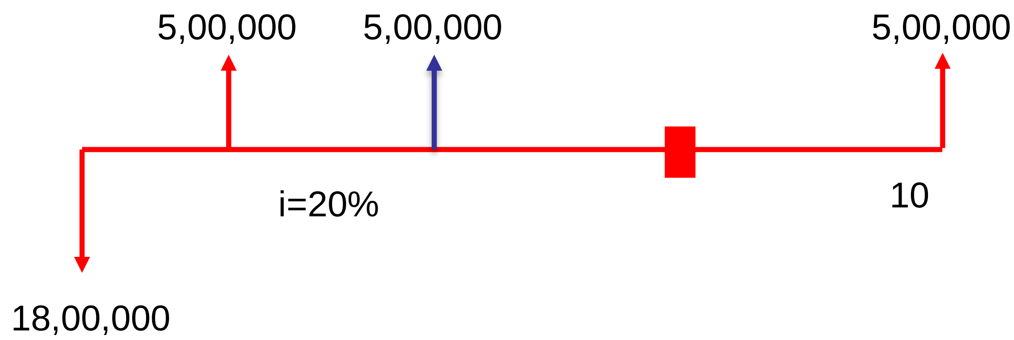
$$PW(20\%)_2 = -20,00,000 + 6,00,000 \text{ X } (\frac{P}{A}, 20\%, 10)$$

$$= -20,00,000 + 6,00,000 \text{ X }$$

$$(\frac{(1+0.2)^{10}-1}{0.2(1+0.2)^{10}})$$

$$= \text{Rs } 5,15,483.3/=$$

# Technology-3




Present Worth for this technology is

$$PW(20\%)_3 = -18{,}00{,}000 + 6{,}00{,}000 \text{ X } (\frac{P}{A}, 20\%, 10)$$

$$= -18{,}00{,}000 + 5{,}00{,}000 \text{ X } (\frac{(1+0.2)^{10}-1}{0.2(1+0.2)^{10}})$$

$$= \text{Rs } 2{,}96{,}236/=$$

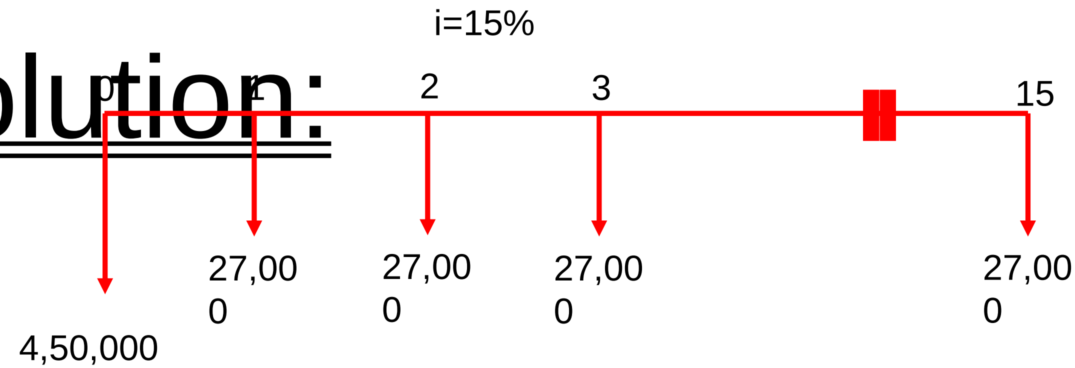
It is clear that the present worth of technology 2 is the highest. Among all the technologies ∴ ***Technology 2*** is suggested for implementation to expand production .

(2) An Engineer has two bids for an elevator to be installed in a new building . The details of the bids for the elevators are as follows.

| Bid | Initial Cost Rs | Engineer's Estimates | |
|---|---|---|---|
| | | Service Life (years) | Annual Operations & Maintenance Cost (Rs) |
| Alpha Elevator Inc. | 4,50,000 | 15 | 27,000 |
| Beta Elevator Inc. | 5,40,000 | 15 | 28,500 |

Determine which bid should be accepted , based on the present worth of Comparison assuming 15% interest rate, Compounded annually.

# Solution:




## Bid-1 : Alpha Elevator Inc.,

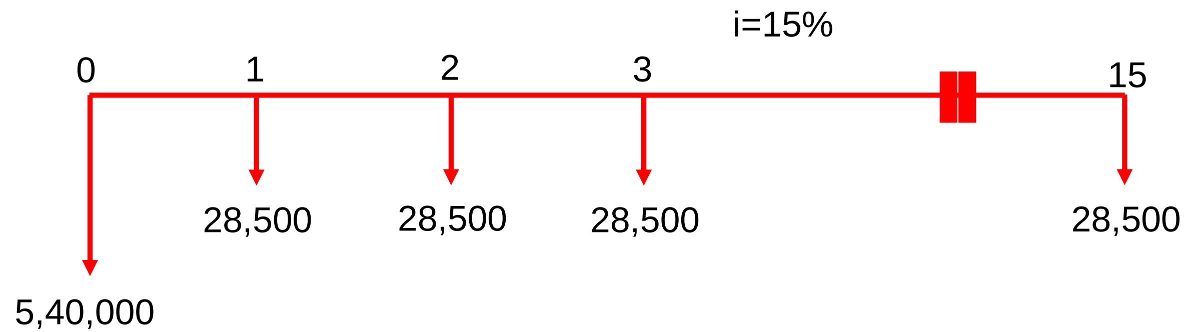
$$PW(15\%)Alpha = 4,50,000 + 27,000 \text{ X } (\frac{P}{A}, 15\%, 15)$$

$$= 4,50,000 + 27,000 \text{ X } (\frac{(1+i)^n - 1}{i(1+i)^n})$$

$$= 4,50,000 + 27,000 \text{ X } (\frac{(1+0.15)^{15} - 1}{0.15(1+0.15)^{15}})$$

$$= \text{Rs } 6,07,979/=$$

# Bid-2




$$PW(15\%)Beta = 5{,}40{,}000 + 28{,}500 \text{ X } \left(\frac{P}{A}, 15\%, 15\right)$$

$$= 5{,}40{,}000 + 28{,}500 \text{ X } \left(\frac{(1+0.15)^{15}-1}{0.15(1+0.15)^{15}}\right)$$

$$= \text{Rs } 7{,}06{,}650/=$$

Hence the total present Worth Cost of bid 1 is less than that of bid 2 . Hence bid 1 is to be selected for implementation.

∴ Elevator from ***Alpha Elevator Inc.*** is to be purchased and installed in the new building.

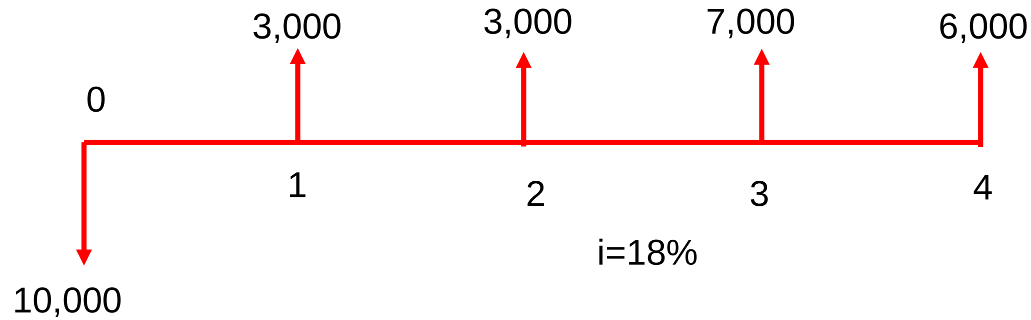
(3) Investment proposals ‘A’ and ‘B’ have the net Cash Flows as follows :-

| Proposal | End of years | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| A (Rs) | -10,000 | 3,000 | 3,000 | 7,000 | 6,000 |
| B (Rs) | -10,000 | 6,000 | 6,000 | 3,000 | 3,000 |

Compare the present worth of ‘A’ with that of ‘B’ at **I = 18%** . Which proposal should be selected ?

# Solution:

**Present Worth of A**




Present Worth of the above Cash Flow diagram is

PW(18%)$_A$ = -10,000 + 3,000 X $(\frac{\mathbf{P}}{\mathbf{F}}, 18\%, 1)$ + 3,000 X

$(\frac{P}{F}, 18\%, 2)$

+ 7,000 X $(\frac{P}{F}, 18\%, 3)$ + 6,000 X

$(\frac{P}{F}, 18\%, 4)$

= -10,000 + 3,000 X $(\frac{1}{(1+i)^n})$ + 3,000 X $(\frac{1}{(1+i)^n})$

$$= -10{,}000 + 3{,}000 \times \left(\frac{1}{(1+0.18)^1}\right) + 3{,}000 \times \left(\frac{1}{(1+0.18)^2}\right)$$

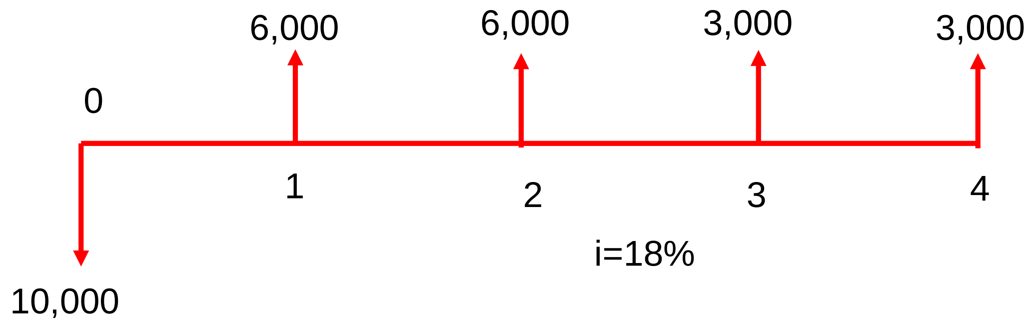
$$+ 7{,}000 \times \left(\frac{1}{(1+0.18)^3}\right) + 6{,}000 \times \left(\frac{1}{(1+0.18)^4}\right)$$

$$= -10{,}000 + 3{,}000 \times (0.8475) + 3{,}000 \times (0.7182)$$

$$+ 7{,}000 \times (0.6086) + 6{,}000 \times (0.5158)$$

= Rs 2,052.10

# Present Worth of B




Present Worth of the above Cash Flow diagram is

$PW(18\%)_B$ = -10,000 + 6,000 X $(\frac{P}{F}, 18\%, 1)$ + 6,000 X $(\frac{P}{F}, 18\%, 2)$

+ 3,000 X $(\frac{P}{F}, 18\%, 3)$ + 3,000 X $(\frac{P}{F}, 18\%, 4)$

= -10,000 + 6,000 X $(\frac{1}{\quad})$ + 6,000 X $(\frac{1}{\quad})$

$$= -10{,}000 + 6{,}000 \text{ X } \left(\frac{1}{(1+0.18)^1}\right) + 6{,}000 \text{ X } \left(\frac{1}{(1+0.18)^2}\right)$$

$$+ 3{,}000 \text{ X } \left(\frac{1}{(1+0.18)^3}\right) + 3{,}000 \text{ X } \left(\frac{1}{(1+0.18)^4}\right)$$

$$= -10{,}000 + 6{,}000 \text{ X } (0.8475) + 6{,}000 \text{ X } (0.7182)$$

$$+ 3{,}000 \text{ X } (0.6086) + 3{,}000 \text{ X } (0.5158)$$

= Rs 2,767.4

At **i = 18%** , the present worth of proposal 'B' is higher than that of proposal 'A' . Therefore , select ***Proposal B***

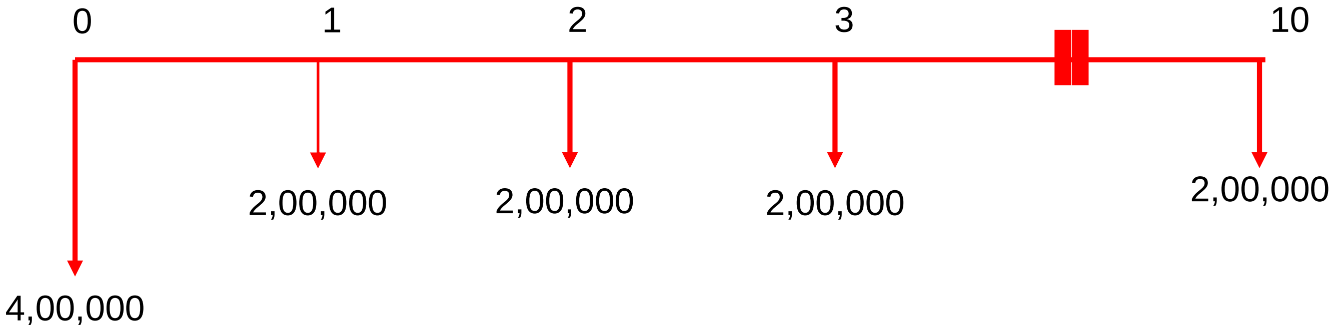
(4) A granite company is planning to buy a fully automated granite cutting machine. If it is purchased under down payment ,the cost of the machine is Rs. 16,00,000 . If it is purchased under the installment basis, the company has to pay 25% of the cost at the time of purchase and the remaining amount in 10 annual equal installments of Rs 2,00,000 each. Suggest the best alternative for the company using the present worth basis at **i =18%** compounded annually .

# Solution:

There are two alternatives available for the Company

1. Down payment of Rs. 16,00,000/=

2. Down payment of Rs. 4,00,000/= and 10 Annual Equal Installments of Rs 2,00,000 each.




The Present Worth of the above Cash Flow diagram is

$$PW(18\%) = 4,00,000 + 2,00,000 \text{ X } \left(\frac{P}{A}, 18\%, 10\right)$$

$$= 4,00,000 + 2,00,000 \text{ X } \left(\frac{(1+i)^n - 1}{i(1+i)^n}\right)$$

$$= 4,00,000 + 2,00,000 \text{ X } \left(\frac{(1+0.18)^{10} - 1}{0.18(1+0.18)^{10}}\right)$$

PW(18%) = Rs 12,98,817/=

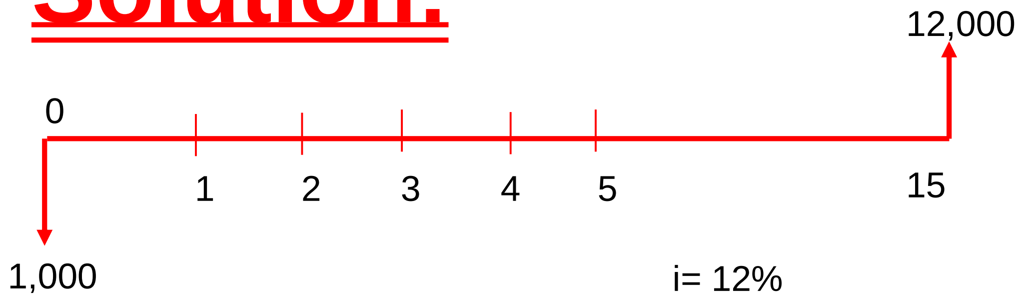
The present worth of this option is Rs 12,98,817 /=, which is less than the first option of complete down payment of Rs 16,00,00 /=.

Hence the company should select the ***Second alternative*** to buy the fully automated granite cutting machine.

(5) A finance Company advertises two investment plans . In plan 1, the company pays Rs 12,000 after 15years for every Rs 1000 invested now. In plan 2 , for every Rs 1000 invested the company pays Rs 4,000 at the end of the 10th year and Rs 4,000 at the end of 15th year. Select the best investment plan from the investment point of view at **i =12%** compounded annually.

# Solution:

Plan-1




The Present Worth of the above Cash Flow diagram is

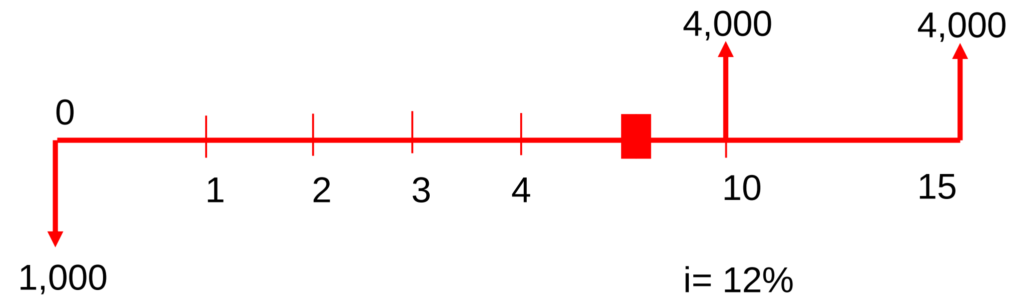
$$PW(12\%) = -1{,}000 + 12{,}000 \text{ X } (\frac{P}{F}, 12\%, 15)$$

$$= -1{,}000 + 12{,}000 \text{ X } (\frac{1}{(1+i)^n})$$

$$= -1{,}000 + 12{,}000 \text{ X } (\frac{1}{(1+0.12)^{15}})$$

PW(12%) = Rs 1,192.35

## Plan-2




The Present Worth of the above Cash Flow diagram is

$$PW(12\%) = -1{,}000 + 4{,}000 \times \left(\frac{P}{F}, 12\%, 10\right) + 4{,}000 \times \left(\frac{P}{F}, 12\%, 15\right)$$

$$= -1{,}000 + 4{,}000 \times \left(\frac{1}{(1+0.12)^{10}}\right) + 4{,}000 \times \left(\frac{1}{(1+0.12)^{15}}\right)$$

PW(12%) = Rs 1,018.80

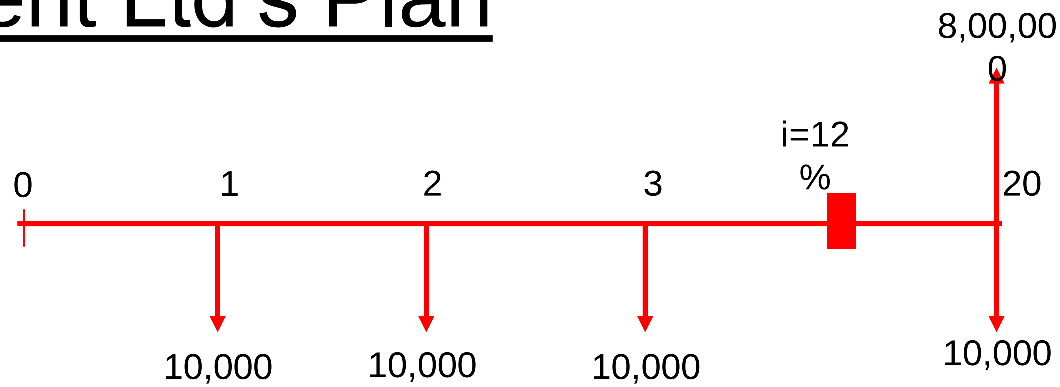
The Present Worth of plan 1 is more than that of plan2.

∴ ***Plan 1*** is the best plan from the investors point of view .

(6) Novel investments Ltd accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs 8,00,000 at the end of the $20^{th}$ year . Innovative Investments Ltd accepts Rs 10,000 at the end of every year for 20 years and pays the investor Rs 15,00,000 at the end of the $25^{th}$ year . Which is the best investment alternative ? Use present worth base with **i = 12% .**

# Solution:

## Novel Investment Ltd's Plan




The Present Worth of the above Cash Flow diagram is

PW(12%) = -10,000 X $(\frac{P}{A}, 12\%, 20)$ + 8,00,000 X $(\frac{P}{F}, 12\%, 20)$

= -10,000 X $\left(\frac{(1+i)^n - 1}{i(1+i)^n}\right)$ + 8,00,000 X

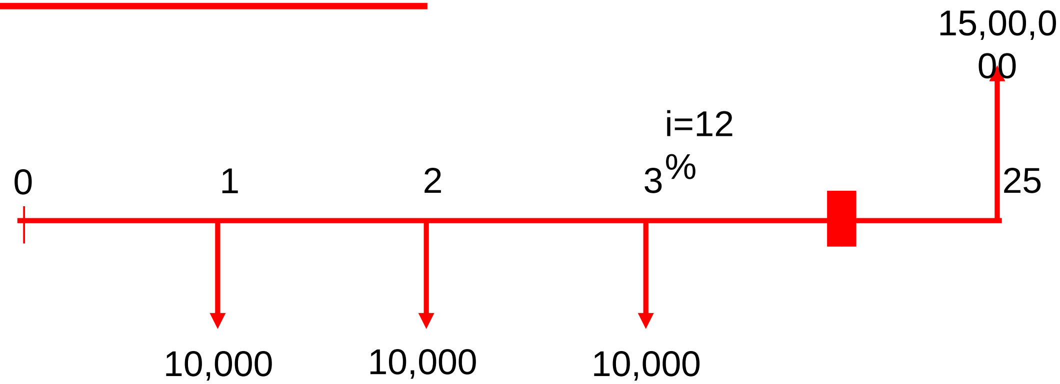
$$= -10{,}000 \text{ X } \left(\frac{(1+i)^n - 1}{i(1+i)^n}\right) + 8{,}00{,}000 \text{ X } \left(\frac{\mathbf{1}}{(\mathbf{1}+\boldsymbol{i})^{\boldsymbol{n}}}\right)$$

$$= -10{,}000 \text{ X } \left(\frac{(1+0.12)^{20} - 1}{0.12(1+0.12)^{20}}\right) + 8{,}00{,}000 \text{ X } \left(\frac{1}{(1+0.12)^{20}}\right)$$

$$= -10{,}000 \text{ X } (7.4694) + 8{,}00{,}000 \text{ X } (0.1037)$$

**PW(12%) = Rs 8,266**

# Innovative Investment Ltd's Plan




The Present Worth of the above Cash Flow diagram is

$$PW(12\%) = -10{,}000 \times (\frac{P}{A}, 12\%, 20) + 15{,}00{,}000 \times (\frac{P}{F}, 12\%, 25)$$

$$= -10{,}000 \times \left(\frac{(1+i)^n - 1}{i(1+i)^n}\right) + 15{,}00{,}000 \times \left(\frac{1}{(1+i)^n}\right)$$

$$= -10{,}000 \text{ X } \left(\frac{(1+i)^n - 1}{i(1+i)^n}\right) \quad + 15{,}00{,}000 \text{ X } \left(\frac{\mathbf{1}}{(\mathbf{1}+\boldsymbol{i})^{\boldsymbol{n}}}\right)$$

$$= -10{,}000 \text{ X } \left(\frac{(1+0.12)^{20} - 1}{0.12(1+0.12)^{20}}\right) \quad + 15{,}00{,}000 \text{ X } \left(\frac{1}{(1+0.12)^{25}}\right)$$

$$= -10{,}000 \text{ X } (7.4694) \quad + 8{,}00{,}000 \text{ X } (0.0588)$$

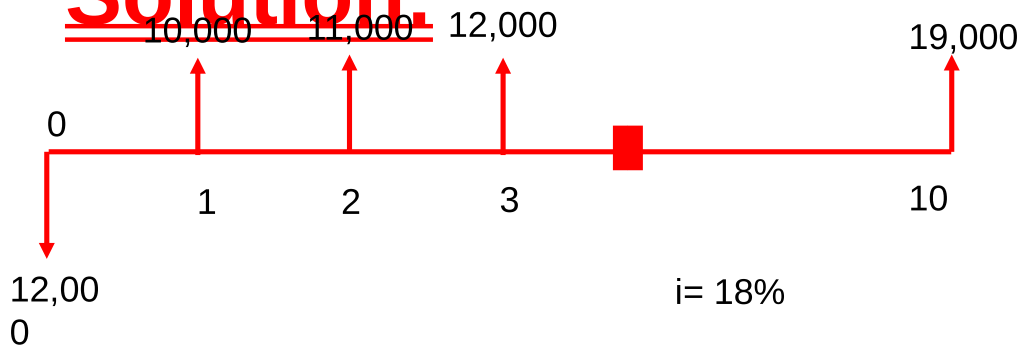
**PW(12%) = Rs 13,506/-**

The present Worth of Innovative Investment Ltd's plan is more than that of Novel Investment Ltd's plan .

∴ ***Innovative Investment Ltd's*** plan is the best from Investor's point of view .

(7) A small business with an initial outlay of Rs 12,000 yields Rs 10,000 during the first year of its operation and the yield increases by Rs 1000 from its second year of operation up to its 10th year of operation . At the end of the life of the business, the Salvage value is zero . Find the present worth of the business by assuming an interest rate of **18%** Compounded annually .

# Solution:




P =12,000 Initial Investment

**i = 18%** compounded annually

Income during the First year = $A_1$ = Rs 10,000

Annual increase in income = G = Rs 1,000 ,

n= 10 years

The Present Worth of the above Cash Flow diagram is

PW(18%) = -12,000 + (10,000 + 1,000X

$(\frac{A}{G}, 18\%, 10))$ X

$(\frac{P}{A}, 18\%, 10)$

= -12,000 + (10,000 + 1,000X

$(\frac{(1+0.18)^{10}-0.18x10-1}{0.18(1+0.18)^{10}-0.18}))$

X $(\frac{(1+0.18)^{10}-1}{\phantom{x}})$

# Future Worth Method

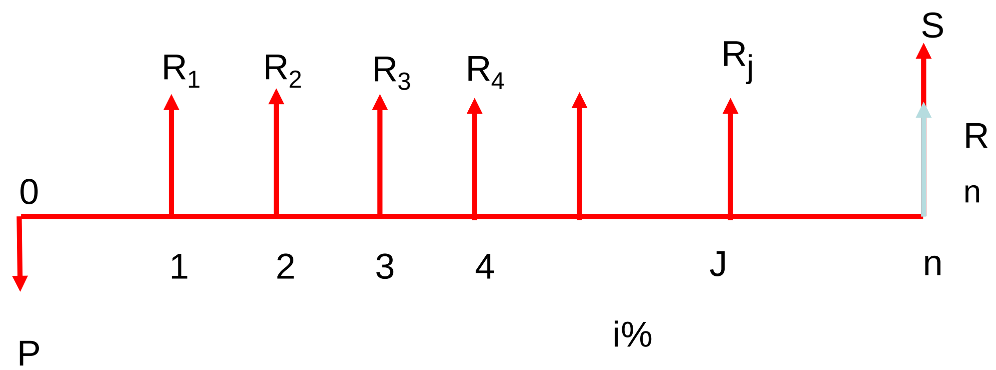
In the future worth method of Comparison of alternatives, the future worth of various alternatives will be Computed.

The alternative with the maximum future worth of net revenue or with minimum future worth of net cost will be selected as the best alternative for implementation.

The sign of various amounts at different points in time in a cash flow diagram is to be decided based on the type of the decision problem.

- ✓Cash dominated Cash flow diagram
- ✓Revenue/Profit dominated Cash flow diagram

# Revenue Dominated Cash Flow Diagram

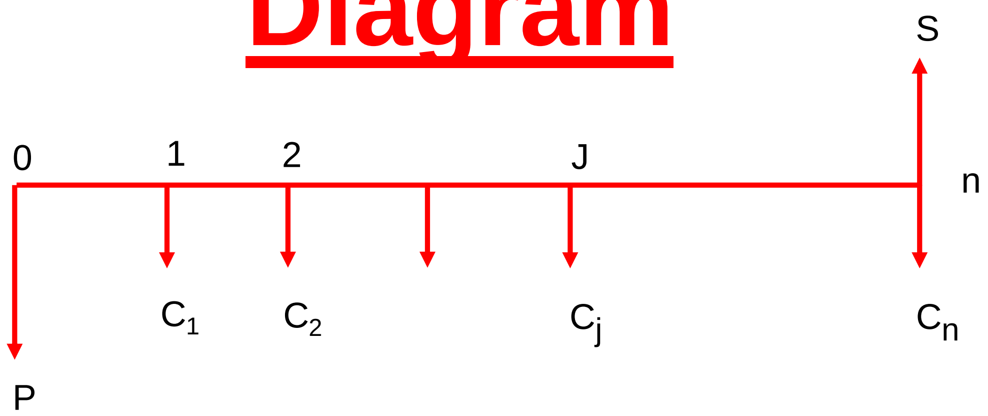



$$\textbf{FW(i)} = -P(1+i)^n + R_1(1+i)^{n-1} + R_2(1+i)^{n-2} + \ldots\ldots + R_j(1+i)^{n-j} + \ldots\ldots + R_n + S$$

Expenditure is assigned with negative sign and the revenues are assigned with positive sign .

Alternative with the maximum future worth amount should be selected as the best alternative.

# Cost Dominated Cash Flow Diagram




$$\mathbf{FW(i)= P}(1+i)^{n} + \mathbf{C_1}\,(1+i)^{n-1} + \mathbf{C_2}\,(1+i)^{n-2} + \ldots + \mathbf{C_j}\,(1+i)^{n-j} + \mathbf{C - S}$$

Expenditure is assigned with positive sign and the revenues are assigned with negative sign .

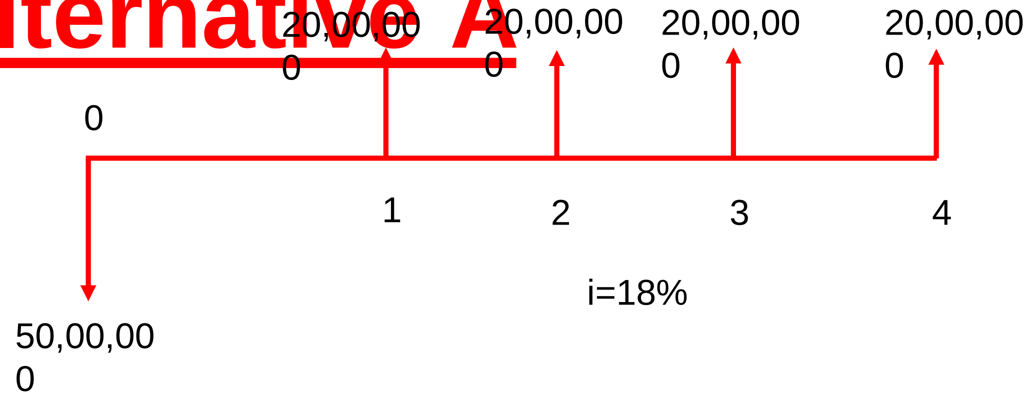
Alternative with the minimum future worth amount should be selected as the best alternative.

① Consider the following two mutually exclusive alternatives

| Alternative | End of year | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| A (Rs) | -50,00,000 | 20,00,000 | 20,00,000 | 20,00,000 | 20,00,000 |
| B (Rs) | -45,00,000 | 18,00,000 | 18,00,000 | 18,00,000 | 18,00,000 |

At **I = 18%** Select the best alternatives based on Future Worth Method of Comparison

# Alternative A



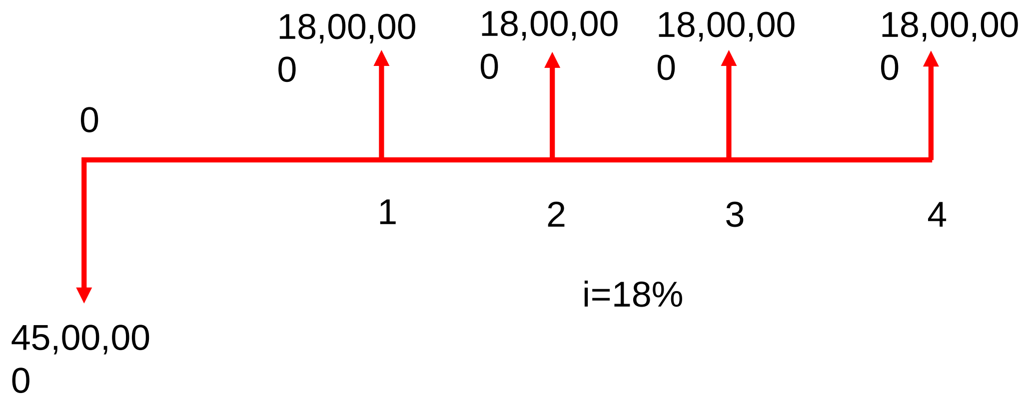

$$FW(18\%)_A = -50{,}00{,}000 \times \left(\frac{F}{P}, 18\%, 4\right) + 20{,}00{,}000 \times \left(\frac{F}{A}, 18\%, 4\right)$$

$$= -50{,}00{,}000 \times (1+i)^n + 20{,}00{,}000 \times \left(\frac{(1+i)^n - 1}{i}\right)$$

$$= -50{,}00{,}000 \times (1+0.18)^4 + 20{,}00{,}000 \times \left(\frac{(1+0.18)^4 - 1}{0.18}\right)$$

= Rs 7,27,200

# Alternative B



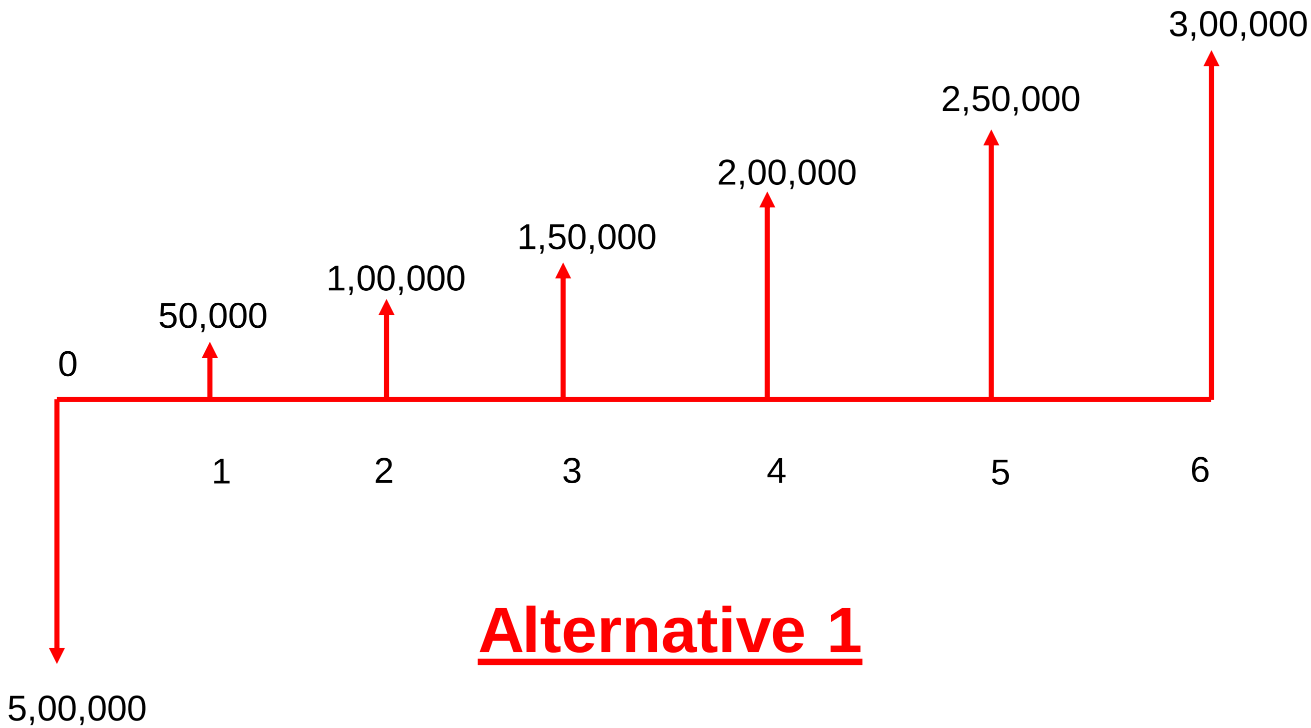

$FW(18\%)_B$ = -45,00,000 X $(\frac{F}{P}, 18\%, 4)$ + 18,00,000 X $(\frac{F}{A}, 18\%, 4)$

= -45,00,000 X $(1 + 0.18)^{4}$ + 18,00,000 X $(\frac{(1+0.18)^{4}-1}{0.18})$

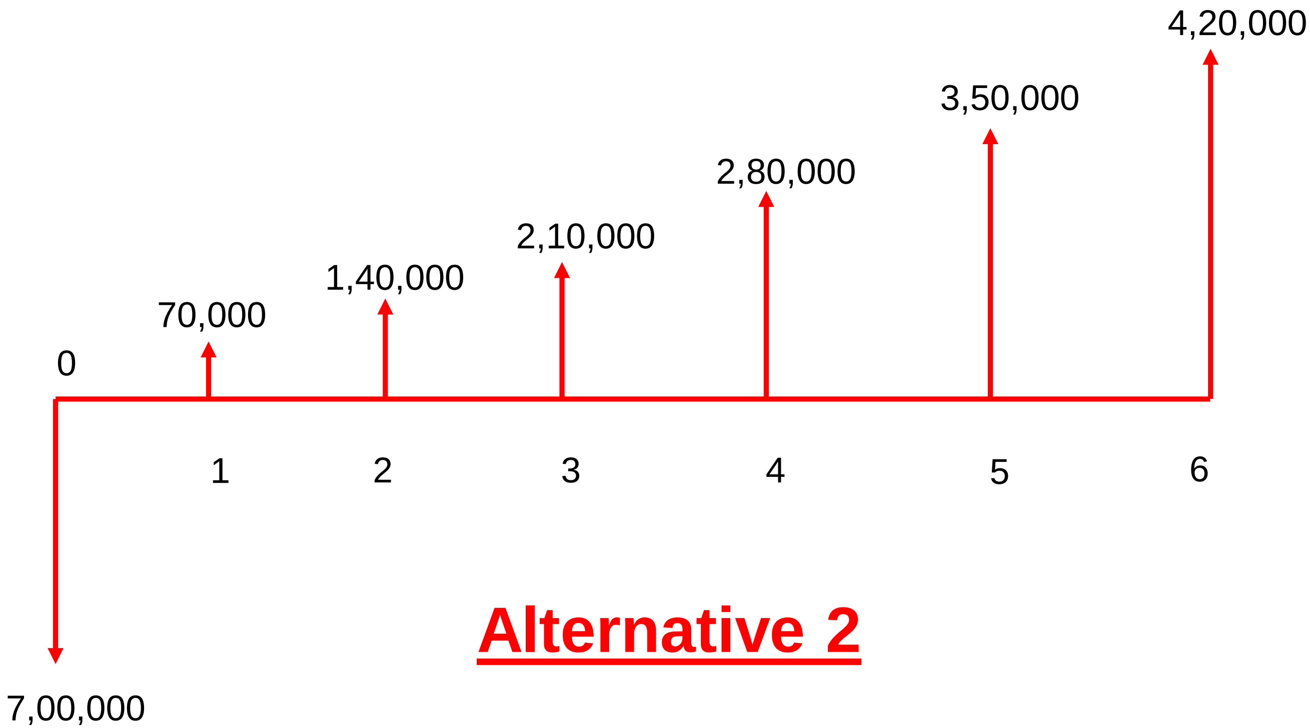
= Rs 6,63,480

The future Worth of alternative ‘A’ is greater than that of

② The Cash Flow Diagram of two mutually exclusive alternatives are given below




**Alternative 1**




**Alternative 2**

(a)Rework part (a) with **i = 9%** and **i = 20%**

(b)Select the best alternative based on Future worth method at i **= 8%**

# Solution:

***Alternative 1***

$$FW_1\,(8\%) = -P(\tfrac{F}{P}, 8\%, 6) + [\, A_1 + G(\tfrac{A}{G}, 8\%, 6) \text{x} (\tfrac{F}{A}, 8\%, 6)]$$

$$= -5{,}00{,}000(1 + i)^n + [\,50{,}000 + 50{,}000 \quad \{\frac{(1+\mathrm{i})^{\mathrm{n}} - \mathrm{in} - 1}{\mathrm{i}(1+\mathrm{i})^{\mathrm{n}} - i}\} \quad \text{X}$$

$$\frac{(1+\mathrm{i})^{\mathrm{n}} - 1}{\mathrm{i}}\,]$$

$$= -5{,}00{,}000(1 + 0.08)^6 + [50{,}000 + 50{,}000\{\frac{(1+0.08)^6 - 0.08\text{x}6 - 1}{0.08(1+0.08)^6 - 0.08}\}\text{X}$$

$$\frac{(1+0.08)^6 - 1}{0.08}\,]$$

$FW_1$ (8%) = Rs 4,08,464

# Alternative 2

This comes under equal payment gradient services

P = Rs 7,00,000 G= Rs 70,000 n=6 Years

$A_1$= Rs 70,000 i=8%

$$FW_2\ (8\%) = -P\ (\tfrac{F}{P}, 8\%, 6) + [\ A_1 + G\ (\tfrac{A}{G}, 8\%, 6)\ x\ \ (\tfrac{F}{A}, 8\%, 6)\ ]$$

$$= -7{,}00{,}000(1+0.08)^6 + [70{,}000+70{,}000\{\frac{(1+0.08)^6-0.08x6-1}{0.08(1+0.08)^6-0.08}\}X \frac{(1+0.08)^6-1}{0.08}]$$

= -7,00,000 x 1.5869 + 2,29,376x7.3359

$FW_2$ (8%) = Rs 5,71,597

The Future worth of alternative ‘2’ is more than that of alternative ‘1’.

∴ ***Alternative ’2’*** must be selected.

**Alternative 1**

**(b) (i) Evaluation at i = 9%**

$FW_1$ (9%) = Rs 3,83,912.27

$FW_2$ (9%) = Rs 5,37,477.18

Alternative 2

**(b) (ii) Evaluation at i=20%**

$FW_1$ (20%) = Rs -14,025.8

$FW_2$ (20%) = Rs -19,636.12

The negative sign of the above Future worth amount indicates that alternative ‘2’ incur loss . Thus , none of the two alternatives should be selected

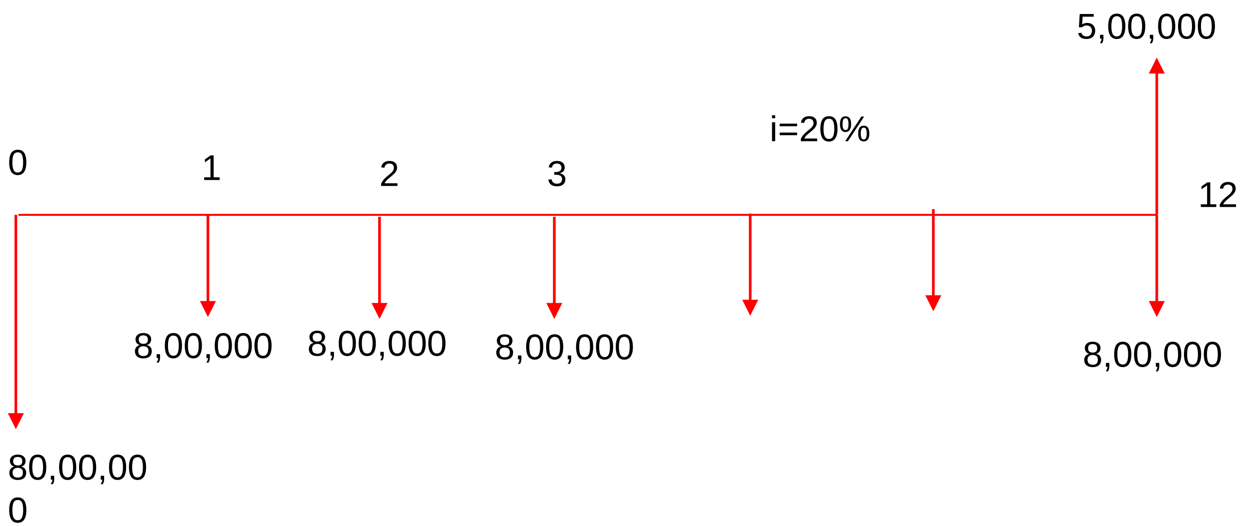
③ M/s Krishna castings Ltd is planning to replace its annealing furnace . It has received tenders from three different original manufacturers of annealing furnace . The details are as follows

| | Manufacturer | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| Initial Cost (Rs) | 80,00,000 | 70,00,000 | 90,00,000 |
| Life (Years) | 12 | 12 | 12 |
| Annual operation and maintenance cost (Rs) | 8,00,000 | 9,00,000 | 8,50,000 |
| Salvage Value after Life (Rs) | 5,00,000 | 4,00,000 | 7,00,000 |

Which is the best alternative based on Future Worth method at **i=20%** .

## Solution:

# Alternative 1 -Manufacturer 1

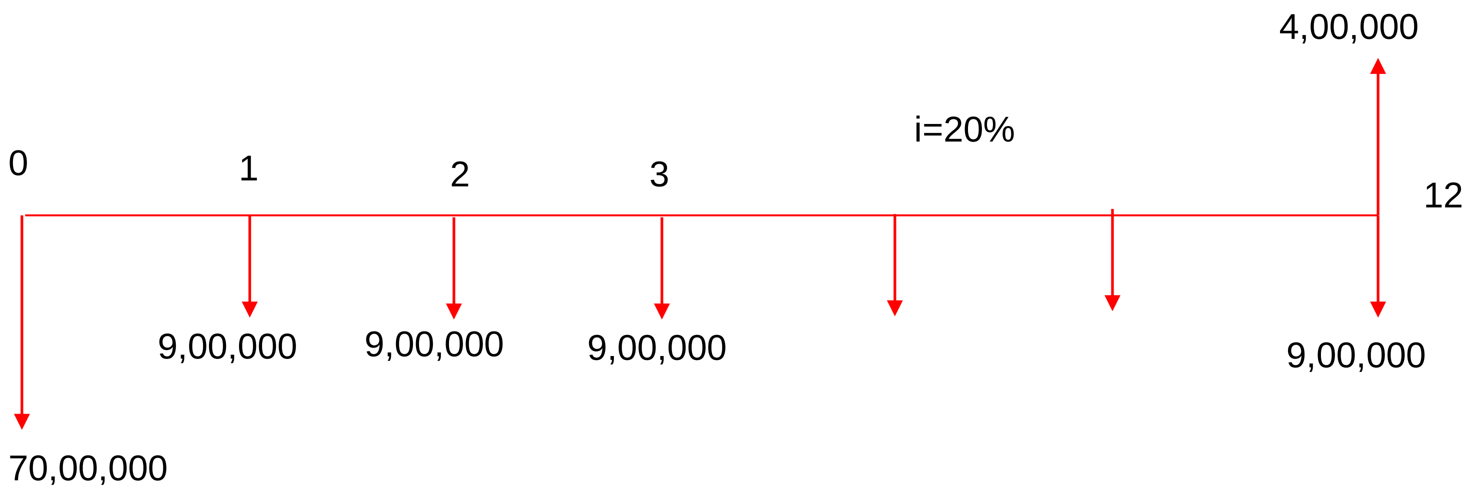



$FW_1$ (20%) = 80,00,000 $(\frac{F}{P}, 20\%, 12)$ + 8,00,00 $(\frac{F}{A}$,20%,12) - 5,00,000

=80,00,000 (8.916) + 8,00,00 (39.581) - 5,00,000

$FW_1$ (20%) = Rs 10,24,92,800

# Alternative 2 -Manufacturer 2

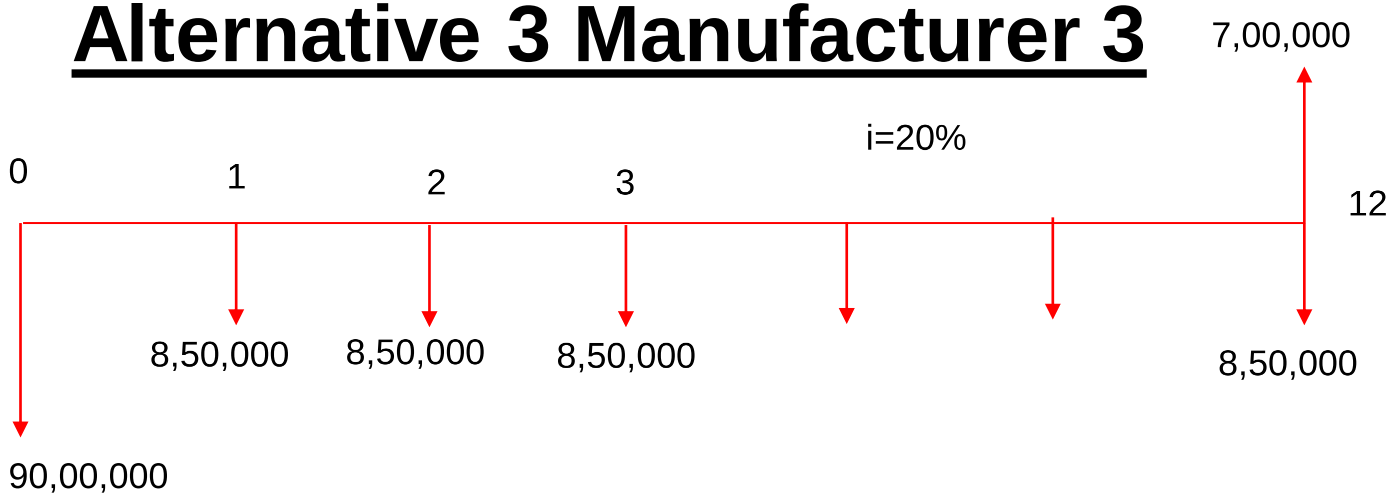



$$FW_2\ (20\%) = 70,00,000\ (\tfrac{F}{P}, 20\%, 12) + 9,00,00\ (\tfrac{F}{A}, 20\%, 12) - 4,00,000$$

$$= 70,00,000\ (8.916) + 9,00,00\ (39.581) - 4,00,000$$

$$FW_2\ (20\%) = \text{Rs } 9,76,34,900$$

# <u>Alternative 3 Manufacturer 3</u>




$$FW_3\ (20\%) = 90{,}00{,}000\left(\frac{F}{P}, 20\%, 12\right) + 8{,}50{,}00\left(\frac{F}{A}, 20\%, 12\right) - 7{,}00{,}000$$

$$= 90{,}00{,}000\ (8.916) + 8{,}50{,}00\ (39.581) - 7{,}00{,}000$$

$$FW_3\ (20\%) = \text{Rs } 11{,}31{,}87{,}850$$

The Future worth Cost of alternative '2' is less than that of the other two alternatives. ∴M/s Krishna Castings should buy from ***Manufacturer '2'***

# Annual Equivalent Method

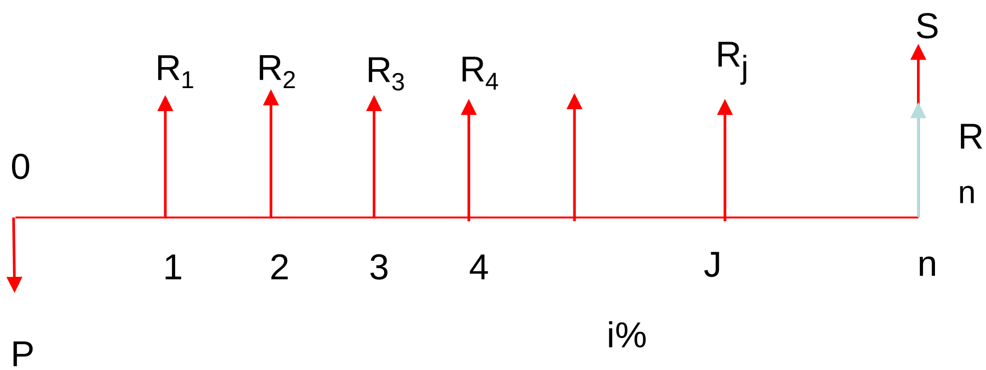
Annual Equivalent Cost or the revenue of each alternative will be Computed .

Then the alternative with the maximum annual equivalent revenue in the case of the revenue based Comparison or with the minimum annual equivalent cost in the Cost of Cost based comparison will be selected as the best alternative.

The sign of various amounts at different points in time in a cash flow diagram is to be decided based on the type of the decision problem.

- ✓Cash dominated Cash flow diagram
- ✓Revenue/Profit dominated Cash flow diagram

# Revenue Dominated Cash Flow Diagram




Step 1

$$\textbf{PW(i)} = -P + R_1 \left(\frac{1}{(1+i)}\right) + R_2 \left(\frac{1}{(1+i)^2}\right) + \ldots\ldots + R_n \left(\frac{1}{(1+i)^n}\right) + S \left(\frac{1}{(1+i)^n}\right)$$

**(or)**

$$\textbf{FW(i)} = -P(1+i)^n + R_1 (1+i)^{n-1} + R_2 (1+i)^{n-2} + \ldots\ldots + R_j (1+i)^{n-j}) + \ldots\ldots$$

$$+ R_n + S$$

**Step 2**

$$A = PW(i) \times \left(\frac{(1+i)^n}{i(1+i)^n - i}\right)$$

$$= PW(i) \times \left(\frac{A}{P}, i, n\right)$$

(or)

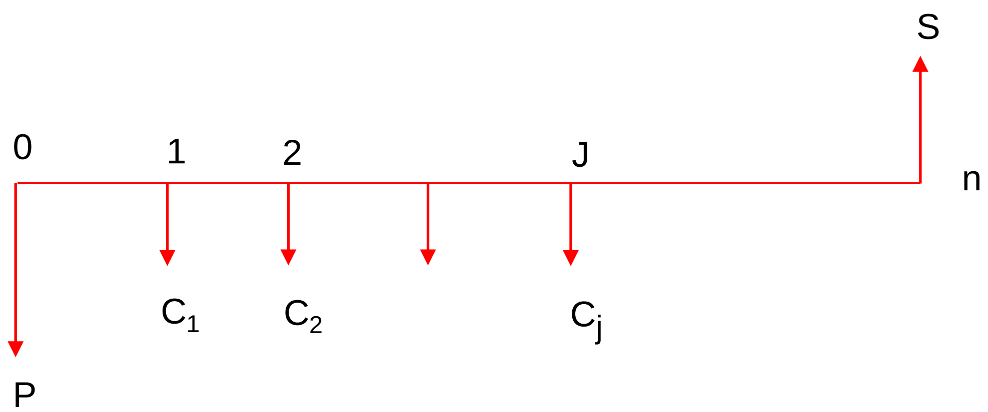
$$A = FW(i) \times \left(\frac{i}{(1+i)^n - 1}\right)$$

$$= FW(i) \times \left(\frac{A}{F}, i, n\right)$$

Note :-

Here the alternative with the maximum annual equivalent revenue should be selected as the best alternative .

# Cost Dominated Cash flow diagram




Step 1

**PW(i)= P + $C_1$ $\left(\frac{1}{(1+i)}\right)$ + $C_2$ $\left(\frac{1}{(1+i)^2}\right)$ + ….. + $C_j$ $\left(\frac{1}{(1+i)^j}\right)$ + $C_n$ $\left(\frac{1}{(1+i)^n}\right)$ - S $\left(\frac{1}{(1+i)^n}\right)$**

(or)

**PW(i)= P$(1+i)^n$ + $C_1$ $(1+i)^{n-1}$ + $C_2$ $(1+i)^{n-2}$ + …. + $C_j$ $(1+i)^{n-j}$+ $C_n$ - S**

**Step 2**

$$A = PW(i) \times \left(\frac{(1+i)^n}{i(1+i)^n - i}\right)$$

$$= PW(i) \times \left(\frac{A}{P}, i, n\right)$$

(or)

$$A = FW(i) \times \left(\frac{i}{(1+i)^n - 1}\right)$$

$$= FW(i) \times \left(\frac{A}{F}, i, n\right)$$

Note :-

Here the alternative with the minimum annual equivalent cost should be selected as the best alternative .

# Problems

1 A Company provides a car to its Chief Executive . The owner of the Company is concerned about the increasing cost of petrol. The cost per litre of petrol for the first year of operation is Rs 21 . He feels that cost of petrol will be increasing by Re 1 every year . His experience with the company car indicates that it averages 9 km per litre of petrol . The executive expects to drive an average of 20,000 km each year for the next 4 years . What is the annual equivalent cost of fuel over this period of time ? If he is offered similar service with the same quality on rental basis at Rs 60,000 per year , should the owner continue to provide company car to his executive or alternatively provide a rental car to his executive ? Assume **i=18%** . If the rental car is preferred , then the company car will find some other use within the company ?

# Solution:

Average Number of km run/year = 20,000 km

Number of km/litre of petrol = 9 km

$\therefore$ Petrol Consumption / year $= \frac{20,000}{9} =$ 2,222.2 litres

Cost / litre of petrol for the 1st year = Rs 21

Cost / litre of petrol for the 2nd year = Rs 21+1 = Rs 22

Cost / litre of petrol for the 3rd year = Rs 23

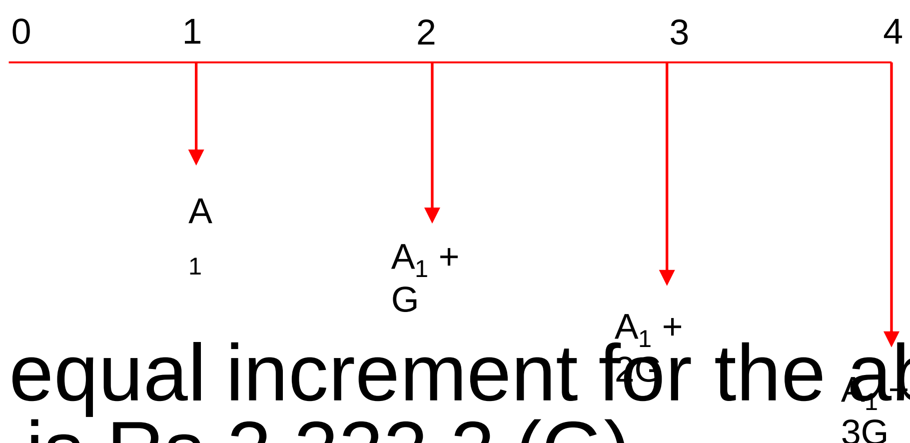
Cost / litre of petrol for the 4th year = Rs 24

∴ Fuel expenditure for the 1st year = 2,222.3 x 21 = Rs 46,666.20

Fuel expenditure for the 2nd year = 2,222.2 x 22 = Rs 48,888.40

Fuel expenditure for the 3rd year = 2,222.2 x 23 = Rs 51,110.60

Fuel expenditure for the 4th year = 2,222.2 x 24 = Rs 53,332.80




The annual equal increment for the above expenditure is Rs 2,222.2 (G)

$A_1 = 46,666.20$

$A_1 = 46,666.20 + 2,222,2\ (\frac{A}{G}, 18\%\ , 4\ )$

$= 46,666.20 + 2,222,2\ (\frac{(1+i)^n - in - 1}{i(1+i)^n - i})$

$= 46,666.20 + 2,222,2\ (\frac{(1+0.18)^4 - 0.18x4 - 1}{0.18(1+0.18)^4 - 0.18})$

$A_1 =$ Rs 49,542

The proposal of using the Company Car by spending for petrol by Company will cost an annual equivalent amount of Rs 49,542 for four years .

This cost is less than the annual return value of Rs 60,000

∴The company should continue to provide its Car to its executive .

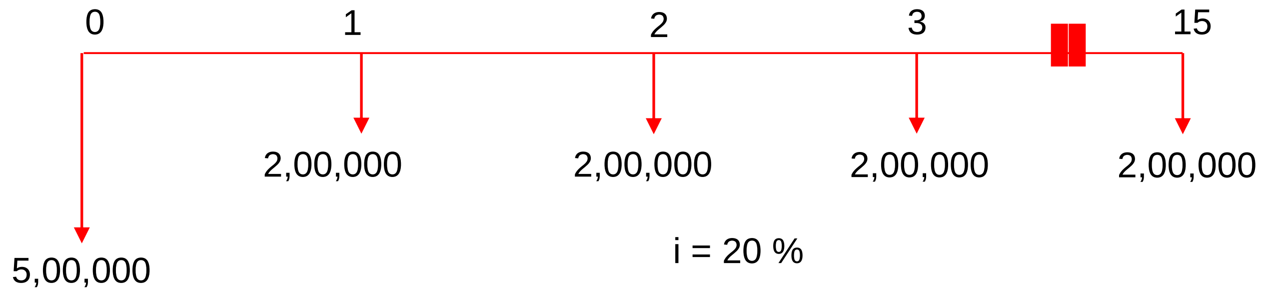
② A Company is planning to purchase an advanced machine centre . Three original manufacturers have responded to its tender notice whose particulars are tabulated as follows

| **Manufacturer** | **Down payment (Rs)** | **Yearly Equal Installment (Rs)** | **No. of Installments** |
|---|---|---|---|
| 1 | 5,00,000 | 2,00,000 | 15 |
| 2 | 4,00,000 | 3,00,000 | 15 |
| 3 | 6,00,000 | 1,50,000 | 15 |

Determine the best alternative based on the annual equivalent method by assuming **i = 20%** compounded annually .

# Solution:

**Alternative 1**



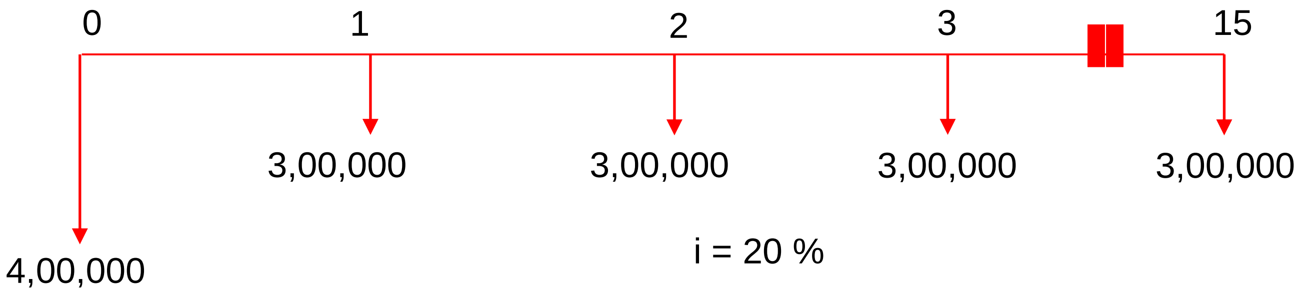

$$AE_1\ (20\%) = 5{,}00{,}000 \times \left(\frac{A}{P}, i, n\right) + 2{,}00{,}000$$

$$= 5{,}00{,}000 \times \left(\frac{i(1+i)^n}{(1+i)^n - 1}\right) + 2{,}00{,}000$$

$$= 5{,}00{,}000 \times \left(\frac{0.2\ (1+0.2)^{15}}{(1+0.2)^{15} - 1}\right) + 2{,}00{,}000$$

$$AE_1\ (20\%) = \text{Rs } 3{,}06{,}950$$

## Alternative 2



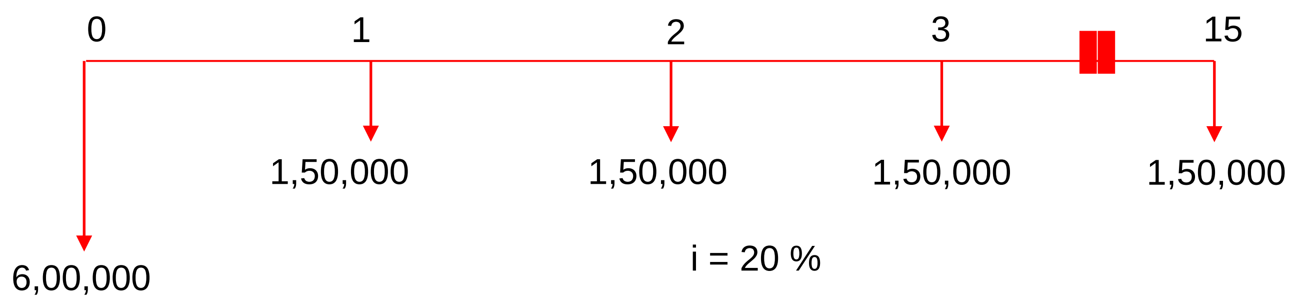

$$AE_2\ (20\%) = 4{,}00{,}000 \times \left(\frac{A}{P}, i, n\right) + 3{,}00{,}000$$

$$= 4{,}00{,}000 \times \left(\frac{i(1+i)^n}{(1+i)^n - 1}\right) + 3{,}00{,}000$$

$$= 4{,}00{,}000 \times \left(\frac{0.2\ (1+0.2)^{15}}{(1+0.2)^{15} - 1}\right) + 3{,}00{,}000$$

$$AE_2\ (20\%) = \text{Rs } 3{,}85{,}560$$

**Alternative 3**




$AE_3$ (20%) $= 6{,}00{,}000 \times (\frac{A}{P}, i, n) + 1{,}50{,}000$

$= 6{,}00{,}000 \times (\frac{0.2\,(1+0.2)^{15}}{(1+0.2)^{15}-1}) + 1{,}50{,}000$

$AE_3$ (20%) = Rs 2,78,340

The annual Equivalent Cost of manufacturer '3' is less than that of manufacturers '1' and '2'.

∴ The company should buy the advanced M/c centre from ***Manufacturer '3'***

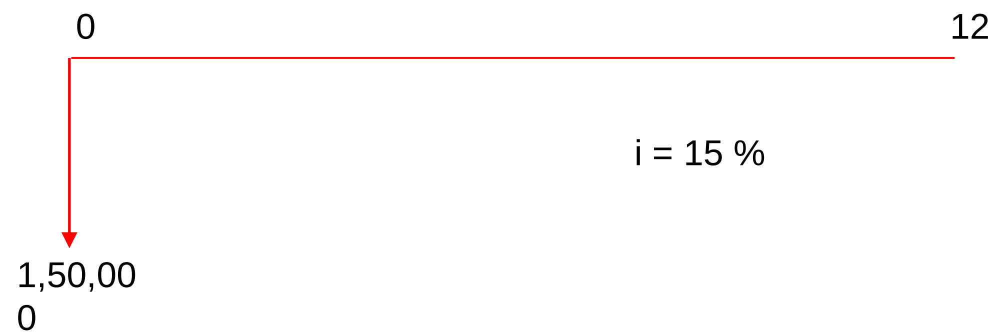
③ A certain individual firm desires an economic analysis to determine which of the two machines is attractive in a given interval of time . The minimum attractive rate of return for the firm is **15%** . The following data are to be used in the analysis.

| | Machine X | Machine Y |
|---|---|---|
| First Cost (Rs) | 1,50,000 | 2,40,000 |
| Estimated life (years) | 12 | 12 |
| Salvage Value (Rs) | 0 | 6,000 |
| Annual Maintenance Cost (Rs) | 0 | 4,500 |

Which machine would you choose based on annual equivalent cost ?

# Solution:

**Machine X**



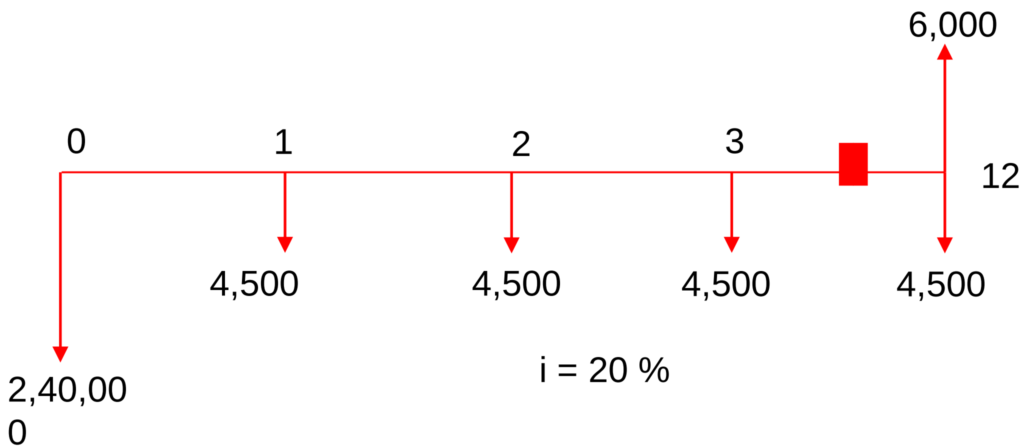

$$AE_X\ (15\%) = 1{,}50{,}000 \times \left(\frac{A}{P}, 15\%, 12\right)$$

$$= 1{,}50{,}000 \times \left(\frac{i(1+i)^n}{(1+i)^n - 1}\right)$$

$$= 1{,}50{,}000 \times \left(\frac{0.15(1+0.15)^{12}}{(1+0.15)^{12} - 1}\right)$$

$AE_X$ (15%) = Rs 27,675

## Machine Y




$$AE_Y\ (15\%) = 2{,}40{,}000 \times \left(\frac{A}{P}, 15\%, 12\right) + 4{,}500 - 6{,}000 \times \left(\frac{A}{F}, 15\%, 12\right)$$

$$= 2{,}40{,}000 \times \left(\frac{0.15\ (1+.15)^{12}}{(1+0.15)^{12}-1}\right) + 4{,}500 - 6{,}000 \times \left(\frac{0.15}{(1+0.15)^{12}-1}\right)$$

$$AE_Y\ (15\%) = \text{Rs } 48{,}575$$

The annual equivalent cost of machine 'X' is less than that of machine 'Y' . Hence ***Machine 'X'*** is more cost effective machine .

④ Two possible routes for laying a power line are under study . Data on the routes are as follows.

| | **Around the lake** | **Under the lake** |
|---|---|---|
| Length | 15 km | 5 km |
| First cost (Rs) | 1,50,000 / km | 7,50,000 / km |
| Useful life (years) | 15 | 15 |
| Maintenance Cost (Rs) | 6,000 / km / year | 12,000 / km / year |
| Salvage value (Rs) | 90,000 / km | 1,50,000 / km |
| Yearly power loss (Rs) | 15,000 / km | 15,000 / km |

If 15% interest is used, should the power line be routed around or under the lake .

# Solution:

## Around the lake

Initial cost = 1,50,000 x 15 = Rs 22,50,000

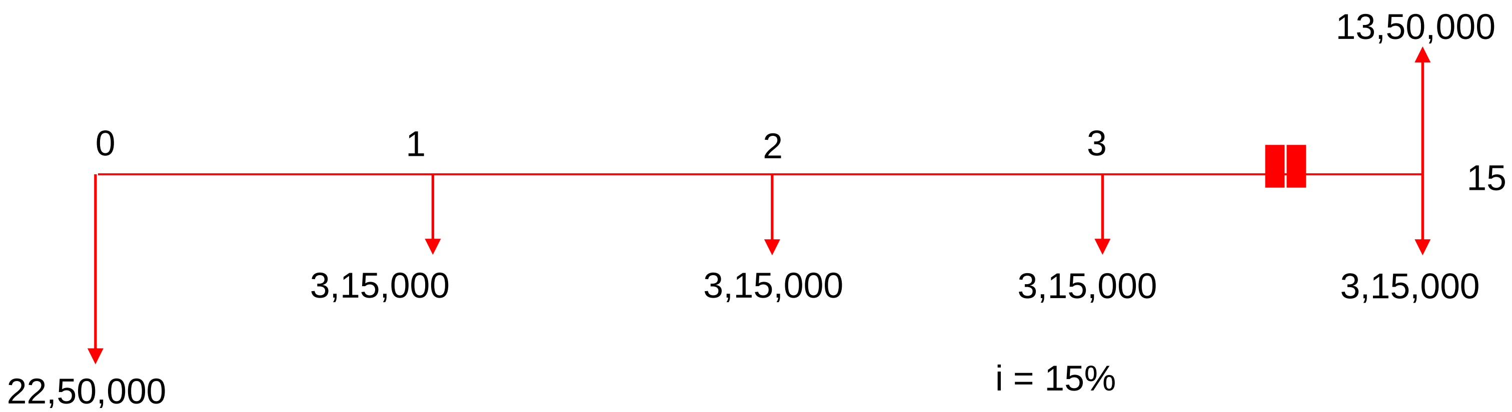
Maintenance Cost/year = 6,000x15 = Rs 90,000

Power loss /year = 15,000x15 = Rs 2,25,000

Maintenance cost & Power loss/ year = Rs 90,000 + Rs 2,25,000

= Rs 3,15,000

Salvage value = 90,000x 15 = Rs 13,50,000




$$AE_1\ (15\%) = 22{,}50{,}000 \times \left(\frac{A}{P}, 15\%, 15\right) + 3{,}15{,}000 - 13{,}50{,}000 \times \left(\frac{A}{F}, 15\%, 15\right)$$

$$= 22{,}50{,}000 \times \left(\frac{0.15\ (1+.15)^{15}}{(1+0.15)^{15}-1}\right) + 3{,}15{,}000 - 13{,}50{,}000 \times \left(\frac{0.15}{(1+0.15)^{15}-1}\right)$$

$AE_1$ (15%) = Rs 6,71,400

# Alternative 2

**<u>Under the lake</u>**

Initial cost = 7,50,000 x 5 = Rs 37,50,000

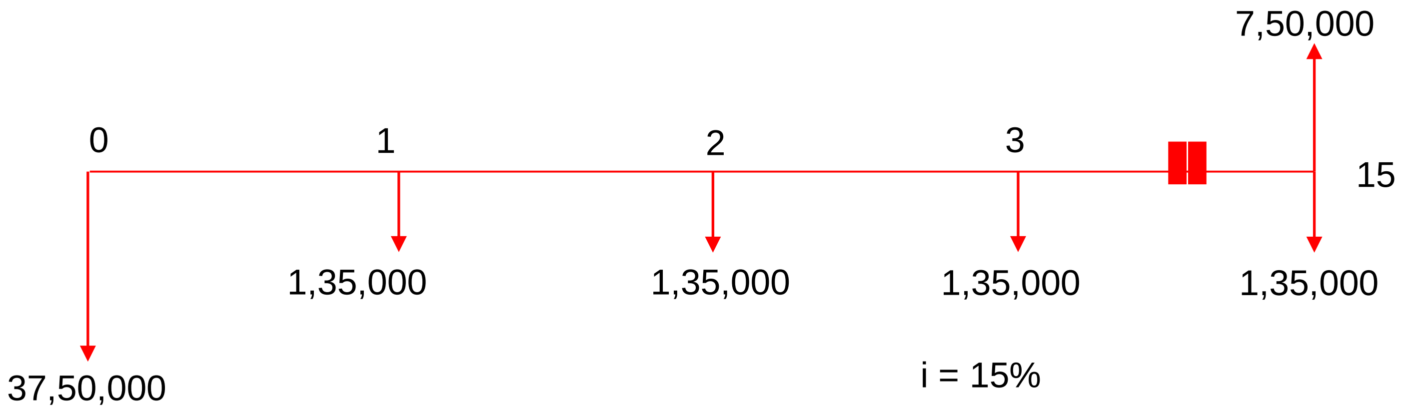
Maintenance Cost/year = 12,000x5 = Rs 60,000

Power loss /year = 15,000x5 = Rs 75,000

Maintenance cost & Power loss/ year = Rs 60,000 + Rs 75,000

= Rs 1,35,000

Salvage value = 1,50,000x 5 = Rs




$$AE_2\ (15\%) = 37{,}50{,}000 \times \left(\frac{A}{P}, 15\%, 15\right) + 1{,}35{,}000 - 7{,}50{,}000 \times \left(\frac{A}{F}, 15\%, 15\right)$$

$$= 37{,}50{,}000 \times \left(\frac{0.15\ (1+.15)^{15}}{(1+0.15)^{15}-1}\right) + 1{,}35{,}000 - 7{,}50{,}000 \times \left(\frac{0.15}{(1+0.15)^{15}-1}\right)$$

$AE_2$ (15%) = Rs 7,60,500

The annual equivalent cost of alternative ‘1’ is less than that of alternative ‘2’

∴ Select the route ***Around the Lake*** for laying the power line

⑤ A suburban taxi company is analyzing the proposal of buying car with diesel & petrol engines. The cars averages 60,000 km a year with a useful life of 3 years for petrol taxi and 4 years for diesel taxi . Other details are as follows.

| | **Diesel** | **Petrol** |
|---|---|---|
| Vehicle cost (Rs) | 3,90,000 | 3,60,000 |
| Fuel cost / litre | 8 | 20 |
| Mileage in km/litre | 30 | 20 |
| Annual Repairs (Rs) | 9,000 | 6,000 |
| Annual insurance premium (Rs) | 15,000 | 15,000 |
| Resale value at end of vehicle life (Rs) | 60,000 | 90,000 |

Determine the more economical choice if interest is 20% compounded annually.

## Solution:

**Alternative 1**

**Purchase of Diesel taxi**

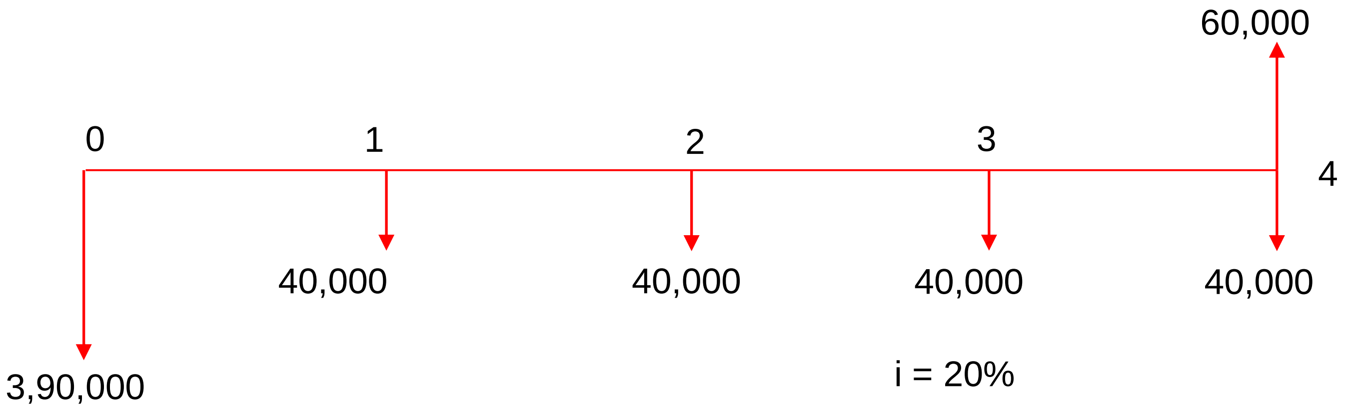
Vehicle cost = Rs 3,90,000

Life = 4 years

No. of litres /year = $\frac{60,000}{30}$ = 2,000 litres

Fuel cost /year = 2,000 x 8 = Rs 16,000

Fuel cost , annual repairs & Insurance premium/year




$$AE_1\ (20\%) = 3{,}90{,}000 \times \left(\frac{A}{P}, 20\%, 4\right) + 40{,}000 - 60{,}000 \times \left(\frac{A}{F}, 20\%, 4\right)$$

$$= 3{,}90{,}000 \times \left(\frac{0.2\ (1+0.2)^4}{(1+0.2)^4-1}\right) + 40{,}000 - 60{,}000 \times \left(\frac{0.2}{(1+0.2)^4-1}\right)$$

$$AE_1\ (20\%) = \text{Rs } 1{,}79{,}479$$

**Alternative 2**
**Purchase of petrol taxi**

Vehicle cost = Rs 3,60,000

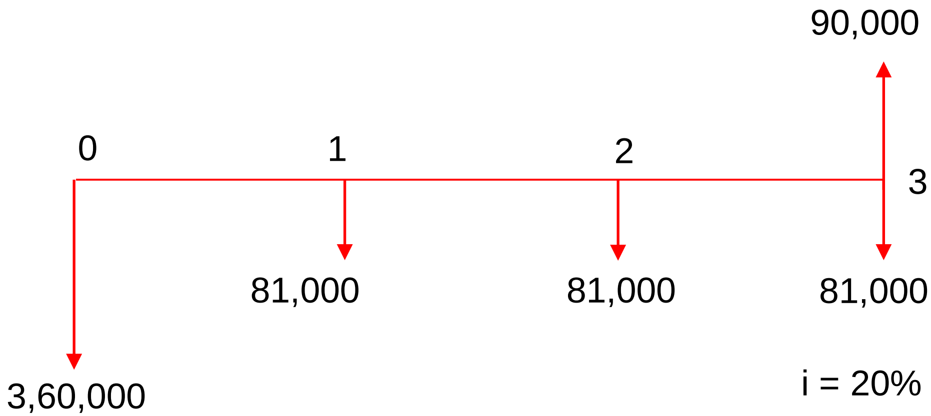
Life = 3 years

No. of litres /year = $\frac{60,000}{20}$= 3,000 litres

Fuel cost /year = 3,000 x 20 = Rs 60,000

Fuel cost , annual repairs & Insurance premium/year

= Rs 60,000 + Rs 6,000 +




$$AE_2\ (20\%) = 3{,}60{,}000 \times \left(\frac{A}{P}, 20\%, 3\right) + 81{,}000 - 90{,}000 \times \left(\frac{A}{F}, 20\%, 3\right)$$

$$= 3{,}90{,}000 \times \left(\frac{0.2\ (1+.2)^3}{(1+0.2)^3-1}\right) + 81{,}000 - 90{,}000 \times \left(\frac{0.2}{(1+0.2)^3-1}\right)$$

$$AE_2\ (20\%) = \text{Rs } 2{,}27{,}169$$

**The annual equivalent cost of diesel taxi is lesser than that of petrol taxi.**

**∴The purchase of diesel car is cost effective.**

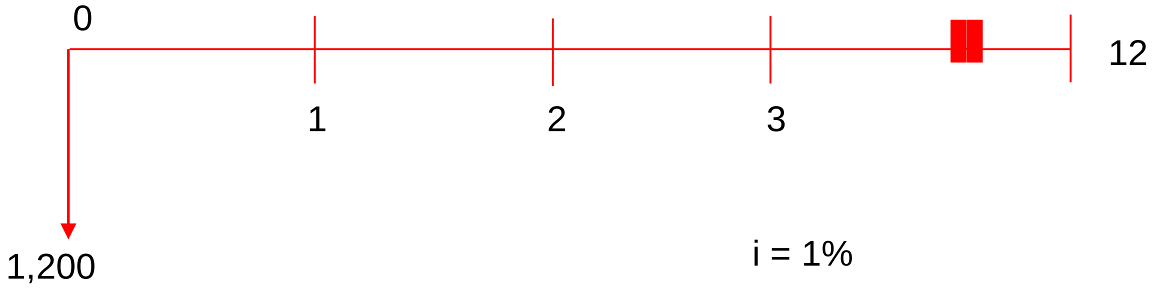
⑥ A transport company has been looking for a new tyre for a truck and has located the following alternatives .

| Brand | Tyre Warranty (Months) | Pirce per tyre (Rs) |
|---|---|---|
| A | 12 | 1,200 |
| B | 24 | 1,800 |
| C | 36 | 2,100 |
| D | 48 | 2,700 |

Determine the more economical choice if interest is 12% compounded annually.

# Solution:

Brand A

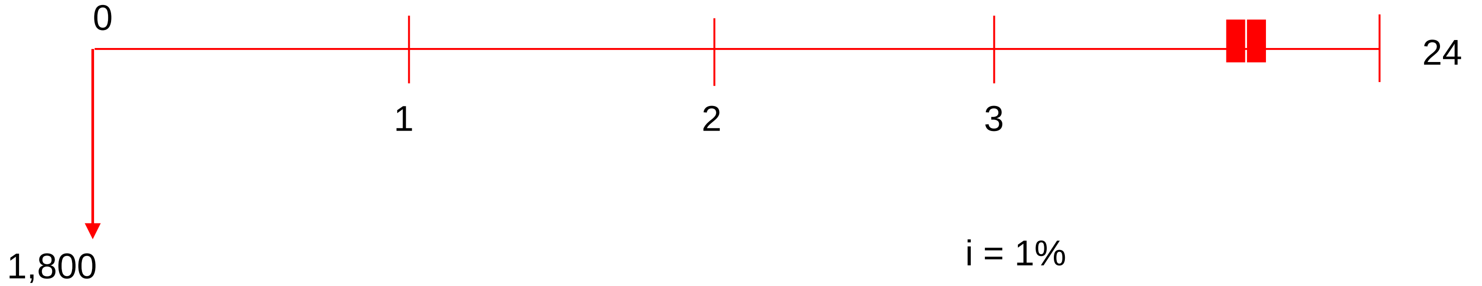



$$AE_A\,(1\%) \;=\; 1{,}200 \times \left(\frac{A}{P}, 1\%, 12\right)$$

$$= 1{,}200 \times \left(\frac{0.01\,(1+.01)^{12}}{(1+0.01)^{12}-1}\right)$$

$$AE_A\,(1\%) \;=\; \text{Rs } 107$$

# Brand B

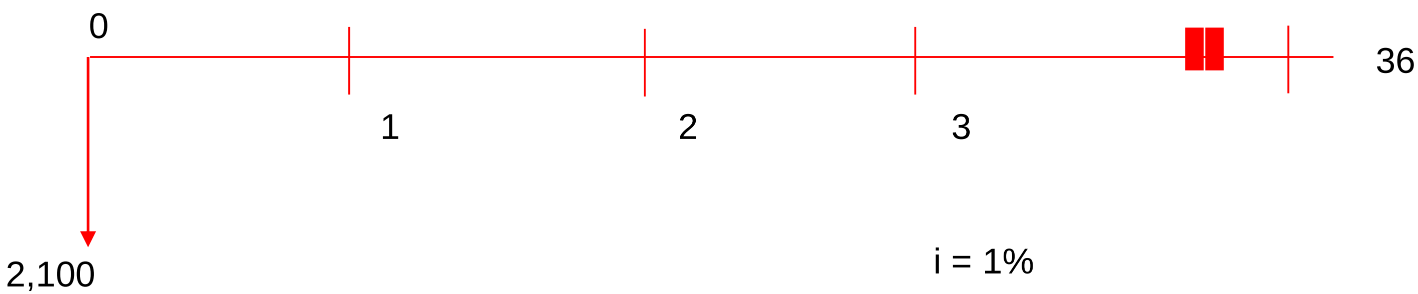



$$AE_B\ (1\%) = 1{,}800 \times \left(\frac{A}{P}, 1\%, 24\right)$$

$$= 1{,}800 \times \left(\frac{0.01\ (1+.01)^{24}}{(1+0.01)^{24}-1}\right)$$

$$AE_B\ (1\%) = \text{Rs } 85$$

# Brand C

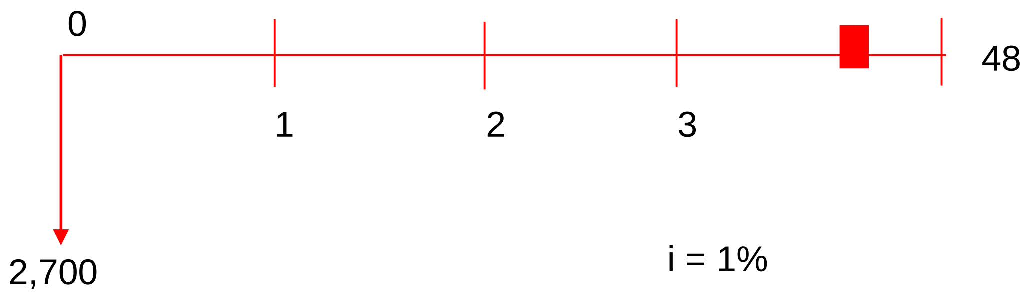



$$AE_C\,(1\%) = 2{,}100 \times \left(\frac{A}{P}, 1\%, 36\right)$$

$$= 2{,}100 \times \left(\frac{0.0.1\,(1+.01)^{36}}{(1+0.01)^{36}-1}\right)$$

$$AE_C\,(1\%) = \text{Rs } 70$$

# Brand D




$$AE_D\ (1\%) = 2{,}700 \times \left(\frac{A}{P}, 1\%, 48\right)$$

$$= 2{,}700 \times \left(\frac{0.0.1\ (1+.01)^{48}}{(1+0.01)^{48}-1}\right)$$

$$AE_D\ (1\%) = \text{Rs } 71$$

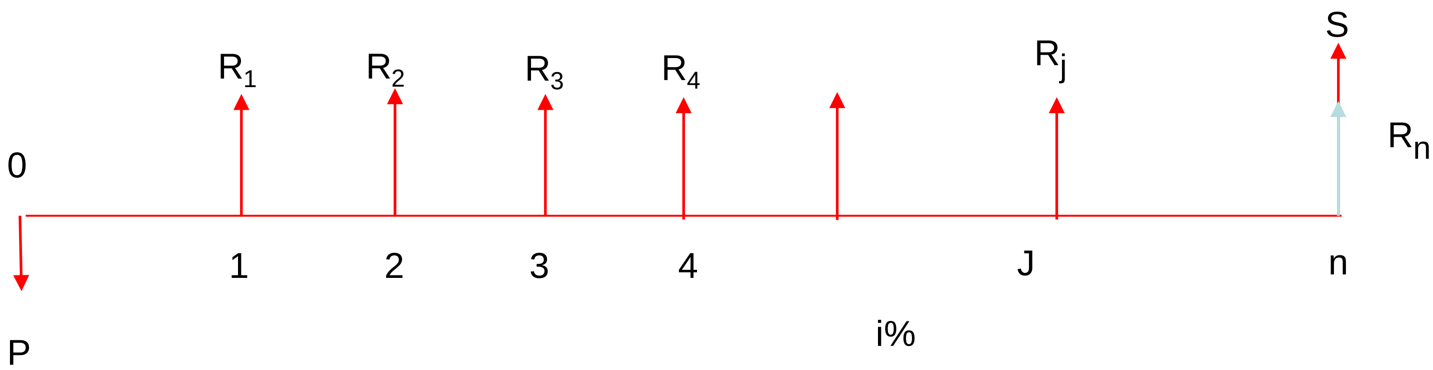
The annual equivalent cost of ***Brand 'C'*** is less than that of other brands . Hence it should be used in the vehicles of the trucking company.

# Rate of Return Method

The rate of return of a cash flow pattern is the interest rate at which the present worth of that cash flow pattern reduces to zero .

In this method of comparison , the rate of return for each alternative is computed .

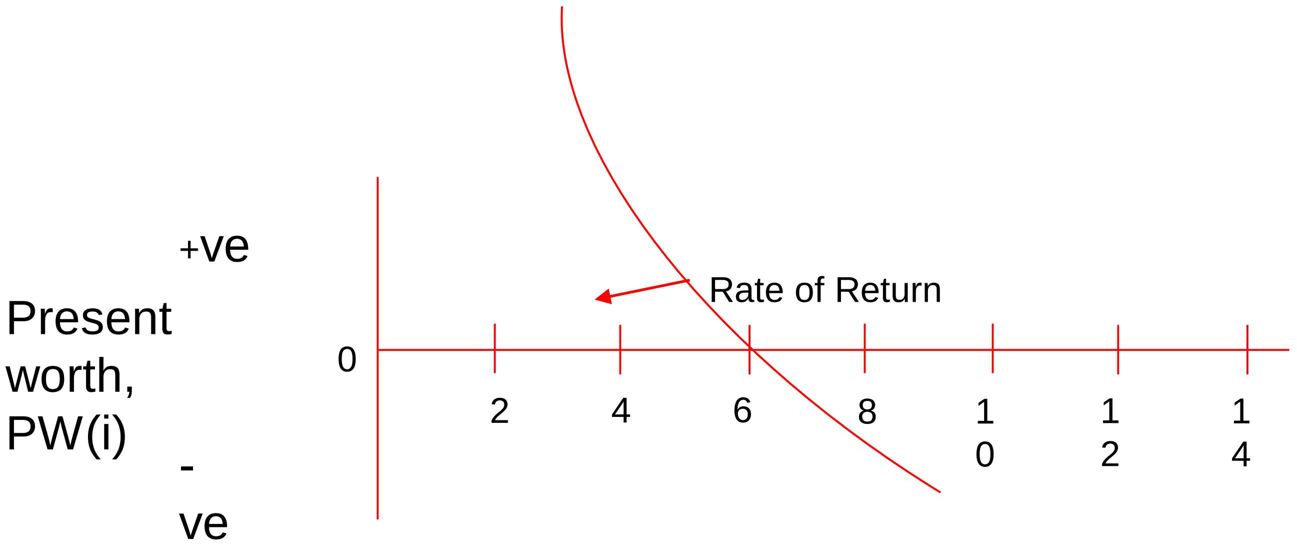
Then the alternative which has the highest rate of return is selected as the best alternative .




The first step is to find the net present worth of the cash flow diagram using the following expression at a given interest rate.

$$\mathbf{PW(i)= \; -P + R_1 \left(\frac{1}{(1+i)}\right) + R_2 \left(\frac{1}{(1+i)^2}\right) + \ldots.. + R_n \left(\frac{1}{(1+i)^n}\right) + S \left(\frac{1}{(1+i)^n}\right)}$$

The above function is to be evaluated for different values of ‘i’ until the present worth function reduces to zero as shown in fig .




The present worth goes on decreasing when the interest rate is increased .

The value of ‘I’ at which Present Worth Curve cuts the X-axis is the rate of return of the given proposal / project.

# Unit -4

# Replacement and Maintenance Analysis

# INTRODUCTION

- Organizations providing goods/services use several facilities like equipment and machinery which are directly required in their operations.

- All such facilities should be continuously monitored for their efficient functioning.

- Besides the quality of service of the facilities, the operation and maintenance cost would increase with the passage of time.

- Hence, it is necessary to maintain the equipment in good operating conditions with economical cost.

# INTRODUCTION - Contd..

- Thus, we need an integrated approach to minimize the cost of maintenance. In certain cases, the equipment will be obsolete over a period of time.

- If a firm wants to be in the same business competitively, it has to take decision on whether to replace the old equipment or to retain it by taking the cost of maintenance and operation into account.

# Maintenance - Definition

- Maintenance can be defined as a combination of actions carried out to replace, repair, service (or modify) the components in a manufacturing plant so that it will continue to operate at a specified 'availability' for a specified time

# Objectives of Maintenance

- Minimum Breakdown time
- Utilization of optimum capacity
- To keep the life of the equipment
- To ensure highest availability
- To modify the machine tools and other production facilities
- Economy
- Improve Productivity
- Ensure safety

# Reasons for replacement of an equipment

## PHYSICAL IMPAIRMENT

- It refers only to changes in the physical condition of the machine itself.
- This would lead to a decline in the value of the service rendered, increased operating cost, increased maintenance cost or a combination of these.

## OBSOLESCENCE

- Obsolescence is due to improvement of the tools of production.
- It is mainly improvement in technology.

# Functions of Maintenance

- To develop maintenance policies, procedures and standards for the organization

- To schedule planned maintenance work in consultation with the concerned production departments

- To carry out repairs and rectify or overhaul planned equipment and other facilities to ensure good availability and optimum efficiency

- To ensure scheduled inspection and lubrication of plant, machinery and equipment

- To maintain and carry out repairs of buildings, utilities, material handling equipments and other service facilities such as yards, central stores, roadways etc.

# Functions of Maintenance

- To keep record of all maintenance work
- To carryout periodic inspections of equipment and facilities to know conditions which may lead to their breakdown and stoppage
- To prepare inventory of spares and other materials needed for maintenance work
- To carry out alterations, modifications or improvements in existing equipment to minimize breakdown.
- To prepare maintenance budget and to ensure that maintenance expenditure does not exceed the planned budget

# Functions of Maintenance

- To recruit and train personnel to carry out maintenance work effectively and efficiently
- To enforce safety standards as required for use of certain categories of equipment's such as pressure vessels, chemical plants, etc
- To handle disposition of scraps or surplus materials
- Good house keeping
- To develop proper management information system, to provide information to top management about the maintenance activities

# Types of Maintenance

- Corrective or breakdown maintenance
- Preventive Maintenance

# Corrective or Breakdown Maintenance

- Repairs are made after the equipment fails to perform its normal function
  - Failure to replace worn out parts
  - Lack of lubrication
  - Neglected cooling system
  - External factors such as too low or too high line voltage, wrong fuel, etc

# Disadvantages of Corrective or Breakdown Maintenance

- Leads to disruption of production plans
- Impossible to plan work load and distribution of maintenance work for balanced attention of all equipment's
- Increases overtime practice and involves prolonged down time due to non availability of requisite manpower and spares.
- Lead to considerable reduction of output.
- Becomes difficult to maintain the quality of the products
- Spoilage of materials is incrased due to production of more defective parts.
- Increased chances of accidents and less safety to both

# Preventive Maintenance

- Consists of routine actions to be taken in a planned manner to prevent break down and to ensure operational efficiency to the extent it is economically and practicably possible.

- Have the data showing the frequency with which machines have maintenance free performance for a given number of operating hours.

# Advantages of Preventive maintenance

- Reduces the down time during repairs

- Breakdown in minimized and machines run at a higher level of efficiency.

- Pre-determination of date of commencement of work ensures to plan the work load and distribution of maintenance work force for balanced attention of all equipment

# Preventive Maintenance

- **Elements of Preventive Maintenance**
  - Inspection
  - Overhauls
  - Lubrication
  - Planning and scheduling
  - Record and analysis
  - Training of maintenance staff
  - Storage of spare parts.

# Inadequate existing facilities - Alternatives

1. Replacement of the existing equipment with a new one.

2. Augmenting the existing one with an additional equipment.

# Maintenance Cost - Explanation

- Preventive maintenance will reduce expediting cost and downtime cost up to a point.
- Beyond that point, the cost of preventive maintenance will be more when compared to the breakdown maintenance cost.
- The total cost, which is the sum of the preventive maintenance cost and the breakdown maintenance cost, will go on decreasing with an increase in the level of maintenance up to a point.

The level of Maintenance corresponding to the minimum total cost is the **optimal level of maintenance.**

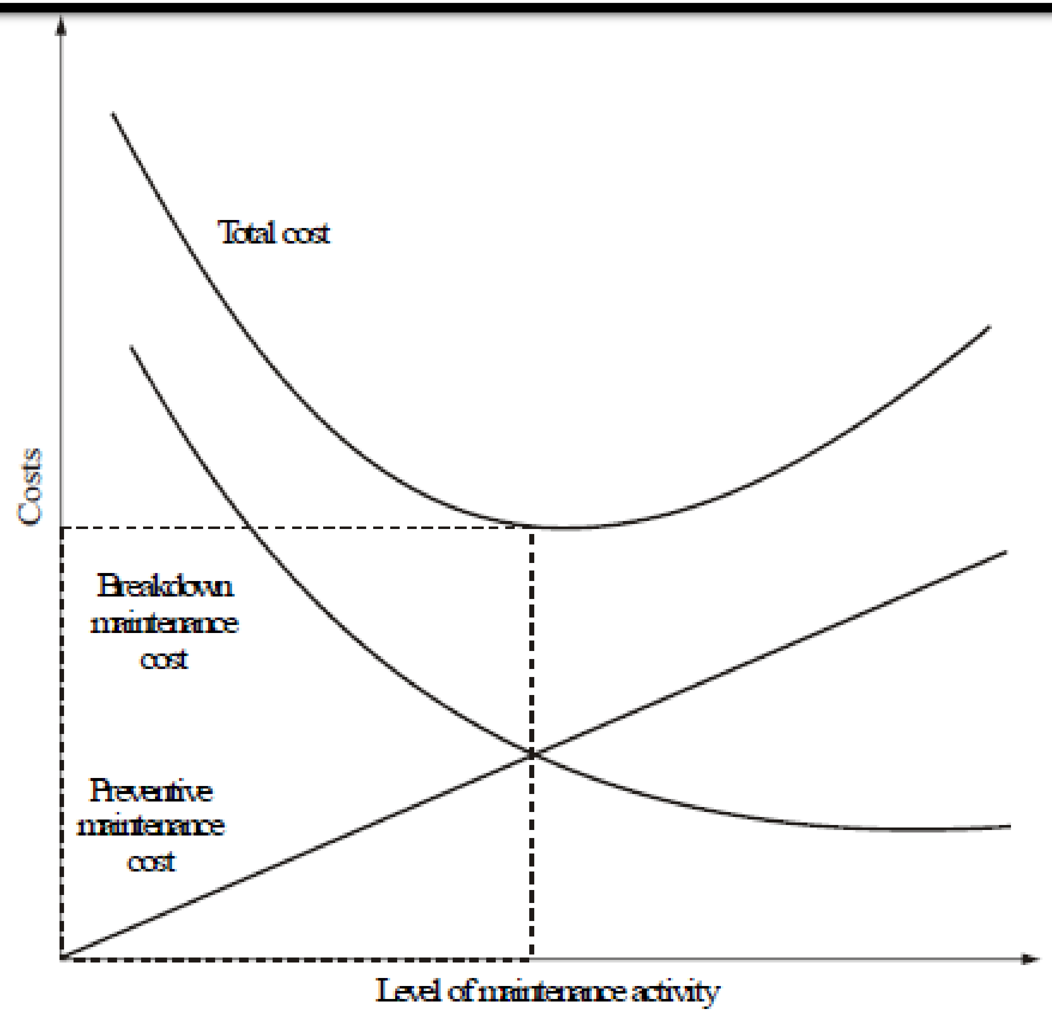



Fig. 8.1 Maintenance costs.

# Types of Replacement problems

- Replacement study can be classified into two categories:

**(a)**Replacement of assets that deteriorate with time (Replacement due to gradual failure or wear and tear of the components of the machines).

**(i)** Determination of economic life of an asset.

**(ii)**Replacement of an existing asset with a new asset.

**(b)**Simple probabilistic model for assets which fail

# Determination of Economic Life of an Asset

- Any asset will have the following cost components:

1. Capital recovery cost (average first cost), computed from the first cost (purchase price) of the machine.

2. Average operating and maintenance cost (O & M cost)

3. Total cost which is the sum of capital recovery cost (average first cost) and average maintenance cost.

# Chart showing Economic life of an Asset

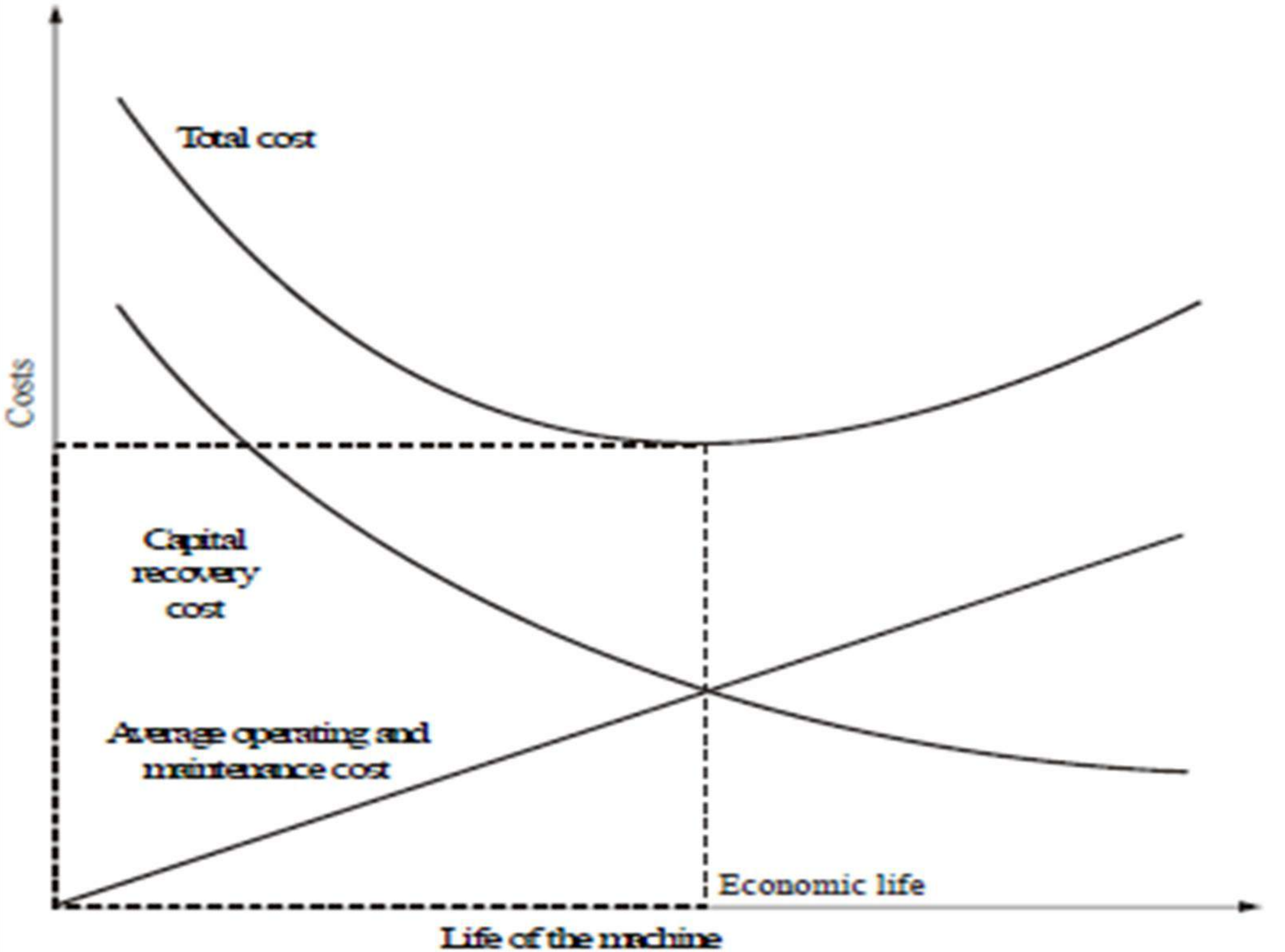


Fig. 8.2 Chart showing economic life.

# Problem 1

- A firm is considering replacement of an equipment, whose first cost is Rs. 4,000 and the scrap value is negligible at the end of any year. Based on experience, it was found that the maintenance cost is zero during the first year and it increases by Rs. 200 every year thereafter.

(a) When should the equipment be replaced if i= 0%?

(b) When should the equipment be replaced if i = 12%?

# Solution: Case (a)

Table 8.1 Calculations to Determine Economic Life (First cost = Rs. 4,000, Interest = 0 %)

| *End of year (n)* | *Maintenance cost at end of year* | *Summation of maintenance costs* | *Average cost of maintenance through year given* | *Average first cost if replaced at year end given* | *Average total cost through year given* |
|---|---|---|---|---|---|
| | | ΣB | C/A | 4,000/A | D + E |
| A | B (Rs.) | C (Rs.) | D (Rs.) | E (Rs.) | F (Rs.) |
| 1 | 0 | 0 | 0 | 4,000.00 | 4,000.00 |
| 2 | 200 | 200 | 100 | 2,000.00 | 2,100.00 |
| 3 | 400 | 600 | 200 | 1,333.33 | 1,533.33 |
| 4 | 600 | 1,200 | 300 | 1,000.00 | 1,300.00 |
| 5 | 800 | 2,000 | 400 | 800.00 | 1,200.00 |
| 6 | 1,000 | 3,000 | 500 | 666.67 | 1,166.67* |
| 7 | 1,200 | 4,200 | 600 | 571.43 | 1,171.43 |

*Economic life of the machine = 6 years

# Solution: Case (b)

**Table 8.2** Calculations to Determine Economic Life (First cost = Rs. 4,000, Interest = 12%)

| *End of year (n)* | *Maintenance cost at end of year* | *P/F, 12%, n* | *Present worth as of beginning of year 1 of maintenance costs* | *Summation of present worth of maintenance costs through year given* | *Present worth of cumulative maintenance cost & first cost* | *A/P, 12%, n* | *Annual equivalent total cost through year given* |
|---|---|---|---|---|---|---|---|
| | | | (B × C) | ΣD | E + Rs. 4,000 | | F × G |
| A | B (Rs) | C | D (Rs.) | E (Rs.) | F (Rs.) | G | H (Rs.) |
| 1 | 0 | 0.8929 | 0.00 | 0.00 | 4,000.00 | 1.1200 | 4,480.00 |
| 2 | 200 | 0.7972 | 159.44 | 159.44 | 4,159.44 | 0.5917 | 2,461.14 |
| 3 | 400 | 0.7118 | 284.72 | 444.16 | 4,444.16 | 0.4163 | 1,850.10 |
| 4 | 600 | 0.6355 | 381.30 | 825.46 | 4,825.46 | 0.3292 | 1,588.54 |
| 5 | 800 | 0.5674 | 453.92 | 1,279.38 | 5,279.38 | 0.2774 | 1,464.50 |
| 6 | 1,000 | 0.5066 | 506.60 | 1,785.98 | 5,785.98 | 0.2432 | 1,407.15 |
| 7 | 1,200 | 0.4524 | 542.88 | 2,328.86 | 6,328.86 | 0.2191 | 1,386.65* |
| 8 | 1,400 | 0.4039 | 565.46 | 2,894.32 | 6,894.32 | 0.2013 | 1,387.83 |
| 9 | 1,600 | 0.3606 | 576.96 | 3,471.28 | 7,471.28 | 0.1877 | 1,402.36 |
| 10 | 1,800 | 0.3220 | 579.60 | 4,050.88 | 8,050.88 | 0.1770 | 1,425.00 |

*Economic life of the machine = 7 years

# Problem 2

- The following table gives the operation cost, maintenance cost and salvage value at the end of every year of a machine whose purchase value is Rs. 20,000. Find the economic life of the machine assuming interest rate, i = 15%.

- **Given :** Purchase Value, Operation, Maintenance and Salvage Values of a Machine

- **To Find:** Economic Life @ i=15%

# Problem 2 – Operation, Maintenance and Salvage value

| *End of year* (*n*) | *Operation cost at the end of year (Rs.)* | *Maintenance cost at the end of year (Rs.)* | *Salvage value at the end of year (Rs.)* |
|---|---|---|---|
| 1 | 3,000 | 300 | 9,000 |
| 2 | 4,000 | 400 | 8,000 |
| 3 | 5,000 | 500 | 7,000 |
| 4 | 6,000 | 600 | 6,000 |
| 5 | 7,000 | 700 | 5,000 |
| 6 | 8,000 | 800 | 4,000 |
| 7 | 9,000 | 900 | 3,000 |
| 8 | 10,000 | 1,000 | 2,000 |
| 9 | 11,000 | 1,100 | 1,000 |
| 10 | 12,000 | 1,200 | 0 |

# Solution

Table 8.3 Calculations to Determine Economic Life

(First Cost = Rs. 20,000, Interest Rate = 15%)

| *End of year* (*n*) | *Operation cost at the end of year* | *Maintenance cost at the end of year* | *Sum of operation and maintenance costs at the end of year* | *P/F, 15%, n* | *Present worth as of beginning of year 1 of sum of operation & maintenance cost* | *Cumulative sum of column F through year designated* | *Salvage value at the end of year* | *Present worth as of beginning of year 1 of salvage value* | *Total present worth* | *A/P, 15%, n* | *Annual equlant. total cost through year given* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | B + C | | D × E | ΣF | | H × E | G + 20,000 – I | | J × K |
| A | B (Rs.) | C (Rs.) | D (Rs.) | E | F (Rs.) | G (Rs.) | H (Rs.) | I (Rs.) | J (Rs.) | K | L (Rs.) |
| 1 | 3,000 | 300 | 3,300 | 0.8696 | 2,869.68 | 2,869.68 | 9,000 | 7,826.40 | 15,043.28 | 1.1500 | 17,299.77 |
| 2 | 4,000 | 400 | 4,400 | 0.7562 | 3,326.84 | 6,196.52 | 8,000 | 6,048.80 | 20,147.72 | 0.6151 | 12,392.86 |
| 3 | 5,000 | 500 | 5,500 | 0.6575 | 3,616.25 | 9,812.77 | 7,000 | 4,602.50 | 25,210.27 | 0.4380 | 11,042.01 |
| 4 | 6,000 | 600 | 6,600 | 0.5718 | 3,773.88 | 13,586.65 | 6,000 | 3,430.80 | 30,155.85 | 0.3503 | 10,563.59 |
| 5 | 7,000 | 700 | 7,700 | 0.4972 | 3,828.44 | 17,415.09 | 5,000 | 2,486.00 | 34,929.09 | 0.2983 | 10,419.35 * |
| 6 | 8,000 | 800 | 8,800 | 0.4323 | 3,804.24 | 21,219.33 | 4,000 | 1,729.20 | 39,490.13 | 0.2642 | 10,433.29 |
| 7 | 9,000 | 900 | 9,900 | 0.3759 | 3,721.41 | 24,940.74 | 3,000 | 1,127.70 | 43,813.04 | 0.2404 | 10,532.66 |

*Economic Life = 5 years

# Solution

(First Cost = Rs. 20,000, Interest Rate = 15%)

| End of year (n) | Operation cost at the end of year | Maintenance cost at the end of year | Sum of operation and maintenance costs at the end of year | P/F, 15%, n | Present worth as of beginning of year 1 of sum of operation & maintenance cost | Cumulative sum of column F through year designated | Salvage value at the end of year | Present worth as of beginning of year 1 of salvage value | Total present worth | A/P, 15%, n | Annual equivalent total cost through year given |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | B + C | | D × E | ΣF | | H × E | G + 20,000 − I | | J × K |
| A | B (Rs.) | C (Rs.) | D (Rs.) | E | F (Rs.) | G (Rs.) | H (Rs.) | I (Rs.) | J (Rs.) | K | L (Rs.) |
| 1 | 3,000 | 300 | 3,300 | 0.8696 | 2,869.68 | 2,869.68 | 9,000 | 7,826.40 | 15,043.28 | 1.1500 | 17,299.77 |
| 2 | 4,000 | 400 | 4,400 | 0.7562 | 3,326.84 | 6,196.52 | 8,000 | 6,048.80 | 20,147.72 | 0.6151 | 12,392.86 |
| 3 | 5,000 | 500 | 5,500 | 0.6575 | 3,616.25 | 9,812.77 | 7,000 | 4,602.50 | 25,210.27 | 0.4380 | 11,042.01 |
| 4 | 6,000 | 600 | 6,600 | 0.5718 | 3,773.88 | 13,586.65 | 6,000 | 3,430.80 | 30,155.85 | 0.3503 | 10,563.59 |
| 5 | 7,000 | 700 | 7,700 | 0.4972 | 3,828.44 | 17,415.09 | 5,000 | 2,486.00 | 34,929.09 | 0.2983 | 10,419.35 * |
| 6 | 8,000 | 800 | 8,800 | 0.4323 | 3,804.24 | 21,219.33 | 4,000 | 1,729.20 | 39,490.13 | 0.2642 | 10,433.29 |
| 7 | 9,000 | 900 | 9,900 | 0.3759 | 3,721.41 | 24,940.74 | 3,000 | 1,127.70 | 43,813.04 | 0.2404 | 10,532.66 |

*Economic Life = 5 years

# Problem 3

- A company has already identified machine A and determined the economic life as four years by assuming 15% interest rate. The annual equivalent total cost corresponding to the economic life is Rs. 2,780.

- Now, the manufacturer of machine B has approached the company. Machine B, which has the same capacity as that of machine A, is priced at Rs. 6,000. The maintenance cost of machine B is estimated at Rs. 1,500 for the first year and an equal yearly increment of Rs. 300 thereafter. If the money is worth 15% per year, which machine should be purchased? (Assume that the scrap value of each of the machines is negligible at any year.)

- **Given:**

**MACHINE – A:**

Economic Life of 4 years @ i=15%

Annual equivalent total cost = Rs. 2780

**MACHINE – B:**

Price = Rs. 6000.

The maintenance cost of machine B is estimated at Rs. 1,500 for the first year and an equal yearly increment of Rs. 300 thereafter.

- **To Find:**

Which machine should be purchased?

# Solution:

**Table 8.4** Calculations to Determine Economic Life (First Cost = Rs. 6,000, Interest = 15%)

| *End of year (n)* | *Maintenance cost for end of year* | *P/F, 15%, n* | *Present worth as of beginning of year 1 of maintenance costs* | *Summation of present worth of maintenance costs through year given* | *Column E + Rs. 6000* | *A/P, 15%, n* | *Annual equivalent total cost through year given* |
|---|---|---|---|---|---|---|---|
| | | | B × C | Σ D | | | F × G |
| A | B (Rs) | C | D (Rs.) | E (Rs.) | F (Rs.) | G | H (Rs.) |
| 1 | 1,500 | 0.8696 | 1,304.40 | 1,304.40 | 7,304.40 | 1.1500 | 8,400.06 |
| 2 | 1,800 | 0.7561 | 1,360.98 | 2,665.38 | 8,665.38 | 0.6151 | 5,330.08 |
| 3 | 2,100 | 0.6575 | 1,380.75 | 4,046.13 | 10,046.13 | 0.4380 | 4,400.21 |
| 4 | 2,400 | 0.5718 | 1,372.32 | 5,418.45 | 11,418.45 | 0.3503 | 3,999.88 |
| 5 | 2,700 | 0.4972 | 1,342.44 | 6,760.89 | 12,760.89 | 0.2983 | 3,806.57 |
| 6 | 3,000 | 0.4323 | 1,296.90 | 8,057.79 | 14,057.79 | 0.2642 | 3,714.07 |
| 7 | 3,300 | 0.3759 | 1,240.47 | 9,298.26 | 15,298.26 | 0.2404 | 3,677.70 |
| 8 | 3,600 | 0.3269 | 1,176.84 | 10,475.10 | 16,475.10 | 0.2229 | 3,672.30 * |
| 9 | 3,900 | 0.2843 | 1,108.77 | 11,583.87 | 17,583.87 | 0.2096 | 3,685.58 |
| 10 | 4,200 | 0.2472 | 1,038.24 | 12,622.11 | 18,622.11 | 0.1993 | 3,711.39 |

*Economic life of the machine = 8 years

# Result

- In Column H, the minimum annual equivalent total cost occurs when n is equal to 8. Hence the economic life of machine B is 8 years and the corresponding annual equivalent total cost is Rs. 3,672.30.

## RESULT

- Minimum annual equivalent total cost for machine A = Rs. 2,780
- Minimum annual equivalent total cost for machine B = Rs. 3,672.30
- Since the minimum annual equivalent total cost of machine A is less than that of machine B, machine A is selected as the best machine which has the economic life of four years.

## Replacement of Existing Asset with New Asset

- In this analysis, the annual equivalent cost of each alternative should be computed first.

- Then the alternative which has the least cost should be selected as the best alternative.

# Capital Recovery with Return

Consider the following data of a machine. Let

P = purchase price of the machine,

F = salvage value of the machine at the end of machine life,

n = life of the machine in years, and

i = interest rate, compounded annually

- The equation for the annual equivalent amount for the cash flow diagram is

**AE(i) = P * (A/P, i, n) – F * (A/F, i, n)**

- This equation represents the capital recovery with return.

# Cash Flow diagram of a Machine

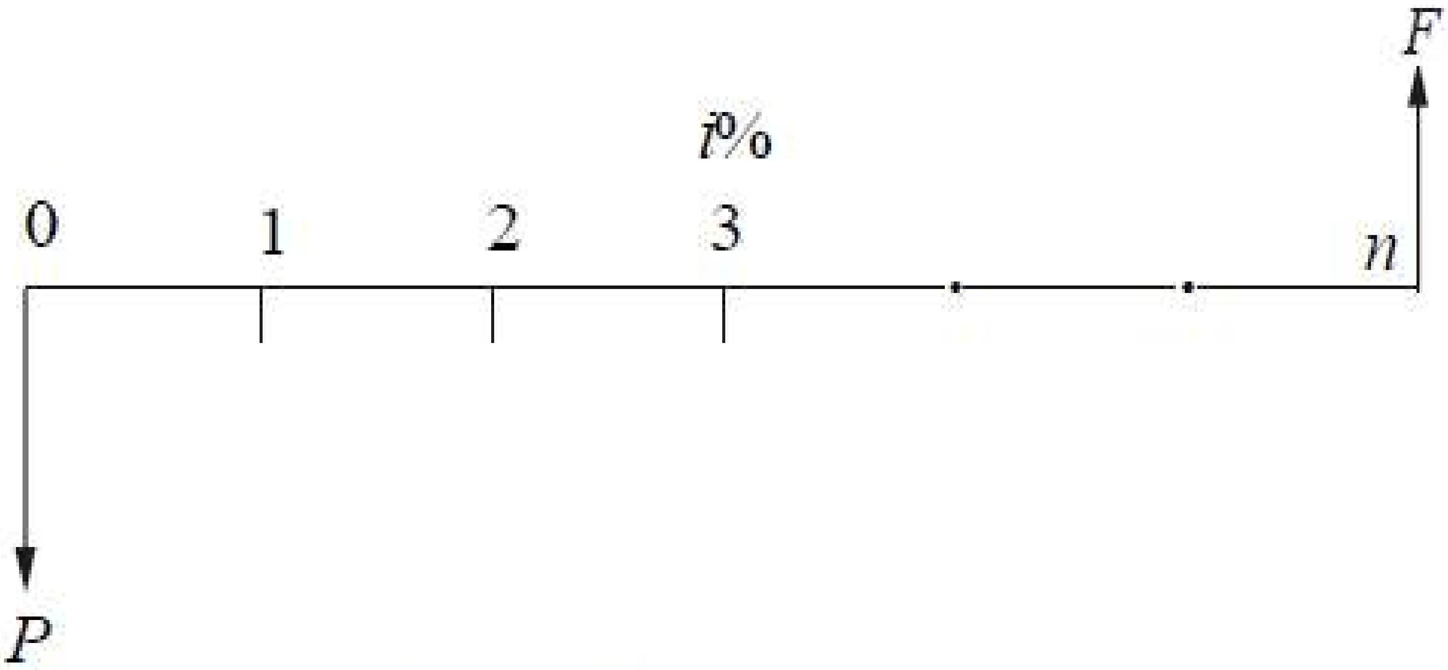



**Fig. 8.3** Cash flow diagram of machine.

# Concept of Challenger and Defender

- If an existing equipment is considered for replacement with a new equipment, then the **existing equipment is known as the defender and the new equipment is known as challenger.**

- Assume that an equipment has been purchased about three years back for Rs. 5,00,000 and it is considered for replacement with a new equipment.

# Problem 4

- Two years ago, a machine was purchased at a cost of Rs. 2,00,000 to be useful for eight years. Its salvage value at the end of its life is Rs. 25,000. The annual maintenance cost is Rs. 25,000. The market value of the present machine is Rs. 1,20,000. Now, a new machine to cater to the need of the present machine is available at Rs. 1,50,000 to be useful for six years. Its annual maintenance cost is Rs. 14,000. The salvage value of the new machine is Rs. 20,000. Using an interest rate of 12%, find whether it is worth replacing the present machine with the new machine.

# Alternative 1: Present Machine

**Given:**

Purchase price = Rs. 2,00,000

- Present value (P) = Rs. 1,20,000
- Salvage value (F) = Rs. 25,000
- Annual maintenance cost (A) = Rs. 25,000
- Remaining life = 6 years
- Interest rate = 12%

**To Find:**

Whether it is worth replacing the present machine with the new machine.

# Solution – Cash Flow Diagram for Alternative 1

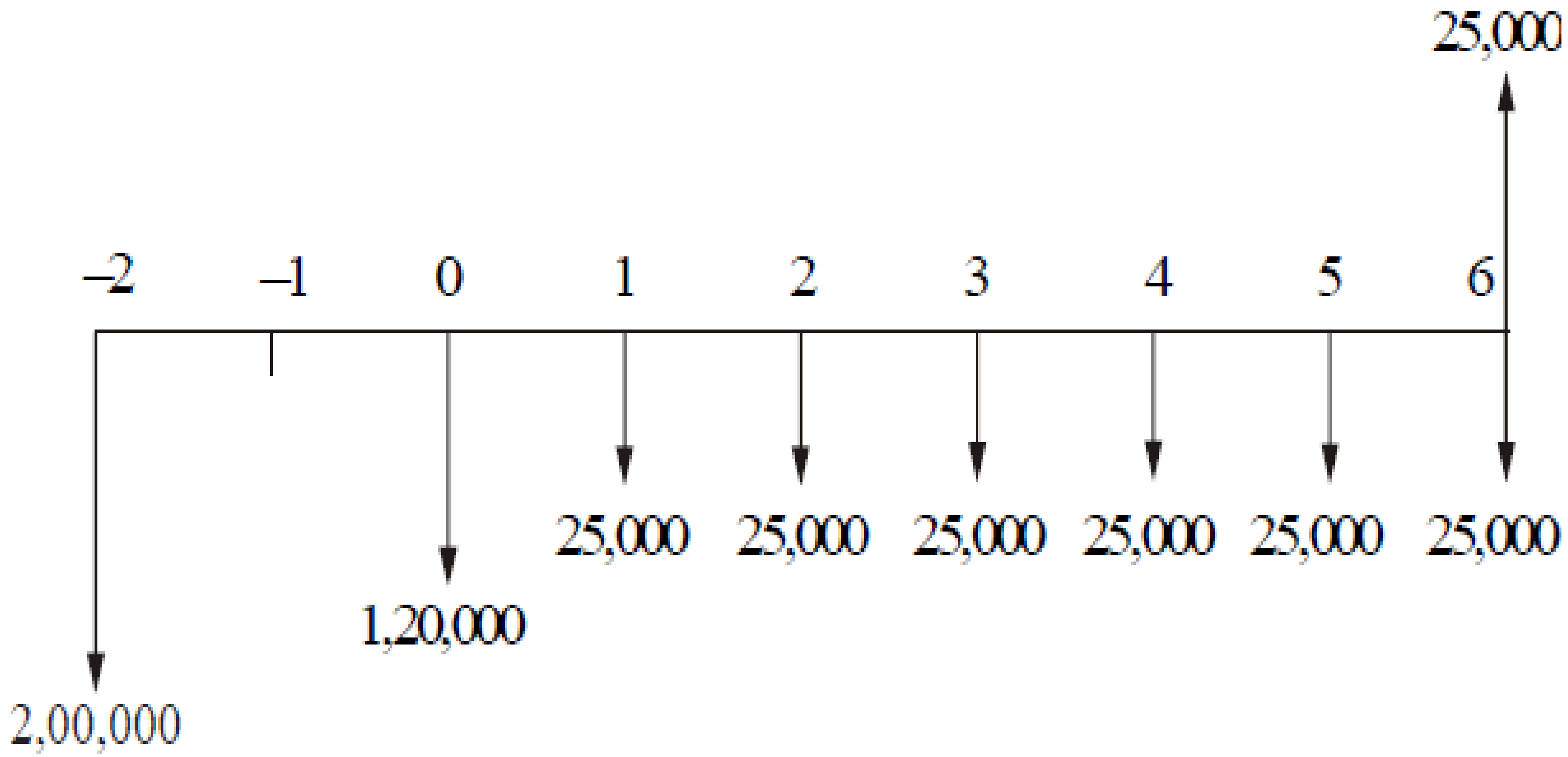



**Fig. 8.4** Cash flow diagram for alternative 1.

# Soln : Alternative 1(Present Machine)

- The annual maintenance cost for the preceding periods are not shown in this figure.

- The annual equivalent cost is computed as

**AE(12%) = P * (A/P, 12%, 6) – F * (A/F, 12%, 6) + A**

= (120000 )(0.2432) – 25000 x 0.1232 + 25,000

**AE(12%) = Rs. 51,104**

# Alternative 2: New Machine

**Given:**

- Purchase price (P) = Rs. 1,50,000
- Salvage value (F) = Rs. 20,000
- Annual maintenance cost (A) = Rs. 14,000
- Life = 6 years
- Interest rate = 12%

# Solution – Cash Flow Diagram for Alternative 2

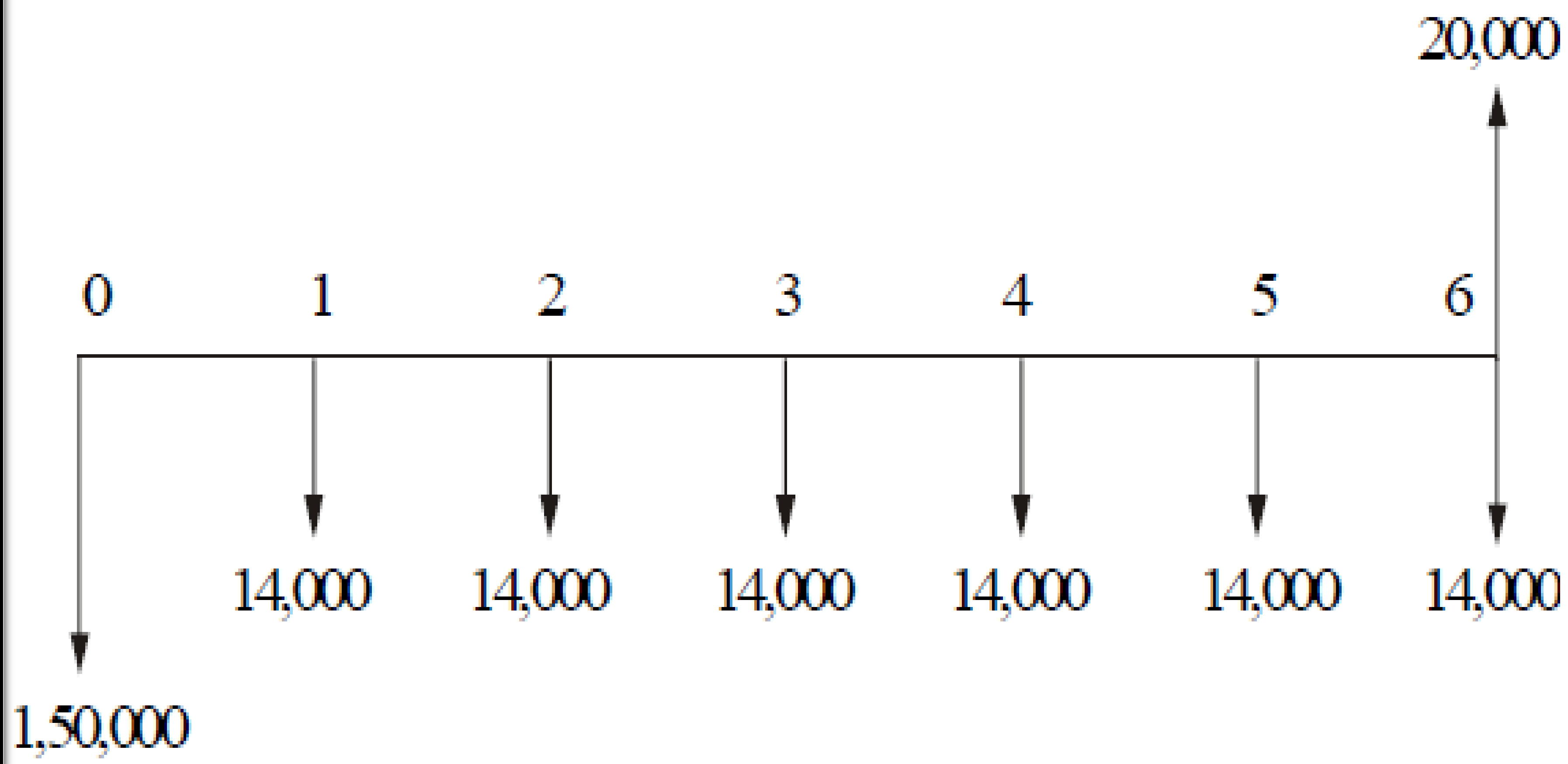



**Fig. 8.5** Cash flow diagram for alternative 2.

# Soln : Alternative 2 (New Machine)

- The formula for the annual equivalent cost is

**AE(12%) = P x (A/P, 12%, 6) – F x (A/F, 12%, 6) + A**

= (1,50,000 )(0.2432) - 20,000 x 0.1232 + 14,000

**AE(12%) = Rs. 48,016**

**RESULT:**

- Since the annual equivalent cost of the new machine is less than that of the present machine, it is suggested that the present machine be replaced with the new machine.

# Problem 5

- A diesel engine was installed 10 years ago at a cost of Rs. 50,000. It has a present realizable market value of Rs. 15,000. If kept, it can be expected to last five years more, with operating and maintenance cost of Rs. 14,000 per year and to have a salvage value of Rs. 8,000 at the end of the fifth year. This engine can be replaced with an improved version costing Rs. 65,000 which has an expected life of 20 years. This improved version will have an estimated annual operating and maintenance cost of Rs. 9,000 and ultimate salvage value of Rs. 13,000. Using an interest rate of 15%, make an annual equivalent cost analysis to determine

# Alternative 1 – Old Diesel Engine

**Given:**

- Purchase price = Rs. 50,000
- Present value (P) = Rs. 15,000
- Salvage value (F) = Rs. 8,000
- Annual operating and maintenance cost (A) = Rs. 14,000
- Remaining life (n) = 5 years
- Interest rate = 15%

**To Find**:

Whether to keep or replace the old engine.

# Cash Flow Diagram for Old Diesel Engine

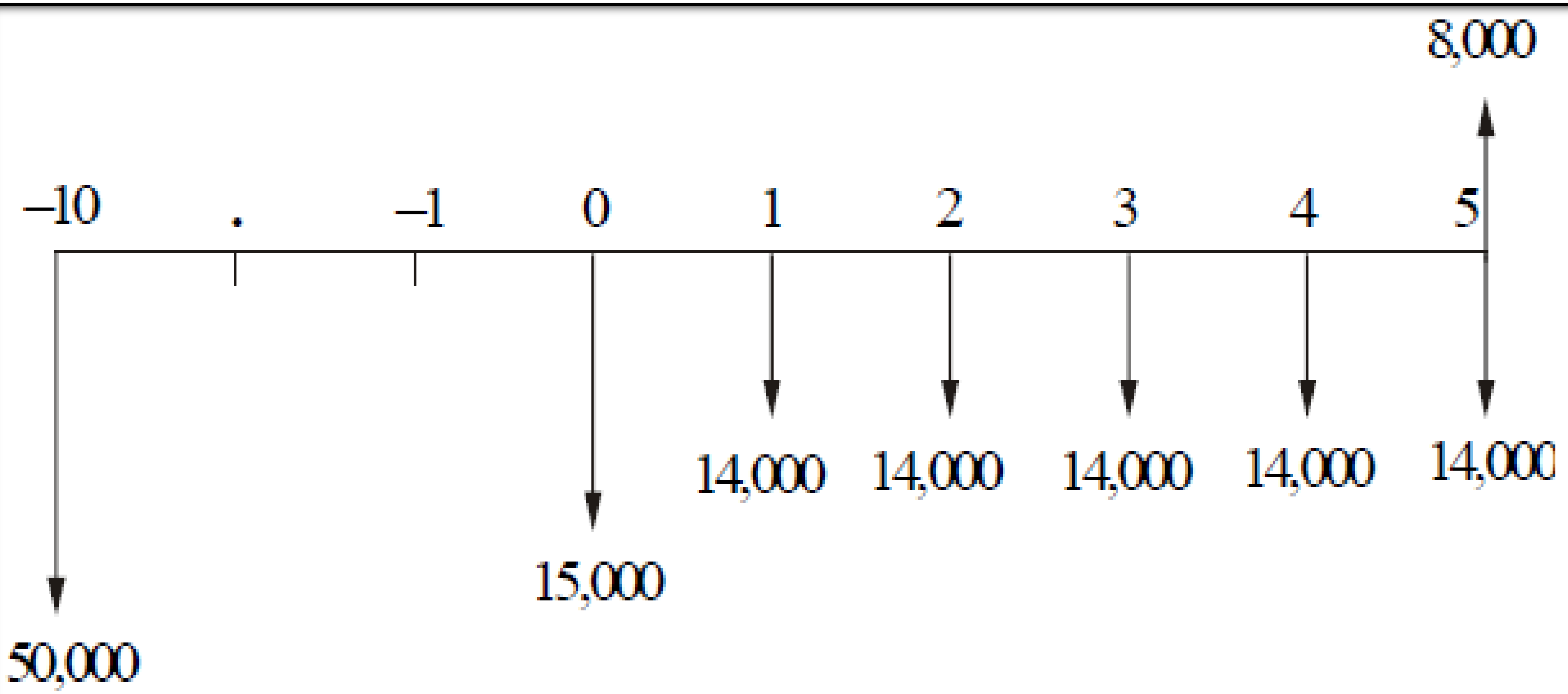



**Fig. 8.6** Cash flow diagram for alternative 1.

# Soln : Alternative 1(Old Diesel Engine)

- The formula for the annual equivalent cost is

- **AE(15%) = P x (A/P, 15%, 5) – F x (A/F, 15%, 5) + A**

= (15,000 x 0.2983) - 8,000 x 0.1483 + 14,000

**AE(15%) = Rs. 17,288.10**

# Alternative 2 – New Diesel Engine

**Given:**

- Present value (P) = Rs. 65,000
- Salvage value (F) = Rs. 13,000
- Annual operating and maintenance cost (A) = Rs. 9,000
- Life (n) = 20 years
- Interest rate = 15%

# Cash Flow Diagram for New Diesel Engine

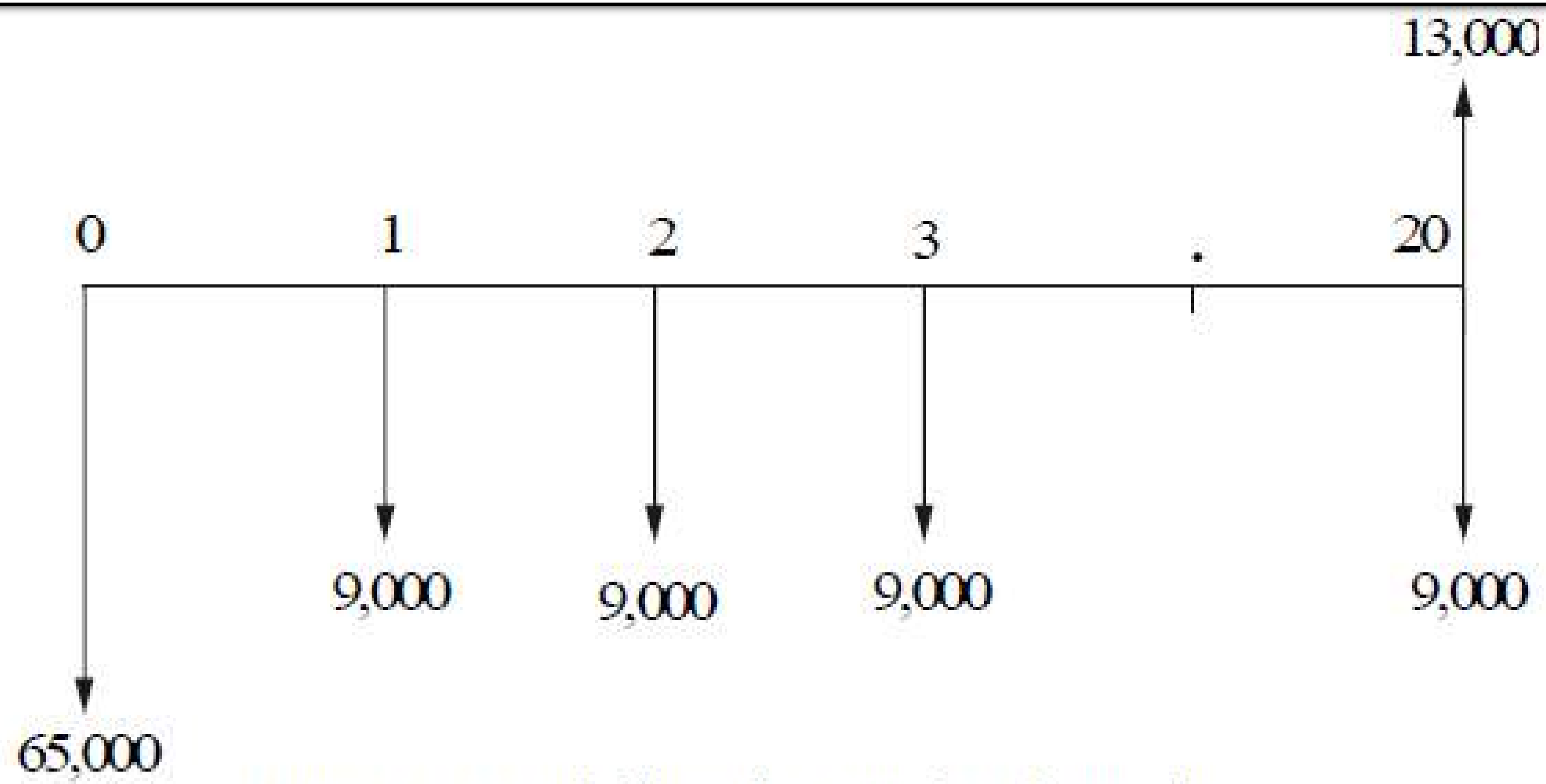



**Fig. 8.7** Cash flow diagram for alternative 2.

# Soln : Alternative 2(New Diesel Engine)

- **AE(15%) = P x (A/P, 15%, 20) – F x (A/F, 15%, 20) + A**

  = (65,000)(0.1598) - 13,000 x 0.0098 + 9,000

  **AE(15%) = Rs. 19,259.60**

- For comparing the engines based on equal lives (20 years), the annual equivalent figures are given in following fig.

- Equal lives are nothing but the least common multiple of the lives of the alternatives.

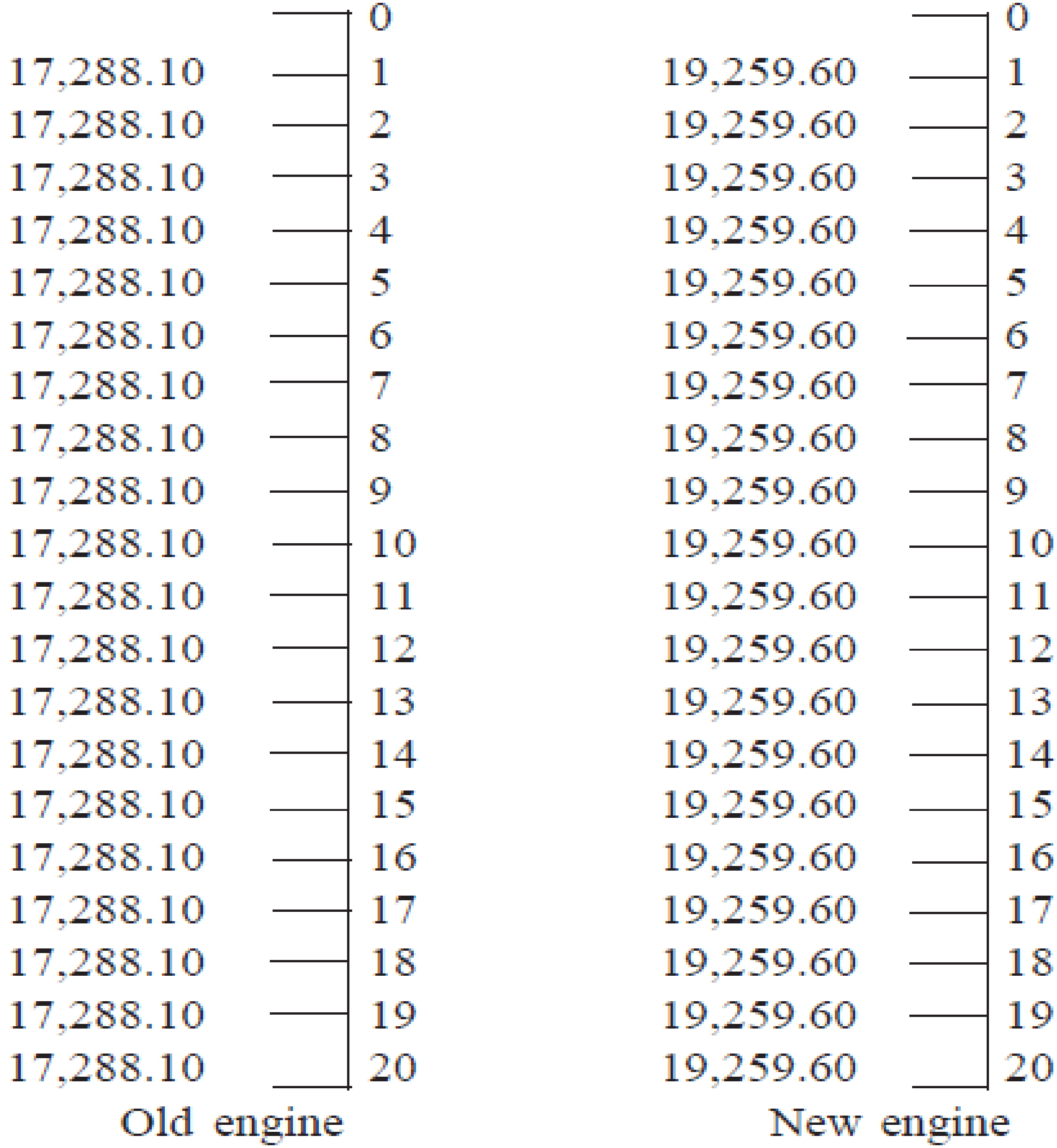



**Fig. 8.8** Cash flow diagram of alternatives based on common lives.

# Result

- Since the annual equivalent cost of the old diesel engine is less than that of the new diesel engine, it is suggested to keep the old diesel engine.

- Here, an important assumption is that the old engine will be replaced four times during the 20 years period of comparison.

# Problem 6

- A steel highway bridge must either be reinforced or replaced. Reinforcement would cost Rs. 6,60,000 and would make the bridge fit for an additional five years of service. If it is reinforced, it is estimated that its net salvage value would be Rs. 4,00,000 at the time it is retired from service. The new pre-stressed concrete bridge would cost Rs. 15,00,000 and would meet the foreseeable requirements of the next 40 years. Such a bridge would have no salvage value. It is estimated that the annual maintenance cost of the reinforced bridge would exceed that of the concrete bridge by Rs. 96,000. If the bridge is replaced by a new pre-stressed concrete bridge, the scrap value of the steel would exceed the demolition cost by Rs. 4,20,000. Assume that the money

# Alternative 1-Reinforce the Existing Bridge

**Given:**

- Cost of reinforcement (P) = Rs. 6,60,000
- Salvage value after 5 years (F) = Rs. 4,00,000
- The excess annual maintenance cost over pre-stressed concrete bridge (A)= Rs. 96,000
- Life (n) = 5 years
- Interest rate (i) = 10%

# Cash Flow Diagram for Alternative 1

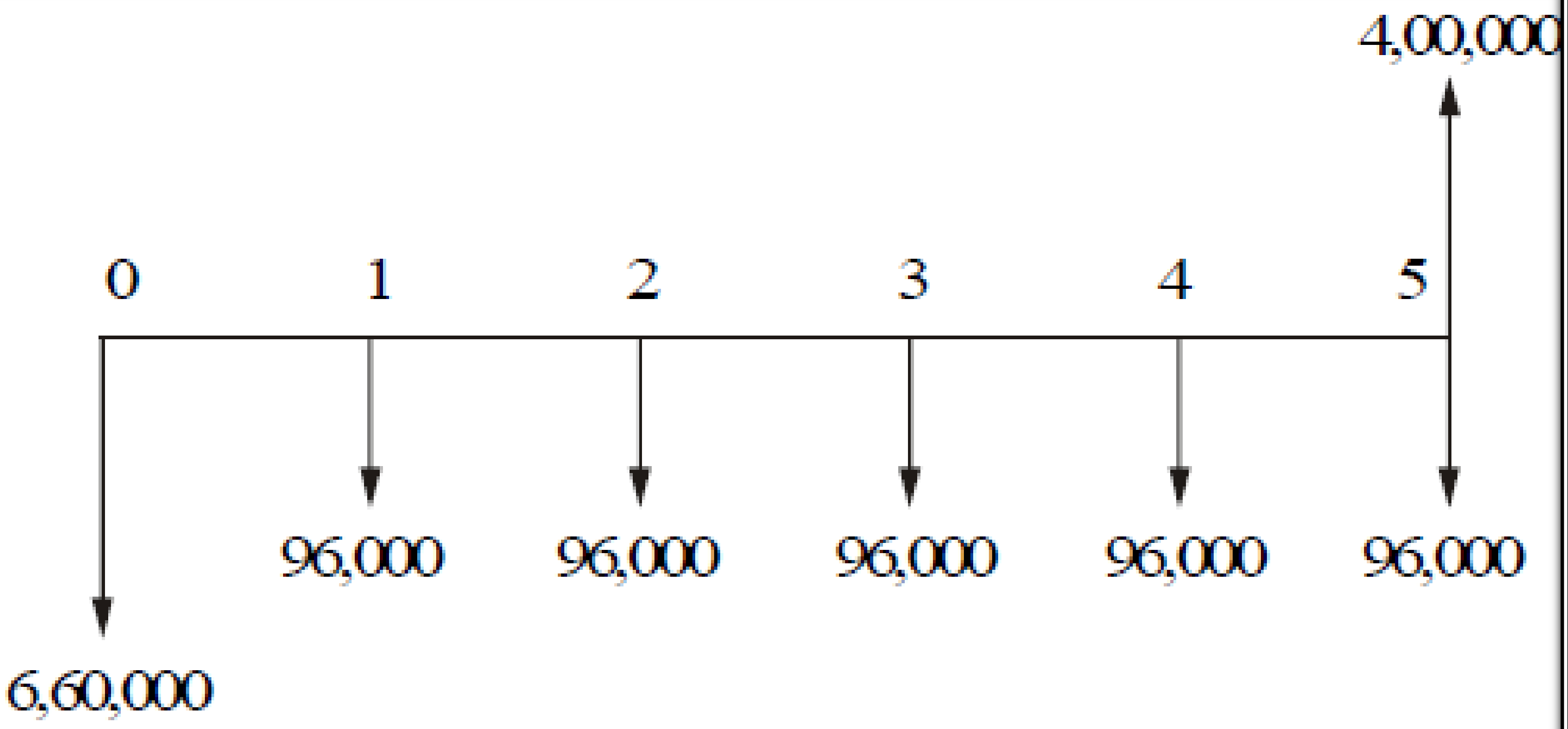



**Fig. 8.9** Cash flow diagram for alternative 1.

# Soln : Alternative 1

**AE(10%) = P x (A/P, 10%, 5) – F x (A/F, 10%, 5) + A**

= (6,60,000 x 0.2638 + – 4,00,000 x 0.1638) + 96,000

AE(10%) = Rs. 2,04,588

# Alternative 2-Replace the Existing Bridge by a new pre-stressed concrete bridge

**Given:**

- Cost of pre-stressed concrete bridge (P)

  = Rs. 15,00,000

- Excess scrap value of steel over the demolition cost of the current bridge

  (X) = Rs. 4,20,000

- Life (n) = 40 years
- Interest rate (i) = 10%

# Cash Flow Diagram for alternative 2

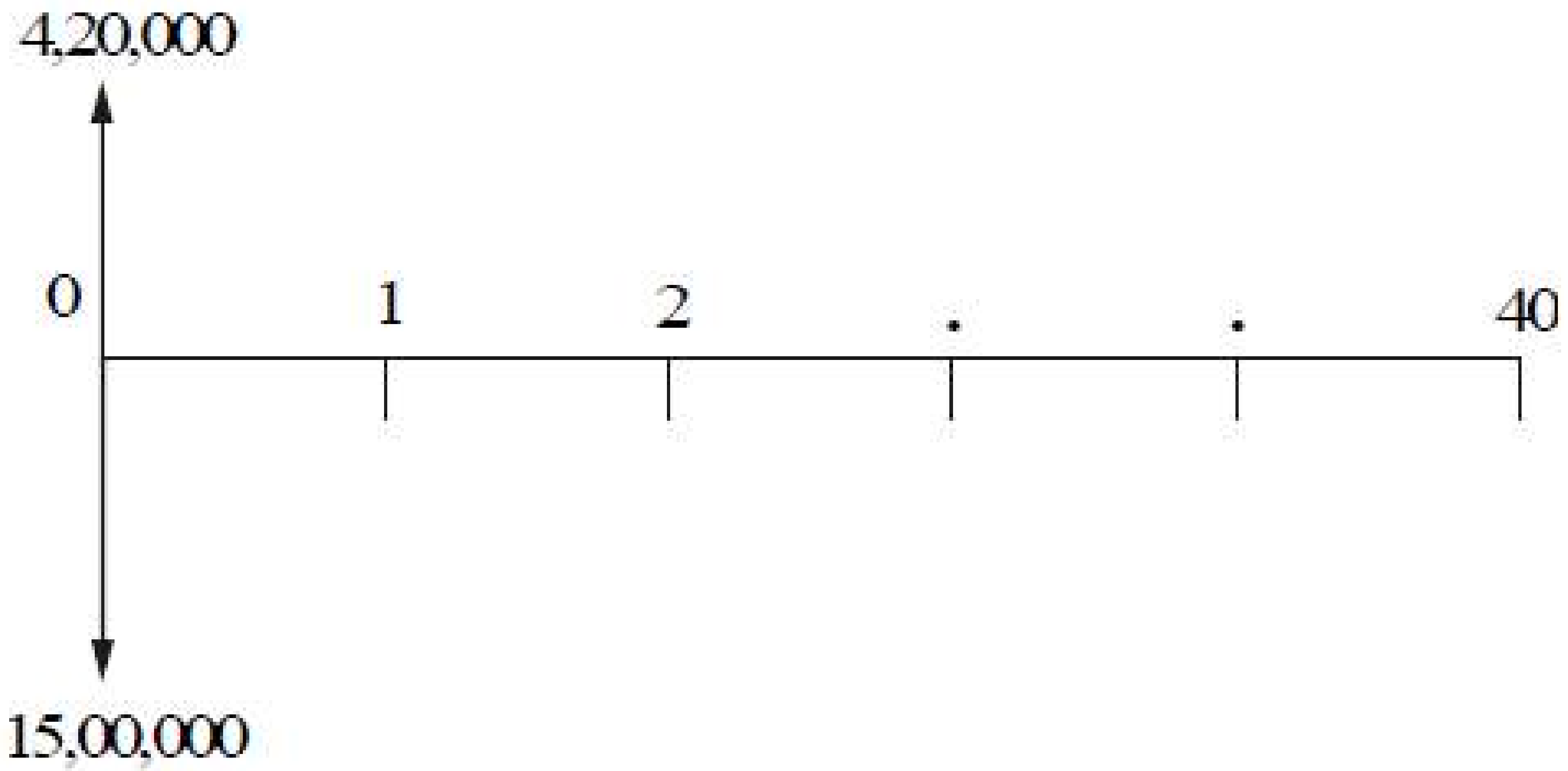



Fig. 8.10 Cash flow diagram for alternative 2.

# Soln : Alternative 2

**AE(10%) = (P – X) (A/P, 10%, 40)**

= (15,00,000 – 4,20,000)  0.1023

**AE(10%)= Rs. 1,10,484**

- The annual equivalent cost of alternative 2 is less than that of alternative 1.
- Based on equal lives comparison over 40 years, alternative 2 is selected as the best alternative.
- Thus, it is suggested to go in for pre-stressed concrete bridge.

# Problem 7

- Three years back, a municipality purchased a 10 hp motor for pumping drinking water. Its useful life was estimated to be 10 years. Due to the fast development of that locality, the municipality is unable to meet the current demand for water with the existing motor. The municipality can cope with the situation either by augmenting an additional 5 hp motor or replacing the existing10 hp motor with a new 15 hp motor. The details of these motors are now tabulated. The current market value of the 10 hp motor is Rs. 10,000. Using an interest rate of 15%, find the best alternative.

# Details of the Motor

| | *Old 10 hp motor* | *New 5 hp motor* | *New 15 hp motor* |
|---|---|---|---|
| Purchase cost ($P$) Rs. | 25,000 | 10,000 | 35,000 |
| Life in years ($n$) | 10 | 7 | 7 |
| Salvage value at the end of machine life (Rs.) | 1,500 | 800 | 4,000 |
| Annual operating & maintenance cost (Rs.) | 1,600 | 1,000 | 500 |

The current market value of the 10 hp motor is Rs. 10,000. Using an interest rate of 15%, find the best alternative.

# Soln : Alternative 1—Augmenting the present 10 hp motor with an additional 5 hp motor

- Total annual equivalent cost = Annual equivalent cost of 10 hp motor + Annual equivalent cost of 5 hp motor

**Calculation of annual equivalent cost of 10 hp Motor:**

- Present market value of the 10 hp motor (P) = Rs. 10,000
- Remaining life (n) = 7 years
- Salvage value at the end of motor life (F) = Rs. 1,500
- Annual operation and maintenance cost (A) = Rs. 1,600 and Interest rate, i = 15%

# Cash Flow Diagram for 10hp Motor

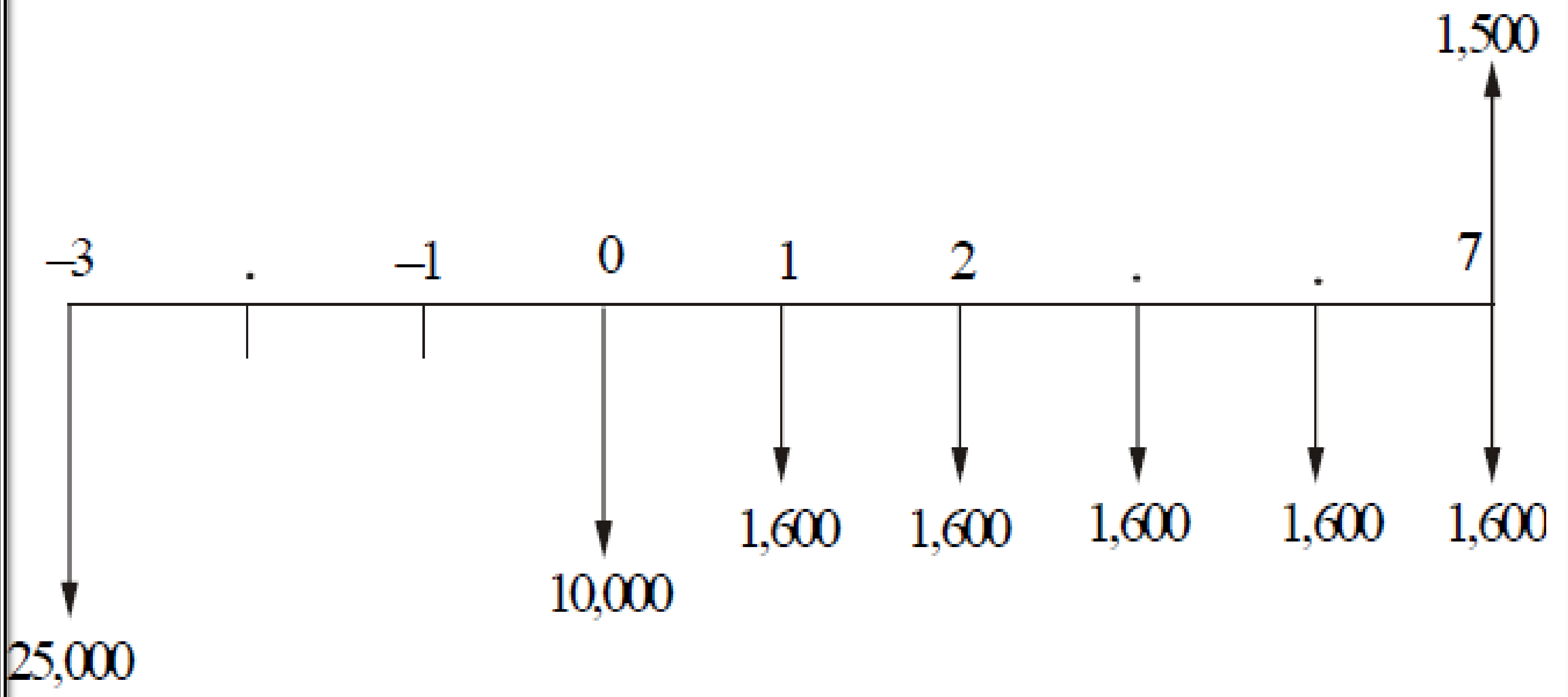



**Fig. 8.11** Cash flow diagram for 10 hp motor.

# Annual Equiv. Cost of 10hp motor

- The annual equivalent cost of the 10 hp motor is calculated as

**AE(15%) = P x (A/P, 15%, 7)– F x (A/F, 15%, 7) + A**

= 10000 x 0.2404 –1500 x 0.0904 + 1600

**AE(15%) = Rs. 3,868.40**

# Calculation of annual equivalent cost of 5 hp Motor

- Purchase value of the 5 hp motor (P) = Rs. 10,000
- Life (n) = 7 years
- Salvage value at the end of motor life (F) = Rs. 800
- Annual operation and maintenance cost (A) = Rs. 1,000
- Interest rate, i = 15%

# Cash flow diagram of 5hp Motor

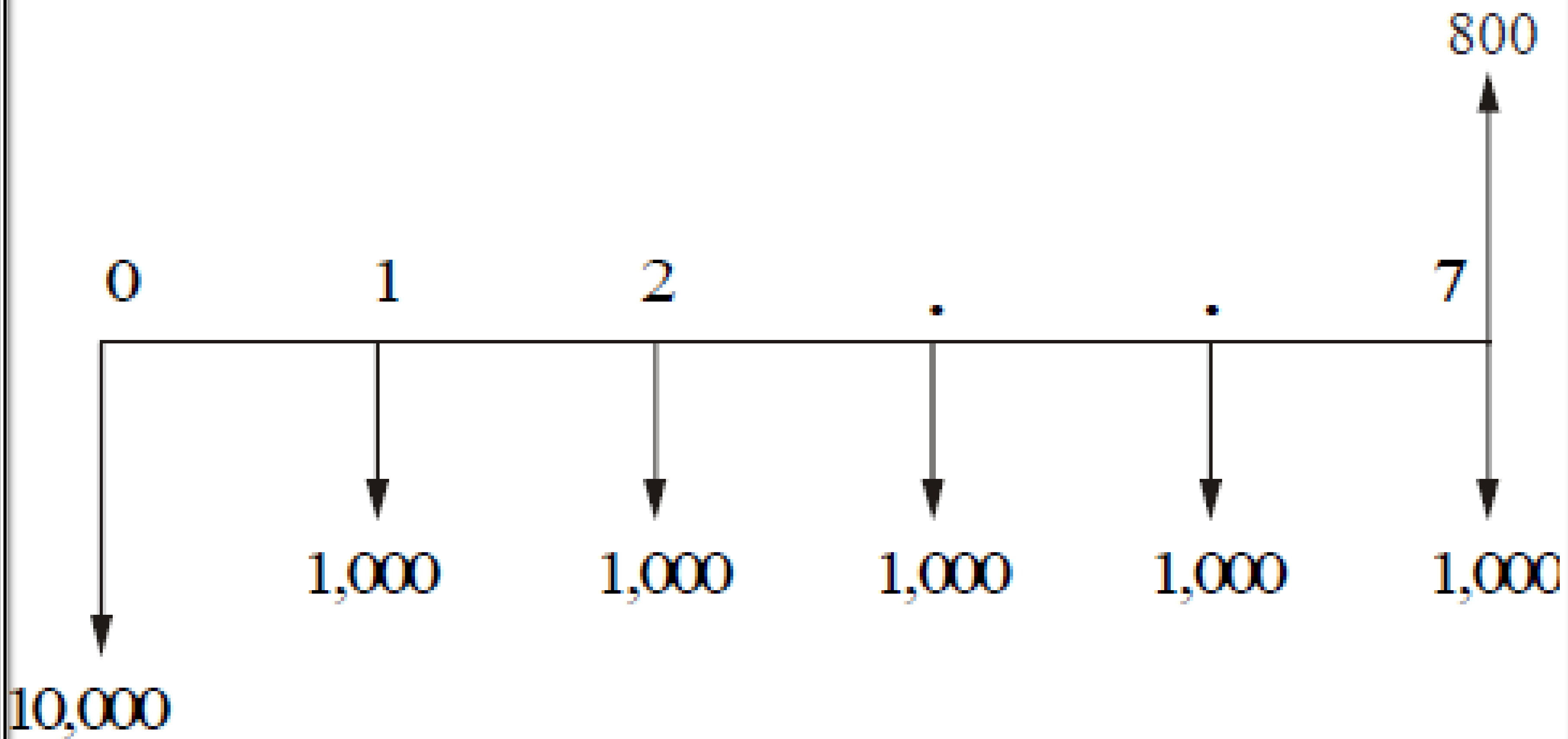



**Fig. 8.12** Cash flow diagram for 5 hp motor.

# Total Annual Equivalent Cost for Alternative 1

- The annual equivalent cost of the 5 hp motor is computed as

**AE(15%) = P x (A/P, 15%, 7) – F x (A/F, 15%, 7) + A**

= 10000 x 0.2404 -800 x 0.0904 + 1000

**AE(15%) = Rs. 3,331.68**

**Total annual equivalent cost:**

Total annual equivalent cost of the alternative1

= Rs. 3,868.40+ Rs. 3,331.68

**Total Equiv. cost of alter.1 = Rs. 7,200.08**

# Soln : Alternative 2—Replacing the present 10 hp motor with a new 15 hp motor

- Purchase value of the 15 hp motor (P) = Rs. 35,000
- Life (n) = 7 years
- Salvage value at the end of motor life (F) = Rs. 4,000
- Annual operation and maintenance cost (A) = Rs. 500
- Interest rate, i = 15%

# Cash flow diagram for Alternative 2

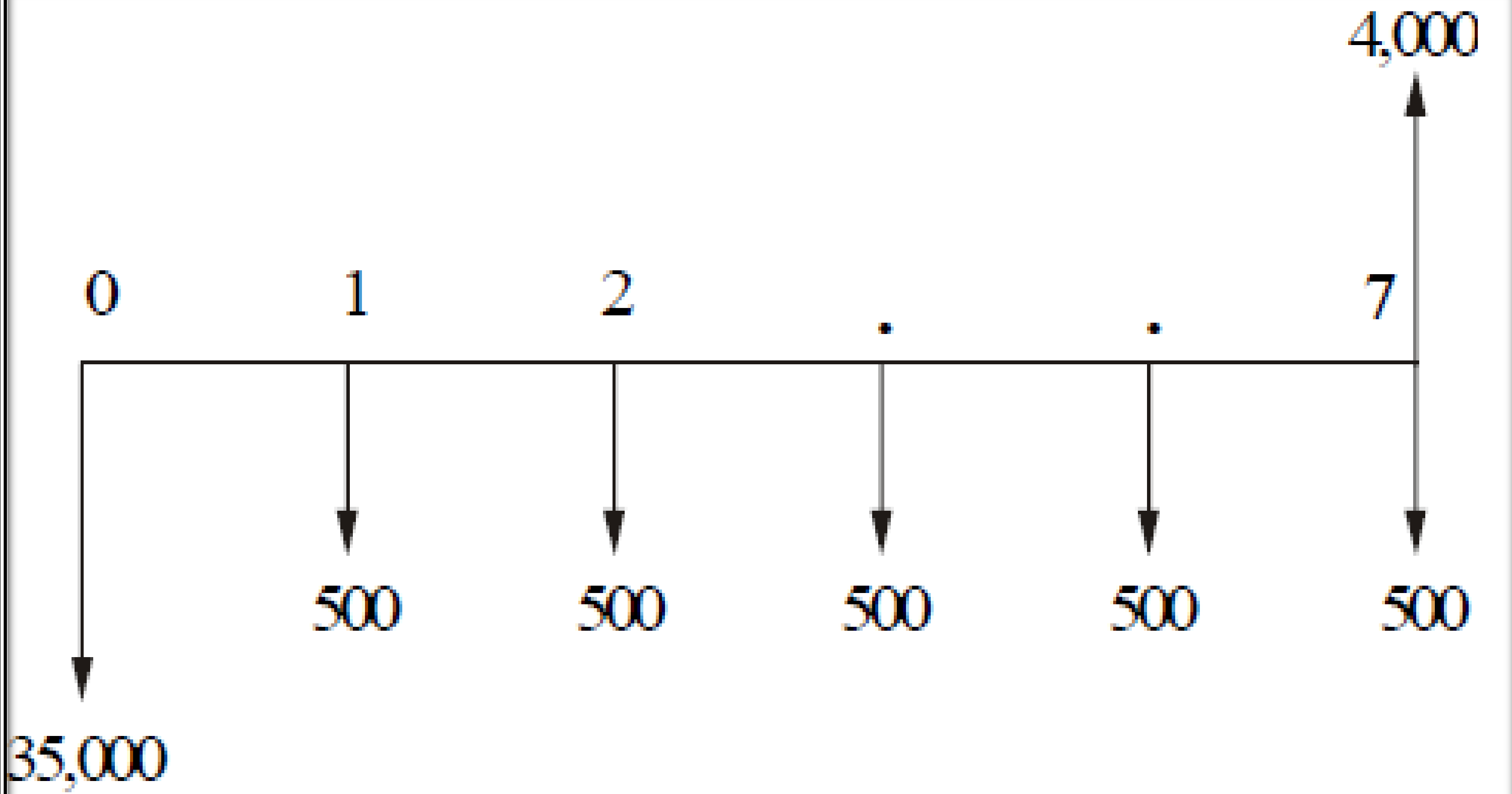



Fig. 8.13 Cash flow diagram for alternative 2.

# Soln : Alternative 2

- The annual equivalent cost of alternative 2 is

**AE(15%) = P x (A/P, 15%, 7) – F x (A/F, 15%, 7) + A**

= 35000 x 0.2404 –4000 x 0.0904 + 500

**AE(15%) = Rs. 8,552.40**

**RESULT:**

- The total annual equivalent cost of alternative 1 is less than that of alternative 2.

- Therefore, it is suggested that the present 10 hp motor be augmented with a new 5 hp motor.

# Problem 8

- A machine was purchased two years ago for Rs. 10,000. Its annual maintenance cost is Rs. 750. Its life is six years and its salvage value at the end of its life is Rs. 1,000. Now, a company is offering a new machine at a cost of Rs. 10,000. Its life is four years and its salvage value at the end of its life is Rs. 4,000. The annual maintenance cost of the new machine is Rs. 500. The company which is supplying the new machine is willing to take the old machine for Rs. 8,000 if it is replaced by the new machine. Assume an interest rate of 12%, compounded annually.

(a)Find the comparative use value of the old machine.

# Cash flow Diagram for Old Machine

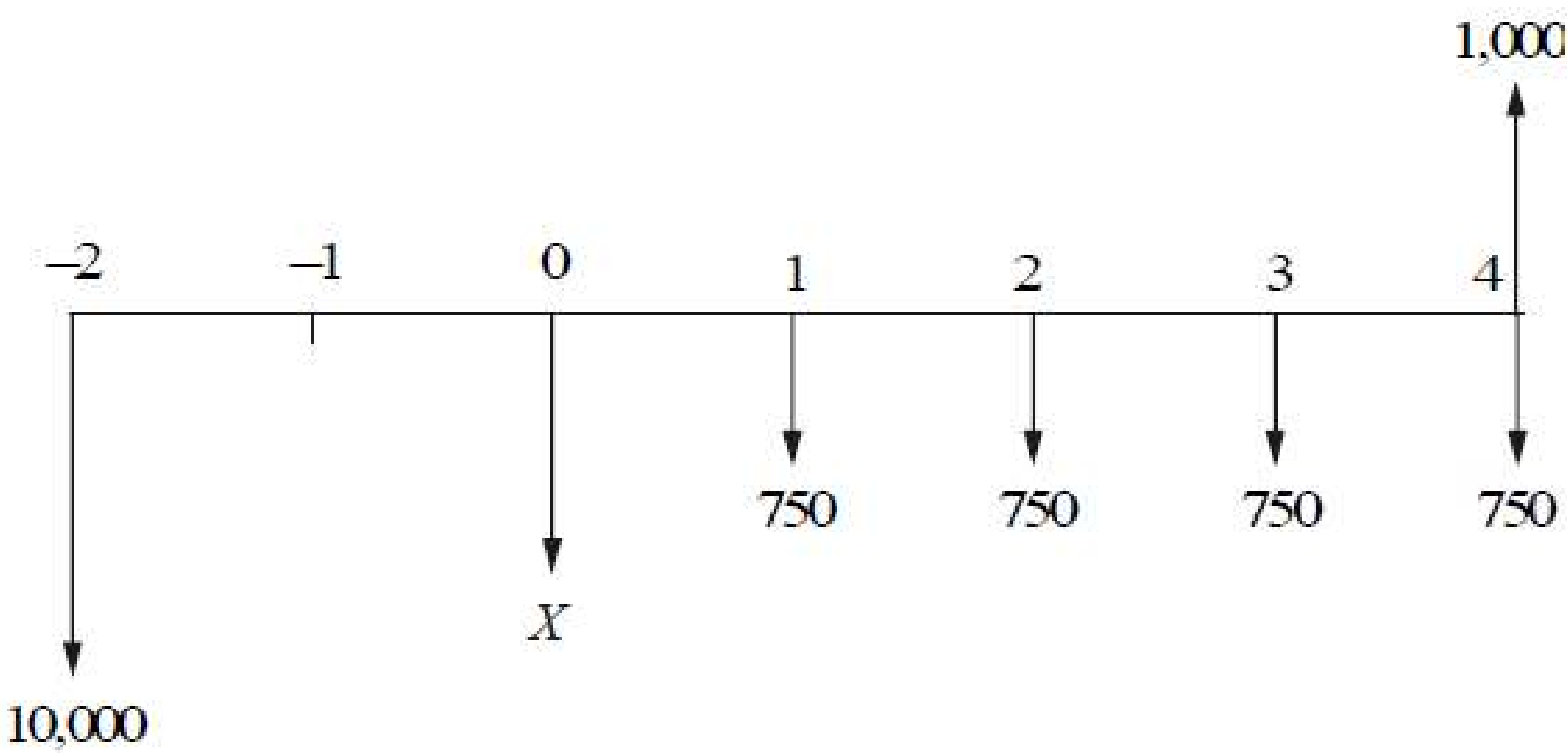



**Fig. 8.14** Cash flow diagram for old machine.

# Soln: Alternative 1

AE(12%) = X x (A/P, 12%, 4) – F x (A/F, 12%, 4) + A

AE(12%)= X * 0.3292 –1000 * 0.2092 + 750

# Soln: Alternative 2

- Cost of the new Machine (P) = Rs. 10,000
- Life (n) = 4 years.
- Salvage value of the new machine (F) = Rs. 4,000
- Annual Maintenance cost (A) = Rs. 500
- Interest rate, i = 12%

# Cash Flow Diagram for Alternative 2

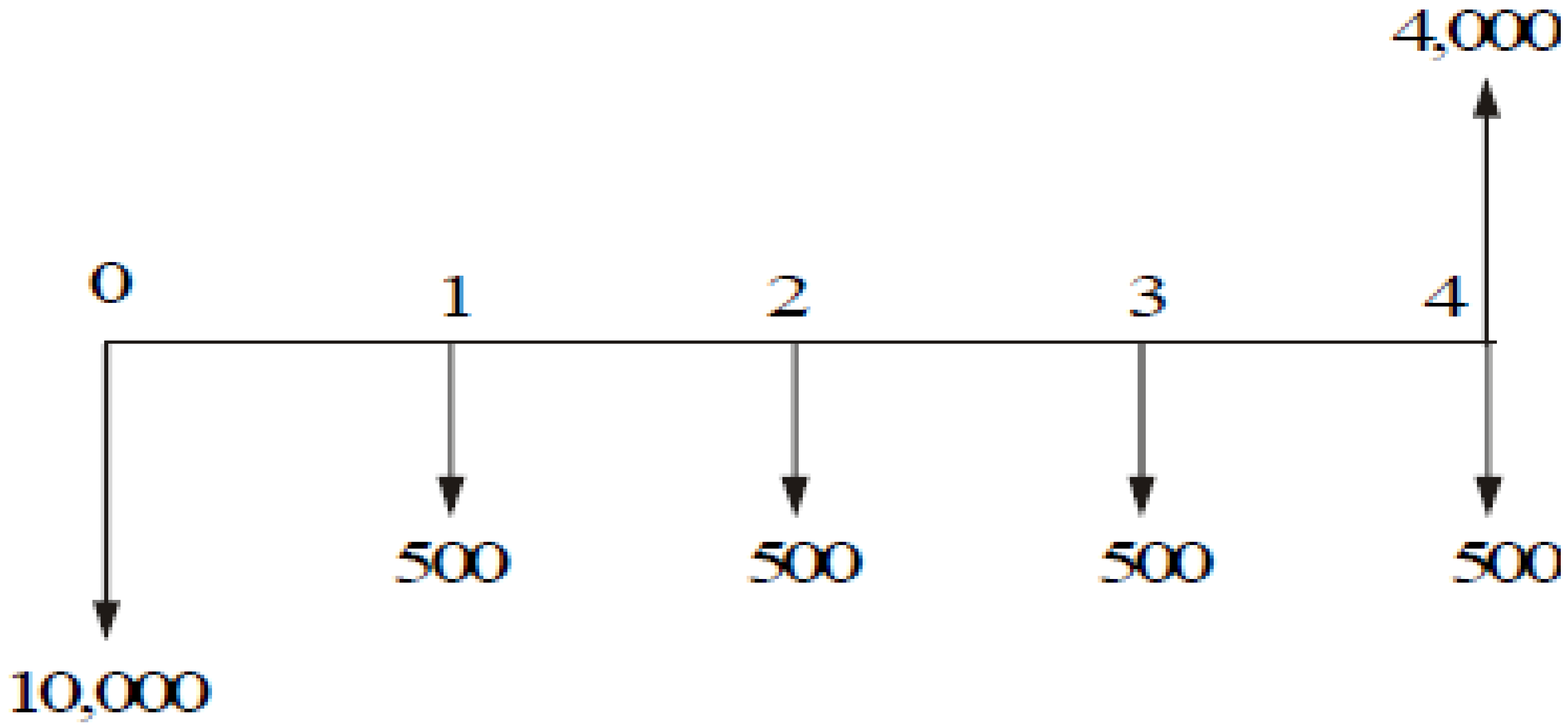



**Fig. 8.15** Cash flow diagram for new machine.

# Solution:

**AE(12%) = P x (A/P, 12%, 4) – F x (A/F, 12%, 4) + A**

= 10000 x 0.3292 – 4000 x .2092 + 500

**AE(12%) = Rs. 2,955.20**

**To Find X:**

Now, equate the annual equivalent costs of the two alternatives and solve for *X.*

(X–1000)(0.3292)+1000x0.12+750=2955.20

**X = Rs. 7,334.14**

# Result

- The comparative use value of the old machine is Rs. 7,334.14, which is less than the price (Rs. 8,000) offered by the company which is supplying the new machine in the event of replacing the old machine by the new machine.
- Therefore, it is advisable to replace the old machine with the new one.

# Simple Probabilistic Model for Items which Fail completely

- Electronic items like transistors, resistors, tube lights, bulbs, etc. could **fail all of a sudden, instead of gradual deterioration.**

- The failure of the item may result in complete breakdown of the system. The system may contain a collection of such items or just one item, say a tube light.

# Various replacement policies

- The following are the replacement policies which are applicable for the above situation.

**(i) Individual replacement policy:**

Under this policy, an item is replaced immediately after its failure.

**(ii) Group replacement policy:**

- At what equal intervals are all the items to be replaced simultaneously with a provision to replace the items individually which fail during a fixed group replacement period?

- There is a trade-off between the individual replacement policy and the group replacement policy.

- Hence, for a given problem, each of the replacement policies is evaluated and the **most economical policy is selected** for implementation.

# Problem 9

The failure rates of transistors in a computer are summarized

| End of Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Probability of Failure to Date | 0.07 | 0.18 | 0.30 | 0.48 | 0.69 | 0.89 | 1.00 |

The cost of replacing an individual failed transistor is Rs. 9. If all the transistors are replaced simultaneously, it would cost Rs. 3.00 per transistor.

# Problem 9 – Cntd..

- Any one of the following two options can be followed to replace the transistors:

(a) Replace the transistors individually when they fail (individual replacement policy).

(b) Replace all the transistors simultaneously at fixed intervals and replace the individual transistors as they fail in service during the fixed interval (group replacement policy).

- Find out the optimal replacement policy, i.e. individual replacement policy or group replacement policy. If group replacement policy is optimal, then find at what equal intervals should all the transistors be replaced.

# Solution

- Assume that there are 100 transistors in use.

- Let $p_i$ be the probability that a transistor which was new when placed in position for use, fails during the i'th week of its life. Hence,

$p_1 = 0.07$, $p_2 = 0.11$, $p_3 = 0.12$, $p_4 = 0.18$, $p_5 = 0.21$, $p_6 = 0.20$, $p_7 = 0.11$

- Since the sum of $p_i$ is equal to 1 at the end of the $7^{th}$ week, the transistors are sure to fail during the seventh week.

- **Assume that**

(a) transistors that fail during a week are replaced just before the end of the week, and

(b) the actual percentage of failures during a week for a sub-group of transistors with the same age is same as the expected percentage of failures during the week for that sub-group of transistors.

- Let

$N_i$ = the number of transistors replaced at the end of the ith week

$N_0$= number of transistors replaced at the end of the week 0 (or at the beginning of the first week).

= 100

N1 = Number of transistors replaced at the end of the 1st week

= N0 x p1 = 100 x 0.07 = 7

N2 = Number of transistors replaced at the end of the 2nd week

= N0 x p2 + N1 x p1 = 100 x 0.11 + 7 x 0.07 = 12

N3 = N0 x p3 + N1 x p2 + N2 x p1 = 100 x 0.12 + 7 x 0.11 + 12 x 0.07 = 14

N4 = N0 x p4 +N1xp3 + N2 x p2 + N3 x p1

= 100 x 0.18 + 7 x 0.12 + 12 x 0.11 + 14 x 0.07 = 21

N5 = N0 x p5 + N1 x p4 + N2 x p3 + N3 x p2 +N4 x p1

= 100 x 0.21 + 7 x 0.18 + 12 x 0.12 + 14 x 0.11 + 21 x 0.07 = 27

N6 = N0 x p6 + N1 x p5 + N2 x p4 + N3 x p3 + N4 x p2 + N5 x p1

= 100 x 0.2 + 7 x 0.21 + 12 x 0.18 + 14 x 0.12 + 21 x 0.11 + 27x0.07=30

N7 = N0xp7 +N1 x p6 + N2 x p5 + N3 x p4 + N4 x p3 + N5 x p2 + N6xp1

= 100x0.11 +7 x 0.2+12x 0.21+14 x 0.18 +21 x 0.12 + 27 x 0.11 + 30x 0.07

=25

# Calculation of Individual Replacement Cost

$$\text{Expected life of each transistor} = \sum_{i=1}^{7} i \times p_i$$

$$= 1 \times 0.07 + 2 \times 0.11 + 3 \times 0.12 + 4 \times 0.18$$

$$+ 5 \times 0.21 + 6 \times 0.2 + 7 \times 0.11$$

$$= 4.39 \text{ weeks}$$

Average No. of failures/week = 100/4.39 = 23 (approx.)

Therefore,

Cost of individual replacement

= (No. of failures/week × Individual replacement cost/transistor)

= 23 × 9 = Rs. 207.

**Table 8.6** Calculations of Cost for Preventive Maintenance

| *End of week* A | *Cost of replacing 100 transistors at a time* B (Rs.) | *Cost of replacing transistors individually during given replacement period* C (Rs.) | *Total cost* (B + C) D (Rs.) | *Average cost/week* (D/A) E (Rs.) |
|---|---|---|---|---|
| 1 | 300 | $7 \times 9 = 63$ | 363 | 363.00 |
| 2 | 300 | $(7 + 12) \times 9 = 171$ | 471 | 235.50 |
| 3 | 300 | $(7 + 12 + 14) \times 9 = 297$ | 597 | 199.00 |
| 4 | 300 | $(7 + 12 + 14 + 21) \times 9 = 486$ | 786 | 196.50* |
| 5 | 300 | $(7 + 12 + 14 + 21 + 27) \times 9 = 729$ | 1,029 | 205.80 |
| 6 | 300 | $(7 + 12 + 14 + 21 + 27 + 30) \times 9 = 999$ | 1,299 | 216.50 |
| 7 | 300 | $(7 + 12 + 14 + 21 + 27 + 30 + 25) \times 9 = 1,224$ | 1,524 | 217.71 |

*Indicates the minimum average cost/week.

# Determination of group replacement cost

- Cost of transistor when replaced simultaneously = Rs. 3
- Cost of transistor when replaced individually = Rs. 9
- It is clear that the average cost/week is minimum for the fourth week. Hence, the group replacement period is four weeks.
- Individual replacement cost/week = Rs. 207
- Minimum group replacement cost/week = Rs. 196.50

# Result

- Since the minimum group replacement cost/week is less than the individual replacement cost/week, the group replacement policy is the best, and hence all the transistors should be replaced once in four weeks and the transistors which fail during this four-week period are to be replaced individually.

# Problem 10

An electronic equipment contains 1,000 resistors. When any resistor fails, it is replaced. The cost of replacing a resistor individually is Rs. 10. If all the resistors are replaced at the same time, the cost per resistor is Rs. 4. The percent surviving, S(i) at the end of month i is tabulated as follows:

| I | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| S(i) | 100 | 96 | 89 | 68 | 37 | 13 | 0 |

What is the optimum replacement plan?

# Solution

- Let $p_i$ be the probability of failure during the month i. Then,

$p_1 = (100 – 96)/100 = 0.04$

$p_2 = (96 – 89)/100 = 0.07$

$p_3 = (89 – 68)/100 = 0.21$

$p_4 = (68 – 37)/100 = 0.31$

$p_5 = (37 – 13)/100 = 0.24$

$p_6 = (13 – 0)/100 = 0.13$

- It is clear that no resistor can survive beyond six months. Hence, a resistor which has survived for five months would certainly fail during the sixth month.

- We assume that the resistors failing during a month are accounted at the end of the month.

Let $N_i$ = No. of resistors replaced at the end of the ith month.

$N_0 = 1000$

$N_1$ = Number of resistors replaced at the end of the 1st week

$= N_0 \times p_1 = 1000 \times 0.04 = 40$

$N_2$ = Number of resistors replaced at the end of the 2nd week

$= N_0 \times p_2 + N_1 \times p_1 = 1000 \times 0.07 + 40 \times 0.04 = 72$

$N_3 = N_0 \times p_3 + N_1 \times p_2 + N_2 \times p_1 = 1000 \times 0.21 + 40 \times 0.07 + 72 \times 0.04 = 216$

$N_4 = N_0 \times p_4 + N_1 \times p_3 + N_2 \times p_2 + N_3 \times p_1$

$= 1000 \times 0.31 + 40 \times 0.21 + 72 \times 0.07 + 216 \times 0.04 = 332$

$N_5 = N_0 \times p_5 + N_1 \times p_4 + N_2 \times p_3 + N_3 \times p_2 + N_4 \times p_1$

$= 1000 \times 0.24 + 40 \times 0.31 + 72 \times 0.21 + 216 \times 0.07 + 332 \times 0.04 = 296$

$N_6 = N_0 \times p_6 + N_1 \times p_5 + N_2 \times p_4 + N_3 \times p_3 + N_4 \times p_2 + N_5 \times p_1$

$= 1000 \times 0.13 + 40 \times 0.24 + 72 \times 0.31 + 216 \times 0.21 + 332 \times 0.07 + 296 \times 0.04 = 242$

# Determination of Individual replacement cost

$$\text{Expected life of each resistor} = \sum_{i=1}^{6} i \times p_i$$

$$= 1 \times 0.04 + 2 \times 0.07 + 3 \times 0.21 + 4 \times 0.31$$

$$+ 5 \times 0.24 + 6 \times 0.13$$

$$= 4.03 \text{ months.}$$

Average number of failures/month = 1,000/4.03 = 248 (approx.)

Therefore,

Cost of individual replacement

= (No. of failures/month × individual replacement cost/resistor)

= 248 × 10 = Rs. 2,480.

**Table 8.7** Calculations of Costs for Preventive Maintenance

| *End of month* | *Cost of replacing 1,000 resistors at a time* | *Cost of replacing resistors individually during given replacement period* | *Total cost (B + C)* | *Average cost/ month (D/A)* |
|---|---|---|---|---|
| A | B (Rs.) | C (Rs.) | D (Rs.) | E (Rs.) |
| 1 | 4,000 | $40 \times 10 = 400$ | 4,400 | 4,400.00 |
| 2 | 4,000 | $(40 + 72)10 = 1,120$ | 5,120 | 2,560.00 |
| 3 | 4,000 | $(40 + 72 + 216)10 = 3,280$ | 7,280 | 2,426.67* |
| 4 | 4,000 | $(40 + 72 + 216 + 332)10 = 6,600$ | 10,600 | 2,650.00 |

*Indicates the minimum average cost/month.

## Determination of Group replacement Cost

Cost/resistor when replaced simultaneously = Rs. 4.00

Cost/resistor when replaced individually = Rs. 10.00

It is clear that the average cost/month is minimum for the third month. Thus, the group replacement period is three months.

# Result

- Individual replacement cost/month=Rs. 2,480.00
- Minimum group replacement cost/month = Rs. 2,426.67

**RESULT:**

- Since the minimum group replacement cost/month is less than the individual replacement cost/month, the group replacement policy is the best and hence all the resistors are to be replaced once in three months and the resistors which fail during this three months period are to be replaced individually.

# Evaluation of Public Alternatives

- Public alternatives are constructing bridges, roads, dams, establishing public utilities, etc.

- The main objective of any public alternative is to provide goods/services to the public at the minimum cost.

- BC ratio = $\frac{\text{Equivalent Benefits}}{\text{Equivalent Costs}}$

# Evaluation of Public Alternatives

- $B_P$ = present worth of the total benefits
- $B_F$ = future worth of the total benefits
- $B_A$ = annual equivalent of the total benefits
- P = initial investment
- $P_F$ = future worth of the initial investment
- $P_A$ = annual equivalent of the initial investment
- C = yearly cost of operation and maintenance
- $C_P$ = present worth of yearly cost of operation and maintenance
- $C_F$ = future worth of yearly cost of operation and maintenance

# Evaluation of Public Alternatives

- BC ratio $= \dfrac{B_P}{P + CP}$

- BC ratio $= \dfrac{B_F}{P_F + CF}$

- BC ratio $= \dfrac{B_A}{P_A + C}$

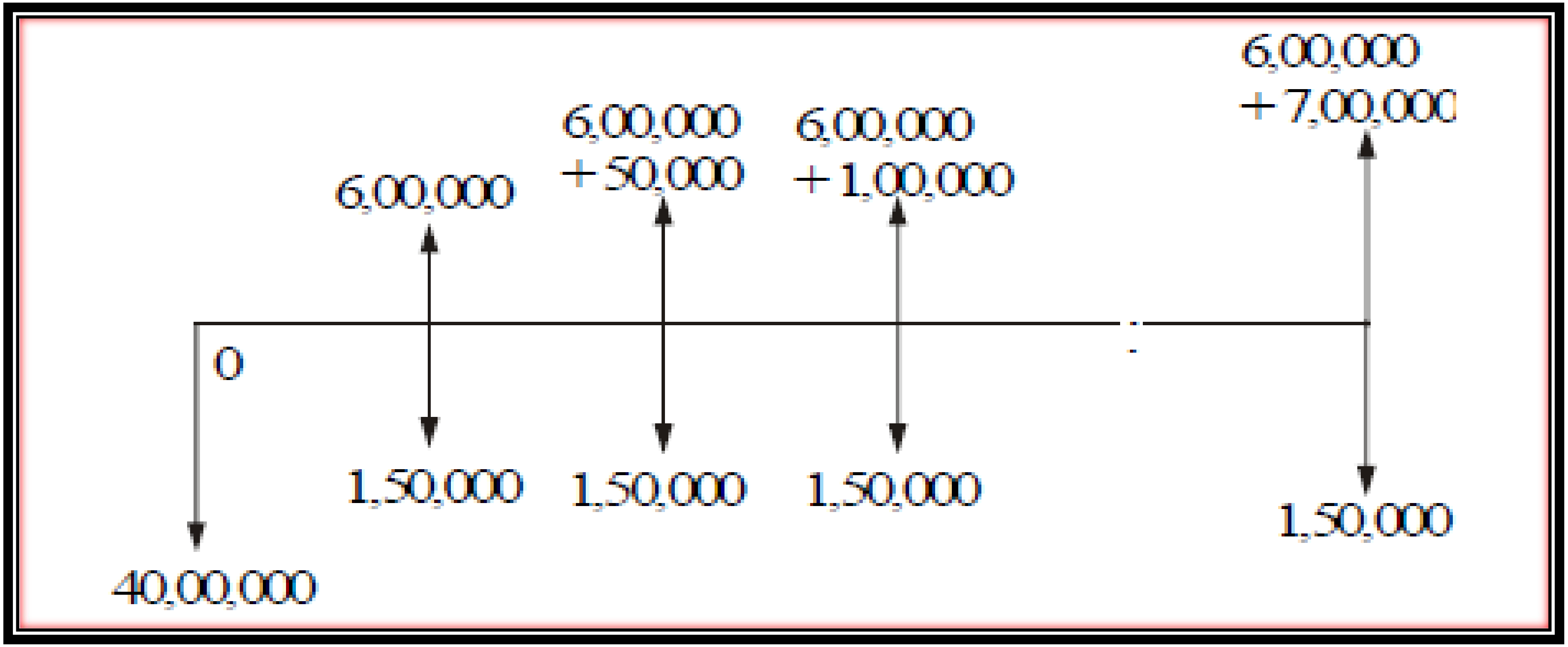
# Problem - 1

- In a particular locality of a state, the vehicle users take a roundabout route to reach certain places because of the presence of a river. This results in excessive travel time and increased fuel cost. So, the state government is planning to construct a bridge across the river. The estimated initial investment for constructing the bridge is Rs. 40,00,000. The estimated life of the bridge is 15 years. The annual operation and maintenance cost is Rs. 1,50,000. The value of fuel savings due to the construction of the bridge is Rs. 6,00,000 in the first year and it increases by Rs. 50,000 every year thereafter till the end of the life of the bridge. Check whether the project is justified based on BC ratio by assuming an interest rate of 12%, compounded annually.

# Solution- 1

- Initial investment = Rs. 40,00,000
- Annual operation and maintenance = Rs. 1,50,000
- Annual fuel savings during the first year = Rs. 6,00,000
- Equal increment in fuel savings in the following years = Rs. 50,000
- Life of the project = 15 years
- Interest rate = 12%

# Solution- 1




Total present worth of costs = Initial investment (*P*) + Present worth of annual operating and maintenance cost

# Solution- 1

= Rs. 40,00,000 + 1,50,000 (*P*/*A*, 12%, 15)

= Rs. 40,00,000 + 1,50,000 6.8109

= Rs. 50,21,635

- *Total present worth of fuel savings* (*BP*):
- $A1$ = Rs. 6,00,000
- $G$ = Rs. 50,000
- $n$ = 15 years
- $i$ = 12%