# UNIT-I

# BASIC CONCEPTS AND FIRST LAW

## Thermodynamics

Thermodynamics is a branch of science that deals with the relationship between the heat and mechanical energy.

## Thermodynamics Laws and Applications

Thermodynamics laws are formulated based on the experience and the results of experiments. These laws govern the principles of energy transfer. These laws are used in the field of energy transfer, like, steam and nuclear power plants, gas turbines, internal combustion engines, air conditioning plants, refrigeration, jet propulsion, compressors, chemical process plants, etc.

## Macroscopic and Microscopic concepts

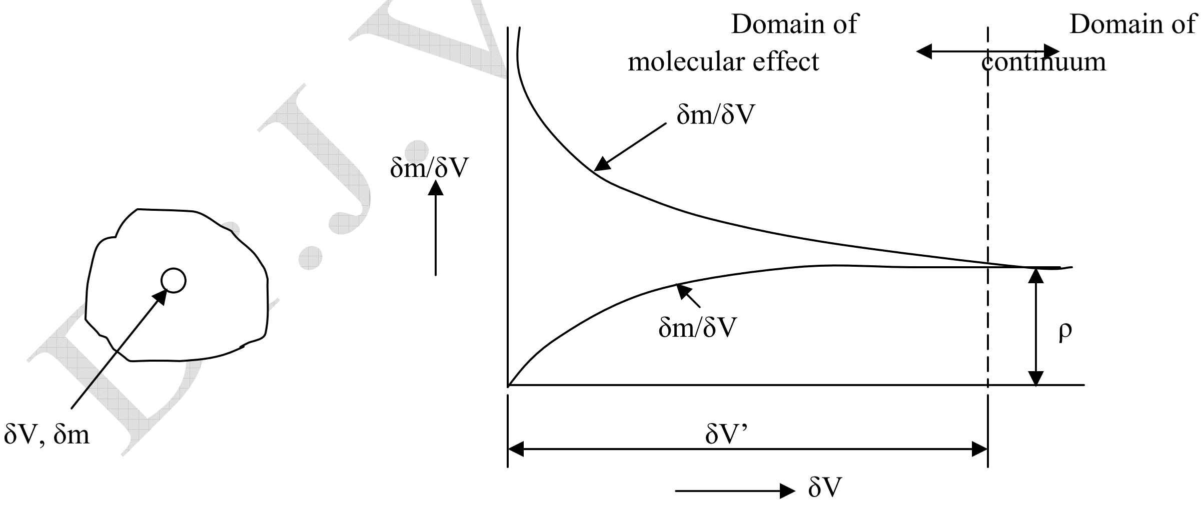
In macroscopic approach, the behaviour of the matter is studied considering a certain quantity of matter (consisting of many molecules), without the events occurring in the molecular level. Action of many molecules can be perceived by human senses.

In microscopic approach, the matter is composed of myriads of the molecules and the behaviour of the matter is described by summing up behaviour of the individual molecules.

## Concept of Continuum

In macroscopic analysis we consider a small amount (small volume) of matter. This small volume is very large compared to molecular dimensions. Even a very small volume of a matter contains large number of molecules.

Consider the mass δm in a volume δV. The average mass density is defined as δm/δV.



If $\delta V = 0$, $\delta m/\delta V = \infty$

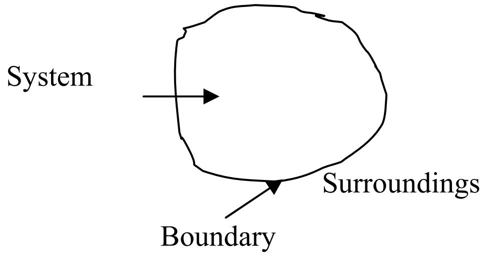
If $\delta m = 0$, $\delta m/\delta V = 0$

The smallest volume which may be regarded as continuous is δV'. The density ρ of the system at a point is defined as

$$\rho = \lim_{\delta V \to \delta V'} \frac{\delta m}{\delta V}$$

## Thermodynamic System, Surroundings and Boundary

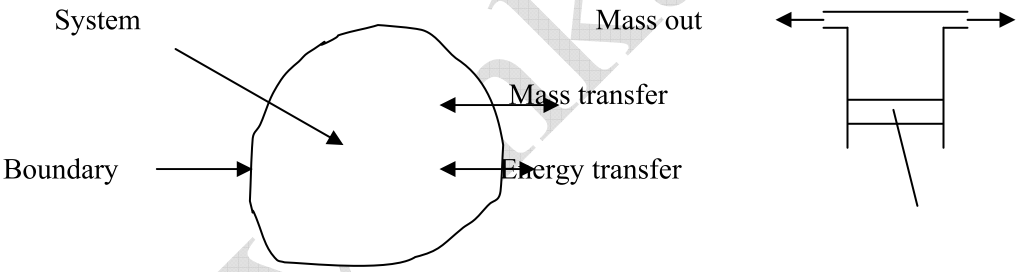
A thermodynamic system is a specified quantity of matter or a region in a space upon which the attention is focused on to analyse the problem.



Everything outside the system is called surroundings. The system and surroundings is separated by a boundary. The boundary may be real or imaginary, fixed or moving.
There are three types of systems:

1. Open system
2. Closed system
3. Isolated system

### Open system



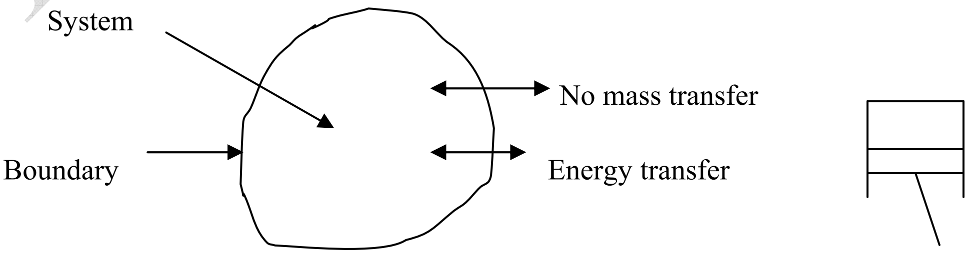
The open system is one in which both matter (mass) and energy (work or heat) cross the boundary of the system.

Examples: Steam turbine, Gas turbine, Rotary compressors, Heat exchangers, Centrifugal pump, Water turbine, etc.

Work Transfer = $W = -\int V\,dp$

### Closed system

The closed system is a system of fixed mass. In closed system, there is no mass transfer, but energy transfer takes place.

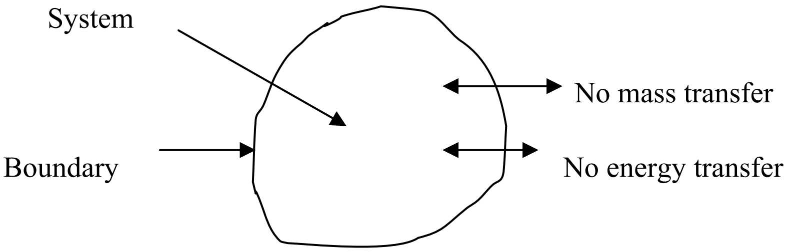


Examples: I.C.engine cylinder with the both valves closed, Reciprocating air compressor with both the valves closed.

Work transfer = $W = \int p\, dV$

**Isolated system**

In isolated system, there is no mass or energy transfer across the boundary of the system.



**Universe**

The system and surroundings together is called "Universe".

**Thermodynamic Equilibrium**

The system under Mechanical, Thermal and Chemical equilibrium is said to be under Thermodynamic equilibrium.

A system is said to be in mechanical equilibrium when there is no unbalanced force within the system. (Uniformity of force).

A system is said to be in chemical equilibrium when there is no chemical reaction between different parts of the system. (Absence of chemical reaction).

A system is said to be in thermal equilibrium when there is no temperature change between different parts of the system. (Uniformity of temperature).

The system under mechanical and thermal equilibrium but not under chemical equilibrium is said to be under meta-stable equilibrium.

**Thermodynamic Property**

Any observable characteristic of the system under thermodynamic equilibrium is called a property.

Examples: Pressure, Temperature, Volume, Density.

**Intensive and Extensive property**

The property which is not depending on the mass of the system is called intensive or intrinsic property.

Examples: Pressure, Temperature, Density, Specific volume ($m^3$/kg), Specific enthalpy (J/kg)

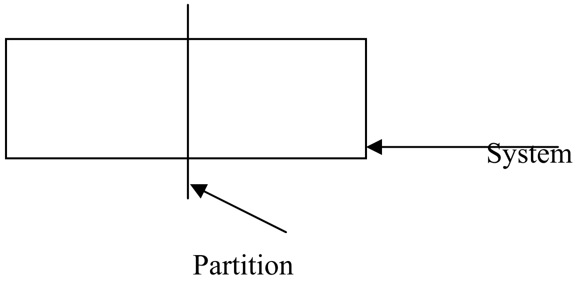
The property which is depending on the mass of the system is called extensive or extrinsic property.

Examples: Volume ($m^3$), Mass (kg), Enthalpy (J)

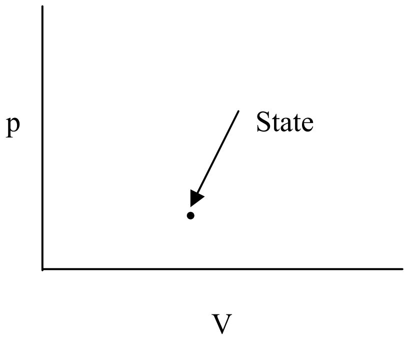
Consider a system composed of some fluid. Measure all the properties of the system, like, pressure ($p_1$), volume ($V_1$), temperature ($T_1$), density ($\rho_1$), enthalpy ($H_1$), mass ($m_1$), specific volume ($v_1$), specific enthalpy ($h_1$), etc. Now, disturb the mass by placing a partition and measure all the properties of a part ($p_2$, $V_2$, $T_2$, $\rho_2$, $H_2$, $m_2$, $v_2$, $h_2$).

It can be seen that,

$p_1 = p_2$ ; $T_1 = T_2$ ; $V_1 \neq V_2$ ; $\rho_1 = \rho_2$ ; $H_1 \neq H_2$ ; $m_1 \neq m_2$ ; $v_1 = v_2$ ; $h_1 = h_2$



### Thermodynamic State

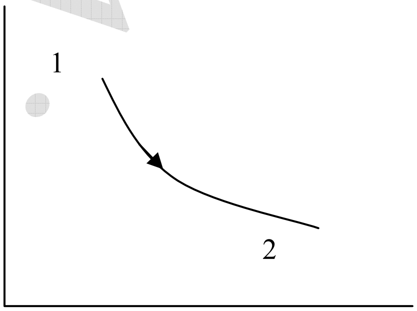


A state is the condition of the system at an instant of time as described or measured by its properties. Or each unique condition of the system is called state.

Example: $p = 8$ bar & $V = 0.5\ m^3$

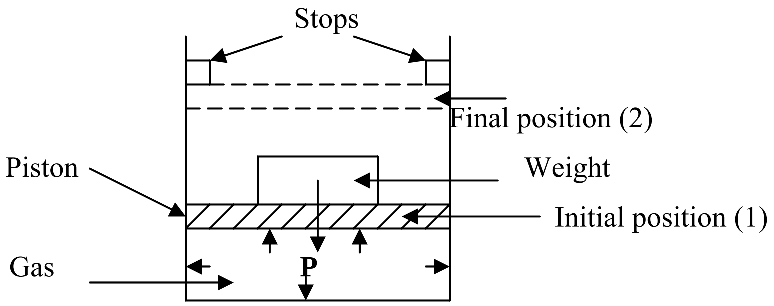
### Thermodynamic Path and Process

The succession of states passed through during a change of state is called the "Path". When the path is completely specified, the change of state is called a process.

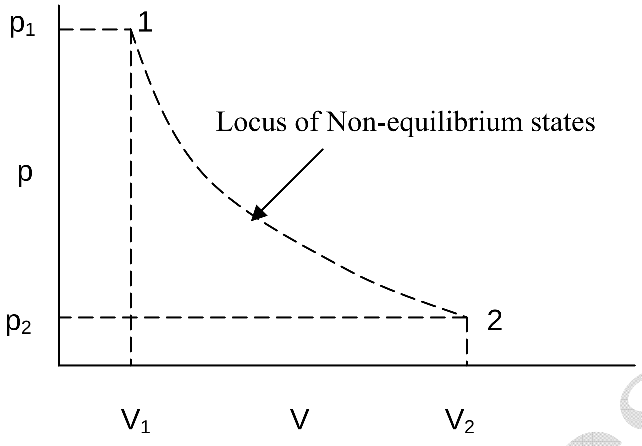


### Quasi-static process

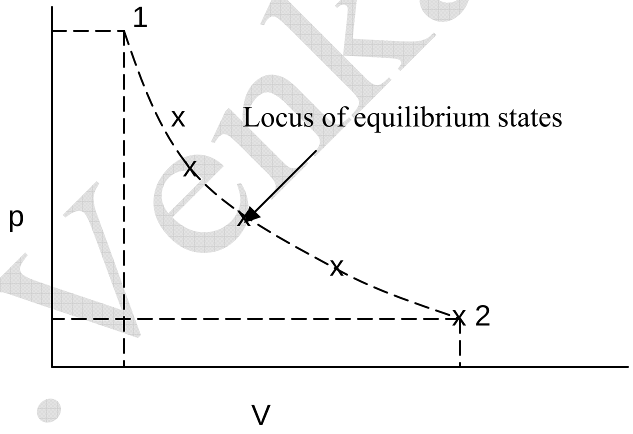
Quasi means 'almost'. A quasi-static process is a succession of equilibrium states and infinite slowness is its characteristic feature.



Consider a system of gas contained in a cylinder. The force exerted by the gas is balanced by the weight. The gas is in equilibrium in the initial state. If the weight is removed, the piston will move up until it hits the stops. The system again comes to equilibrium state. The intermediate states between 1 and 2 are non-equilibrium states and cannot be represented by the thermodynamic coordinates.



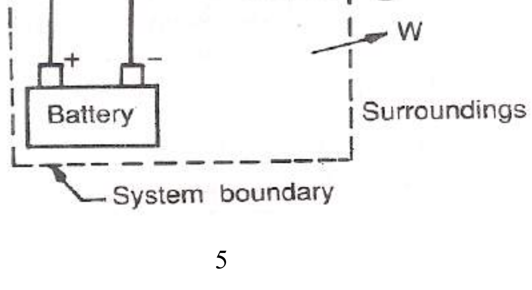
Now if single weight on the piston is made up of very small pieces of weights and these weights are removed one by one very slowly from the top of the piston, at any instant of the upward travel of the piston, the departure of the state of the system from equilibrium state will be infinitesimally small. So, each intermediate state will be an equilibrium state. Such a process which is locus of all the equilibrium points is called “Quasi-static process”.



## Thermodynamic Cycle

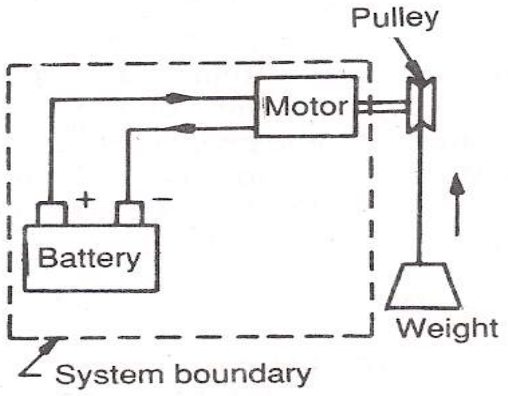
A single process cannot take place continuously without end. To get continuous work, a set of processes has to be repeated again and again. The set of processes which brings the system to the original state is called a cycle. Continuous work transfer is possible only with a cycle.
Examples: Otto cycle, Diesel cycle, Dual cycle, etc.

## Work



Work is one form of energy transfer. Work is said to be done by a system if the sole effect on things external to the system can be reduced to the raising of weight. The weight may not actually be raised, but the net effect external to the system would be the raising of a weight. Consider a Battery-motor system driving a fan as shown in fig. The system is doing work upon the surroundings.

When the fan is replaced by a pulley and a weight as shown, the weight may be raised. The sole effect on the things external to the system is then raising of a weight.



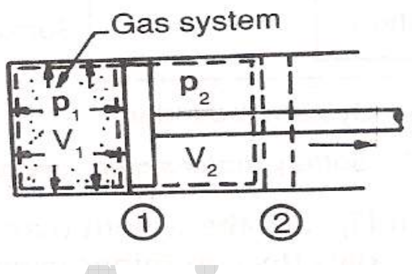
Work done by a system → +ve

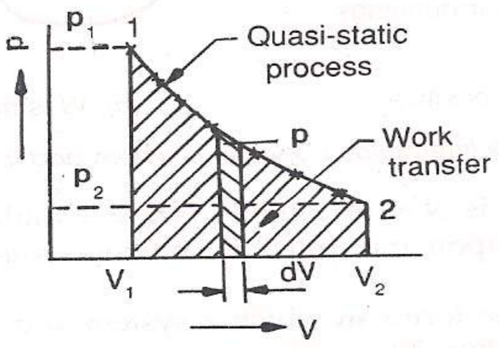
Work done on the system → - ve

Unit of work → Nm (or) J

**pdV work (or) Displacement work**

Consider a cylinder-piston arrangement. Let $p_1$ and $V_1$ be pressure and volume of the gas in the cylinder at initial position 1. Let the piston moves to a new final position 2 specified by pressure $p_2$ and volume $V_2$. At any point in the travel of the piston, let the pressure be p and the volume V.





Work done = Force x Distance moved

dW = F x dl = p A dl = p dV

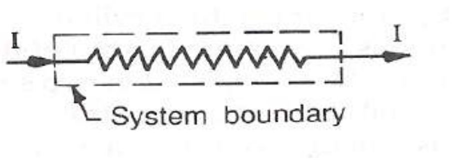
When the piston moves out from position 1 to position 2 with the volume changing from $V_1$ to $V_2$, the amount of work W done by the system will be

$$W_{1-2} = \int_{V_1}^{V_2} p\,dV$$

Work transfer = Area under the curve on a p-V diagram

**Electrical work**

When a current flows through a resistor, there is work transfer into the system. This is because the current can drive a motor, the motor can drive a pulley and the pulley can raise a weight.

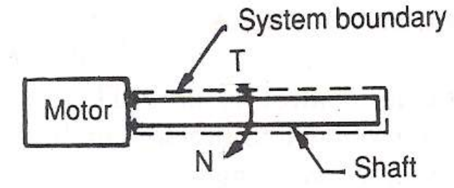


Work transfer, W = E I

E → Potential difference (Voltage)

I → Current

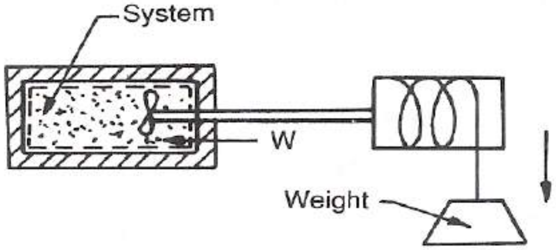
**Shaft work**



When the shaft is rotated by a motor, there is work transfer into the system. This is because the shaft can rotate a pulley which can raise a weight. If T is the torque applied to the shaft and ω is the angular velocity, work transfer can be written as

$$W = T\,\omega$$

**Paddle-wheel work or stirring work**



As the weight is lowered, and the paddle wheel turns, there is work transfer into the fluid system which gets stirred. Since the volume of the system remains constant, pdV work is zero. If m is the mass of the weight lowered through a distance dZ and T is the torque transmitted by the shaft in rotating through an angle of dθ, the work transfer is given by,

$$W = \int_1^2 W'\,dz = \int_1^2 T\,d\theta$$

W' → Weight lowered

**Flow work**

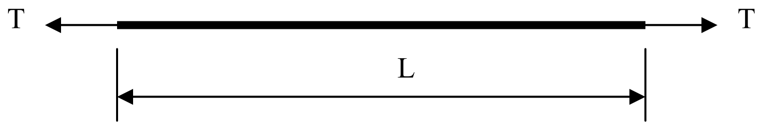
The flow is significant in open system. This work represents the energy transferred across the system boundary as result of the energy imparted to the fluid by a pump, blower or compressor to make the fluid to flow across the control volume.

Flow work is given by, W = p V

**Work done in stretching a wire**

Let us consider a wire of length L, subjected to a tension force T. The infinitesimal amount of work is done on the wire which makes the wire to stretch to a length L + dL.

$$W = -\int_1^2 T\,dL$$



Negative work → The work is done on the wire.

**Work done in changing the area of a surface film**

A film on the surface of a liquid has a surface tension, which is a property of the liquid and the surroundings. The surface tension acts to make the surface are of the liquid a minimum. The work done on a homogeneous liquid film in changing its surface area by an infinitesimal amount dA is

$$W = -\int_1^2 \sigma \, dA$$

σ → Surface tension (N/m)

**Magnetization of a paramagnetic solid**

The work done per unit volume on a magnetic material through which the magnetic and magnetization fields are uniform is

$$W = \int_{I_1}^{I_2} H \, dI$$

H → Field strength

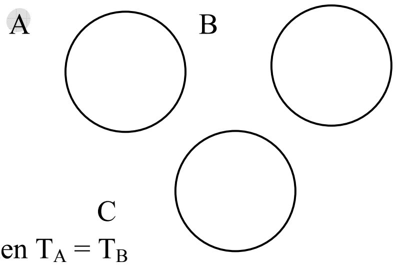
I → Component of the magnetization field in the direction of the field

**Heat**

Heat is defined as the form of energy that is transferred across the boundary by virtue of a temperature difference. The temperature difference is 'potential' and heat transfer per unit area is 'flux'.

**Zeroth Law of Thermodynamics**

If the bodies A and B are in thermal equilibrium with the third body C, then these two bodies A and B will be in thermal equilibrium with each other. This law is the basis for measurement of temperature.



If $T_A = T_C$ & $T_B = T_C$ then $T_A = T_B$

**Application of Zeroth law**

It is the basis for temperature measurement.

**Ideal and Real gases**

An ideal gas or perfect gas is a hypothetical gas consisting of identical particles of zero volume, with no intermolecular forces. Additionally, the constituent atoms or molecules undergo perfectly elastic collisions with the walls of the container. Real gases do not exhibit these exact properties. Gases are most ideal at high temperatures and low pressure.

An ideal gas obeys the perfect gas law. The specific heats are constant.

Perfect gas law, $p V = n R_u T = m R T$

p → Pressure in Pa

V → Volume in $m^3$

n → Amount of gas in kg-mole = m / M

M → Molecular weight

$R_u$ → Universal gas constant = 8314 J/kg-mole K

m → Amount of gas in kg

R → Characteristic gas constant in J/kg K

T → Temperature of gas in K

In reality there is no ideal or perfect gas. At a very low pressure and at a very high temperature, real gases like nitrogen, hydrogen, oxygen, helium, etc., behave as perfect gases. These gases are called 'Semi perfect' or 'Permanent' gases. For real gases specific heats vary appreciably with temperature and little with pressure.

**Internal energy and enthalpy**

*Internal Energy* → Energy possessed by the system.

Joule's law states that the specific internal energy of a gas depends only on the temperature of the gas and is independent of both pressure and volume.

$$u = f(T)$$

*Enthalpy* → Total heat content of the flowing fluid

From definition of enthalpy, $h = u + pv$

But $pv = RT$

Therefore, $h = u + RT$ and h also function of temperature only.

$$h = f(T)$$

**Specific heats**

*Specific heat at constant volume* is defined as the amount of heat required to rise the temperature of 1 kg of fluid through 1°C when the volume is kept constant.

The specific heat capacity at constant volume is defined as $C_v = \left(\frac{\partial u}{\partial T}\right)_v$

We know that u = f(T). Therefore $C_v = \frac{du}{dT}$

*Specific heat at constant pressure* is defined as the amount of heat required to rise the temperature of 1 kg of fluid through 1°C when the pressure is kept constant.

The specific heat capacity at constant pressure is defined as $C_p = \left(\frac{\partial h}{\partial T}\right)_p$

We know that h = f(T). Therefore $C_p = \frac{dh}{dT}$

**First Law of thermodynamics**

I-Corollary:

*For a cycle:*

Whenever a system undergoes a cyclic change, the algebraic sum of heat transfer is proportional to the algebraic sum of work transfer.

Net heat transfer = Net work transfer

$$\oint dQ = \oint dW$$

*For a process:*

$$dQ = dW + dU$$

or $$Q = W + \Delta U$$

$\Delta U$ = Change in internal energy = $m\, C_v\, \Delta T$

m = Mass of the working fluid

$C_v$ = Specific heat at constant volume

$\Delta T$ = Change in temperature

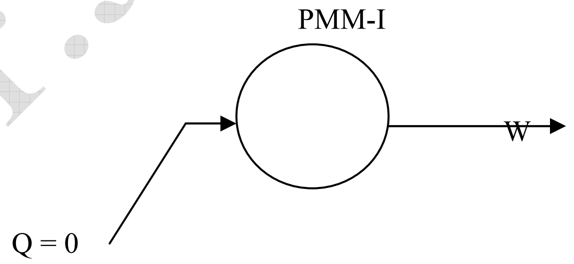
II-Corollary: (Law of Conservation of Energy)

In an isolated system, the energy of the system remains constant.

$Q = W = 0$ & $\Delta U = 0$

III-Corollary: (PMM-I)

PMM-I is impossible. (Perpetual Motion Machine of First kind produces work continuously without any input).



**Application of I-Law to Non-Flow or Closed system**

Let,

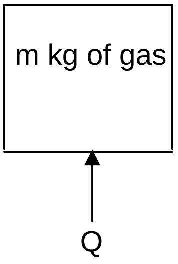
m = Mass of the working fluid
$p_1$ = Initial pressure of the working fluid
$p_2$ = Final pressure of the working fluid
$T_1$ = Initial temperature of the working fluid
$T_2$ = Final temperature of the working fluid
$U_1$ = Initial internal energy of the fluid

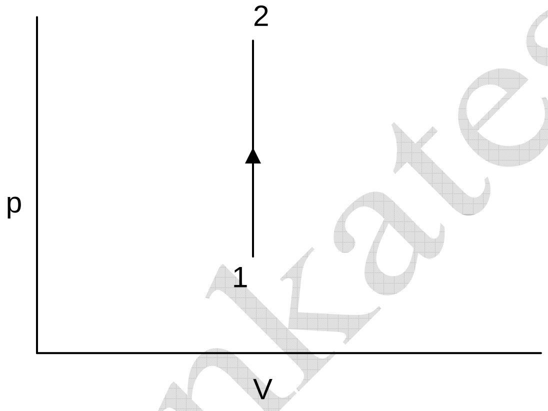
$U_2$ = Final internal energy of the fluid
W = Work transfer
Q = Heat transfer
$C_p$ = Specific heat at constant pressure
$C_v$ = Specific heat at constant volume

**Constant volume process (Isometric process)**

In a constant volume process the working fluid is contained in a closed vessel. The boundary of the system is immovable and hence no work transfer is possible through boundary of the system.

Consider a vessel containing m kg of certain gas. Q J of heat is supplied to the gas and there will be pressure rise, but the volume remains constant.





During constant volume process, dV = 0

General gas equation → $\frac{pV}{T} = C$

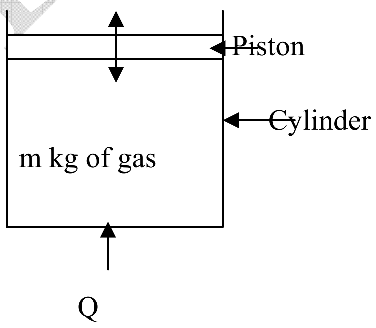
For constant volume process, $\frac{p}{T} = C$

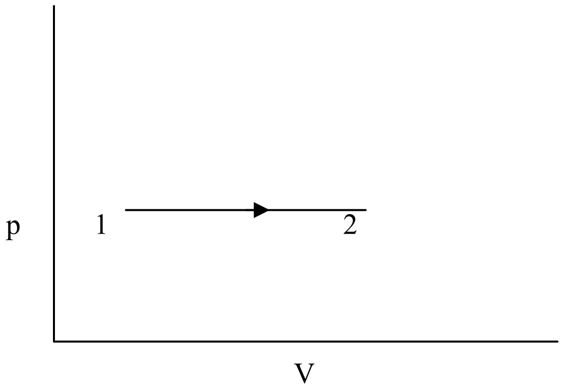
Work transfer = W = $\int p\, dV = 0$ ;

As per I-law for a process, $Q = W + \Delta U$

$$Q = \Delta U = m\, C_v\, (T_2 - T_1)$$

**Constant pressure process (Isobaric process)**





Consider a cylinder-piston arrangement as shown in fig. The piston is free to move up and down. Let the cylinder contain m kg of certain gas. The heat is added to the gas. Since the piston is free to move, the pressure remains constant and there is increase of volume. In this case the boundary of the system is movable.

During constant pressure process, $p = C$

General gas equation → $\frac{pV}{T} = C$

For constant pressure process, $\frac{V}{T} = C$

Work transfer $$W = \int p\,dV = p\int_1^2 dV = p\,(V_2 - V_1)$$

As per I-law for a process, $Q = W + \Delta U$

$$Q = p\,(V_2 - V_1) + m\,C_v\,(T_2 - T_1)$$
$$= m\,R\,(T_2 - T_1) + m\,C_v\,(T_2 - T_1)$$
$$= m\,(R + C_v)\,(T_2 - T_1)$$
$$= m\,C_p\,(T_2 - T_1)$$

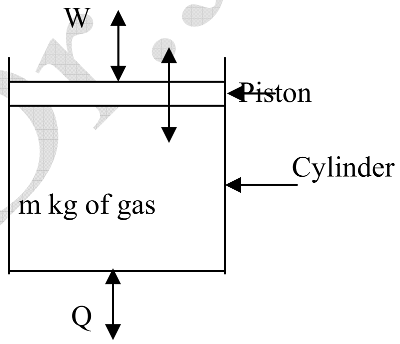
**Constant temperature process (Isothermal process or Hyperbolic process)**

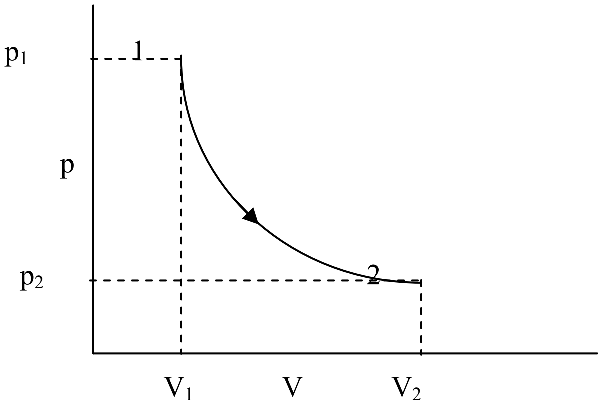
A process at a constant temperature is called an isothermal process. When a working substance in a cylinder behind a piston expands from a high pressure to a low pressure there is a tendency for the temperature to fall. In an isothermal expansion heat must be added continuously in order to keep the temperature at the initial value. Similarly in an isothermal compression heat must be removed from working fluid continuously during the process.

During constant temperature process, $T = C$

General gas equation → $\frac{pV}{T} = C$

For constant temperature process, $pV = C$





Work transfer, $W = \int p dV = \int \frac{C}{V} dV = C\int \frac{dV}{V} = pV \ln\left[\frac{V_2}{V_1}\right]$

$$= p_1V_1 \ln\left[\frac{V_2}{V_1}\right] = p_2V_2 \ln\left[\frac{V_2}{V_1}\right]$$

As per I-law for a process, $Q = W + \Delta U$

$$= pV \ln\left[\frac{V_2}{V_1}\right] + mC_v(T_2 - T_1)$$

$$= pV \ln\left[\frac{V_2}{V_1}\right] + 0 = p_1V_1 \ln\left[\frac{V_2}{V_1}\right]$$

During isothermal process, $Q = W$

*Note*: The isothermal and hyperbolic processes are identical only in the case of a perfect gas and not for a vapour. For example the isothermal expansion of wet steam is not hyperbolic.

**Constant entropy process (Isentropic process)**

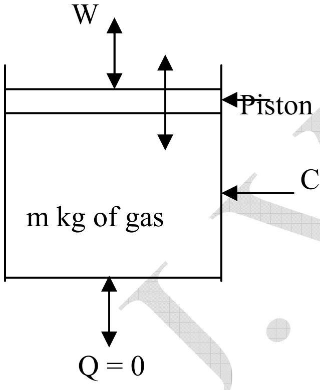
In an isentropic process, the heat transfer between the working fluid and surroundings is zero.

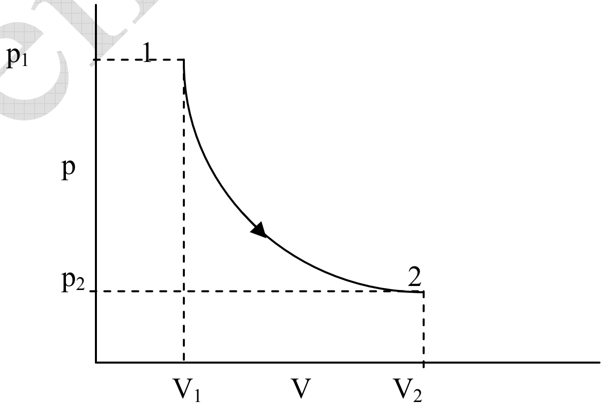
During isentropic process, $Q = 0$

The governing equation for isentropic or reversible adiabatic process is,

$$p\,V^{\gamma} = C$$

$\gamma$ → Specific heat ratio = $C_p/C_v$





Work transfer

$$W = \int pdV = \int_1^2 \frac{C}{V^{\gamma}} dV = C\int_1^2 \frac{dV}{V^{\gamma}}$$

$$= C\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_1}^{V_2}$$

$$= C\left[\frac{V_2^{-\gamma+1} - V_1^{-\gamma+1}}{-\gamma+1}\right]$$

$$= pV^{\gamma}\left[\frac{V_2^{-\gamma+1} - V_1^{-\gamma+1}}{-\gamma+1}\right]$$

$pV^{\gamma} = p_1V_1^{\gamma} = p_2V_2^{\gamma} = C$

$$W = \left[\frac{p_2V_2 - p_1V_1}{1-\gamma}\right]$$

Heat transfer = Q = W + ΔU = 0

Therefore we can write, $W = -\Delta U = -m\,C_v\,(T_2 - T_1)$

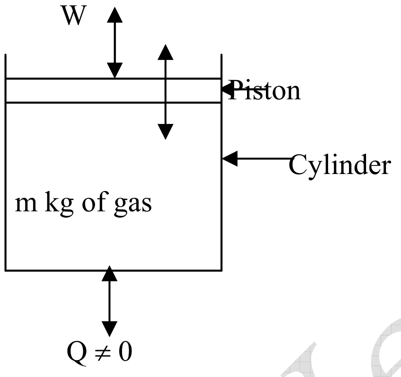
**Polytropic process**

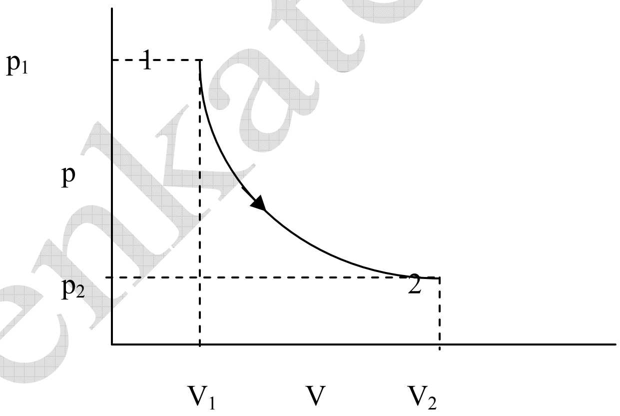
In polytropic process, there is heat transfer between the working fluid and surroundings. Pressure, volume and temperature are variables during a process.

The governing equation for isentropic or reversible adiabatic process is,

$$p\,V^n = C$$

n → Polytropic index





Work transfer

$$W = \int p\,dV = \int_1^2 \frac{C}{V^n}\,dV = C\int_1^2 \frac{dV}{V^n}$$

$$= C\left[\frac{V^{-n+1}}{-n+1}\right]_1^2$$

$$= C\left[\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1}\right]$$

$$= pV^n\left[\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1}\right]$$

$pV^n = p_1V_1^n = p_2V_2^n$

$$W = \left[\frac{p_2V_2 - p_1V_1}{1-n}\right] = \left[\frac{mR\,(T_2 - T_1)}{1-n}\right]$$

Heat transfer = Q = W + Δ U

$$Q = \left[\frac{mR\,(T_2 - T_1)}{1-n}\right] + m\,C_v\,(T_2 - T_1)$$

$$= \left[\frac{mR\,(T_2 - T_1)}{1-n}\left(1 + \frac{C_v\,(1-n)}{R}\right)\right]$$

$$= \left[\frac{mR\,(T_2 - T_1)}{1-n}\left(\frac{R + C_v - C_v n}{R}\right)\right]$$

$$= \left[\frac{mR\,(T_2 - T_1)}{1-n}\left(\frac{C_p - C_v + C_v - C_v n}{C_p - C_v}\right)\right]$$

$$= \left[\frac{mR\,(T_2 - T_1)}{1-n}\left(\frac{C_p - C_v n}{C_p - C_v}\right)\right]$$

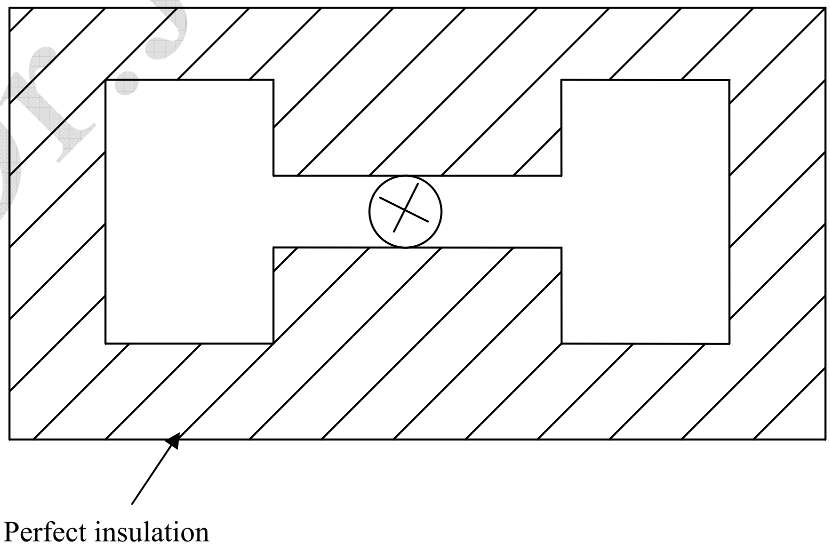
Dividing both numerator and denominator by $C_v$

$$Q = \left[\frac{mR\,(T_2 - T_1)}{1-n}\left(\frac{\gamma - n}{\gamma - 1}\right)\right]$$

$$Q = \left[\left(\frac{\gamma - n}{\gamma - 1}\right)W\right]$$

**Free expansion process (Constant internal energy process)**

Consider two vessels A and B interconnected by a short pipe with a valve 'V', and perfectly thermally insulated. Initially let the vessel A be filled with a fluid at a certain pressure and let vessel B be completely evacuated. When the valve V is opened, the fluid in vessel A will expand rapidly to fill the vessel B. The pressure finally will be lower than the initial pressure in the vessel A. This is known as "Free or Unrestricted expansion". The process is highly irreversible, since the fluid is eddying continuously during the process.



In free expansion process, W = 0;  Q = 0;

As per I-Law, $Q = W + \Delta U$

$\Delta U = 0$

$\therefore \quad U_2 = U_1$

### Steady Flow Energy Equation (SFEE) (I-Law for open system)

Consider a system as shown in fig. Let m kg/s of working fluid is entering and leaving the system.

Let, m = Mass flow rate of fluid through the system --- kg/s

$U_1$ = Internal energy of the fluid at inlet --- W

$C_1$ = Velocity of the fluid at inlet --- m/s

$p_1$ = Pressure of the fluid at inlet -- Pa

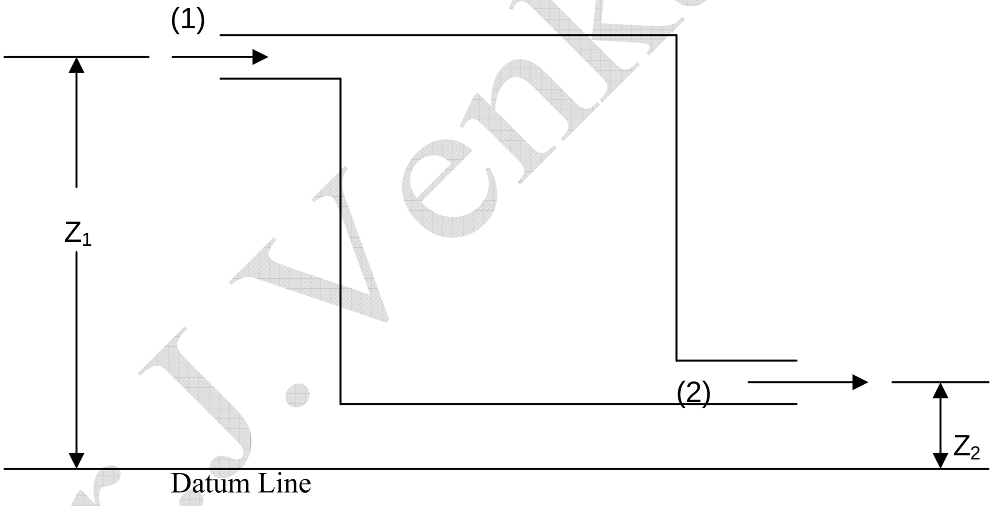
$V_1$ = Volume flow rate of fluid at inlet --- $m^3/s$

$Z_1$ = Height of the inlet section from datum --- m

Q = Heat transfer through the system --- W

W = Work transfer through the system --- W

$U_2$, $C_2$, $p_2$, $V_2$ and $Z_2$ are corresponding values at outlet



In addition to internal energy, other forms of energy associated with the mass entering and leaving the system is considered here.

Kinetic energy of fluid = m $C^2/2$

Potential energy of the fluid = m g Z

Flow energy of fluid = pV

Between inlet and outlet, the energy equation can be written as,

$$U_1 + p_1V_1 + m\frac{C_1^2}{2} + mgZ_1 + Q = U_2 + p_2V_2 + m\frac{C_2^2}{2} + mgZ_2 + W$$

$$mu_1 + mp_1v_1 + m\frac{C_1^2}{2} + mgZ_1 + Q = mu_2 + mp_2v_2 + m\frac{C_2^2}{2} + mgZ_2 + W$$

$$mh_1 + m\frac{C_1^2}{2} + mgZ_1 + Q = mh_2 + m\frac{C_2^2}{2} + mgZ_2 + W$$

Where, v → Specific volume = V/m

u → Specific internal energy = U/m

h → Specific enthalpy = u + pv

**Application of I-Law to open system**

***Water turbine***

In a water turbine water is supplied from a height. The potential energy of water is converted into kinetic energy when it enters into the turbine and part of it is converted into useful work which is used to generate electricity.

SFEE is given by,

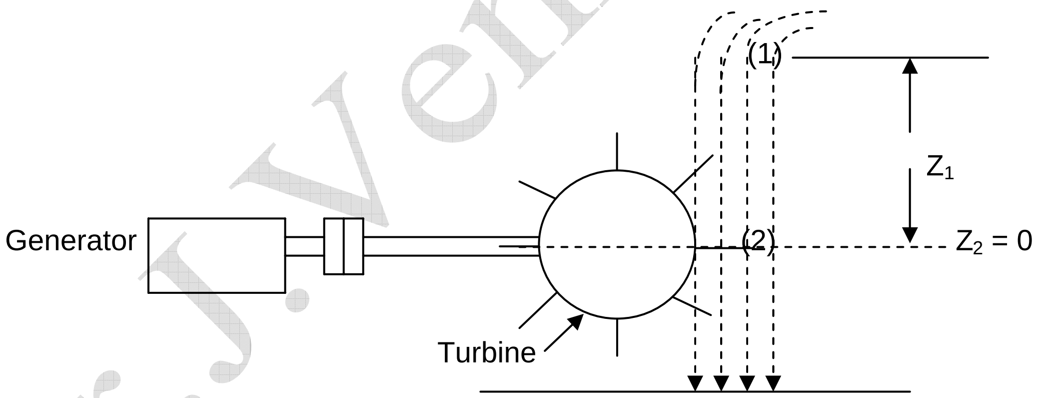
$$mu_1 + mp_1v_1 + m\frac{C_1^2}{2} + mgZ_1 + Q = mu_2 + mp_2v_2 + m\frac{C_2^2}{2} + mgZ_2 + W$$

Generally in water turbine,

Heat transfer (Q) = 0;   $Z_2 = 0$;  $v_1 = v_2$;  $T_1 = T_2$ or $U_1 = U_2$

Energy equation becomes,

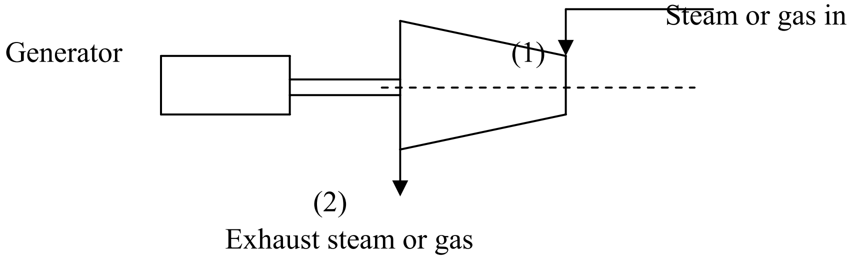
$$mp_1v_1 + m\frac{C_1^2}{2} + mgZ_1 = mp_2v_2 + m\frac{C_2^2}{2} + W$$



*Note*: W is positive since work is done by the system.

***Steam or Gas turbine***

In a steam or gas turbine, steam or gas is passed through the turbine and part of its energy is converted into work in the turbine. The output of the turbine runs a generator to produce electricity.



Generally in steam or gas turbine,

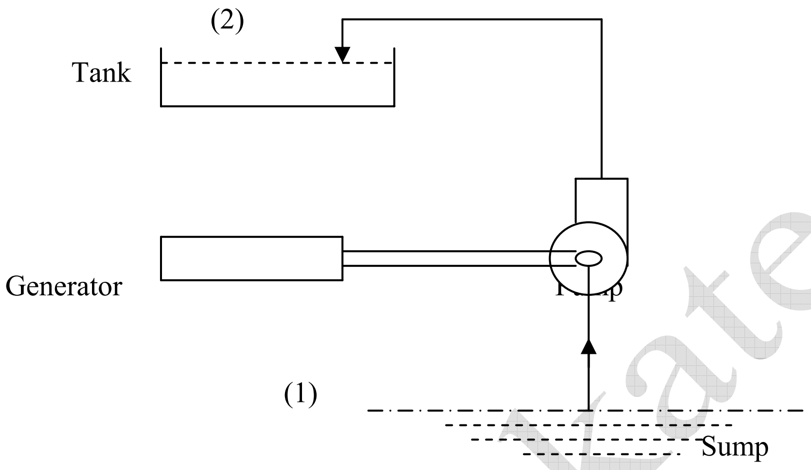
$Z_2 - Z_1 = 0$; $Q = 0$ (insulated turbine); $Q = -$ ve if not perfectly insulated.

Energy equation becomes,

$$mh_1 + m\frac{C_1^2}{2} - Q = mh_2 + m\frac{C_2^2}{2} + W$$

*Note*: W is positive since work is done by the system. Q is negative since the heat is transferred from the hot casing to low temperature surroundings.

***Water pump***



A water pump draws water from a lower level and pumps it to higher level. Work is required to run the pump and this may be supplied from an external source such as an electric motor or a diesel engine.

Generally in water pump,

Heat transfer (Q) = 0; $v_1 = v_2$; $T_1 = T_2$ or $U_1 = U_2$

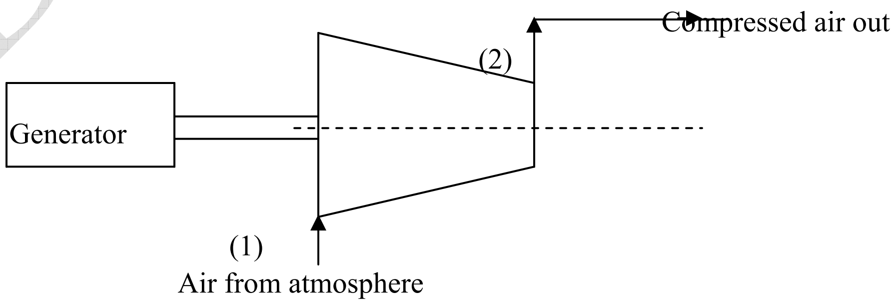
Energy equation becomes,

$$mp_1v_1 + m\frac{C_1^2}{2} + mgZ_1 = mp_2v_2 + m\frac{C_2^2}{2} + mgZ_2 - W$$

*Note*: W is negative since work is done on the system.

***Centrifugal air compressor***

A centrifugal compressor compresses air and supplies the same at moderate pressure and in large quantity.



Generally in centrifugal compressor,

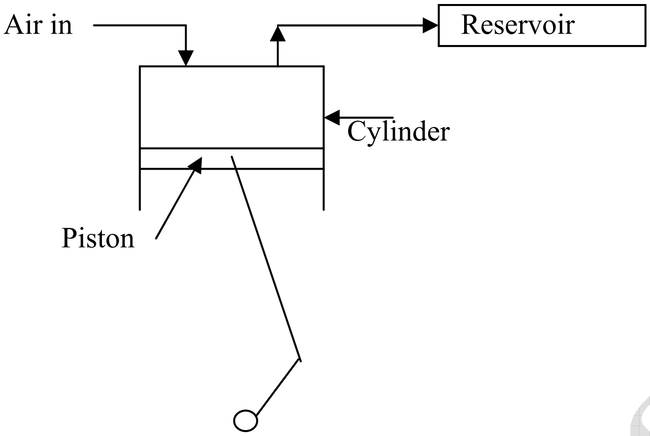
$Z_2 - Z_1 = 0$; $Q = -$ ve if not perfectly insulated.

Energy equation becomes,

$$mh_1 + m\frac{C_1^2}{2} - Q = mh_2 + m\frac{C_2^2}{2} - W$$

*Note*: W is negative since work is done on the system. Q also is negative since the heat is transferred from the hot casing to low temperature surroundings.

***Reciprocating air compressor***



The reciprocating air compressor draws in air from atmosphere and supply it at relatively higher pressure and in small quantity. The velocity of air entering and leaving the compressor is generally very small and is neglected.

Generally in reciprocating compressor,

$Z_2 - Z_1 = 0$; $C_1 = C_2$; Q = - ve if not perfectly insulated.

Energy equation becomes,

$$m\frac{C_1^2}{2} - Q = m\frac{C_2^2}{2} - W$$

*Note*: W is negative since work is done on the system. Q also is negative since the heat is transferred from the hot casing to low temperature surroundings.

***Heat Exchangers***

A heat exchanger is a device to transfer heat from one fluid to another fluid through walls. There is enthalpy change.

Examples: Boiler, Condenser, Evaporator.

Generally in heat exchangers,

$Z_2 - Z_1 = 0$; $C_1 = C_2$; Q = + ve in boilers and evaporators
Q = - ve in evaporators

*Boilers:*

A boiler produces high temperature vapour absorbing heat from the external source. The heat is transferred to liquid.

Energy equation becomes, $mh_1 + Q = mh_2$

*Condensers:*

A condenser is a device to condense the vapour by rejecting its heat to the cooling medium. Here the heat is rejected by the system.

Energy equation becomes, $mh_1 - Q = mh_2$

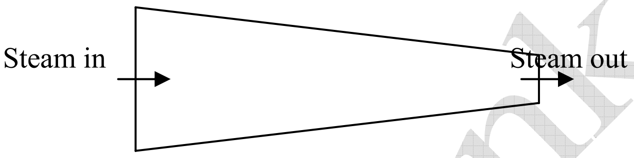
*Evaporators:*

An evaporator is device which produces low temperature vapour by absorbing heat from relatively hot source. Here the heat is absorbed by the system.

Energy equation becomes, $mh_1 + Q = mh_2$

***Steam nozzle***

Steam nozzles are used in steam engines and steam power plants. It converts the pressure energy into kinetic energy. Generally the nozzles are insulated. In steam nozzles the steam is expanded isentropically.



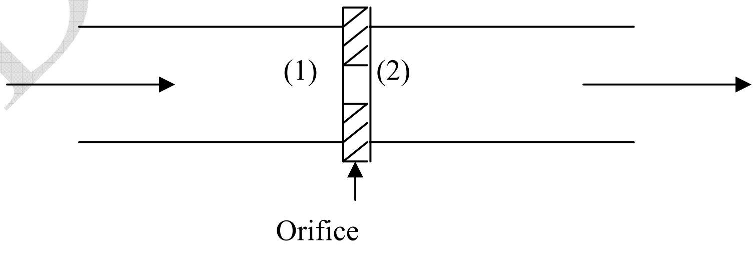
Generally in nozzles, $Z_2 - Z_1 = 0$; $Q = 0$; $W = 0$;

Energy equation becomes, $mh_1 + m\frac{C_1^2}{2} = mh_2 + m\frac{C_2^2}{2}$

The exit velocity of the steam can be written as, $C_2 = \sqrt{C_1^2 + 2(h_1 - h_2)}$

***Throttling process (Constant enthalpy process)***

When a fluid flows through a constricted passage, like a partially opened valve, an orifice, or a porous plug, there is an appreciable drop in pressure and the flow is said to be throttled. The fig shows the process of throttling by an orifice. The pipe is perfectly insulated.



Heat transfer (Q) = 0

Work transfer (W) = 0

$C_1 = C_2$; $Z_1 = Z_2$

Energy equation becomes, $h_1 = h_2$

The throttling process is commonly used for the following process:

(i) For determining the condition of steam (dryness fraction)

(ii) For controlling the speed of the turbine

(iii) Used in refrigeration plant for reducing the pressure of the refrigerant before entry into the evaporator

### Path and Point function

Consider a process 1-2 as shown in fig (a). The process 1-2 may follow a path A, B or C. The area under a curve in p-V diagram represents the work transfer and the area under a curve in a T-s diagram represents the heat transfer as shown in fig (b). Work or heat depends on the path of the system, but not on the end states. For this reason, work or heat is called a path function. $W_{1-2}$ or $Q_{1-2}$ is an inexact or imperfect differential.

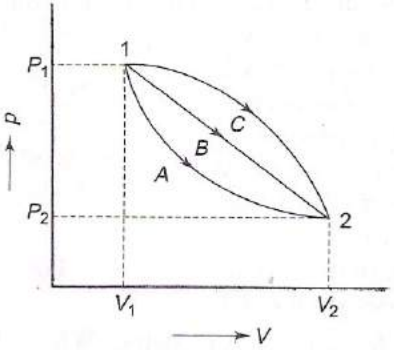
$$\int_1^2 dW = W_2 - W_1$$

Thermodynamic properties are point functions, since for a given state, there is a definite value for each property. The change in a thermodynamic property of a system in a change of state is independent of the path of the system and depends only on the end states of the system. The differentials of point functions are exact or perfect differentials.

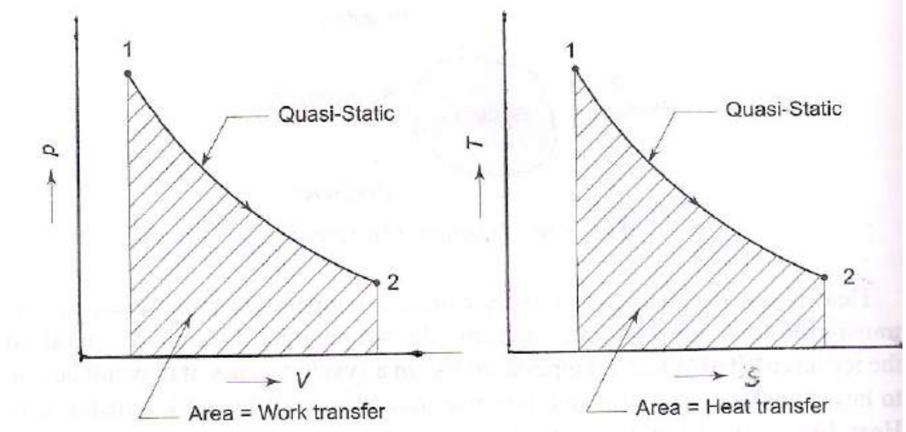
$$\int_{V_1}^{V_2} dV = V_2 - V_1$$

*Note:* In a cyclic process, the change in any property is zero.

$$\oint dV = 0, \quad \oint dp = 0, \quad \oint dT = 0$$



(a)



(b)

## Internal energy – A property of the system

Consider a system which changes its state from state 1 to state 2 by following the path A and returns to its original state 1 following (i) the path B and (ii) the path C.

Consider a cycle 1-A-2-B-1.

As per I-law for a cycle, $\oint dQ = \oint dW$

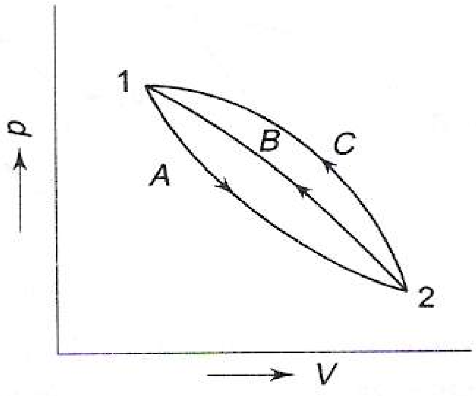
$$Q_A + Q_B = W_A + W_B$$

$$Q_A - W_A = W_B - Q_B \qquad \text{---------- (1)}$$

For a process 1-A-2,

$$Q_A = W_A + \Delta U_A$$

$$Q_A - W_A = \Delta U_A \qquad \text{--------- (2)}$$



For a process 2-B-1,

$$Q_B = W_B + \Delta U_B$$

$$W_B - Q_B = - \Delta U_B \qquad \text{--------- (3)}$$

Therefore, from (1), (2) and (3), we can write,

$$\Delta U_A = - \Delta U_B \qquad \text{--------- (4)}$$

Consider a cycle 1-A-2-C-1.

$$Q_A + Q_C = W_A + W_C$$

$$Q_A - W_A = W_C - Q_C \qquad \text{--------- (5)}$$

For a process 2-C-1,

$$Q_C = W_C + \Delta U_C$$

$$W_C - Q_C = - \Delta U_C \qquad \text{--------- (6)}$$

Therefore, from (2), (5) and (6), we can write,

$$\Delta U_A = - \Delta U_C \qquad \text{--------- (7)}$$

From (4) and (7), $\Delta U_C = \Delta U_B$

From the above, it can be concluded that the internal energy depends on end states and independent of path. Therefore the internal energy is a property.

## Molar volume ($V_m$)

Molar volume $V_n$ of a substance is the ratio of the volume $V$ of a sample of that substance to the number of moles in the sample.

The molar volume for an ideal gas at 298.15 K and 1 bar is 0.024788 $m^3 mol^{-1}$ or 24.788 l/mol.

$V_m$ = V / Number of moles (n) ------ $m^3$/kmol

V → Volume in $m^3$

M → Molar mass in kg/kmol

$\rho$ → Density in kg/$m^3$

$$p\,V = m\,R\,T = n\,R_u\,T$$

$$p\,V_m = R_u\,T$$

R → Characteristic gas constant in J/kg-K

$R_u$ → Universal gas constant in J/kmol-K

**Characteristic and Universal Gas constant**

Characteristic gas constant (R) = $C_p - C_v$ J/kg-K

Universal gas constant ($R_u$) = $C_{p(mole)} - C_{v(mole)}$ J/kmol-K

$C_{p(mole)}$ → Molar specific heat at constant pressure = M $C_p$ J/kmol-K

$C_{v(mole)}$ → Molar specific heat at constant volume = M $C_v$ J/kmol-K

**Molar mass and Molecular mass (M)**

Molar mass is the mass of one mole of the substance and is defined in kg/kmol. Molecular mass (M) is the mass of one molecule of the substance and is defined in amu (automic mass unit). Molar mass and molecular mass differs only in units.

1 amu ≈ $1.660538782(83) \times 10^{-27}$ kg

**Note:**

It is advisable to use appropriate units while numerical problems are being solved.

| | |
|---|---|
| Pressure (p) | N/$m^2$ (or) Pa |
| Temperature (T) | K |
| Volume (V) | $m^3$ |
| Volume flow rate ($\overset{*}{V}$) | $m^3$/s |
| Mass (m) | kg |
| Mass flow rate ($\overset{*}{m}$) | kg/s |
| Velocity (C) | m/s |
| Angular velocity ($\omega$) | rad/s |
| Speed (N) | rpm |
| Energy | Nm (or) J |
| Force (F) | N |
| Power (P) | J/s (or) W |
| Specific heat ($C_p$ or $C_v$) | J/kg-K (or) J/kg °C |
| Characteristic Gas constant (R) | J/kg-K |
| Universal gas constant ($R_u$) | 8134 J/kmol-K |

| | |
|---|---|
| Molar volume or Molecular volume ($V_m$) | $m^3$/kmol |
| Molecular mass (M) | amu |
| Enthalpy (H) | J |
| Specific enthalpy (h) | J/kg |
| Entropy (S) | J/K |
| Specific entropy (s) | J/kg-K |
| Molar specific heat ($C_{p(mole)}$ or $C_{v(mole)}$) | J/kmol-K |
| Gravitational acceleration (g) | 9.81 $m/s^2$ |

### Limitations of I-law of thermodynamics

The I-law states that, in carrying out a process, heat & work are mutually convertible, a balance of energy must hold as internal energy, the energy is neither gained nor lost in a process, it only transforms.

But, this law does not place any distinction on the direction of the process, under consideration.

According to the I-law, it is assumed that any change of thermodynamic state can take place in either direction. But it has been found that this is not the case particularly in the inter-conversion of heat and work.

The processes naturally proceed in certain directions and not in the opposite directions, even though the reversal of the processes does not violate the I-law.

Example: If two metal blocks at temperatures $T_1$ & $T_2$ ($T_1 > T_2$) are brought into contact with each other, the heat flows from the high temperature block to the low temperature block till the temperature of both the blocks are equal. The heat flows from low temperature block to the high temperature block is impossible, i.e., the original temperatures $T_1$ & $T_2$ cannot be restored.

Therefore the I-law is a necessary but not sufficient due to the following restrictions:

(i) No restriction on the direction of energy flow

(ii) It does not deal with the portion of heat that may be converted into useful work

# PROBLEMS

1. **Following amount of heat transfer occurs during a cycle comprising of four processes. Calculate the workdone of the cycle and indicate about the type of work. +120 kJ, -20 kJ, +16 kJ and +24 kJ.**

*Given*: Cycle with four processes

Heat transfer during process 1 – 2 = $Q_{1\text{-}2}$ = 120 kJ

Heat transfer during process 2 – 3 = $Q_{2\text{-}3}$ = - 20 kJ

Heat transfer during process 3 – 4 = $Q_{3\text{-}4}$ = 16 kJ

Heat transfer during process 4 – 1 = $Q_{4\text{-}1}$ = 24 kJ

*Required:* Workdone and type of work

*Solution:*

First law for cycle → Net heat transfer = Net work transfer

$$\oint dQ = \oint dW$$

$$\oint dW = 120 + (-20) + 16 + 24 = 140\,kJ$$

The workdone is positive, therefore the work is done by the system.

2. **The following data refer to a closed system which undergoes a thermodynamics cycle consisting of four processes. Show that the data is consistent with the I-law of thermodynamics and calculate, (a) Net rate of work output in kW and (b) Change in internal energy.**

| Process | Heat Transfer (kJ/min) | Work Transfer (kJ/min) |
|---|---|---|
| 1 – 2 | Nil | - 1000 |
| 2 – 3 | 40,000 | Nil |
| 3 – 4 | - 4,000 | 26,000 |
| 4 – 5 | - 12,000 | - 1000 |

*Given:* $Q_{1\text{-}2}$ = 0, $Q_{2\text{-}3}$ = 40000 kJ/min, $Q_{3\text{-}4}$ = -4000 kJ/min, $Q_{4\text{-}1}$ = -12000 kJ/min

$W_{1\text{-}2}$ = -1000 kJ/min, $W_{2\text{-}3}$ = 0, $W_{3\text{-}4}$ = -4000 kJ/min, $W_{4\text{-}1}$ = -12000 kJ/min

*Required:* (a) Net work in kW (b) $\Delta$ U

*Solution:*

(a) First law for cycle → Net heat transfer = Net work transfer

$$\oint dQ = \oint dW$$

$$\oint dW = -1000 + 0 + 26000 - 1000 = 24000\,kJ/\text{min}$$

$$= 24000 / 60 = 400 \text{ kW}$$

$$\oint dQ = 40000 - 4000 - 120000 = 24000 \, kJ/\mathbf{min}$$

$$= 24000 / 60 = 400 \text{ kW}$$

$$\oint dQ = \oint dW$$

Therefore, the data is consistent with the I-law of thermodynamics.

(b) Change in internal energy during process 1 – 2 = $U_2 - U_1$

I-law for a process

$$Q = W + \Delta U$$

$$U_2 - U_1 = Q_{1\text{-}2} - W_{1\text{-}2}$$

$$= 0 - (-1000) = 1000 \text{ kJ/min}$$

$$U_3 - U_2 = Q_{2\text{-}3} - W_{2\text{-}3}$$

$$= 40000 - 0 = 40000 \text{ kJ/min}$$

$$U_4 - U_3 = Q_{3\text{-}4} - W_{3\text{-}4}$$

$$= -4000 - 26000 = -30000 \text{ kJ/min}$$

$$U_1 - U_4 = Q_{4\text{-}1} - W_{4\text{-}1}$$

$$= -12000 - (-1000) = -11000 \text{ kJ/min}$$

3. **Calculate the workdone when the volume changes from 4 m$^3$ to 8 m$^3$ through a non-flow quasi-static process in which the pressure p is given by, p = (4 V – 5) bar.**

*Given:* A process

Initial volume ($V_1$) = 4 m$^3$

Final volume ($V_2$) = 8 m$^3$

p = (4 V – 5) bar

*Required:* Workdone

*Solution*

Work transfer in non-flow process is $W = \int_1^2 p\,dV = \int_1^2 (4V - 5)\,dV \times 10^5$

$$= \left[\frac{4V^2}{2} - 5V\right]_1$$

$$= [\,4\,(V_2^2 - V_1^2)/2 - 5\,(V_2 - V_1)] \times 10^5$$

$$= [4 \times (8^2 - 4^2)/2 - 5 \times (8 - 4)] \times 10^5$$

$$= \mathbf{76 \times 10^5 \text{ J} - Ans}$$

4. **The internal energy of a certain substance is given by the equation u = 3.56 p v + 84, where u is given in kJ/kg, p is in kPa and v is in m$^3$/kg. A system composed of 3 kg of this substance expands from an initial pressure of 500 kPa and a volume of 0.22 m$^3$ to a final pressure of 100 kPa in a process in which pressure and volume are related by pV$^{1.2}$ = C. (a) If the expansion is quasi-static, find heat transfer, workdone and change in internal energy for the process (b) In another process the same system expands according to the same pressure-volume relationship as in part (a), and from the same initial state to the same final**

**state as in part (a), but the heat transfer in this case is 30 kJ. Find the work transfer for this process (c) Explain the difference in work transfer in part (a) and part (b).**

*Given:*

Specific internal energy = u = 3.56 p v + 84 kJ/kg

Mass of the working fluid (m) = 3 kg

Initial pressure ($p_1$) = 500 kPa

Initial volume ($V_1$) = 0.22 $m^3$

Final pressure ($p_2$) = 100 kPa

Index of expansion (n) = 1.2

*Required:* (a) Q, W, Δ U (b) W (c) Difference W in (a) and (b)

*Solution:*

(a) Heat transfer during a process, $Q = W + \Delta U$

$$u = 3.56\, p\, v + 84$$

$$u_1 = 3.56\, p_1\, v_1 + 84$$

$$u_2 = 3.56\, p_2\, v_2 + 84$$

$$\Delta u = 3.56\,(p_2\, v_2 - p_1\, v_1)$$

$$\Delta U = m\, \Delta u \quad \text{and} \quad V = m\, v$$

$$\Delta U = 3.56\,(p_2\, V_2 - p_1\, V_1)$$

*To find $V_2$*

$$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{1/n}$$

$$\frac{V_2}{0.22} = \left(\frac{5}{1}\right)^{1/1.2}$$

$$V_2 = 0.841\ m^3$$

$$\therefore \quad \Delta U = 3.56\,(100 \times 0.841 - 500 \times 0.22)$$

= **- 92.204 kJ --- Ans**

Process follows the law $pV^{1.2} = C$.

$$W = (p_2V_2 - p_1V_1) / (1 - n)$$

$$= (100 \times 10^3 \times 0.841 - 500 \times 10^3 \times 0.22)\ (1 - 1.2)$$

= 129500 J **= 129.5 kJ --- Ans**

$$Q_{1\text{-}2} = W_{1\text{-}2} + \Delta U$$

= 129.5 – 92.204 = **37.296 kJ --- Ans**

(b) Heat transfer = 30 kJ

$$W_{1\text{-}2} = Q_{1\text{-}2} - \Delta U$$

= 30 – (- 92.204) = **122.204 kJ --- Ans**

(c) The work in part (b) is not equal to ∫p dV. The process in part (b) is not quasi-static.

5. **A fluid is confined in a cylinder by a spring loaded, frictionless piston so that the pressure in the fluid is a linear function of the volume ($p = a + bV$). The internal energy of the fluid is given by the equation, $U = 34 + 3.15\ p\ V$, where, U is in kJ, p is in kPa and V is in $m^3$. If the fluid changes from an initial state of 170 kPa, 0.03 $m^3$ to a final state of 400 kPa, 0.06 $m^3$, with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer.**

*Given:*

Pressure $p = a + bV$

Internal energy (U) $= 34 + 3.15\ pV$ kJ

Initial pressure ($p_1$) = 170 kPa

Initial volume ($V_1$) = 0.03 $m^3$

Final pressure ($p_2$) = 400 kPa

Final volume ($V_2$) = 0.06 $m^3$

*Required:* W and Q

*Solution:*

$$U_1 = 34 + 3.15\ p_1V_1$$

$$U_2 = 34 + 3.15\ p_2V_2$$

$$\Delta U = 3.15\ (p_2V_2 - p_1V_1)$$

$$= 3.15 \times (400 \times 10^3 \times 0.06 - 170 \times 10^3 \times 0.03)$$

$$= 59535 \text{ J}$$

$$p_1 = a + b\ V_1$$

$$170 = a + 0.03\ b \quad \text{--- (1)}$$

$$p_2 = a + bV_2$$

$$400 = a + 0.06\ b \quad \text{--- (2)}$$

From (1) and (2) $230 = 0.03\ b$, $b = 7666.67$ kPa

Substituting in (1), $a = -60$ kPa

Work transfer is $W = \int_1^2 p\,dV$

$$= \int_1^2 (a + bV)\,dV$$

$$= [a\ V + b\ V^2/2] = a\ (V_2 - V_1) + b\ (V_2^2 - V_1^2)/2$$

$$= -60 \times (0.06 - 0.03) + 7666.67 \times (0.06^2 - 0.03^2)/2$$

**= 8.55 kJ = 8550 J --- Ans**

Work is positive, i.e., Work is done by the system.

$$Q_{1\text{-}2} = W_{1\text{-}2} + \Delta U$$

= 8550 + 59535 **= 68085 J --- Ans**

Heat transfer is positive, i.e., Heat is supplied to the system.

**6. A steam turbine operates under steady flow conditions. It receives 7200 kg/h of steam from the boiler. The steam enters the turbine at enthalpy of 2800 kJ/kg, a velocity of 4000 m/min and an elevation of 4 m. The steam leaves the turbine at enthalpy of 2000 kJ/kg, a velocity of 8000 m/min and an elevation of 1 m. Due to radiation, heat losses from the turbine to the surroundings amount to 1580 kJ/h. Calculate the output of the turbine.**

*Given:*

Mass flow rate of steam (m) = 7200 kg/h = 2 kg/s

Initial specific enthalpy ($h_1$) = 2800 kJ/kg

Initial velocity ($C_1$) = 4000 m/min = 66.67 m/s

Elevation of inlet ($Z_1$) = 1 m

Final specific enthalpy ($h_2$) = 2000 kJ/kg

Final velocity ($C_2$) = 8000 m/min = 133.3 m/s

Elevation of outlet ($Z_2$) = 4 m

Heat losses from turbine (Q) = -1580 kJ/h = 0.4388 kJ/s

*Required:* Work output

*Solution:*

SFEE is given by, $m (h_1 – h_2) + m (C_1^2 – C_2^2)/2 + m g (Z_1 – Z_2) + Q = W$

$$W = 2 (2800 – 2000) \times 10^3 + 2 \times (66.67^2 – 133.3^2)/2$$
$$+ 2 \times 9.81 \times (1 – 4) – 0.4388 \times 10^3$$

**= 1586178.3 W --- Ans**

**7. Steam enters a nozzle at a pressure of 7 bar and 20°C (initial enthalpy = 2850 kJ/kg) and leaves at a pressure of 1.5 bar. The initial velocity of steam at the entrance is 40 m/s and the exit velocity of steam from nozzle is 700 m/s. The mass flow rate through the nozzle is 1400 kg/h. The heat loss from the nozzle is 11705 kJ/h. Determine the final enthalpy of steam and nozzle area if the specific volume at outlet is 1.24 m³/kg.**

*Given:*

Initial pressure ($p_1$) = 7 bar

Initial velocity ($C_1$) = 40 m/s

Initial enthalpy ($h_1$) = 2850 kJ/kg

Final velocity ($C_2$) = 700 m/s

Specific volume at outlet ($v_2$) = 1.24 $m^3$/kg

Mass flow rate (m) = 1400 kg/h = 0.389 kg/s

Heat loss from nozzle (Q) = -11105 kJ/h = -3251.4 J/s

*Required:* $h_2$ and $A_2$

*Solution:*

SFEE is given by, $m (h_1 – h_2) + m (C_1^2 – C_2^2) / 2 + m g (Z_1 – Z_2) + Q = W$

Assume $Z_1 – Z_2 = 0$ if not given. For nozzle W = 0

$$m (h_1 – h_2) + m (C_1^2 – C_2^2) + Q = 0$$

$$0.389 \times (2850 - h_2) \times 10^3 + 0.389 \times (40^2 – 700^2) / 2 – 3251.4 = 0$$

$h_2$ = **2614.16 kJ/kg ---- Ans**

Mass flow rate (m) = $A_2 C_2 / v_2 = A_1 C_1 / v_1$

$0.389 = A_2 \times 700 / 1.24$

$A_2$ = **0.000689 $m^2$ --- Ans**

8. **Air flows at the rate of 0.5 kg/s through an air compressor, entering at 7 m/s, 100 kPa and 0.95 $m^3$/kg and leaving at 5 m/s, 700 kPa, and 0.19 $m^3$/kg. The internal energy of air leaving is 90 kJ/kg greater than that of the air entering. Cooling water in the compressor jackets absorbs heat from the air at the rate of 58 kW. (a) Compute the rate of shaft work input to the air in kW (b) Find the ratio of the inlet pipe diameter to outer pipe diameter.**

*Given:*

Mass flow rate (m) = 0.5 kg/s

Initial velocity ($C_1$) = 7 m/s

Initial specific volume ($v_1$) = 0.95 $m^3$/kg

Initial pressure ($p_1$) = 100 kPa = 1 bar

Final velocity ($C_2$) = 5 m/s

Final pressure ($p_2$) = 700 kPa = 7 bar

Final specific volume ($v_2$) = 0.19 $m^3$/kg

$u_2 - u_1$ = 90 kJ/kg

Heat liberated by the air (Q) = 58 kJ/s ( - ve)

*Required:* (a) W (b) $d_1 / d_2$

*Solution:*

(a) SFEE is given by,

$$m (u_1 - u_2) + m (p_1v_1 - p_2v_2) + m (C_1^2 - C_2^2) / 2 + m\, g\, (Z_1 - Z_2) + Q = W$$

Assume $Z_2 - Z_1 = 0$ if not given.

$$0.5 \times (-90) \times 10^3 + 0.5 \times (1 \times 0.95 \times 10^5 - 7 \times 0.19 \times 10^5) + 0.5\ (7^2 - 5^2) / 2 - 58000 = W$$

$$W = -\ 121994\ W$$

∴ Work input to the compressor = **121994 W --- Ans**

(b) Mass flow rate is given by,

$$m = A_1 C_1 / v_1 = A_2 C_2 / v_2$$

$$A_1 / A_2 = C_2 v_1 / (C_1 v_2)$$

$$= 5 \times 0.95 / (7 \times 0.19) = 3.57$$

$$A_1 / A_2 = d_1^2 / d_2^2$$

$d_1 / d_2 = \sqrt{3.57}$ = **1.8898 --- Ans**

9. **In a steam power station, steam flows steadily through a 0.2 diameter pipeline from the boiler to the turbine. At the boiler end, the steam conditions are found to be 4 MPa, 400°C, specific enthalpy 3214.6 kJ/kg and specific volume 0.073 $m^3$/kg. At the turbine end, the conditions are found to be 3.5 MPa, 392°C, specific enthalpy 3202.6 kJ/kg and specific volume 0.084 $m^3$/kg. There is a heat loss of 8.5 kJ/kg from the pipeline. Calculate the steam flow rate.**

*Given:*

Diameter of pipeline (d) = 0.2 m

Pressure of the steam at inlet ($p_1$) = 4 MPa = 40 bar

Initial temperature ($T_1$) = $400^o$ C = 673 K

Initial specific enthalpy ($h_1$) = 3213.6 kJ/kg

Initial specific volume ($v_1$) = 0.073 $m^3$/kg

Pressure of the steam at outlet ($p_2$) = 3.5 MPa = 35 bar

Final temperature ($T_2$) = $392^o$ C = 563 K

Final specific enthalpy ($h_2$) = 3202.6 kJ/kg

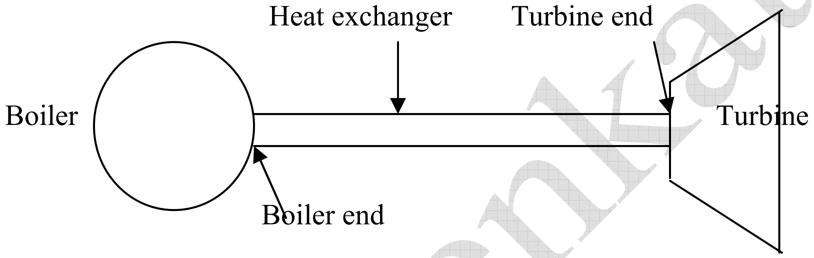
Initial specific volume ($v_1$) = 0.073 $m^3$/kg

Heat losses from turbine (q) = -8.5 kJ/kg

*Required:* Steam flow rate

*Solution:*

This is the problem of heat exchanger.



SFEE is given by, $m (h_1 - h_2) + m (C_1^2 - C_2^2) / 2 + m g (Z_1 - Z_2) + Q = W$

Assume $Z_1 - Z_2 = 0$ if not given. For heat exchanger, W = 0 ;

$$\text{Mass flow rate } (m) = A_1 C_1 / v_1 = A_2 C_2 / v_2$$

$$A_1 = A_2$$

$$\therefore C_1 = C_2 v_1 / v_2 = C_2\ 0.073 / 0.084 = 0.869\ C_2 \quad \text{--- (1)}$$

SFEE for heat exchanger becomes,

$$m (h_1 - h_2) + m (C_1^2 - C_2^2) / 2 + m\,q = 0$$

$$(h_1 - h_2) + (C_1^2 - C_2^2) / 2 + q = 0$$

$$(3213.6 - 3202.6) \times 10^3 + ((0.869C_2)^2 - C_2^2)/2 - 8.5 \times 10^3 = 0$$

$$C_2 = 142.9 \text{ m/s}$$

$$m = A_2 C_2 / v_2 = (\pi / 4) d^2 C_2 / v_2$$

$$= \pi \times 0.2^2 \times 142.9 / (4 \times 0.084) = \mathbf{53.45\ kg/s} \quad \text{--- } \mathbf{Ans}$$

**10. A piston and cylinder machine contains a fluid system which passes through a complete cycle of four operations.**

| Process | Q (kJ/min) | W (kJ/min) | ΔU (kJ/min) |
|---|---|---|---|
| a – b | 0 | 2170 | --- |

|  |  |  |  |
|---|---|---|---|
| b – c | 21000 | 0 | --- |
| c – d | -2100 | --- | -36600 |
| d – a | --- | --- | --- |

**During cycle the sum of all heat transfers is -170 kJ. The system operates 100 cycles per min. Complete the following table showing the method for each item and compute the net rate if output in kW.**

*Given:*

Net heat transfer = -170 kJ

No of cycles = 100 / min

*Required:* To complete the table and to determine the net work,

*Solution:*

$\Delta U$ → Change in internal energy

I-law for a cycle, Net work transfer = Net heat transfer

$$\therefore \quad \text{Net work output} = -170\text{ kJ}$$

$$= -170 \text{ x no of cycles per second}$$

$$= -170 \times 100 / 60 = \mathbf{-283.33\ kW} \text{ --- } \mathbf{Ans}$$

I-law for a process is $Q = W + \Delta U$

$$(\Delta U)_{a\text{-}b} = Q_{a\text{-}b} - W_{a\text{-}b}$$

$$= 0 - 2170 = \mathbf{-2170\ kJ/min\ \text{---}\ Ans}$$

$$(\Delta U)_{b\text{-}c} = Q_{b\text{-}c} - W_{b\text{-}c}$$

$$= 21000 - 0 = \mathbf{-21000\ kJ/min\ \text{---}\ Ans}$$

$$W_{c\text{-}d} = Q_{c\text{-}d} - (\Delta U)_{c\text{-}d}$$

$$= -2100 + 36600 = 34500\ \mathbf{kJ/min\ \text{---}\ Ans}$$

Net Heat transfer, $Q_{d\text{-}a} = Q_{a\text{-}b} + Q_{b\text{-}c} + Q_{c\text{-}d} + Q_{d\text{-}a}$

$$= -17000 - 0 - 21000 + 2100$$

$$= \mathbf{-35900\ kJ/min\ \text{---}\ Ans}$$

Net work transfer, $W_{d\text{-}a} = W_{a\text{-}b} + W_{b\text{-}c} + W_{c\text{-}d} + W_{d\text{-}a}$

$$= -17000 - 2170 - 0 - 34500$$

$$= \mathbf{-53670\ kJ/min\ \text{---}\ Ans}$$

For a cycle, sum of $\Delta U = 0$

$$(\Delta U)_{a\text{-}b} + (\Delta U)_{b\text{-}c} + (\Delta U)_{c\text{-}d} + (\Delta U)_{d\text{-}a} = 0$$

$$-2170 + 21000 - 36600 + (\Delta U)_{d\text{-}a} = 0$$

$$(\Delta U)_{d\text{-}a} = \mathbf{17770\ kJ/min\ \text{----}\ Ans}$$

| Process | Q (kJ/min) | W (kJ/min) | ΔU (kJ/min) |
|---|---|---|---|
| a – b | 0 | 2170 | -2170 |

| | | | |
|---|---|---|---|
| b – c | 21000 | 0 | 21000 |
| c – d | -2100 | 34500 | -36600 |
| d – a | -35900 | -53670 | 17770 |

11. **12 kg of air per min is delivered by a centrifugal compressor. The inlet and outlet conditions of air are $C_1 = 12$ m/s, $p_1 = 1$ bar, $v_1 = 0.5$ m$^3$/kg and $C_2 = 90$ m/s, $p_2 = 8$ bar, $v_2 = 0.14$ m$^3$/kg. The increase in enthalpy of air passing through compressor is 150 kJ/kg and heat loss to the surroundings is 700 kJ/min. Find motor power required to drive the compressor and ratio of inlet and outlet pipe diameters. Assume that inlet and discharge lines are at the same level.**

*Given:*

Mass flow rate of air (m) = 12 kg/min = 0.2 kg/s

Increase in enthalpy $(h_2 – h_1)$ = 150 kJ/kg

Heat loss to the surroundings (Q) = 700 kJ/min = 11666.7 J/s (-ve)

$Z_1 – Z_2 = 0$

*Required:* Power and Diameter ratio

*Solution:*

SFEE is given by, $m (h_1 – h_2) + m (C_1^2 – C_2^2) / 2 + m g (Z_1 – Z_2) + Q = W$

$$0.2 \text{ x } (-150 \text{ x } 10^3) + 0.2 \text{ x } (12^2 – 90^2) /2 – 11666.7 = W$$

$$W = -42462.3 \text{ W}$$

Work in W is Power. ∴ P = -42462.3 W

Power required to drive the compressor = **42262.3 W --- Ans**

$$\text{Mass flow rate (m)} = A_1 C_1 / v_1 = A_2 C_2 / v_2$$

$$A_1 / A_2 = v_1 C_2 / (v_2 C_1)$$

$$= 0.5 \text{ x } 90 / (0.14 \text{ x } 12) = 26.7857$$

$d_1 / d_2 = \sqrt{(A_1 / A_2)} = \sqrt{26.7857} =$ **5.175 --- Ans**

12. **At the inlet of a certain nozzle, the enthalpy of the fluid passing is 3000 kJ/kg and the velocity is 60 m/s. At the discharge end, the enthalpy is 2762 kJ/kg. The nozzle is horizontal and there is negligible heat loss from it. (a) Find the velocity of fluid at exit (b) If the inlet area is 0.1 m$^2$ and the specific volume at inlet is 0.187 m$^3$/kg, find the mass flow rate (c) If the specific volume at the nozzle exit is 0.498 m$^3$/kg, find the exit area of the nozzle.**

*Given:*

Nozzle

Initial enthalpy $(h_1)$ = 3000 kJ/kg

Initial velocity $(C_1)$ = 60 m/s

Heat transfer (Q) = 0

*Required:* (a) $C_2$ (b) m (c) $A_2$

*Solution:*

(a) In nozzle flow, W = 0; $Z_1 – Z_2 = 0$ (if not given);

SFEE is given by, $m (h_1 – h_2) + m (C_1^2 – C_2^2) / 2 + m g (Z_1 – Z_2) + Q = W$

For given nozzle, $m\,(h_1 - h_2) + m\,(C_1^2 - C_2^2) / 2 = 0$

$$(h_1 - h_2) + (C_1^2 - C_2^2) / 2 = 0$$

$$C_2^2 = C_1^2 + 2\,(h_1 - h_2)$$

$$= 60^2 + 2 \times (3000 - 2762) \times 10^3$$

$C_2$ = **692.5 m/s --- Ans**

(b) $A_1 = 0.1\ \text{m}^2$ ; $v_1 = 0.187\ \text{m}^3/\text{kg}$

Mass flow rate $(m) = A_1 C_1 / v_1 = A_2 C_2 / v_2$

$= 0.1 \times 60 / 0.187 =$ **32.08 kg/s --- Ans**

(c) $v_2 = 0.498\ \text{m}^3/\text{kg}$ ; $m = A_2 C_2 / v_2$

$$32.08 = A_2 \times 692.5 / 0.498$$

$A_2$ = **0.02307 m$^2$ --- Ans**

**13. 85 kJ of heat is supplied to a system at constant volume. The system rejects 90 kJ of heat at constant pressure and 20 kJ of work is done on it. The system is brought to its original state by adiabatic process. Determine the adiabatic work. Determine also the values of internal energy at all end states if initial value is 100 kJ.**

*Given:*

There are three processes.

1 – 2 → Constant volume process → HS = 85 kJ

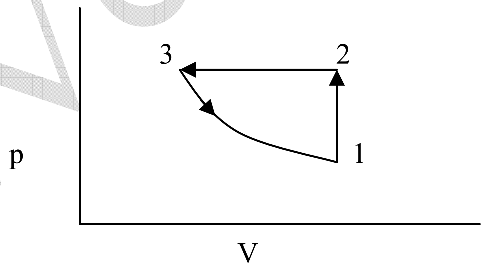
2 – 3 → Constant pressure process → HR = 90 kJ & W = -20 kJ

3 – 1 → Adiabatic process

Initial internal energy $(U_1) = 100$ kJ

*Required:* $W_{3\text{-}1}$, $U_2$ & $U_3$

*Solution:*



I-law for a process, $Q = W + \Delta U$

1 – 2 → Constant volume process, $W_{1\text{-}2} = 0$

$$Q_{1\text{-}2} = W_{1\text{-}2} + U_2 - U_1$$

$$85 = 0 + U_2 - 100$$

$U_2$ = **85 kJ --- Ans**

2 – 3 → Constant pressure process

$$Q_{2\text{-}3} = W_{2\text{-}3} + U_3 - U_2$$

$$-90 = -20 + U_3 - 185$$

$U_3$ = **115 kJ --- Ans**

3 – 1 → Adiabatic process, $Q_{3\text{-}1} = 0$

$$Q_{3\text{-}1} = W_{3\text{-}1} + U_1 - U_3$$

$$0 = W_{3-1} + 100 - 115$$

$$W_{3-1} = \mathbf{15\ kJ} \quad \text{--- Ans}$$

**14. 2 m$^3$ of hydrogen at a pressure of 1 bar and 20$^o$C is compressed isentropically to 4 bar. The same gas is expanded to original volume by constant temperature process and reached initial pressure and temperature by constant volume heat rejection process. Determine (a) pressure, volume and temperature at each end of operation (b) the heat transferred during the isothermal process (c) the heat rejected during constant volume process and (d) change in internal energy during each process. Assume R = 4.206 kJ/kg-K and C$_p$ = 14.25 kJ/kg-K.**

*Given:*

There are three processes.

1 – 2 → Isentropic process

2 – 3 → Constant temperature process

3 – 1 → Constant volume process

Initial volume of hydrogen ($V_1$) = 2 m$^3$ = $V_3$

Initial pressure ($p_1$) = 1 bar

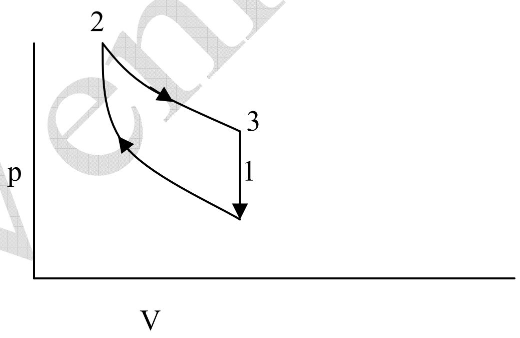
Initial temperature ($T_1$) = 20$^o$C = 293 K

Pressure after isentropic compression ($p_2$) = 4 bar

Volume after isothermal expansion ($V_3$) = 2 m$^3$

*Required:* (a) $V_2$, $T_2$, $p_3$

*Solution:*



(a) $R = C_p - C_v$

$$C_v = 14.25 - 4.206 = 10.044 \text{ kJ/kg-K}$$

$$\gamma = C_p / C_v = 14.25 / 10.044 = 1.419$$

For adiabatic process,

$$\frac{T_2}{T_1} = \left[\frac{p_2}{p_1}\right]^{(\gamma-1)/\gamma}$$

$$\frac{T_2}{293} = \left[\frac{4}{1}\right]^{(1.419-1)/1.419}$$ **$T_2$ = 441.2 K --- Ans = $T_3$**

$$\frac{V_2}{V_1} = \left[\frac{p_1}{p_2}\right]^{1/\gamma}$$

$$\frac{V_2}{2} = \left[\frac{1}{4}\right]^{1/1.4}$$ **$V_2 = 0.753\ m^3$ --- Ans**

2 – 3 → Constant temperature process

$$p_2\ V_2 = p_3\ V_3$$

$$4 \times 0.753 = p_3 \times 2$$

$p_3$ = **1.506 bar --- Ans**

(b) Heat transferred during isothermal process ($Q_{2-3}$)

$$Q_{2-3} = p_2V_2 \ln[V_3/V_2]$$

$$= 4 \times 10^5 \times 0.753 \times \ln[2/0.753]$$

= **294223.4 J --- Ans**

(c) Heat rejected during constant volume process ($Q_{3-1}$)

$$Q_{3-1} = m\ C_v\ (T_1 - T_3)$$

*To find m*

$$p_1\ V_1 = m\ R\ T_1$$

$$1 \times 10^5 \times 2 = m \times 4206 \times 293$$

$$m = 0.1623\ kg$$

$\therefore$ $Q_{3-1} = 0.1623 \times 1004.4 \times (293 - 441.2)$

**= -241586.9 J --- Ans**

(d) $U_2 - U_1 = -W_{1-2} = (p_2V_2 - p_1V_1) / (1 - \gamma)$

$$= -(4 \times 10^5 \times 0.753 - 1 \times 10^5 \times 2) / (1 - 1.419)$$

= **241527.4 J --- Ans**

$$U_3 - U_2 = 0$$

$U_1 - U_3 = Q_{3-1}$ = **-241586.9 J ---- Ans**

**15. 0.5 $m^3$ of air at 30°C and 1 bar is compressed polytropically to 0.08 $m^3$. Find the final pressure and temperature and workdone, change in internal energy and enthalpy, when the index of compression has the value of 1.5. Take for air $C_p$ = 1.005 kJ/kg-K and $C_v$ = 0.718 kJ/kg-K.**

*Given:*

Polytropic process

Initial volume ($V_1$) = 0.5 $m^3$

Initial temperature ($T_1$) = 30°C = 303 K

Final volume ($V_2$) = 0.08 $m^3$

Index of compression (n) = 1.5

*Required:* $p_2$, $T_2$, W, $\Delta$ U, $\Delta$ H

*Solution:*

$$\frac{T_2}{T_1} = \left[\frac{V_1}{V_2}\right]^{(1-n)}$$

$$\frac{T_2}{303} = \left[\frac{0.5}{0.08}\right]^{(1-1.5)}$$

$T_2$ = **757.5 K --- Ans**

$$\frac{p_2}{p_1} = \left[\frac{V_1}{V_2}\right]^{n}$$

$$\frac{p_2}{1} = \left[\frac{0.5}{0.08}\right]^{1.5}$$ $p_2$ = **15.625 bar --- Ans**

$W_{1\text{-}2} = (p_2V_2 - p_1V_1) / (1 - n)$

$= (15.625 \times 10^5 \times 0.08 - 1 \times 10^5 \times 0.5) / (1 - 1.5)$

= **-150000 J ---- Ans**

$U_2 - U_1 = m\, C_v\, (T_2 - T_1)$

*To find m*

$p_1\, V_1 = m\, R\, T_1$

$R = C_p - C_v = 1.005 - 0.718 = 0.387$ kJ/kg-K

$\therefore$ $1 \times 10^5 = m \times 287 \times 303$

$m = 0.5749$ kg

$\therefore$ $\Delta U = 0.5749 \times 0.718 \times (757.5 - 303)$ **= 187.6 KJ --- Ans**

$\Delta H = H_2 - H_1 = m\, C_p\, (T_2 - T_1)$

$= 0.5749 \times 1.005 \times (757.5 - 303) =$ **262.6 kJ --- Ans**

**16. 3 kg of an ideal gas is expanded from a pressure of 8 bar and volume of 1.5 $m^3$ to a pressure of 1.6 bar and volume of 4.5 $m^3$. The change in internal energy is 450 kJ. The specific heat at constant volume for the gas is 0.7 kJ/kg-K. Determine (a) Gas constant (b) Index of polytropic expansion (c) Workdone during polytropic expansion and (e) Initial and final temperatures.**

*Given:*

Mass of gas (m) = 3 kg

Initial pressure ($p_1$) = 8 bar

Initial Volume ($V_1$) = 1.5 $m^3$

Final pressure ($p_2$) = 1.6 bar

Final volume ($V_2$) = 4.5 $m^3$

Change in internal energy ($U_2 - U_1$) = 450 kJ

Specific heat at constant volume ($C_v$) = 0.7 kJ/kg-K

*Required:* (a) R (b) n (c) W (d) $T_1$ & $T_2$

*Solution:*

(a) $p_1\, V_1 = m\, R\, T_1$

$8 \times 10^5 \times 1.5 = 3 \times R\, T_1$

$R\,T_1 = 400000$ --- (1)

$p_2\,V_2 = m\,R\,T_2$

$1.6 \times 10^5 \times 4.5 = 3 \times R\,T_2$

$R\,T_2 = 240000$ --- (2)

From (1) and (2) $R\,(T_2 - T_1) = -160000$

$T_2 - T_1 = -160000 / R$ --- (3)

Also $U_2 - U_1 = -450$ kJ (-ve on expansion)

$U_2 - U_1 = m\,C_v\,(T_2 - T_1)$

$-450 = 3 \times 0.7 \times (T_2 - T_1)$

Substituting (3), $-450 = 3 \times 0.7 \times -160000 / R$

$R =$ **746.6 J/kg-K ---- Ans**

(b) $p_1V_1^n = p_2V_2^n$

$\therefore$ $n = \ln\,[p_1/p_2] / \ln\,[V_2/V_1]$

$= \ln\,[8/1.6] / \ln\,[4.5/1.5] =$ **1.465 ---- Ans**

(c) $W = (p_2V_2 - p_1V_1) / (1 - n)$

$= (1.6 \times 4.5 \times 10^5 - 8 \times 1.5 \times 10^5) / (1 - 1.465)$

= **1032258 J ---- Ans**

(d) $p_1\,V_1 = m\,R\,T_1$

$8 \times 10^5 \times 1.5 = 3 \times 746.6 \times T_1$

$T_1 =$ **535.8 K ---- Ans**

$p_2\,V_2 = m\,R\,T_2$

$1.6 \times 10^5 \times 4.5 = 3 \times 746.6 \times T_2$

$T_2 =$ **321.5 K ---- Ans**

**17. A gas mixture obeying perfect gas law has molar mass of 26.7 kg/kmol. The gas mixture is compressed to a compression ratio of 12 according to the law $pV^{1.25} = C$, from initial conditions of 0.9 bar and 333 K. Assume a mean molar specific heat at a constant volume of 21.1 kJ/kmolK, find per kg of mass, the workdone and heat flow across the cylinder walls. For the above gas, determine the value of characteristic gas constant, molar specific heat at constant pressure and ratio of specific heats.**

*Given:*

Polytropic process

Index of compression (n) = 1.25

Molar mass (M) = 26.7

Compression ratio ($p_2/p_1$) = 12

Initial pressure ($p_1$) = 0.9 bar

Initial temperature ($T_1$) = 333 K

$C_{v(\text{mole})}$ = 21.1 kJ/kmolK

Mass of gas (m) = 1 kg

*Required:* W, Q, R, $C_{p(\text{mole})}$, $C_p/C_v$

*Solution:*

Work done during polytropic process is given by,

$$W = \frac{p_2V_2 - p_1V_1}{1-n} = \frac{m\,R\,(T_2 - T_1)}{1-n}$$

*To find R*

Universal gas constant ($R_u$) = 8314 J/kg-K

Characteristic gas constant (R) = $R_u$/M = 8314/26.7 = **311.38 J/kg-K --- Ans**

*To find $T_2$*

$$\frac{T_2}{T_1} = \left[\frac{p_2}{p_1}\right]^{(n-1)/n}$$

$$\frac{T_2}{333} = [12]^{(1.25-1)/1.25}$$

$T_2$ = 547.4 K

$$W = \frac{1x\,313.38\,x\,(547.4 - 333)}{1-1.25}$$

**= - 268754.7 J --- Ans**

*Heat transfer (Q)* $Q = \frac{\gamma - n}{1-n}\,x\,W \;\; = \frac{1.4-1.25}{1-1.25}\,x - 268754\,.7$

**= 100783 J --- Ans**

*Gas constant R*

$R = C_p - C_v$

$C_v = C_{v(mole)} / M$

$C_p = C_{p(mole)} / M$

$\therefore\; C_v = 21100 / 26.7 = 790.26$ J/kg-K

$313.38 = C_p - 790.26$

$C_p = 1101.64$ J/kg-K

$\therefore\; C_{p(mole)} = 1101.64 \text{ x } 26.7 =$ **29413.8 J/kmolK ---- Ans**

*Specific heat ratio*

$C_p/C_v = 1101.64/790.26 =$ **1.394 --- Ans**

**18. A centrifugal pump delivers 2750 kg of water per min from initial pressure of 0.8 bar absolute to a final pressure of 2.8 bar absolute. The suction is 2 m below and the delivery is 5 m above the centre of pump. If the suction and delivery pipes are of 15 cm and 10 cm diameters respectively, make calculation for power required to run the pump.**

*Given:*

Centrifugal pump

Flow process

Mass flow rate of water (m) = 2750 kg/min = 2750/60 kg/s

Initial pressure ($p_1$) = 0.8 bar (abs)

Final pressure ($p_2$) = 2.8 bar (abs)

Suction below the centre of the pump = 2 m

Delivery above the centre of the pump = 5 m

Suction pipe diameter ($d_1$) = 15 cm = 0.15 m

Delivery pipe diameter ($d_2$) = 10 cm = 0.1 m

*Required:* W

*Solution:*

SFEE is given by,

$$\overset{*}{m}\left[h_1 + \frac{C_1^2}{2} + g\,Z_1\right] + Q = \overset{*}{m}\left[h_2 + \frac{C_2^2}{2} + g\,Z_2\right] + W$$

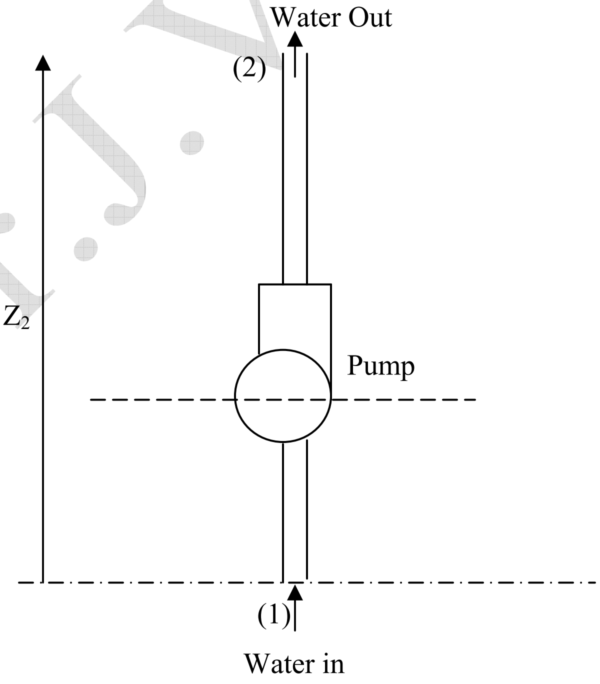
$$\overset{*}{m}\left[u_1 + p_1 v_1 + \frac{C_1^2}{2} + g\,Z_1\right] + Q = \overset{*}{m}\left[u_2 + p_2 v_2 + \frac{C_2^2}{2} + g\,Z_2\right] + W$$

Consider datum from suction (1), $Z_1 = 0$ & $Z_2 = 2 + 5 = 7$ m

Generally for any liquid, $u_1 = u_2$, $v_1 = v_2 = v$ and $Q = 0$

Therefore, SFEE is reduced to,

$$\overset{*}{m}\left[p_1 v + \frac{C_1^2}{2} + g\,Z_1\right] = \overset{*}{m}\left[p_2 v + \frac{C_2^2}{2} + g\,Z_2\right] + W$$



*To find $C_1$ and $C_2$*

$$\overset{*}{m} = \frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2}$$

For water ρ = 1000 kg/$m^3$ and v = 1/ρ = 1/1000 $m^3$/kg

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} x 0.15^2$$

$$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} x 0.1^2$$

$$2750/60 = \frac{\pi/4 x 0.15^2 C_1}{1/1000}$$

$C_1$ = 2.593 m/s

$$2750/60 = \frac{\pi/4 x 0.1^2 C_2}{1/1000}$$

$C_2$ = 5.835 m/s

Therefore,

$$(2750/60)\left[0.8 x 10^5 x (1/1000) + \frac{2.593^2}{2} + 9.81 x 0\right]$$

$$= (2750/60)\left[2.8 x 10^5 x (1/1000) + \frac{5.835^2}{2} + (9.81 x 7)\right] + W$$

**W = 12833 W --- Ans**

19. **In an isentropic flow through nozzle, air flows at the rate of 600 kg/h. At the inlet to the nozzle , pressure is 2 MPa and temperature is 127°C. The exit pressure is 0.5 MPa. Initial velocity is 300 m/s. Determine, (i) Exit velocity of air and (ii) Inlet and exit area of nozzle.**

*Given:*

Nozzle

Flow process

Fluid → Air

Mass flow rate of air (m) = 600 kg/h = 600/3600 kg/s

Initial pressure ($p_1$) = 2 MPa = 2 x $10^6$ Pa

Initial temperature ($T_1$) = $127^0$C = 400 K

Final pressure ($p_2$) = 0.5 MPa = 0.5 x $10^6$ Pa

Initial velocity ($C_1$) = 300 m/s

*Required:* (i) $C_2$ (ii) $A_1$ & $A_2$

*Solution:*

**(i)**

SFEE is given by,

$$\overset{*}{m}\left[h_1 + \frac{C_1^2}{2} + g Z_1\right] + Q = \overset{*}{m}\left[h_2 + \frac{C_2^2}{2} + g Z_2\right] + W$$

Take $Z_1 = Z_2$

The flow is isentropic, Q = 0

For nozzle, W = 0

Therefore, SFEE is reduced to,

$$\left[h_1 + \frac{C_1^2}{2}\right] = \left[h_2 + \frac{C_2^2}{2}\right]$$

$$C_2 = \sqrt{C_1^2 + 2(h_1 - h_2)}$$

$$C_2 = \sqrt{C_1^2 + 2\,C_p(T_1 - T_2)}$$

*To find* $T_2$

$$\frac{T_2}{T_1} = \left[\frac{p_2}{p_1}\right]^{(\gamma-1)/\gamma}$$

For air, $\gamma = 1.4$, R = 287 J/kg-K & $C_p$ = 1005 J/kg-K

$$\frac{T_2}{400} = \left[\frac{0.5}{2}\right]^{(1.4-1)/1.4}$$

$T_2$ = 269.2 K

Therefore,

$$C_2 = \sqrt{300^2 + 2\,x1005(400 - 269.2)}$$

**$C_2$ = 593.06 m/s ---- Ans**

**(ii)**

$$\overset{*}{m} = \frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2}$$

*To fin* $v_1$ & $v_2$

$p_1\, v_1 = R\, T_1$

$2 \times 10^6 \times v_1 = 287 \times 400$

$v_1 = 0.0574$ m³/kg

$p_2\, v_2 = R\, T_2$

$0.5 \times 10^6 \times v_2 = 287 \times 269.2$

$V_2 = 0.1545$ m³/kg

$$600/3600 = \frac{A_1\, x300}{0.0574}$$

$A_1 = 3.1889 \times 10^{-5}$ m² --- **Ans**

$$600/3600 = \frac{A_2\, x593.06}{0.1545}$$

$A_2 = 4.3419 \times 10^{-5}$ m² --- **Ans**

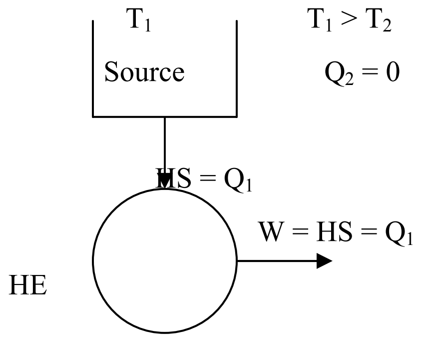
# UNIT-II

# SECOND LAW AND AVAILABILITY ANALYSIS

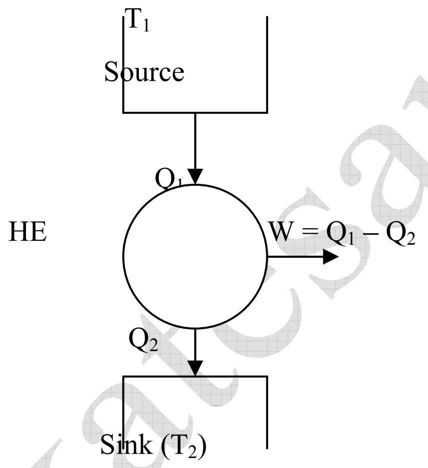
## Second law of Thermodynamics

### *Kelvin-Planck statement*

It is impossible to construct an engine to work in a cyclic process whose sloe effect is to convert all the heat supplied into an equivalent of work.



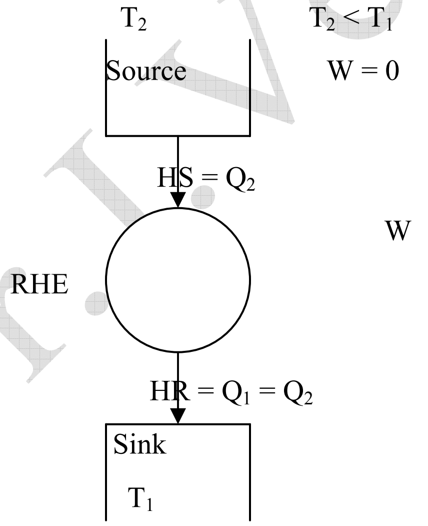
Impossible Kelvin Planck statement



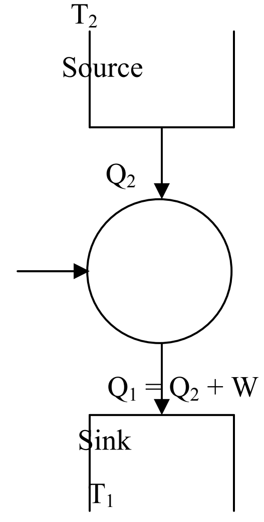
Possible Kelvin statement

### *Clausius statement*

It is impossible to construct a device to work in a cyclic process whose sole effect is the transfer of heat from a body at a lower temperature to a body at a higher temperature.



Impossible Clausius statement



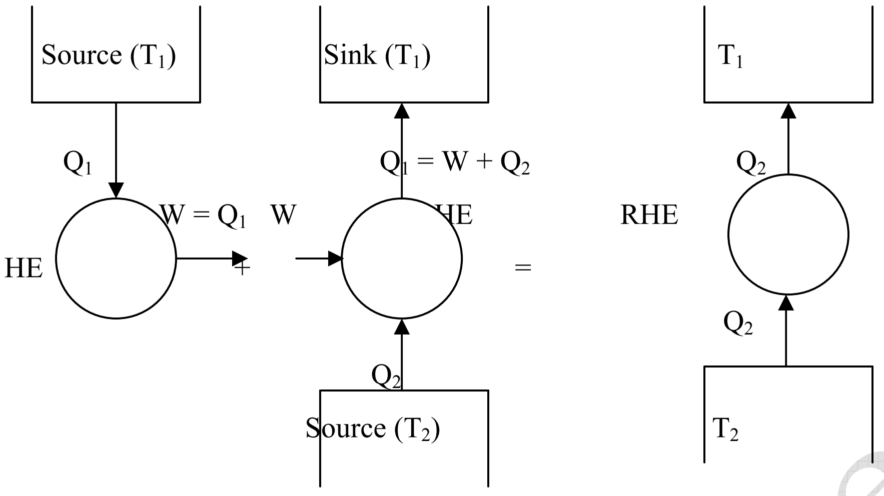
Possible Clausius statement

## Equivalence of Kelvin and Clausius statements

Consider the combination as shown in fig. The heat engine contradicted by Kelvin statement is producing work. We assume that it is possible. Now it is combined with the RHE which is a regular heat pump obeying Clausius statement. The combination results in a RHE which is contradicting the Clausius statement. This means that if Kelvin statement is contradicted, Clausius statement is automatically contradicted.

Net effect → $+Q_1 - Q_1 + Q_2 = W = Q_1$

$Q_1 = Q_2$ (impossible)



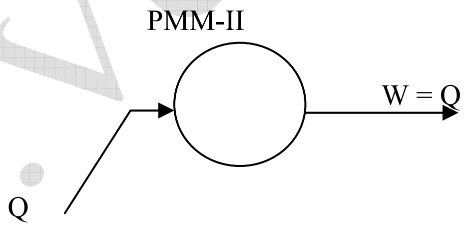
Impossible Kelvin + Possible Clausius = Impossible Clausius

Similarly the impossible Clausius and possible Kelvin can be combined and the resulting engine will contradict the Kelvin statement.

So, if one of the statements is contradicted, the other is automatically contradicted. So, the effect of these two apparently different statements are basically the same.

**Perpetual Motion Machine of Second kind (PMM-II)**

- A machine which continuously absorb heat from a single reservoir and would convert this heat completely into work.



Efficiency of PMM-II is 100 %.
PMM-II obeys I-law but violates the II-law.

**Reversibility and Irreversibility**

The II-law of thermodynamics enables us to divide all processes into two classes:

1. Reversible or ideal process
2. Irreversible or natural process

A reversible process is a process that, after it has taken place, can be reversed and causes no change in either the system or its surroundings. A reversible process is carried out infinitely slowly with an infinitesimal gradient, so that every state passed through by the system is an equilibrium state. So, a reversible process coincides with a quasi-static process.

Examples of reversible processes:

- Frictionless adiabatic process (Isentropic process)

- Frictionless isothermal process
- Condensation and boiling of liquids

A process that is not reversible is termed irreversible. In an irreversible process, finite changes are made; therefore the system is not at equilibrium throughout the process. At the same point in an irreversible cycle, the system will be in the same state, but the surroundings are permanently changed after each cycle.

Examples of irreversible processes:

- Combustion process
- Mixing of two fluids
- Heat flow from a higher temperature to lower temperature
- Flow of electric current through a resistor

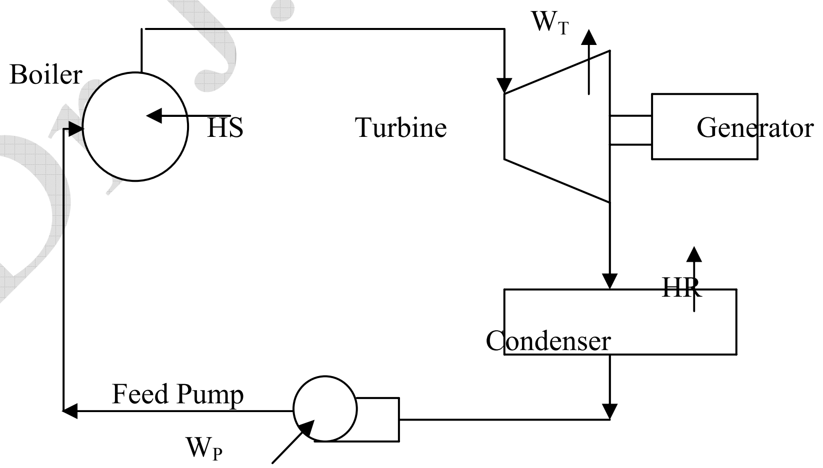
*Conditions for reversibility*

1. The process should not involve friction.
2. Heat transfer should not take place with finite temperature difference.
3. The energy transfer as heat and work during the forward process should be identically equal to energy transfer as heat and work during the reversal of the process.
4. There should be no free or unrestricted expansion.
5. There should be no mixing of the fluids.
6. The process must proceed in a series of equilibrium states.

**Heat engine**

Heat engine is a device used to convert heat energy into useful work when operating in a cyclic process.

Examples: I.C. engines like, Petrol engines, Diesel engines, E.C. engines like, Steam power plants, Gas turbine power plants.



- Boiler produces the steam and supplies to the turbine. Heat is supplied in the boiler.
- Turbine develops the work.
- Condenser condenses the exhaust steam from the turbine. Heat is rejected from the steam.

- Pump takes the condensate to the boiler at high pressure. Work is given to the pump

As per I-law, Net heat transfer is equal to the net work transfer.

$$HS - HR = W_T - W_P$$

Thermal efficiency = Net work transfer / Heat supplied

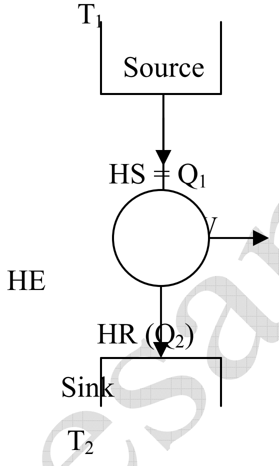
$$= (W_T - W_P) / HS$$

$$= (HS - HR) / HS$$

HS → Heat Supplied

HR → Heat Rejected

HE → Heat Engine



**Reversed heat engine (RHE)**

A RHE is a device to absorb heat from low temperature body with the expenditure of work.

*Heat pump:*

Heat pump is used to take out the heat from low temperature source (atmosphere) at $T_2$ and supplies it to a sink (room) at high temperature $T_1$ for heating of room (Winter air conditioning).

As per I-law, Net heat transfer is equal to the net work transfer.

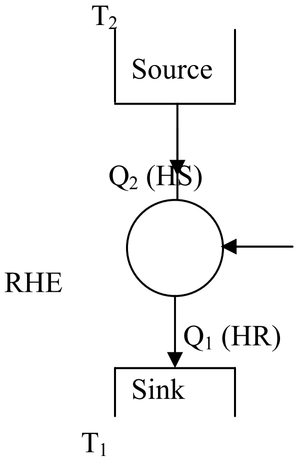
$$HR - HS = W$$

COP → Coefficient of performance

COP = Output / Input

= Heat given to the room / Work

$$= Q_1 / (Q_1 - Q_2)$$



*Refrigerator:*

Refrigerator is used to take out the heat from low temperature source (Room) at $T_2$ and supplies it to a sink (atmosphere) at high temperature $T_1$ for cooling of room (Summer air conditioning).

As per I-law, Net heat transfer is equal to the net work transfer.

$$HR - HS = W$$

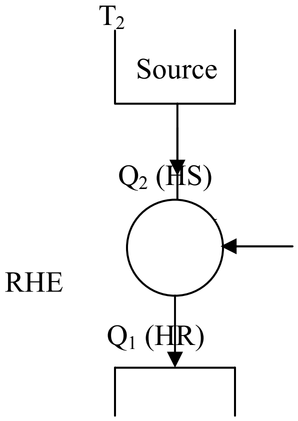
COP → Coefficient of performance

COP = Refrigeration effect / Input

= Heat taken from the room / Work

= Heat supplied to working fluid / Work

$$= Q_2 / (Q_1 - Q_2)$$



Sink

$T_1$

## Air Standard Cycles

**Air Standard cycles:** Cycles using a perfect gas, having the properties of air useful in the study of the I. C. Engine because they represent a limit to which actual cycle may approach and they are subjected to simple mathematical and explanatory treatment.

**Assumptions made for analysis:**

- The properties of the working medium can be calculated by the application of the perfect gas equation.

  i.e., $pV = mRT$

- The Specific heat of the substance remains same during all the processes in the cycle.

  i.e.,$C_p$ & $C_v$ are unchanged.

- The cycles are composed of reversible processes.
- The gas does not undergo any chemical changes.

**Various cycles**

1. Carnot cycle
2. Otto cycle
3. Diesel cycle
4. Stirling cycle
5. Brayton cycle
6. Erricson cycle
7. Dual cycle

**Air standard efficiency of a cycle**

The thermal efficiency of an ideal air standard cycle is called the "Air standard efficiency".

In an ideal air standard cycle, the working fluid is air. The petrol and diesel engines working on Otto cycle and diesel cycle use petrol and diesel oil with air. This air fuel mixture behaves like air before the combustion takes place. The properties of combustion products are also not different from those of air. Therefore the efficiencies of petrol and diesel engines are calculated assuming them working on air standard cycles.

The efficiency of a cycle is given by,

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Net work output}}{\text{Heat Supplied}}$$

$$= \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}}$$

**Carnot Cycle:**

The Carnot cycle is a particular thermodynamic cycle, modeled on the Carnot heat engine, studied by Nicolas Léonard Sadi Carnot in the 1820s and expanded upon by Benoit Paul Émile Clapeyron in the 1830s and 40s.

This cycle has the highest possible efficiency and consists of four simple operations:

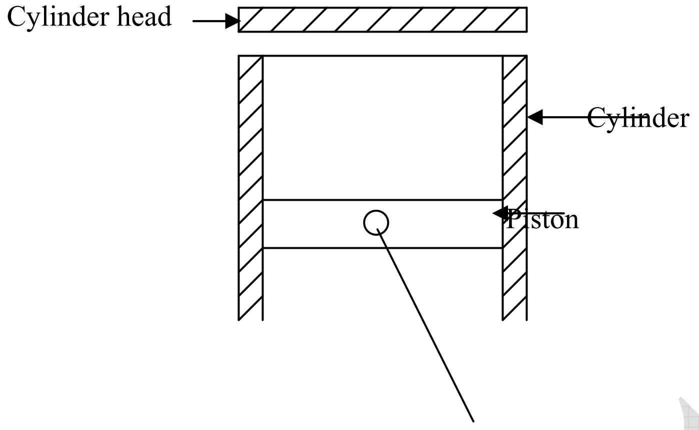
(i) Isothermal expansion

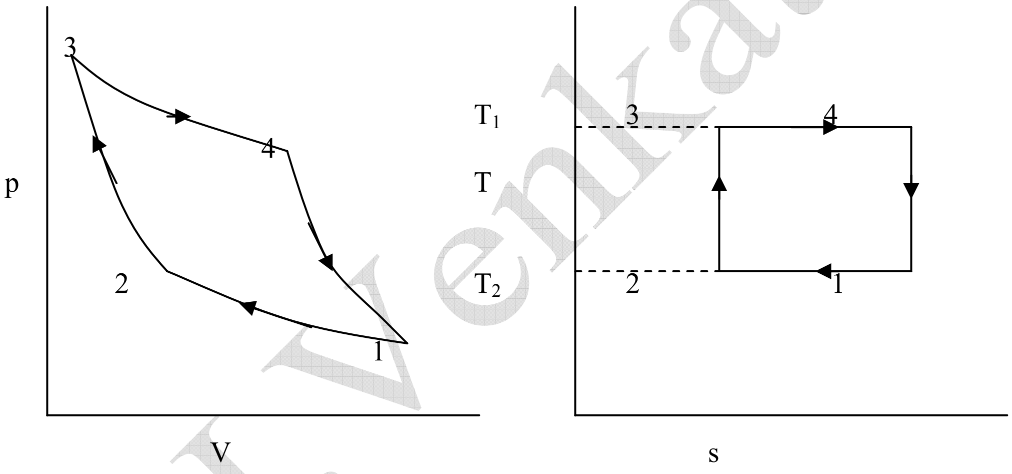
(ii) Isentropic expansion

(iii) Isothermal compression

(iv) Isentropic compression

***Operation:*** Consider a cylinder piston arrangement as shown in the fig. Let m kg of air is enclosed in a cylinder. The cylinder head is made of perfect heat conductor or perfect heat insulator. The cylinder is perfectly insulated.





The cylinder is full of air when the piston is at BDC. The 'perfect heat conductor cylinder head' (Cold body) is brought in contact with the cylinder. The air is compressed at constant temperature from (1) to (2) during its travel towards TDC. During this process the heat is rejected from the air. The 'cold body' is removed and 'perfect heat insulator head' is brought in contact with the cylinder. Now the air is compressed isentropically (2-3) till the piston reaches the TDC. Then the perfect insulator is removed. The perfect conductor (Hot body) is brought in contact with the cylinder and the heat is supplied to the air. Now expansion proceeds at constant temperature (3-4) during its travel towards BDC. The perfect insulator is brought and further expansion proceeds isentropically upto BDC (4-1). Thus the cycle is completed.

**Efficiency of cycle**

Process – 1 – 2 → Isothermal compression

$\therefore$ Heat transferred = $Q_{1-2} = m\, T_2\, (s_2 - s_1)$ = HR

Note: The area under the curve in T-s diagram is heat transferred.

Process – 2 – 3 → Isentropic compression

$\therefore$ Heat transferred = $Q_{2\text{-}3} = 0$;

Process – 3 – 4 → Isothermal expansion

$\therefore$ Heat transferred = $Q_{3\text{-}4} = m\, T_1\,(s_2 - s_1)$ = HS

Process – 4 – 1 → Isentropic expansion

$\therefore$ Heat transferred = $Q_{4\text{-}1} = 0$

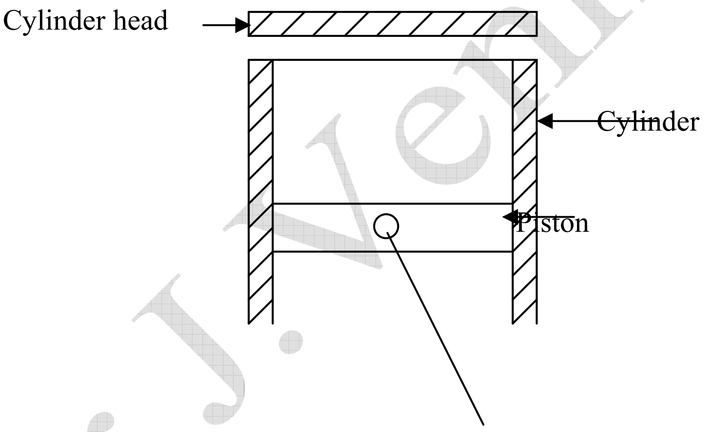
$\therefore$ Cycle efficiency ($\eta$) = [Heat supplied – Heat rejected]/Heat supplied

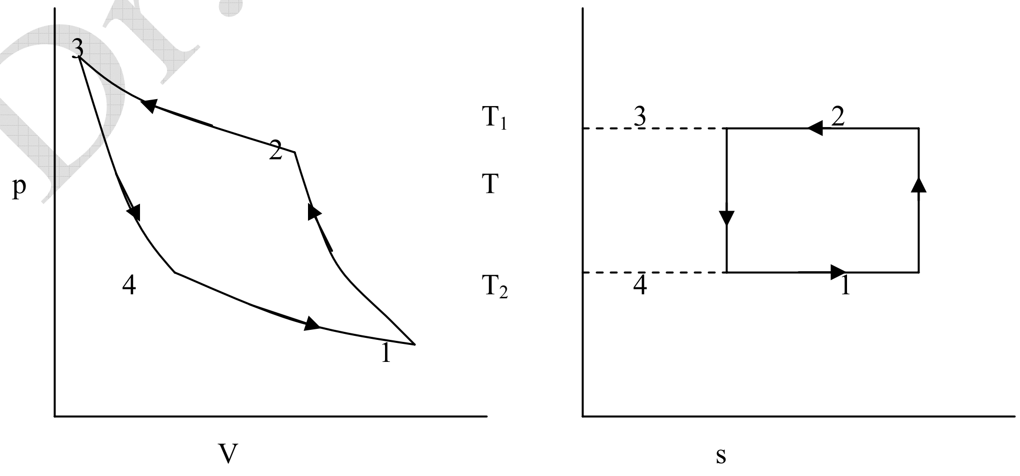
$$= \frac{m\,T_1\,(s_2 - s_1) - m\,T_2\,(s_2 - s_1)}{m\,T_1\,(s_2 - s_1)}$$

$$= \frac{T_1 - T_2}{T_1} = \frac{T_{max} - T_{min}}{T_{min}}$$

The Carnot cycle efficiency is depending only on the temperatures $T_1$ and $T_2$.

**Reversed Carnot cycle**





If a machine working on reversed Carnot cycle is driven from an external source, will work as a refrigerator.

p-V and T-s diagrams of reversed Carnot cycle are shown. The cylinder is full of air at temperature $T_2$ when the piston is at BDC. The 'perfect heat insulator cylinder head' is brought in contact with the cylinder. The air is compressed isentropically from (1) to (2) during its travel towards TDC. During this process the temperature of air is raised to $T_1$. The 'perfect heat insulator is removed and 'perfect heat conductor head' (Cold body) is brought in contact with the cylinder. Now the air is compressed at constant temperature (2-3) till the piston reaches the TDC. Then the cold body is removed. The perfect insulator is brought in contact with the cylinder. Now the air expanded isentropically from (3) to (4) during its travel towards BDC. During this process the temperature of air is lowered to $T_2$. The perfect conductor (Hot body) is brought and further expansion proceeds at constant temperature upto BDC (4-1). Thus the cycle is completed.

**Efficiency of cycle**

Process – 1 – 2 → Isentropic compression

$\therefore$ Heat transferred = $Q_{1\text{-}2} = 0$

Note: The area under the curve in T-s diagram is heat transferred.

Process – 2 – 3 → Isothermal compression

$\therefore$ Heat transferred = $Q_{2\text{-}3} = m\,T_1\,(s_2 - s_3)$ = HR;

Process – 3 – 4 → Isentropic expansion

$\therefore$ Heat transferred = $Q_{3\text{-}4} = 0$;

Process – 4 – 1 → Isothermal expansion

$\therefore$ Heat transferred = $Q_{4\text{-}1} = m\,T_2\,(s_1 - s_4) = m\,T_2\,(s_2 - s_3)$ = HS;

$\therefore$ COP = Refrigeration effect / Work input

= Heat supplied to the air / (Heat rejected – Heat supplied)

$$= \frac{m\,T_2\,(s_2 - s_2)}{m\,T_1\,(s_2 - s_3) - m\,T_2\,(s_2 - s_3)}$$

$$= \frac{T_2}{T_1 - T_2} = \frac{T_{min}}{T_{max} - T_{min}}$$

The Carnot cycle COP is depending only on the temperatures $T_1$ and $T_2$.

**Absolute Thermodynamic temperature scale (Kelvin scale)**

The efficiency of any heat engine receiving heat $Q_1$ and rejecting heat $Q_2$ is given by

$$\eta = \frac{W_{net}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_{1_1}}$$

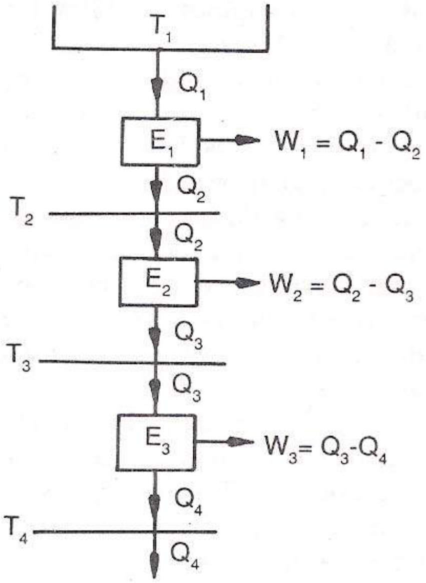
The efficiency of any reversible heat engine is given by

$$\eta_{rev} = \frac{W_{net}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_{1_1}} = 1 - \frac{T_2}{T_1}$$

$$1-\frac{Q_2}{Q_1} = f\,(T_1, T_2)$$

Or in terms of new function F, $\frac{Q_1}{Q_2} = F\,(T_1, T_2)$

Consider reversible heat engines operating in series.



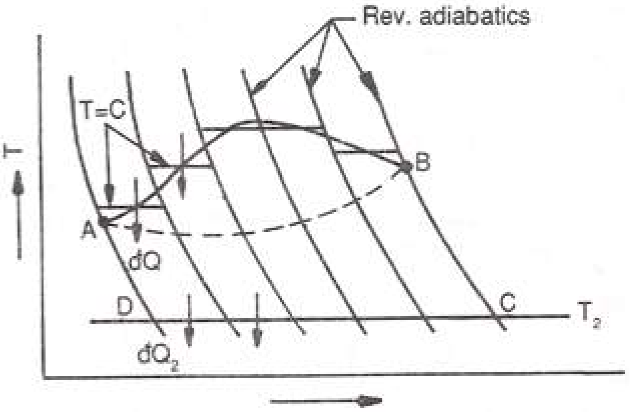
We can write, $\frac{Q_1}{Q_2} = F\,(T_1, T_2)$

$$\frac{Q_2}{Q_3} = F\,(T_2, T_3)$$

$$\frac{Q_1}{Q_2} = \frac{Q_1/Q_3}{Q_2/Q_3} = \frac{F\,(T_1, T_3)}{F\,(T_2, T_3)} = \frac{\phi(T_1)}{\phi(T_2)}$$

$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ → **Kelvin scale**

**Clausius inequality**



Let us consider a cycle ABCD as shown. Let AB be the general process, either reversible or irreversible, while the others are reversible. Let the cycle be divided into a number of small cycles as shown.

For one of the elementary cycles,

$$\eta = 1 - \frac{dQ_2}{dQ}$$

dQ → The heat supplied at T

$dQ_2$ → Heat supplied at $T_2$

The efficiency of the general cycle will be less than or equal to the efficiency of the reversible cycle.

$$1 - \frac{dQ_2}{dQ} \le \left(1 - \frac{dQ_2}{dQ}\right)_{rev}$$

$$\frac{dQ_2}{dQ} \ge \left(\frac{dQ_2}{dQ}\right)_{rev}$$

$$\frac{dQ}{dQ_2} \le \left(\frac{dQ}{dQ_2}\right)_{rev}$$

But $\left(\frac{dQ}{dQ_2}\right)_{rev} = \frac{T}{T_2}$

$$\frac{dQ}{dQ_2} \le \frac{T}{T_2}$$

$\frac{dQ}{T} \le \frac{dQ_2}{T_2}$ for any process

For reversible process, $dS = \frac{dQ_{rev}}{T} = \frac{dQ_2}{T}$

Therefore, $\frac{dQ}{T} \le dS$

By definition, cyclic integral of any property is zero.

Therefore, $$\oint dS = 0$$

$$\oint \frac{dQ}{T} \leq 0 \quad \rightarrow \text{Clausius inequality}$$

$$\oint \frac{dQ}{T} = 0 \rightarrow \text{The cycle is reversible}$$

$$\oint \frac{dQ}{T} < 0 \rightarrow \text{The cycle is irreversible and possible}$$

$$\oint \frac{dQ}{T} > 0 \rightarrow \text{The cycle is impossible}$$

## Entropy

All the heat is not equally valuable for converting into work. Heat that is supplied to a substance at high temperature has a greater possibility of conversion into work than heat supplied to a substance at a lower temperature.

Entropy is a function of a quantity of heat which shows the possibility of conversion of that heat into work. The increase in entropy is small when heat is added at a higher temperature and is greater when heat is added at lower temperature.

The change of entropy is defined mathematically as

$$dS = \frac{dQ}{T}$$

*Characteristics of entropy*

- It increases when heat is supplied irrespective of the fact whether temperature changes or not.
- It decreases when heat is removed irrespective of the fact whether temperature changes or not.
- It remains unchanged in all adiabatic frictionless processes.

## Change in entropy for a closed system and entropy chart

*General expression*

Consider heating of m kg of gas.

Let, $p_1$ → Initial pressure of the gas

$T_1$ → Initial temperature of the gas

$V_1$ → Initial volume of the gas

$C_p$ → Specific heat at constant pressure

$C_v$ → Specific heat at constant volume

R → Gas constant

$p_2$, $T_2$, $V_2$ are corresponding final conditions

I-law for a process → $Q = W + \Delta U$

$$dQ = dW + dU$$

Divide by T → $\frac{dQ}{T} = \frac{dW}{T} + \frac{dU}{T}$

$$dS = \frac{dQ}{T}; \quad W = \int p\,dV; \quad dW = p\,dV; \quad dU = mC_v dT$$

$$dS = \frac{pdV}{T} + mC_v \frac{dT}{T}$$

We know that pV = mRT → $\frac{p}{T} = \frac{mR}{V}$

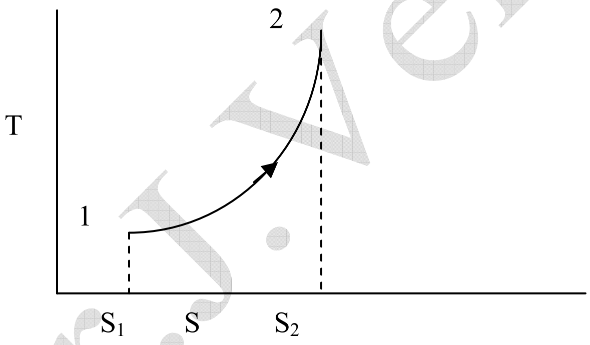
Therefore, $dS = mR\frac{dV}{V} + mC_v \frac{dT}{T}$

Integrating between (1) and (2),

$$\int_1^2 dS = \int_1^2 mR\frac{dV}{V} + \int_1^2 mC_v \frac{dT}{T}$$

$$S_2 - S_1 = mR\,\ln\left[\frac{V_2}{V_1}\right] + mC_v\,\ln\left[\frac{T_2}{T_1}\right]$$

*Constant Volume process*



During constant volume process, $V_1 = V_2$

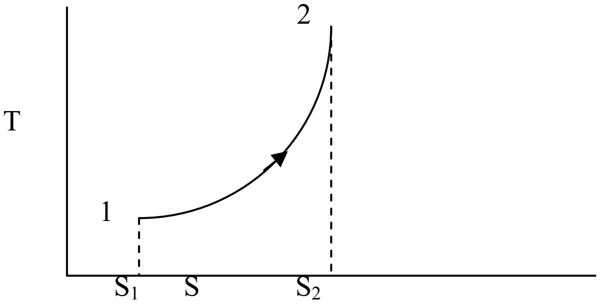
$$S_2 - S_1 = mC_v\,\ln\left[\frac{T_2}{T_1}\right]$$

*Constant pressure process*

During constant pressure process, $p_1 = p_2$

General gas equation can be written as $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

Or $\frac{V_2}{V_1} = \frac{T_2}{T_1}$



$$S_2 - S_1 = mR\ \ln\left[\frac{T_2}{T_1}\right] + mC_v\ \ln\left[\frac{T_2}{T_1}\right]$$

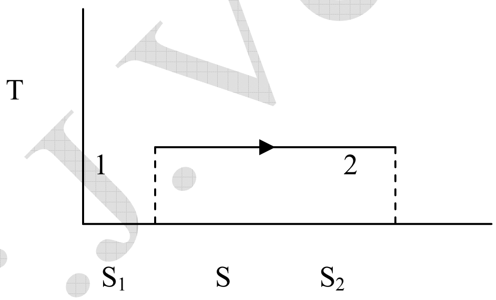
$$= (mR + mC_v)\ \ln\left[\frac{T_2}{T_1}\right]$$

$$= mC_p\ \ln\left[\frac{T_2}{T_1}\right]$$

*Isothermal process*

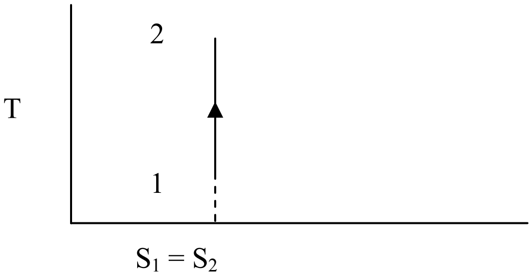
During isothermal process, $T_1 = T_2$

$$S_2 - S_1 = mR\ \ln\left[\frac{V_2}{V_1}\right]$$

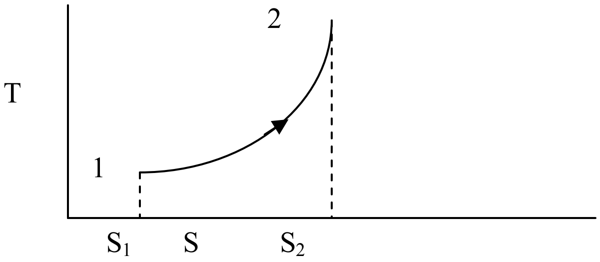


*Isentropic process*

During isentropic process, Q = 0 & $\Delta S = 0$



*Polytropic process*



The governing equation is given by, $pV^n = C$

$$S_2 - S_1 = mR\ \ln\left[\frac{V_2}{V_1}\right] + mC_v\ \ln\left[\frac{T_2}{T_1}\right]$$

We know that, $\left[\frac{V_2}{V_1}\right] = \left[\frac{T_1}{T_2}\right]^{1/(n-1)} = \left[\frac{T_2}{T_1}\right]^{-1/(n-1)}$

Therefore, $S_2 - S_1 = mR\ \ln\left[\frac{T_2}{T_1}\right]^{-1/(n-1)} + mC_v\ \ln\left[\frac{T_2}{T_1}\right]$

$$= m(C_p - C_v)\ \ln\left[\frac{T_2}{T_1}\right]^{-1/(n-1)} + mC_v\ \ln\left[\frac{T_2}{T_1}\right]$$

$$= m(C_p - C_v)\left(\frac{-1}{n-1}\right) \ln\left[\frac{T_2}{T_1}\right] + mC_v\ \ln\left[\frac{T_2}{T_1}\right]$$

$$= mC_v\left(\frac{C_p}{C_v} - 1\right)\left(\frac{-1}{n-1}\right) \ln\left[\frac{T_2}{T_1}\right] + mC_v\ \ln\left[\frac{T_2}{T_1}\right]$$

$$= mC_v(\gamma - 1)\left(\frac{-1}{n-1}\right) \ln\left[\frac{T_2}{T_1}\right] + mC_v\ \ln\left[\frac{T_2}{T_1}\right]$$

$$= mC_v\ \ln\left[\frac{T_2}{T_1}\right]\left[1 - \frac{\gamma - 1}{n-1}\right]$$

$$= mC_v\ \ln\left[\frac{T_2}{T_1}\right]\left[\frac{n - 1 - \gamma + 1}{n-1}\right]$$

$$= mC_v\ \ln\left[\frac{T_2}{T_1}\right]\left[\frac{n-\gamma}{n-1}\right]$$

**Principle of increase of entropy**

For any process, $\frac{dQ}{T} \le dS$ (or) $dS \ge \frac{dQ}{T}$

For isolated system, dQ = 0, $dS_{iso} \ge 0$

For reversible process, $dS_{iso} = 0$; S = constant

For an irreversible process, $dS_{iso} > 0$

It is thus proved that the entropy of an isolated system can never decrease. It always increases and remains constant only when the process is reversible. This is known as the 'principle of increase of entropy' or simply the 'entropy principle'.

$$dS_{iso} = dS_{uni} = dS_{sys} + dS_{surr} \geq 0$$

There is no reversible process in nature. Therefore the entropy of the universe is going on increasing.

**Absolute entropy**

It is possible to find the amount by which the entropy of the system changes in a process, but not the value of absolute entropy. When it is required to find the absolute entropy, a zero value of entropy of the system at an arbitrary chosen standard state is assigned, and the entropy changes are calculated with reference to this standard state.

**Carnot theorem**

It states that of all heat engines operating between a given constant temperature source and a given constant temperature sink, none has a higher efficiency than a reversible engine.

**Available energy (Exergy) and Unavailable energy (Anergy)**

Available energy (AE) → Part of the low grade energy available for conversion into work.

Unavailable energy (UE) → Part of the low grade energy which must be rejected to environment and can not be converted into shaft work.

Heat supplied, $Q_1 = AE + UE$

$W_{max} = AE$

Maximum work can be obtained from ideal or Carnot engine.

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

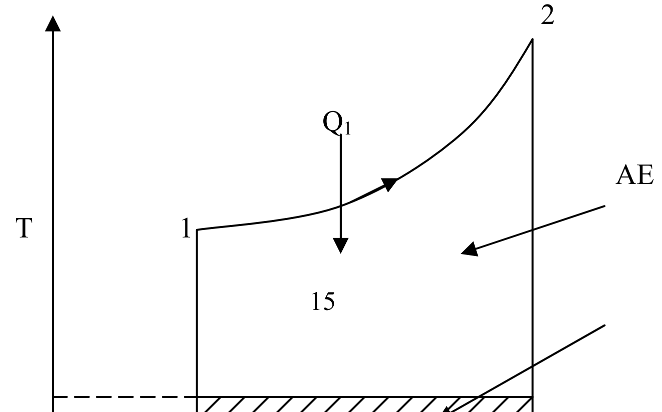
$T_1$ → Higher temperature (Source temperature)

$T_2$ → Lower temperature (Sink temperature)

For maximum efficiency $T_2$ should be equal to atmospheric temperature ($T_a$).

$$\therefore \quad \eta = 1 - \frac{T_a}{T_1} = \frac{W}{Q_1} = \frac{AE}{Q_1}$$

$$AE = Q_1\left(1 - \frac{T_a}{T_1}\right)$$



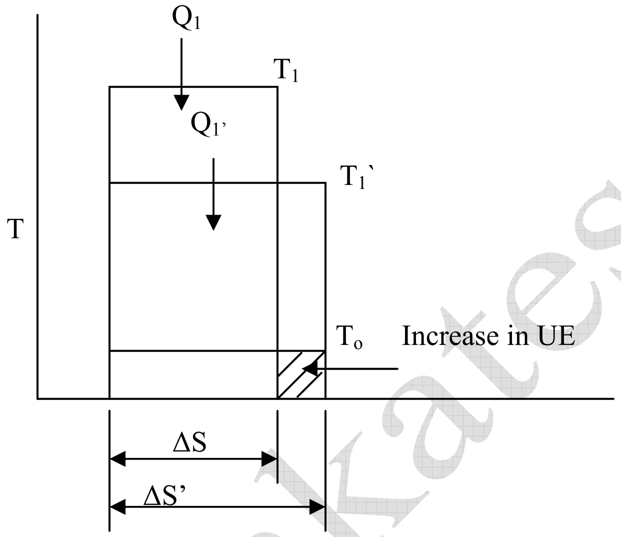
UE

$T_o$

S

Unavailable energy (UE) = $T_o (S_2 - S_1)$

### Decrease in available energy when heat is transferred through a finite temperature difference



Let us consider a reversible heat engine operating between $T_1$ and $T_o$.

$$Q_1 = T_1\,\Delta S; \quad Q_2 = T_{\mathbf{o}}\,\Delta S$$

$$AE = (T_1 - T_o)\,\Delta S = T_1\,\Delta S - T_o\,\Delta S$$

Now assume that the heat $Q_1$ is transferred through a finite temperature difference from the source at $T_1$ to the engine absorbing heat at $T_1$` lower than $T_1$.

$$Q_1 = T_1'\,\Delta S'\,; \quad Q_2' = T_{\mathbf{o}}\,\Delta S'$$

$$AE = (T_1' - T_o)\,\Delta S' = T_1'\,\Delta S' - T_o\,\Delta S'$$

Decrease in AE = $T_1\,\Delta S - T_o\,\Delta S - T_1'\,\Delta S' - T_o\,\Delta S'$

But $T_1\,\Delta S = T_1'\,\Delta S'$

**Availability**

Availability of a given system is defined as the maximum useful work that is obtainable in a process in which the system comes to equilibrium with its surroundings.

*Open system*: $$W_{\max} = (h_1 - h_2) - T_o(s_1 - s_2)$$

$$W_{u\max} = W_{\max}$$

Availability, $\psi_1 - \psi_2 = (h_1 - h_2) - T_o(s_1 - s_2)$

*Closed system*: $$W_{\max} = (u_1 - u_2) - T_o(s_1 - s_2)$$

Availability, $\phi_1 - \phi_2 = (u_1 - u_2) - T_o(s_1 - s_2) + p_o(v_1 - v_2)$

### III-Law of thermodynamics (Nernst theorem)

- States that the entropy of a pure substance at absolute zero temperature (0 K) is zero. Or it is impossible to attain absolute zero temperature.

Consider a reversed heat engine to remove heat from low temperature body to attain 0 K. The work required to run the RHE (refrigerator) will be minimum if it operates on Carnot cycle.

$$\text{Carnot COP} = T_2 / (T_1 - T_2) = \text{Heat extracted / Work input}$$

$$\text{Work input} = \text{Heat extracted x } (T_1 - T_2) / T_2$$

$T_1$ → Sink temperature (high)

$T_2$ → Source temperature (low)

If $T_2$ is 0 K, the work input will become infinitive. Infinitive work input is impossible and therefore attaining absolute zero temperature is impossible.

## Problems

1. **A heat engine receives heat at the rate of 1500 kJ/min and gives an output of 8.2 kW. Determine the thermal efficiency and the rate of heat rejection.**

Given:

Heat received by engine (HS) = 1500 kJ/min = 25 kW

Work output (W) = 8.2 kW

Required: $\eta_{th}$ and HR

Solution:

$\eta_{th}$ = Work output / Heat Supplied = 8.2 / 25 = **0.328 ---- Ans**

Workdone = HS – HR

8.2 = 25 – HR

∴ Heat rejected (HR) = **16.8 kW ---- Ans**

2. **A house requires 2 x $10^5$ kJ/h for heating the house in winter. Heat pump is used to absorb heat from cold air outside in winter and send heat to the house. Work required to operate the heat pump is 3 x $10^4$ kJ/h. Determine heat abstracted from outside and COP.**

Given:

Heat pump

Heat to be supplied to the house (HR) = $2 \times 10^5$ kJ/h

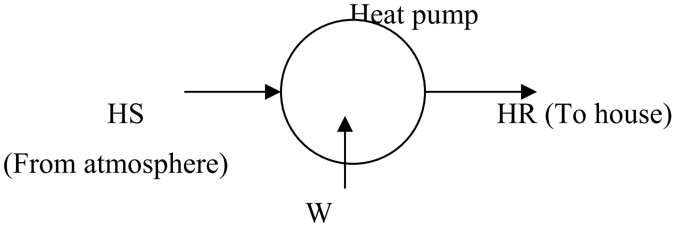
Work input (W) = $3 \times 10^4$ kJ/h

Required: HS and COP

Solution:

HS = Heat absorbed from the atmosphere

HR = Heat rejected by the heat pump = Heat supplied to the room



For heat pump, $- W = HS + HR$

$-3 \times 10^4 = HS - 2 \times 10^5$

HS = **$1.7 \times 10^5$ kJ/h ---- Ans**

COP = Out put / Input

= Heat given to house / work input to heat pump

= $2 \times 10^5 / 3 \times 10^4$ = **6.66 --- Ans**

3. **0.04 kg of $CO_2$ (M = 44) is compressed from 1 bar, 20°C, until the pressure is 9 bar and the volume is then 0.003 $m^3$. Calculate the change of entropy. Take $C_p$ for $CO_2$ as 0.88 kJ/K and assume $CO_2$ to be a perfect gas.**

*Given:*

Mass of gas (m) = 0.04 kg

Initial pressure ($p_1$) = 1.05 bar

Initial temperature ($T_1$) = 20°C = 293 K

Final pressure ($p_2$) = 9 bar

Final volume ($V_2$) = 0.003 $m^3$

*Required*: $\Delta S$

*Solution:*

$S_2 - S_1 = m R \ln [V_2/V_1] + m C_v \ln [T_2/T_1]$ → General equation

Universal gas constant ($R_u$) = 8314 J/kg-K

Characteristic gas constant (R) = $R_u$/M = 8314/44 = 188.95 J/kg-K

To find m

$$p_1 V_1 = m R T_1$$

$$1 \times 10^5 \times V_1 = 0.04 \times 188.95 \times 293$$

$$V_1 = 0.02214494 \text{ m}^3$$

To find $T_2$

$$p_2 V_2 = m R T_2$$

$$9 \times 10^5 \times V_2 = 0.04 \times 188.95 \times T_2$$

$$T_2 = 357.24 \text{ K}$$

To find $C_v$

$$C_p - C_v = R$$

$$880 - C_v = 188.95$$

$$C_v = 691.05 \text{ J/kg-K}$$

$\therefore \quad S_2 - S_1 = (0.04)(188.95) \ln[0.003/0.02214494] + (0.04)(691.05) \ln[357.24/293]$

$= \mathbf{-9.62847 \text{ J/kg-K}}$ **--- Ans**

4. **A perfect gas is compressed according to the law $pV^{1.25} = C$ from an initial pressure of 1 bar and volume of 0.9 $m^3$ to a final volume of 0.6 $m^3$. Determine the final pressure and change of entropy per kg of gas during the process. Take $\gamma = 1.4$ and R = 287 J/kg-K.**

*Given:*

Polytropic process

| | |
|---|---|
| Index of compression (n) | = 1.25 |
| Initial pressure ($p_1$) | = 1 bar |
| Initial volume ($V_1$) | = 0.9 $m^3$ |
| Final volume ($V_2$) | = 0.6 $m^3$ |

*Required:* $p_2$ & $\Delta S$ per kg

*Solution:*

$$S_2 - S_1 = m R \ln[V_2/V_1] + m C_v \ln[T_2/T_1] \rightarrow \text{General equation}$$

To find $p_2$

$$p_2 / p_1 = [V_1/V_2]^n$$

$$p_2 / 1 = [0.9/0.6]^{1.25}$$

$p_2 = 1.66$ bar --- Ans

To find $C_v$

$$C_p - C_v = R \text{ and } C_p/C_v = \gamma$$

$$C_p = \gamma C_v$$

$$\gamma C_v - C_p = R$$

$$1.4 C_v - C_v = 287$$

$$C_v = 717.5 \text{ J/kg-K}$$

$$\therefore \quad C_p = 1.4 \times 717.5 = 1004.5 \text{ J/kg-K}$$

To find $T_1$ & $T_2$

$$m = 1 \text{ kg}$$

$$p_1 V_1 = m R T_1$$

$$1 \times 10^5 \times 0.9 = 1 \times 287 \times T_1$$

$$T_1 = 313.6 \text{ K}$$

$$p_2 V_2 = m R T_2$$

$1.66 \times 10^5 \times 0.6 = 1 \times 287 \times T_2$

$T_2 = 347.04$ K

$\therefore \quad S_2 - S_1 = 1 \times 287 \times \ln [0.6/0.9] + 1 \times 717.5 \times \ln [347.04/313.6]$

= **-43.62 J/kg-K --- Ans**

Note: $\Delta S$ can be found out using, $(S_2 - S_1) = (n - \gamma)/(n - 1) \times m\, C_v \ln [T_2/T_1]$ during polytropic process.

5. **Calculate the change of entropy of 1 kg of air expanding polytropically in a cylinder behind a piston from 7 bar and 600°C to 1.05 bar. The index of expansion is 1.25.**

*Given:*

Polytropic process

| | |
|---|---|
| Index of compression (n) | = 1.25 |
| Mass of air (m) | = 1 kg |
| Initial pressure ($p_1$) | = 7 bar |
| Initial pressure ($p_1$) | = 1.05 bar |
| Initial temperature ($T_1$) | = 600°C = 873 K |

*Required:* $\Delta S$

*Solution:*

$S_2 - S_1 = m R \ln [V_2/V_1] + m C_v \ln [T_2/T_1]$ → General equation

$$\frac{T_2}{T_1} = \left[\frac{p_2}{p_1}\right]^{(n-1)/n}$$

$$\frac{T_2}{T_1} = \left[\frac{1.05}{7}\right]^{(1.25-1)/1.25}$$

= 0.68425

$$\frac{V_2}{V_1} = \left[\frac{p_1}{p_2}\right]^{1/n}$$

$$\frac{V_2}{V_1} = \left[\frac{7}{1.05}\right]^{1/1.25}$$

= 4.5617

$C_v$ = 717 J/kg-K & R = 287 J/kg-K (Taken)

$\therefore \quad S_2 - S_1 = 1 \times 287 \times \ln [4.5617] + 1 \times 717 \times \ln [0.68425]$

= **163.52 J/kg-K --- Ans**

6. **A Carnot engine is operated between two reservoirs at temperatures of 450 K and 325 K. If the engine receives 300 kJ of heat from the source in each cycle, calculate the amount of heart rejected to the sink in each cycle. Calculate the efficiency of the engine and the workdone by the engine in each cycle.**

*Given:*

Carnot engine

Higher temperature ($T_1$) = 450 K

Lower temperature ($T_2$) = 325 K

Heat supplied (HS) = 300 kJ

*Required*: HR, $\eta$, W

*Solution:*

$\eta_c = (T_1 - T_2) / T_1$

$= (450 - 325) / 450 =$ **0.278 ---- Ans**

Also $\eta_c = (HS - HR) / HS = W / HS$

$W = \eta \times HS = 0.278 \times 300 =$ **83.4 kJ --- Ans**

$W = HS - HR$

$83.4 = 300 - HR$

HR = **216.66 kJ --- Ans**

7. **In a Carnot cycle, the maximum pressure and temperature are limited to 18 bar and 410$^o$C. The ratio of isentropic compression is 6 and isothermal expansion is 1.5. Assuming the volume of the air at the beginning of isothermal expansion as 0.18 m$^3$, determine (i) the pressure and temperature at main points (ii) change in entropy during isothermal expansion (iii) mean thermal efficiency of the cycle (iv) mean effective pressure of the cycle and (v) the theoretical power if there are 210 working cycles per min.**

*Given:*

Maximum pressure ($p_3$) = 18 bar

Maximum temperature ($T_3$) = 410$^o$C = 683 K = $T_4$

Ratio of isentropic compression ($V_2/V_3$) = 6

Ratio of isothermal expansion ($V_4/V_3$) = 1.5

Volume at the beginning of isothermal expansion ($V_3$) = 0.18 m$^3$

Working cycle per min (n) = 210

*Required* : (i) $p_1$, $T_1$, $p_2$, $T_2$, $p_4$ (ii) $\Delta$ S (iii) $\eta$ (iv) mep (v) P

*Solution:*

(i) To find $p_2$

$(p_2 / p_3) = [V_3 / V_2]^\gamma$

$\therefore \quad p_2 / 18 = [1/6]^{1.4}$

$\therefore$ **$p_2$ = 1.465 bar --- Ans**

To find $T_2$

$(T_2 / T_3) = [V_3 / V_2]^{\gamma-1}$

$\therefore \quad T_2 / 683 = (1/6)^{1.4-1}$

$\therefore$ **$T_2$ = 333.55 K --- Ans = $T_1$**

To find $p_4$

$p_3V_3 = p_4V_4$

$V_4 = 1.5\ V_3 = 1.5 \times 0.18 = 0.27$ m$^3$

$\therefore\ 18 \times 0.18 = p_4 \times 0.27$

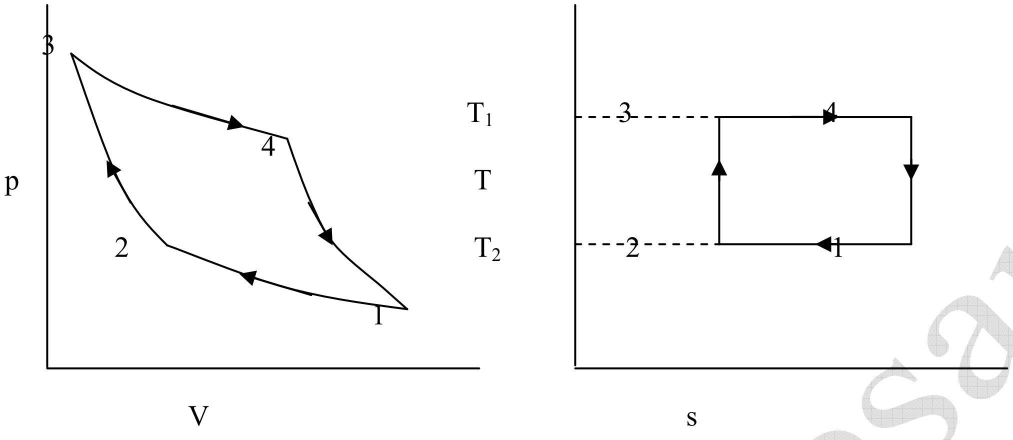
**$p_4$ = 12 bar --- Ans**

To find $p_1$

$$(p_1 / p_4) = [V_4 / V_1]^{\gamma} = [V_3/V_2]^{\gamma}$$

$$\therefore \quad p_1 / 12 = [1/6]^{1.4}$$

$$\therefore \quad \mathbf{p_1 = 0.977\ bar}$$ **--- Ans**



(ii) Change in entropy during isothermal expansion

$$\Delta S = m R \ln [V_4/V_3]$$

To find m

$$p_1 V_1 = m R T_1$$

To find $V_1$

$$V_1 = 6 \times 0.27 = 1.62 \text{ m}^3$$

$$0.977 \times 10^5 \times 1.62 = m \times 287 \times 333.55$$

$$m = 1.653 \text{ kg}$$

$$\therefore \Delta S = 1.653 \times 287 \times \ln [1.5] = \mathbf{192.36\ J/K}$$ **---- Ans**

(iii) $\eta = (T_{max} - T_{min}) / T_{max} = (683 - 333.55) / 683$

$$= \mathbf{0.5116}$$ **---- Ans**

(iv) $p_m = W / V_s$

$$W = \text{Area of T-s chart} = \Delta S\, (T_{max} - T_{min})$$

$$= 192.36 \times (683 - 333.55)$$

$$= 67220.2 \text{ J}$$

$$V_s = V_1 - V_3$$

$$= 1.62 - 0.18 = 1.44 \text{ m}^3$$

$$p_m = 67220.2 / 1.44 = \mathbf{46680.7\ bar}$$ **--- Ans**

(v) Power = W x n / 60 = 67220.2 x 210 / 60 = **2357270.7 W --- Ans**

**8. A domestic food freezer maintains a temperature of -15°C. The ambient air temperature is 30°C. If heat leaks into the freezer at the continuous rate of 1.75 kJ/s, what is the least power necessary to pump this heat out continuously?**

Given:

Freezer (Cold space)

Freezer temperature ($T_2$) = -15°C = 258 K

Ambient temperature ($T_1$) = 30°C = 303 K

Heat into the freezer = Heat absorbed by the refrigerant = $Q_2$ = 1.75 kJ/s

Required: Least power (W)

Solution:

Refrigerator working on Carnot cycle requires least power input for its operation.

$$COP = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

$$COP = \frac{258}{303 - 258} = 5.733$$

$$5.733 = \frac{1.75}{W}$$

W = **0.305 kW --- Ans**

9. **A reversible heat engine operates between two reservoirs at temperatures of 600°C and 40°C. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 40°C and -20°C. The heat transfer to the engine is 2000 kJ and the net work output of the combined engine-refrigerator plant is 360 kJ. (a) Evaluate the heat transfer to the refrigerant and the net heat transfer to the reservoir at 40°C. (b) Reconsider (a) given that the efficiency of the heat engine and the COP of the refrigerator are each 40% of their maximum possible values.**

*Given:*

Combined engine-refrigerator plant

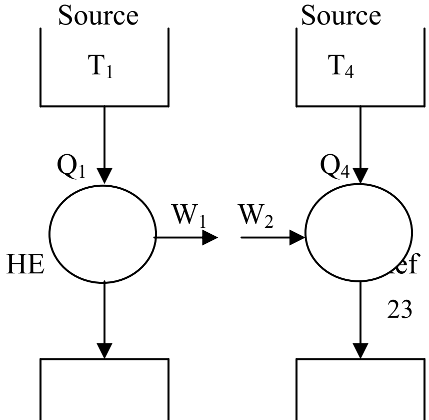
| | |
|---|---|
| Engine source temperature ($T_1$) | = 600°C = 873 K |
| Engine sink temperature ($T_2$) | = 40°C = 313 K |
| Heat transfer to the engine ($Q_1$) | = 2000 kJ |
| Refrigerator source temperature ($T_4$) | = -20°C = 253 K |
| Refrigerator sink temperature ($T_3$) | = 40°C = 313 K |
| Net work transfer ($W_1 - W_2$) | = 360 kJ |

*Required*: (a) For reversible engine, Heat transfer to the refrigerant ($Q_4$) and Net heat transfer to the reservoir at 40°C ($Q_2 + Q_3$)

(b) For irreversible engine with $\eta = 0.4\ (\eta_{rev})$ and COP = 0.4 ($COP_{rev}$), $Q_4$ and $Q_2 + Q_3$

*Solution:*

(a) $$\eta_{rev} = \frac{T_1 - T_2}{T_1} = \frac{W_1}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$



$Q_2$ $\qquad\qquad$ $Q_3$

$T_2$ $\qquad\qquad$ $T_3$

Sink $\qquad\qquad$ Sink

$$\eta_{rev} = \frac{873 - 313}{873} = 0.0414$$

$$0.6414 = \frac{W_1}{2000} \qquad\qquad W_1 = 1282.8 \text{ kJ}$$

$W_1 = Q_1 - Q_2$

$1282.8 = 2000 - Q_2$ $\qquad\qquad$ $Q_2 = 717.2$ kJ

$W_1 - W_2 = 360$

$1282.8 - W_2 = 360$ $\qquad\qquad$ $W_2 = 922.8$ kJ

$$COP_{rev} = \frac{Q_4}{Q_3 - Q_4} = \frac{Q_4}{W_2} = \frac{T_4}{T_3 - T_4}$$

$$COP_{rev} = \frac{253}{313 - 253} = 4.2166$$

$$4.2166 = \frac{Q_4}{922.8}$$ $Q_4$ = **3891 kJ ----- Ans**

$W_2 = Q_3 - Q_4$

$922.8 = Q_3 - 3891$ $\qquad\qquad$ $Q_3 = 4813.8$ kJ

Therefore, $\qquad$ $Q_3 + Q_4 = 4813.8 + 717.2 =$ **5531 kJ ------ Ans**

(b)

$$\eta = 0.4\eta_{rev} = 0.4(0.6414) = 0.256$$

$$COP = 0.4\,COP_{rev} = 0.4(4.2166) = 1.686$$

$$0.256 = \frac{W_1}{2000} \qquad\qquad W_1 = 512 \text{ kJ}$$

$W_1 = Q_1 - Q_2$

$512 = 2000 - Q_2$ $\qquad\qquad$ $Q_2 = 1488$ kJ

$W_1 - W_2 = 360$

$512 - W_2 = 360$ $\qquad\qquad$ $W_2 = 152$ kJ

$$1.686 = \frac{Q_4}{152}$$ $Q_4$ = **256.3 kJ ----- Ans**

$W_2 = Q_3 - Q_4$

$152 = Q_3 - 256.3$ $\qquad\qquad$ $Q_3 = 408.3$ kJ

Therefore, $\qquad$ $Q_3 + Q_4 = 1488 + 408.3 =$ **1896.3 kJ ------ Ans**

**10. A heat engine operating between two reservoirs at 1000 K and 300 K is used to drive a heat pump which extracts heat from the reservoir at 300 K at a rate twice that at which the**

**engine rejects heat to it. If the efficiency of the engine is 40% of the maximum possible and COP of the heat pump is 50% of the maximum possible, what is the temperature of the reservoir to which the heat pump rejects heat? What is the rate of heat rejection from the heat pump if the rate of heat supply to the engine is 50 kW?**

*Given:*

Combined engine-heat pump plant

Engine source temperature ($T_1$) = 1000 K

Engine sink temperature ($T_2$) = 300 K

Heat supply to the engine ($Q_1$) = 50 kW

Heat pump source temperature ($T_4$) = 300 K

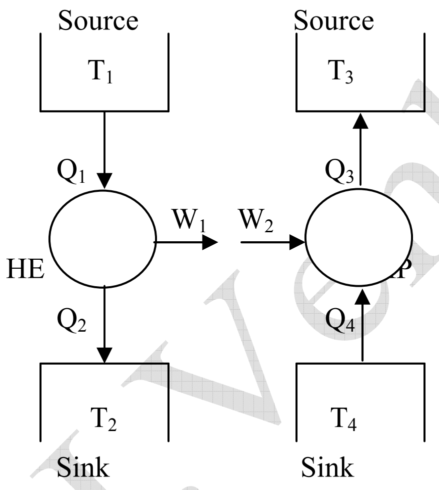
Efficiency of the engine ($\eta$) = $0.4\ \eta_{rev}$

COP of the heat pump = $0.5\ COP_{rev}$

Hear extraction by the heat pump ($Q_4$) = 2 x Heat rejection by the engine ($Q_2$)

*Required:* Temperature of heat pump sink ($T_3$) and Heat rejection from the pump ($Q_3$)

*Solution:*



(a)

$$\eta_{rev} = \frac{T_1 - T_2}{T_1} = \frac{W_1}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta_{rev} = \frac{1000 - 300}{1000} = 0.7$$

$$\eta = 0.4\eta_{rev} = 0.4(0.7) = 0.28$$

$$\eta = \frac{W_1}{Q_1}$$

$$0.28 = \frac{W_1}{50} \qquad W_1 = 14 \text{ kW}$$

$$W_1 = Q_1 - Q_2$$

$$14 = 50 - Q_2 \qquad Q_2 = 36 \text{ kW}$$

$Q_4 = 2\ (36) = 72$ kW

$$COP_{rev} = \frac{Q_3}{Q_3 - Q_4} = \frac{Q_3}{W_2} = \frac{T_3}{T_3 - T_4}$$

$W_2 = Q_3 - Q_4$

$14 = Q_3 - 72$ $\qquad Q_3 =$ **86 kW ---- Ans**

Note: $W_1 = W_2$

$$COP = \frac{Q_3}{Q_3 - Q_4} = \frac{Q_3}{W_2}$$

$$COP = \frac{86}{14} = 6.1428$$

$$COP_{rev} = \frac{COP}{0.5} = \frac{6.1428}{0.5} = 12.286$$

$$12.286 = \frac{T_3}{T_3 - 300}$$

$T_3 =$ **326.6 K ----- Ans**

**11. Two reversible heat engines A and B are arraged in series. Engine-A rejecting heat directly to engine-B and receives 200 kJ at a temperature of 421°C from a hot source, while engine-B is in communication with a cold sink at a temperature of 4.4°C. If the work output of A is twice that of B, find (i) the intermediate temperature between A and B, (ii) the efficiency of each engine and (iii) the heat rejected to the cold sink.**

*Given:*

Two engines in series

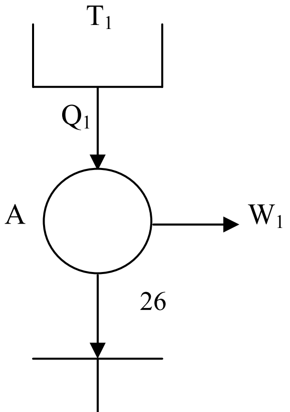
| | |
|---|---|
| Heat received by engine-A ($Q_1$) | = 200 kJ = 200000 J |
| Temperature of source for the engine-A ($T_1$) | = 421°C = 694 K |
| Temperature of cold sink ($T_3$) | = 4.4°C = 277.4 K |
| Work out put of engine-A ($W_1$) | = 2 $W_2$ |

*Required*: (i) $T_2$ (ii) $\eta_A$ and $\eta_B$ (iii) $Q_3$

*Solution:*

$$\eta_A = \frac{T_1 - T_2}{T_1} = \frac{W_1}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\frac{694 - T_2}{694} = \frac{200000 - Q_2}{200000}$$



$Q_2$

$T_2$

$Q_2$

B $W_2$

$Q_3$

$T_3$

$$1-\frac{T_2}{694}=1-\frac{Q_2}{200000}$$

200000 $T_2$ – 694 $Q_2$ = 0

$$T_2=\frac{694\,Q_2}{200000} \quad \text{----- (1)}$$

$$\eta_B=\frac{T_2-T_3}{T_2}=\frac{W_2}{Q_2}=\frac{Q_2-Q_3}{Q_2}$$

$$\frac{T_2-277.4}{T_2}=\frac{0.5(Q_1-Q_2)}{Q_2}$$

$$\frac{T_2-277.4}{T_2}=\frac{0.5\,x\,200000-0.5\,Q_2}{Q_2}$$

$$1-\frac{274}{T_2}=\frac{100000}{Q_2}-0.5$$

$$1-\frac{274\,x\,200000}{694\,Q_2}=\frac{100000}{Q_2}-0.5$$

Solving the above, $Q_2$ = 119961 J

Therefore, $T_2$ = 694 x 119961 / 200000 = **416.26 K ----- Ans**

$$\eta_A=\frac{694-416.26}{694}=0.4 \quad \text{------ } \textbf{Ans}$$

$$\eta_B=\frac{416.26-277.4}{416.26}=0.333 \quad \text{----- } \textbf{Ans}$$

Also, $$\eta_B=\frac{119961-Q_3}{119961}=0.333$$

$Q_3$ = **80014 J --- Ans**

**12. 50 kg of water is at 313 K and enough ice is at -5°C is mixed with water in an adiabatic vessel such that at the end of the process all the ice melts and water at 0°C is obtained. Find the mass of ice required and the entropy change of water and ice. Given $C_p$ of water is 4.2 kJ/kg-K, $C_p$ of ice is 2.1 kJ/kg-K and latent heat of ice is 333.3 kJ/kg.**

*Given:*

Mass of water ($m_w$) = 50 kg

Initial temperature of water ($T_{w,1}$) = 313 K

Initial temperature of ice ($T_{ice,1}$) = -5°C = 268 K

Final temperature of water ($T_{w,2}$) = 0°C = 273 K

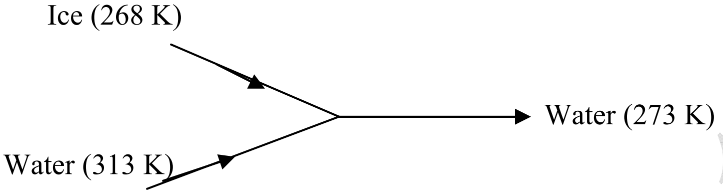
$C_p$ of water = 4200 J/kg-K

$C_p$ of ice = 2100 J/kg-K

Latent heat of ice = $333.3 \times 10^3$ J/kg

*Required:* Mass of ice, Change of entropy of ice and water

*Solution:*



$m_{ice}$ = Mass of ice

Energy of the water before mixing = $m_w\, C_{pw}\, T_{w1}$

= (50) (4200) (313) = 65730000 J

Energy of ice before mixing = $m_{ice}\, C_{pice}\, T_{ice,1}$

= $m_{ice}$ (2100) (268) = 562800 $m_{ice}$ J

Energy of mixture (water at 0°C) = $(m_w + m_{ice})\, C_{pw}\, T_{w,2}$

= $(50 + m_{ice})$ (4200) (273)

Energy of water before mixing + Energy of ice before mixing

= Energy of mixture after mixing

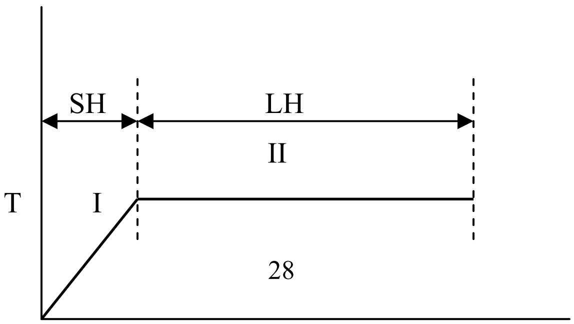
$$65730000 + 562800\, m_{ice} = (50 + m_{ice})\,(4200)\,(273)$$

$m_{ice}$ = **14.388 kg ---- Ans**

Entropy change of water $= m_w\, C_{pw} \ln\left(\dfrac{T_{w,1}}{T_{w,2}}\right) = (50)\,(4200)\ln\left(\dfrac{313}{273}\right)$

= **28713.6 J/K ----- Ans**

Entropy change of ice $(\Delta S)_{ice} = (\Delta S)_I + (\Delta S)_{II}$



Heat (Q)

$$(\Delta S)_I = m_{ice}\, C_{pice}\, \ln\left(\frac{T_{ice,1}}{T_{ice,2}}\right) = (14.388)\,(2100)\ln\left(\frac{268}{273}\right) = -558.5\, J/K$$

$$(\Delta S)_{II} = \frac{dQ}{T} = \frac{m_{ice}\ x\,Latent\,heat\,of\ \ fusion}{T} = \frac{(14.388)(333.3\,x10^3)}{273} = 17566\, J/K$$

$$(\Delta S)_{ice} = (-558.5) + (17566) = 17007.5\ \ J/K$$ ------ **Ans**

**13. 1 kg of air is contained in a piston cylinder assembly at 10 bar pressure and 500 K temperature. The piston moves outwards and the air expands to 2 bar pressure and 350 K temperature. Determine the maximum work obtainable. Assume the environmental conditions to be 1 bar and 290 K. Also make calculations for the availability in the initial and final states.**

*Given:*

Mass of air (m) = 1 kg

Initial pressure of air ($p_1$) = 10 bar = 10 x $10^5$ Pa

Initial temperature of air ($T_1$) = 500 K

Final pressure of air ($p_2$) = 2 bar = 2 x $10^5$ Pa

Final temperature of air ($T_2$) = 350 K

Environment pressure ($p_o$) = 1 bar = 1 x $10^5$ Pa

Environment temperature ($T_o$) = 290 K

*Required:* (a) $W_{max}$ (b) $\Phi_1 - \Phi_o$ and $\Phi_2 - \Phi_o$

*Solution:*

(a)

$$W_{\max} = (U_1 - U_2) - T_o(S_1 - S_2)$$

$$U_1 - U_2 = mC_v(T_1 - T_2) = (1)\,(718)\,(500 - 350) = 107700\, J$$

$$S_1 - S_2 = m\,R\ln\left(\frac{V_1}{V_2}\right) + m\,C_v\,\ln\left(\frac{T_1}{T_2}\right)$$

$$\frac{V_1}{V_2} = \frac{p_2 T_1}{p_1 T_2} = \frac{2\,x500}{10\,x350} = 0.2857$$

$$S_1 - S_2 = (1)\,(287)\ \ln(0.2857) + (1)\,(718)\ln\left(\frac{500}{350}\right) = -103.46\, J/K$$

$$W_{\max} = (107700) - 290(-103.46) = 137703.4\ \ J\ \ ---\ Ans$$

(b)

$$\phi_1 - \phi_o = (U_1 - U_o) - T_o(S_1 - S_o) + p_o(V_1 - V_o)$$

$$U_1 - U_o = mC_v(T_1 - T_o) = (1)\,(718)\,(500 - 290) = 150780\, J$$

$$S_1 - S_o = mR\ln\left(\frac{V_1}{V_o}\right) + mC_v\ln\left(\frac{T_1}{T_o}\right)$$

$$\frac{V_1}{V_o} = \frac{p_oT_1}{p_1T_o} = \frac{1x500}{10\,x\,290} = 0.1724$$

$$S_1 - S_o = (1)(287)\ \ln(0.1724) + (1)(718)\ln\left(\frac{500}{290}\right) = -113.4\ J/K$$

$$V_1 = \frac{mRT_1}{p_1} \qquad V_o = \frac{mRT_o}{p_o}$$

$$V_1 - V_o = mR\left(\frac{T_1}{p_1} - \frac{T_o}{p_o}\right) = (1)(287)\left(\frac{500}{10\,x10^5} - \frac{290}{1x10^5}\right) = -0.6888\,m^3$$

$$\therefore \quad \phi_1 - \phi_o = (150780) - (290)(-113.4) + (1x10^5)(-0.6888)$$
$$= 114786\ J \quad ---- \ Ans$$

$$\phi_2 - \phi_o = (U_2 - U_{o2}) - T_o(S_2 - S_o) + p_o(V_2 - V_o)$$

$$U_2 - U_o = mC_v(T_2 - T_o) = (1)(718)(350 - 290) = 43080\,J$$

$$S_2 - S_o = mR\ln\left(\frac{V_2}{V_o}\right) + mC_v\ln\left(\frac{T_2}{T_o}\right)$$

$$\frac{V_2}{V_o} = \frac{p_oT_2}{p_2T_o} = \frac{1x350}{2\,x\,290} = 0.6034$$

$$S_2 - S_o = (1)(287)\ \ln(0.6034) + (1)(718)\ln\left(\frac{350}{290}\right) = -9.964\ J/K$$

$$V_2 = \frac{mRT_2}{p_2} \qquad V_o = \frac{mRT_o}{p_o}$$

$$V_2 - V_o = mR\left(\frac{T_2}{p_2} - \frac{T_o}{p_o}\right) = (1)(287)\left(\frac{350}{2\,x10^5} - \frac{290}{1x10^5}\right) = -0.33\,m^3$$

$$\therefore \quad \phi_2 - \phi_2 = (43080) - (290)(-9.964) + (1x10^5)(-0.33)$$
$$= 12969.56\ J \quad ---- \ Ans$$

$$\phi_1 - \phi_2 = (\phi_1 - \phi_o) - (\phi_2 - \phi_o)$$

= 114786 – 12969.56 = **101816.44 J --- Ans**

**14. Two kg of air at 500 kPa, 80°C expands adiabatically in a closed system until its volume is doubled and its temperature becomes equal to that of the surroundings which is at 100 kPa, 5°C. For this process, determine (a) the maximum work, (b) the change in availability, and (c) the irreversibility.**

*Given:*

Closed system

Mass of air (m) = 2 kg

Initial pressure of air ($p_1$) = 500 kPa = 500 x $10^3$ Pa

Initial temperature of air ($T_1$) = 80°C = 353 K

Final pressure of air ($p_2$) = 100 kPa = 100 x $10^3$ Pa

Final temperature of air ($T_2$) = 5°C = 278 K

$V_2 = 2\ V_1$

*Required*: (a) $W_{max}$ (b) $\Phi_1 - \Phi_2$ (c) I

*Solution:*

(a)

$$W_{\max} = \left(U_1 - U_2\right) - T_o\left(S_1 - S_2\right)$$

$$U_1 - U_2 = m\,C_v\,(T_1 - T_2) = 2\,(718)\,(353 - 278) = 107700\,J$$

$$S_1 - S_2 = m\,R\ln\left(\frac{V_1}{V_2}\right) + m\,C_v\,\ln\left(\frac{T_1}{T_2}\right)$$

$$= 2\,(287)\ \ln\left(\frac{1}{2}\right) + 2\,(718)\ln\left(\frac{278}{353}\right) = -54.87\,J/K$$

$$\therefore \qquad W_{\max} = (107700) - 278(-54.87) = 122953.86\,J \quad \text{----} \quad \textbf{Ans}$$

(b)

$$\phi_1 - \phi_2 = \left(U_1 - U_2\right) - T_o\left(S_1 - S_2\right) + p_o\left(V_1 - V_2\right)$$

$$p_o\left(V_1 - V_2\right) = p_o\left(V_1 - 2V_1\right) = -p_o V_1$$

$$V_1 = \frac{m\,R\,T_1}{p_1} = \frac{2\,(287)\,(353)}{500\,x\,10^3} = 0.405244\,m^3$$

$$\therefore \qquad \phi_1 - \phi_2 = (107700) - 278(-54.87) - 100\,x\,10^3\,(0.405244)$$

$$= 82429.46\ J \qquad ---- \quad Ans$$

(c) Irreversibility, I = $W_{max}$ - $W_{act}$

$Q = W_{act} + (U_2 - U_1)$

Q = 0 → Adiabatic process

$W_{act} = (U_1 - U_2) = 107700$ J

I = 122953.86 – 107700 = **15253.86 J ---- Ans**

**15. Air expands through a turbine from 500 kPa, 520°C to 100 kPa, 300°C. During expansion 10 kJ/kg of heat is lost to the surroundings which is at 98 kPa, 20°C. Neglecting the KE and PE changes, determine per kg of air (a) the decrease in availability, (b) the maximum work, and (c) the irreversibility.**

Given:

Open system

Initial pressure of air ($p_1$) = 500 kPa = 500 x $10^3$ Pa

Initial temperature of air ($T_1$) = 520°C = 793 K

Final pressure of air ($p_2$) = 100 kPa = 100 x $10^3$ Pa

Final temperature of air ($T_2$) = 300°C = 573 K

Environment pressure ($p_o$) = 98 kPa = 98 x $10^3$ Pa

Environment temperature ($T_o$) = 293 K

Heat loss to the surroundings (q) = 10 kJ/kg = 10 x $10^3$ J/kg

Required: (a) $\Phi_1 - \Phi_2$ (b) $W_{max}$ (c) I

Solution:

(a)

$$\psi_1 - \psi_2 = (H_1 - H_2) - T_o(S_1 - S_2)$$

$$= (h_1 - h_2) - T_o(s_1 - s_2) \text{ per kg}$$

$$h_1 - h_2 = C_p(T_1 - T_2) = (1005)(793 - 573) = 221100\ J/kg$$

$$s_1 - s_2 = R\ln\left(\frac{V_1}{V_2}\right) + C_v \ln\left(\frac{T_1}{T_2}\right)$$

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{p_2T_1}{p_1T_2} = \frac{100\,x\,793}{500\,x\,573} = 0.2768$$

$$\therefore\ s_1 - s_2 = (287)\ \ln(0.2768) + (718)\ln\left(\frac{793}{573}\right) = -135{,}34\ J/kgK$$

$$\psi_1 - \psi_2 = (221100) - 293(-135.34) = 260754.62\ \ J/kg \text{ --- } \textbf{Ans}$$

(b) $\therefore\quad w_{\max} = \psi_1 - \psi_2 = 260754.52\ \ J/kg$ ---- **Ans**

(c) Irreversibility, I = $w_{max}$ - $w_{act}$

SFEE equation is given by,

m $h_1$ + m q = m $h_2$ + m $w_{act}$

$w_{act}$ = q + ($h_1$ – $h_2$) = (-10 x $10^3$) + (221100)

= 211100 J/kg

I = 260754.62 – 211100 = **49654.62 J /kg** ---- **Ans**

# UNIT-III

## PROPERTIES OF PURE SUBSTANCE AND STEAM POWER CYCLE

### Pure Substance

A ‘Pure substance’ is defined as a homogeneous material, which retains its chemical composition even though there may be a change in phase.

Water which is one of the pure substances, is an important engineering fluid. It exists in all three phases, i.e., solid (ice), liquid (water) and gas (steam)

Other examples: Carbon dioxide, Oxygen, Hydrogen, Argon, Ammonia, etc..

### Phases

In general, a pure substance exists in three forms. i) Solid phase, ii) Liquid phase & iii) Vapour or gaseous phase.

When ice melts there is a transformation of phase from solid to liquid. This is called melting or fusion of ice.

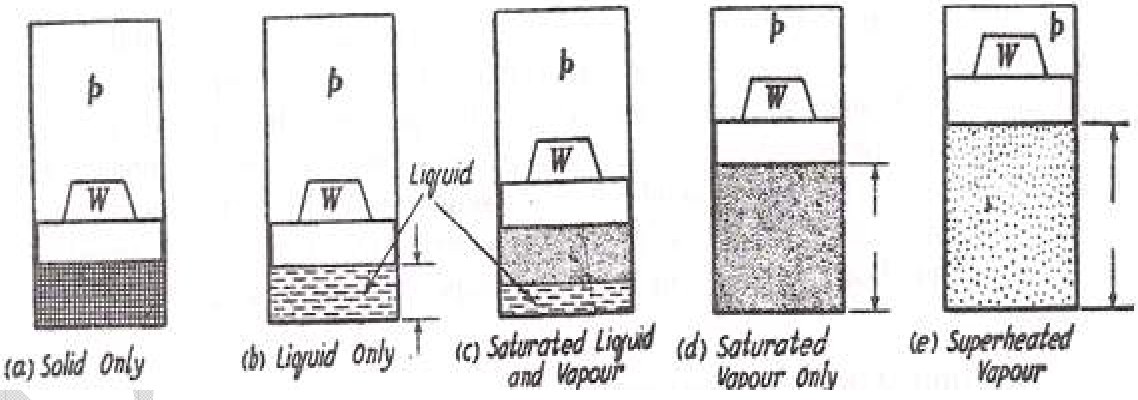
When water freezes there is a transformation of phase from liquid to solid. This is called freezing or solidification of water.

When water vapourises there is transformation of phase from liquid to vapour. This is called vapourisation.

When vapour condenses there is transformation of phase from vapour to liquid. This is called condensation.

### Steam generation

Consider 1 kg of ice contained in the cylinder under a pressure of ‘p’ exerted by the weight ‘W’ placed on the frictionless piston.



Let us assume $p_s = 5$ bar & $T_1 = -10^{o}C$

Let heat be added continuously. First the ice temperature rises and there is no phase change, till the temperature little lower than $0^{o}C$. The fusion temperature of water decreases slightly with increase in pressure.

Fusion or freezing temperature = $0^{o}C$ at atmospheric pressure. The transformation curve is shown below.

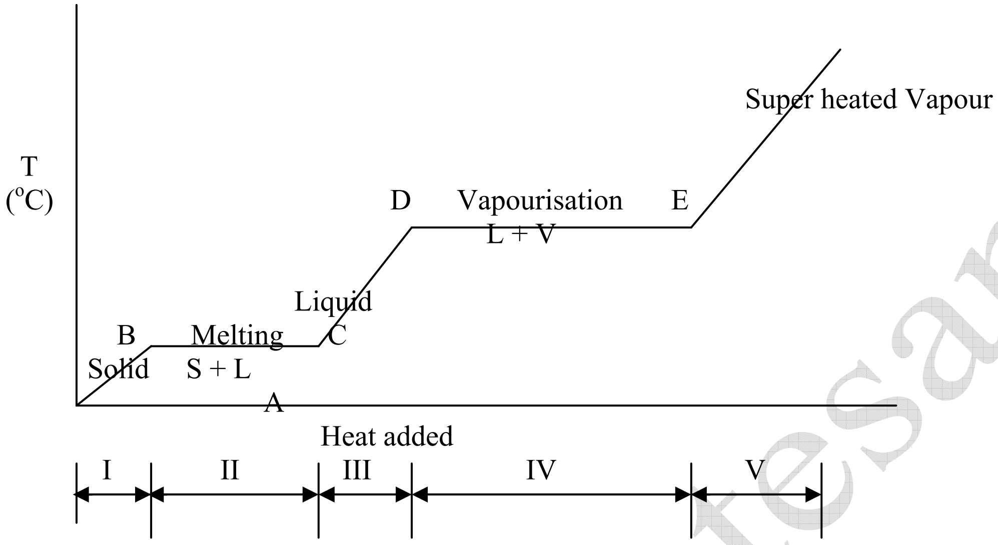
I – Sensible heat addition

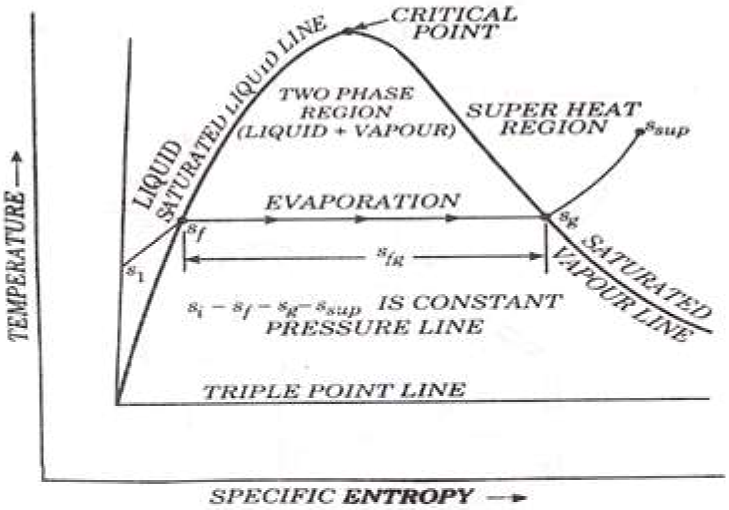
II – Latent heat of fusion

III – Sensible heat addition

IV – Latent heat of vapourisation

V – Heat of super heat





The warming up process is shown by ‘AB’. After this the ice begins to melt and two phase mixture of ice & water remains at the same temperature. No rise in temperature is recorded while heat is being added. This is shown by ‘BC’. At ‘C’, all the ice has melted and there is only one phase-liquid water under the pressure ‘p’.

The quantity of heat required to transform ice into water while there is no change in temperature is called ‘**heat of fusion’.**

If heat is further added to water, its temperature starts rising again and continues to do so till the water reaches the boiling temperature (vapourisation temperature). This process is shown by ‘CD’.

The boiling point is function of pressure.

$T_{boiling}$ = 100°C at 1 atm

= 151.84°C at 5 bar

This temperature is ‘Saturation temperature’ and the pressure corresponding to this temperature is called ‘Saturation pressure’.

After water has reached the saturation temperature, it starts vapourising while temperature remains same and once again the two-phase mixture of water and water vapour is formed.

Two phase system can not be defined only by its temperature and pressure. If pressure is known, the corresponding temperature of the two-phase system is known. Thus the state of vapour is defined by either the pressure or temperature and the quality called ‘dryness fraction’ (x). Dryness fraction is defined as the fraction of vapour present in the whole mass.

The process of vapourisation is shown by ’DE’. At ‘E’ all the water has vapourised and saturated vapour at pressure ‘p’ is obtained as shown. And the heat required to vapourise the liquid to vapour at constant temperature is known as the ‘**Latent heat of vapourisation’.**

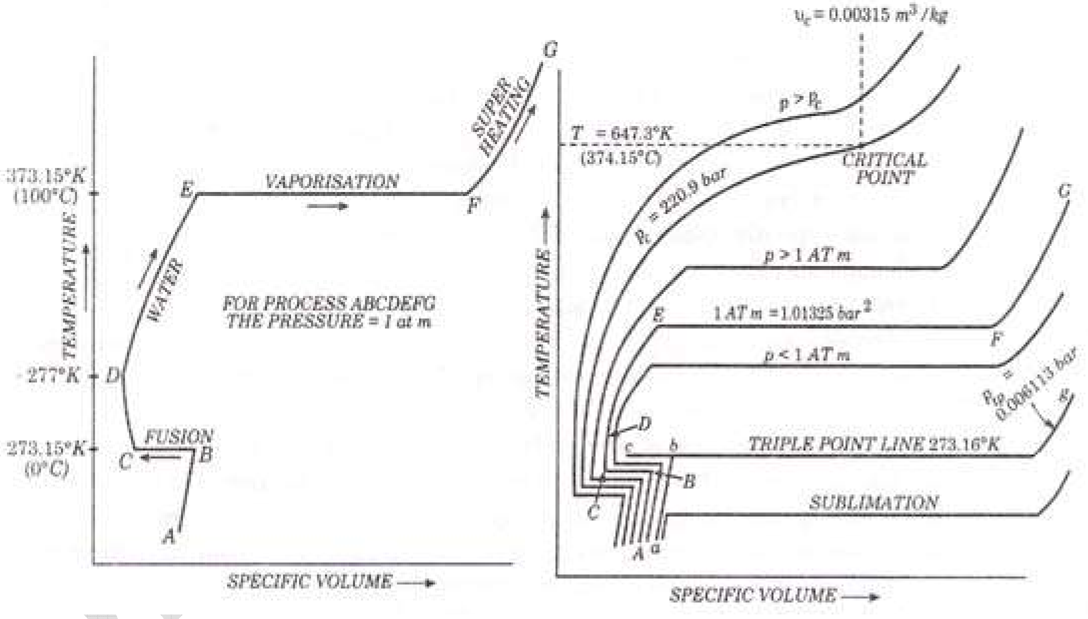
Further heating, i.e., heating after the quality of the vapour is 100 %, a further rise in temperature will rise the volume. The vapour is said to be ‘Superheated’. Therefore, Superheated vapour is vapour at any temperature above the saturation temperature.

**Critical point**

As the pressure increased above atmospheric, the change of volume accompanying evaporation decreases, till at sufficiently higher pressure, it is zero. This is referred as the ‘critical point’ and the properties at this point for water are,

$p_c = 221.2$ bar $\quad T_c = 374.15^oC \quad v_c = 0.00317\ m^3/kg$

Latent heat of vapourisation = 0



**Critical temperature and pressure**

The critical temperature of a liquid is that temperature beyond which the liquid cannot exist, no matter how much pressure is placed on it. The pressure that is needed to cause the gas to condense at the critical temperature is the critical pressure.

**Triple point**

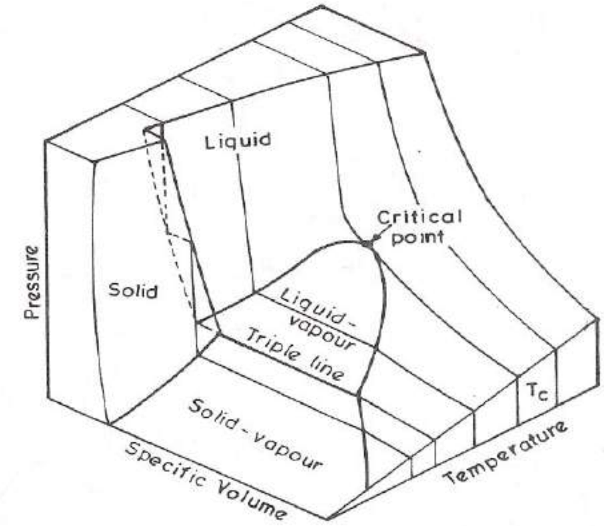
At low pressures there is slight rise in melting points, but a marked drop in boiling points and also very appreciable increase in the change of volume.

At pressure 0.006112 bar, the melting and boiling point temperatures become equal and the change of phase ice-water-steam is shown by single horizontal line’BCF’. This line is called ‘Triple point line’.

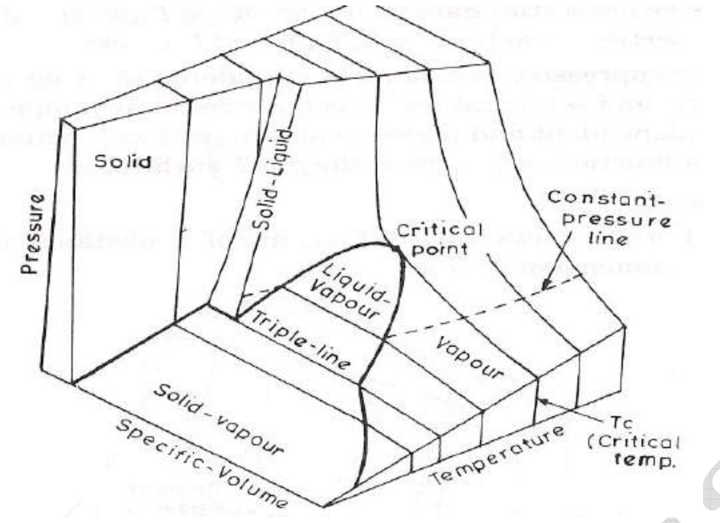
The temperature at this takes place is $0^oC$.

Therefore only at 0°C & 0.006112 bar, ice, water & steam coexist in thermodynamic equilibrium in a closed vessel.

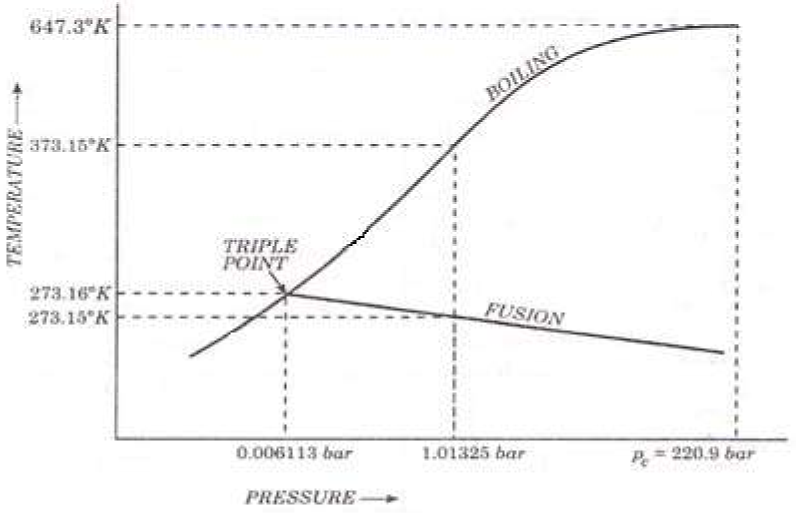
**p-v-T surface**



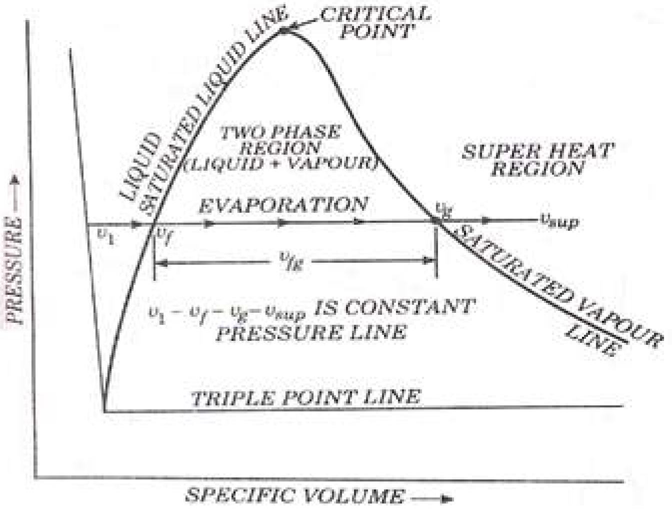
(a)


Solid
Solid-Liquid
Pressure
Critical point
Constant-pressure line
Liquid-Vapour
Triple-line
Vapour
Solid-vapour
Specific-Volume
Temperature
Tc (Critical temp.)

**(b)**


647.3°K
373.15°K
273.16°K
273.15°K
TEMPERATURE
BOILING
TRIPLE POINT
FUSION
0.006113 bar
1.01325 bar
$p_c$ = 220.9 bar
PRESSURE

**(c)**


CRITICAL POINT
SATURATED LIQUID LINE
LIQUID
TWO PHASE REGION (LIQUID + VAPOUR)
SUPER HEAT REGION
EVAPORATION
PRESSURE
$v_1$
$v_f$
$v_g$
$v_{sup}$
$v_{fg}$
SATURATED VAPOUR LINE
$v_1 - v_f - v_g - v_{sup}$ IS CONSTANT PRESSURE LINE
TRIPLE POINT LINE
SPECIFIC VOLUME

**(d)**

Fig (a) shows the p-v-T surface of a substance such as water that expands on freezing, in three-dimensional view.

Fig (b) shows the p-v-T surface of a substance that contracts on freezing. Almost all the substances behave in this manner.

There are regions on the p-v-T surfaces in both the cases labeled, solid, liquid and vapour. In these single phase regions, the state is fixed by any two of the properties, pressure, specific volume and temperature, since all are independent when they are in single phase. Located between the single phase regions are two phase regions, where two phases exist in equilibrium, namely, liquid-vapour, solid-liquid and solid-vapour. In the two phase region, the pressure and temperature are not independent properties. One cannot be changed without changing the other. But, the state can be fixed by specific volume and either pressure or temperature.

### Gibb's Phase rule

Gibbs' phase rule describes the possible number of degrees of freedom (F) in a closed system at equilibrium, in terms of the number of separate phases (P) and the number of chemical components (C) in the system.

Gibbs' rule then follows, as:

$$F = C - P + 2$$

Where F is the number of degrees of freedom, C the number of chemical components, and P is the number of phases that cannot be shared.

Consider water, the $H_2O$ molecule, C = 1.

When three phases are in equilibrium, P = 3, there can be no variation of the (intensive) variables ie. F = 0. Temperature and pressure must be at exactly one point, the 'triple point' (temperature of $0.01^oC$ and pressure of 611.73 Pa). Only at the triple point can three phases of water exist at the same time. At this one point, Gibbs rule states: F = 1 - 3 + 2 = 0.

When two phases are in equilibrium, P = 2, such as along the melting or boiling boundaries, the (intensive) variable pressure is a determined function of (intensive) variable temperature, ie. one degree of freedom. Along these boundaries, Gibbs rule states: F = 1 - 2 + 2 = 1.

Away from the boundaries of the phase diagram of water, only one phase exists (gas, liquid, or solid), P = 1. So there are two degrees of freedom. At these points, Gibbs rule states: F = 1 - 1 + 2 = 2

Note that if you are considering three (intensive) variables: pressure, temperature, and volume of a gas (ie. one phase, P = 1) then only two of the variables can be independent. This fact is illustrated by the universal gas law:

$$p\ V = m\ R\ T$$

Another Example - For instance, a balloon filled with carbon dioxide has one component and one phase, and therefore has two degrees of freedom - in this case temperature and pressure. If you have two phases in the balloon, some solid and some gas, then you lose a degree of freedom - and indeed this is the case, in order to keep this state there is only one possible pressure for any given temperature.

It is important to note that the situation gets more complicated when the (intensive) variables go above critical lines or point in the phase diagram. At temperatures and pressure above the critical point, the physical property differences that differentiate the liquid phase from the gas phase become less defined. This reflects the fact that, at extremely high temperatures and pressures, the liquid and gaseous phases become indistinguishable. In water, the critical point (thermodynamics) occurs at around 647K (374°C or 705°F) and 22.064 MPa .

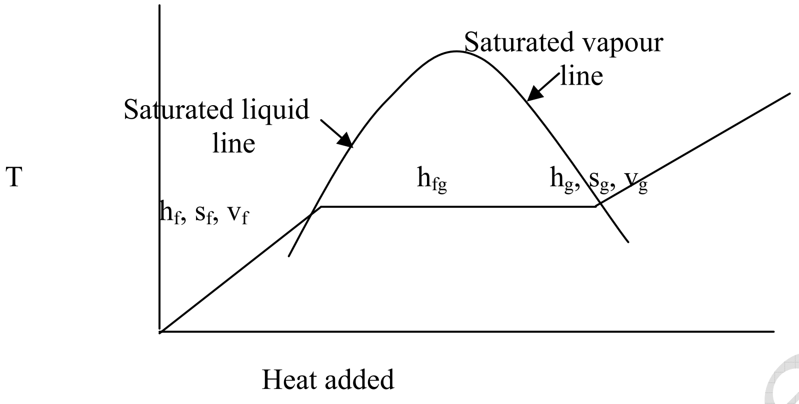
### Type of Steam

*Wet steam* : Wet steam is defined as steam which is partly vapour and partly liquid suspended in it. It means that evaporation of water is not complete.

*Dry saturated steam* : When the wet steam is further heated, and it does not contain any suspended particles of water, it is known as dry saturated steam.

*Superheated steam* : When the dry steam is further heated at constant pressure, thus raising its temperature, it is called superheated steam.

**Steam properties**



The following are the steam properties :

- Dryness fraction
- Specific enthalpy
- Specific volume
- Specific internal energy
- Specific entropy

**Dryness Fraction (Quality of steam)**

It is the fraction of steam that is in the vapour form in the mixture.

Let, $m_g$ = mass of dry steam/kg of mixture

$m_f$ = mass suspended liquid/kg of mixture

x = dryness fraction of steam

Therefore $x = \dfrac{m_g}{m_g + m_f}$

For wet steam, $x < 1$

For dry steam, $x = 1$

**Specific Enthalpy**

*Wet steam* **:** If 1 kg of wet steam is heated $m_f$ kg receives the latent heat of vapourisation and $m_g$ kg is unaffected.

$\therefore$ Latent heat absorbed by the wet steam = $x\,h_{fg}$

$h_{fg}$ = Latent heat of vapourisation

$\therefore$ Specific enthalpy of wet steam ($h_{wet}$) = $h_f + x\,h_{fg}$

$h_f$ = Specific enthalpy of saturated liquid

*Dry steam*

All the '$h_{fg}$' will be absorbed by the $m_g$.

$\therefore$ Specific enthalpy of dry steam ($h_{dry}$) = $h_f + h_{fg} = h_g$ since $[h_{fg} = h_g - h_f]$

*Superheated steam*

Superheated steam is produced by adding sensible heat to dry steam.

$\therefore$ Specific enthalpy of superheated steam ($h_{sup}$) = $h_f + h_{fg} + C_p(T_{sup} - t_s)$

$= h_g + C_p(T_{sup} - t_s)$

$C_p$ = Specific heat at constant pressure = 0.4 to 0.5 kJ/kg-K

$T_{sup}$ = Temperature of superheated steam

$t_s$ = Saturation temperature

$T_{sup} - t_s$ = Degree of superheat

**Specific volume**

*Wet steam*

Let, $v_f$ = specific volume of saturated liquid

$v_g$ = specific volume of dry steam

$\therefore$ specific volume of wet steam ($v_{wet}$) = $v_f + x\, v_{fg}$

$= v_f + x\,(v_g - v_f)$

*Dry saturated steam*

Specific volume of dry steam ($v_{dry}$) = $v_g$

*Superheated steam*

Dry steam & Superheated steams behave like a perfect gas. Since the production of steam is constant pressure process, we can write,

$$\frac{p_s v_g}{t_s} = \frac{p_{\text{sup}}\, v_{\text{sup}}}{T_{\text{sup}}}$$

$$\frac{v_g}{t_s} = \frac{v_{\text{sup}}}{T_{\text{sup}}}$$

$$v_{\text{sup}} = v_g \frac{T_{\text{sup}}}{t_{sat}}$$

**Specific internal energy**

*Wet steam*

We know that specific enthalpy is given by,

$$h = u + pv$$

$$\therefore \quad u = h - pv$$

$$u_{wet} = h_{wet} - pv_{wet}$$

*Dry steam*

$$u_{(dry)} = h_{(dry)} - pv_{(dry)}$$

*Super heated steam*

$$u_{(sup)} = h_{(sup)} - pv_{(sup)}$$

**Specific entropy**

*Wet steam*

$$s_{(wet)} = s_f + x\, s_{fg}$$

*Dry steam*

$$s_{(dry)} = s_f + s_{fg} = s_g$$

*Super heated steam*

$$s_{(sup)} = s_g + C_p \ln [T_{sup}/t_s]$$

$T_{sup}$ = Super heated steam temperature

$t_s$ = Saturation temperature

**External Work of Evaporation**

When he water is evaporated to form saturated steam, its volume increases from $v_f$ to $v_g$ at a constant pressure, and thus external work is done by steam due to increases in volume. The energy for doing the work is obtained during the absorption of latent heat. This work is called external work of evaporation.

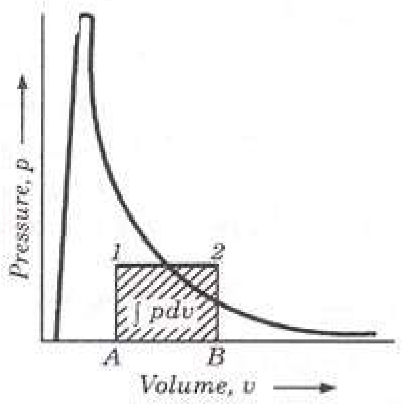
*Wet Steam:* Work of evaporation = $p\,(x\, v_g - v_f)$

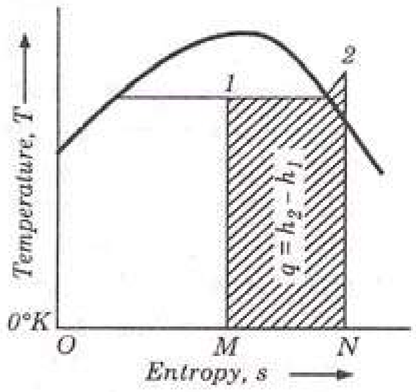
x → Dryness fraction of steam

*Dry Steam:* Work of evaporation = $p\,(v_g - v_f)$

**Processes of vapour**

**Constant pressure process**





Non-Flow process:

$$w = \int_1^2 p\, dv = p(v_2 - v_1)$$

$$\begin{aligned} q &= w + (u_2 - u_1) \\ &= p(v_2 - v_1) + (u_2 - u_1) \\ &= p_2 v_2 - p_1 v_1 + u_2 - u_1 \end{aligned}$$

$$= h_2 - h_1$$

Flow process:

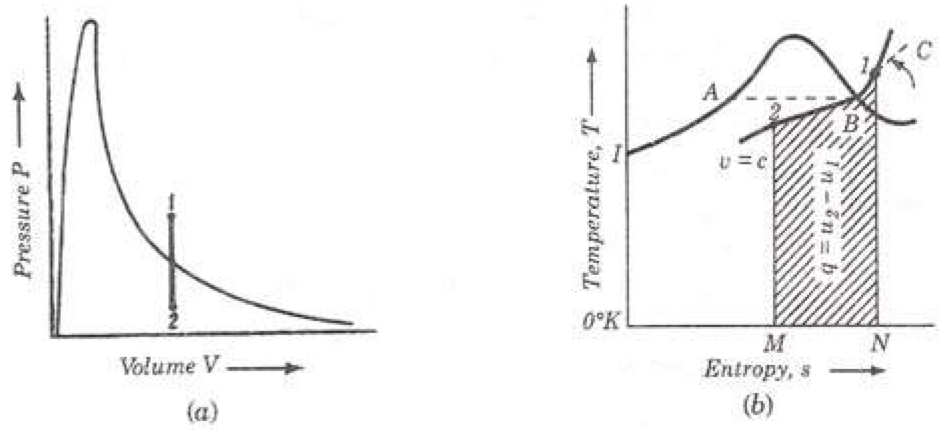
$$w = -\int_1^2 v\,dp = 0$$

$$q = w + (h_2 - h_1)$$

$$= 0 + (h_2 - h_1) = (h_2 - h_1)$$

Therefore the heat transfer is the same in flow and non-flow processes.

**Constant volume process**



Non-Flow process:

$$w = \int_1^2 p\,dv = 0$$

$$q = w + (u_2 - u_1)$$

$$= 0 + (u_2 - u_1)$$

$$= u_2 - u_1$$

Flow process:

$$w = -\int_1^2 v\,dp = v(p_2 - p_1)$$

$$q = w + (h_2 - h_1)$$

$$= v(p_2 - p_1) + (h_2 - h_1)$$

**Constant temperature process**

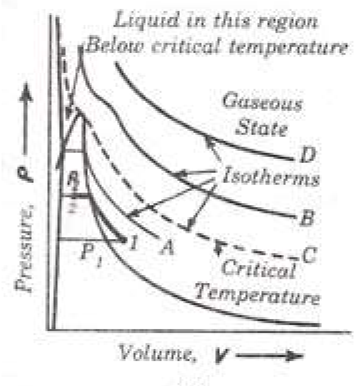
Non-Flow process:

$$q = w + (u_2 - u_1)$$

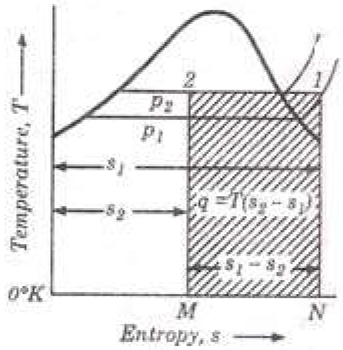
$$w = q + (u_1 - u_2)$$

$$= T(s_2 - s_1) + (u_1 - u_2)$$

$$q = T(s_2 - s_1)$$



(a)



(a)

Flow process:

$$q = w + (h_2 - h_1)$$

$$w = q + (h_1 - h_2)$$

$$= T(s_2 - s_1) + (h_1 - h_2)$$

$$q = T(s_2 - s_1)$$

**Adiabatic process**

Non-Flow process:

q = 0 for adiabatic process

$$q = w + (u_2 - u_1)$$

$w = (u_1 - u_2)$ → Reversible

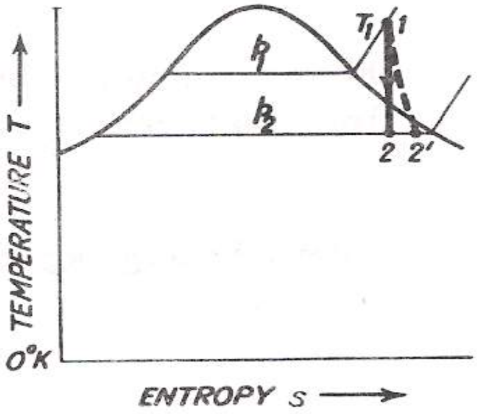
$w = (u_1 - u_{2'})$ → Irreversible

Flow process:

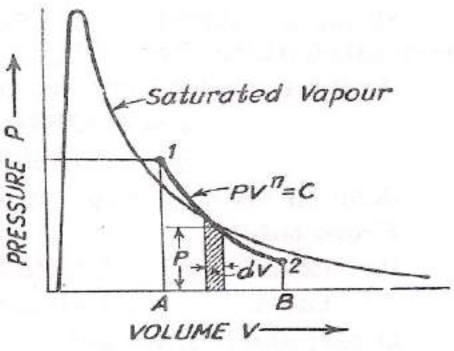
$$q = w + (h_2 - h_1)$$

$w = (h_1 - h_2)$ → Reversible

$w = (h_1 - h_{2'})$ → Irreversible



## Polytropic process



Non-Flow process:

$$w = \int_1^2 p\,dv = \frac{p_2v_2 - p_1v_1}{1-n}$$

$$q = w + (u_2 - u_1)$$

$$= (u_2 - u_1) + \frac{p_2v_2 - p_1v_1}{1-n}$$

Flow process:

$$w = -\int_1^2 v\,dp = \frac{n\,(p_2v_2 - p_1v_1)}{1-n}$$

$$q = w + (h_2 - h_1)$$

$$= (h_2 - h_1) + \frac{n\,(p_2v_2 - p_1v_1)}{1-n}$$

$$= (u_2 - u_1) + \frac{p_2v_2 - p_1v_1}{1-n}$$

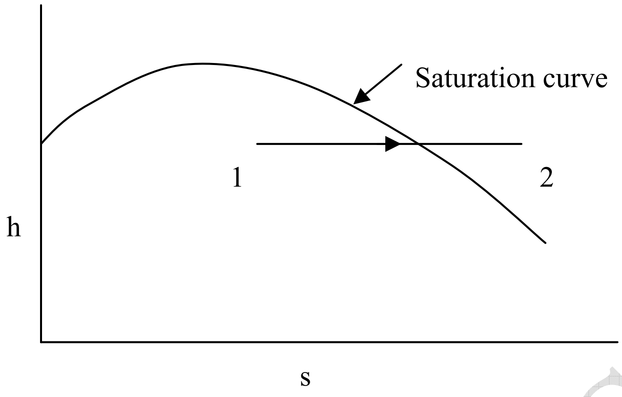
Therefore the heat transfer is the same in flow and non-flow processes.

## Throttling process (Isenthalpic process)

Workdone = 0

Heat transfer = 0

$$h_1 = h_2$$



## Rankine Cycle (Complete expansion cycle)

A power cycle continuously converts heat into work, in which a working fluid repeatedly performs a succession of processes.

In vapour power cycle, the working fluid, which is water, undergoes a change of phase. Heat is transferred to water in the boiler from an external source (furnace) to raise steam. The high pressure, high temperature steam leaving the boiler expands in the turbine to produce shaft work. The steam leaving the turbine condenses into water in the condenser rejecting the heat, and then the water is pumped back to the boiler by feed pump absorbing some work.

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It consists of four ideal processes.

1. Reversible constant pressure process – this is heating process of water to form steam.
2. Reversible adiabatic process – this is expansion process of steam in the steam turbine.
3. Reversible constant pressure process – this is process of condensation of steam till it becomes saturated liquid.
4. Reversible adiabatic process- this is compression process of saturated water in the feed pump.

Since all these four processes are ideal, the cycle is an Ideal cycle, called Rankine cycle. This is a reversible cycle.

For the purpose of analysis the Rankine cycle is assumed to be carried out in a steady flow operation.

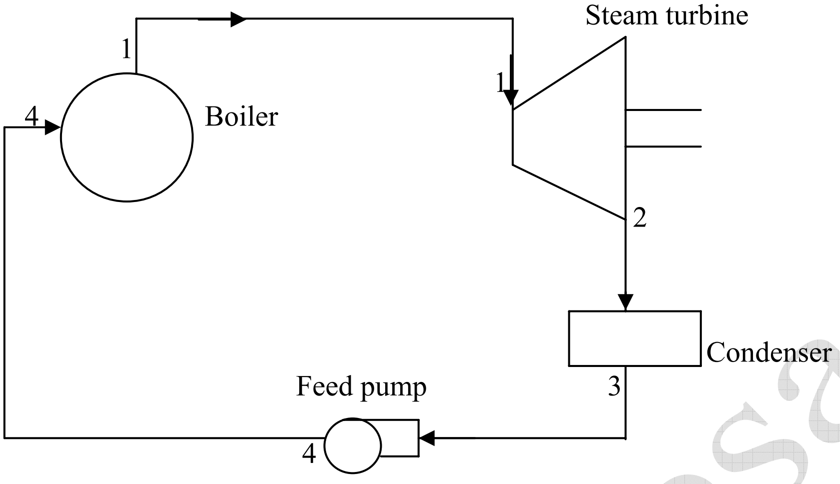
*For Steam turbine* (Process 1-2 – Isentropic expansion)

SFEE is given by, $m h_1 + m C_1^2/2 + Q_{1-2} = m h_2 + m C_2^2 + W_{1-2}$

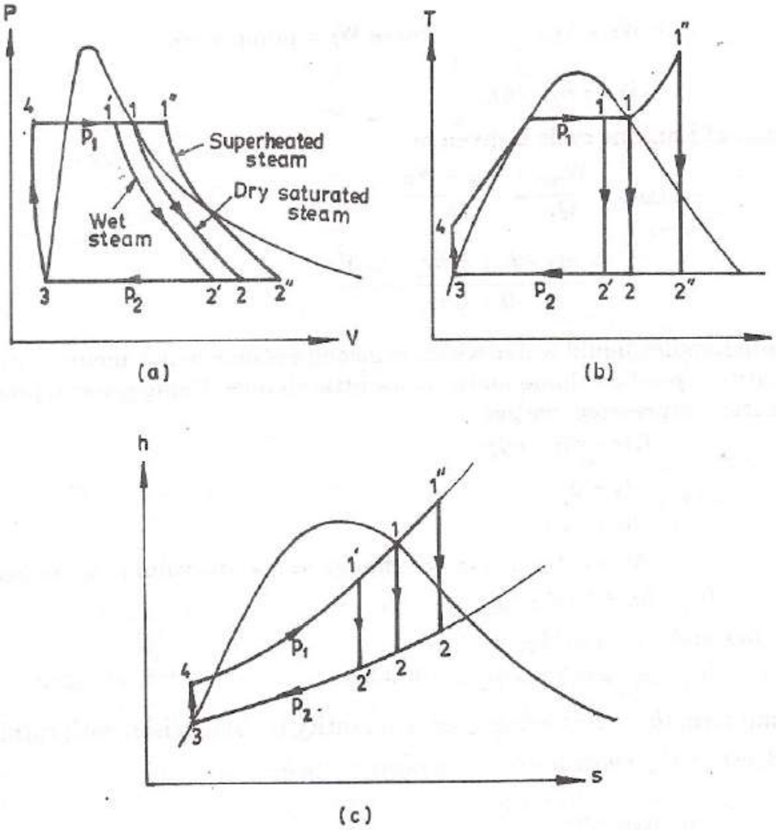
But, $C_1 \approx C_2$ and $Q_{1-2} = 0$

$$\therefore \quad W_{1-2} = m (h_1 - h_2)$$

Workdone by the turbine = $W_T = m\,(h_1 - h_2)$



**Simple steam power plant**



*For Condenser* (Process 2-3 – Constant pressure condensation)

SFEE is given by, $m\,h_2 + m\,C_2^2/2 + Q_{2-3} = m\,h_3 + m\,C_3^2 + W_{2-3}$

But, $C_2 \approx C_3$ and $W_{2-3} = 0$

$\therefore\ Q_{2-3} = m\,(h_3 - h_2)$

Heat rejected in the condenser = $-\,m\,(h_3 - h_2) = m\,(h_2 - h_3)$

*For Feed pump* (Process 3-4 – Isentropic compression)

SFEE is given by, $m\,h_3 + m\,C_3^2/2 + Q_{3-4} = m\,h_4 + m\,C_4^2 + W_{3-4}$

But, $C_3 \approx C_4$ and $Q_{3-4} = 0$

$$\therefore\ W_{3-4} = m\,(h_3 - h_4)$$

Workdone on the Pump = $W_P = -\,m\,(h_3 - h_4) = m\,(h_4 - h_3)$

*For Boiler* (Process 4-1 – Constant pressure heat addition)

SFEE is given by, $m\,h_4 + m\,C_4^2/2 + Q_{4-1} = m\,h_1 + m\,C_1^2 + W_{4-1}$

But, $C_4 \approx C_1$ and $Q_{4-1} = 0$

$$\therefore\quad Q_{4-1} = m\,(h_1 - h_4)$$

Heat supplied in the boiler = $m\,(h_1 - h_4)$

$$\text{Rankine cycle efficiency} = \eta_R = \frac{\text{Output}}{\text{Input}} = \frac{\text{Net work output}}{\text{Heat supplied}}$$

Net work output = $W_T - W_P = m\,(h_1 - h_2) - m\,(h_4 - h_3)$

Heat supplied = $Q_{4-1} = m\,(h_1 - h_4)$

$$\therefore\quad \eta_R = \frac{m\,(h_1 - h_2) - m\,(h_4 - h_3)}{m\,(h_1 - h_4)}$$

$$\therefore\quad \eta_R = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)} \rightarrow \text{considering pump work}$$

But pump work is very small & it can be neglected.

$$= \frac{h_1 - h_2}{h_1 - h_4} \rightarrow \text{neglecting pump work}$$

Note : Here $h_4 \approx h_3 = h_{f3}$ at condenser pressure.

**Other parameters**

**Specific steaming rate (SSC)**– Rate of steam flow required to produce unit shaft power output & usually expressed in ‘kg/kWh’.

$$SSC = \frac{m}{W_T - W_P} \times 3600$$

3600

$$= \frac{}{w_T - w_P}$$

Note : $W_T - W_P = m\,(w_T - w_P)$

**Specific heat rate (SHR)** – Rate of heat input required to produce unit shaft power output. It is usually expressed in ‘kJ/kWh’.

$$SHR = \frac{\text{Heat supplied}}{W_T - W_P} \times 3600$$

$$= \frac{m\,(h_1 - h_4)}{W_T - W_P} \times 3600$$

$$= \frac{h_1 - h_4}{w_T - w_P} \times 3600$$

$$= 3600 / \eta_R$$

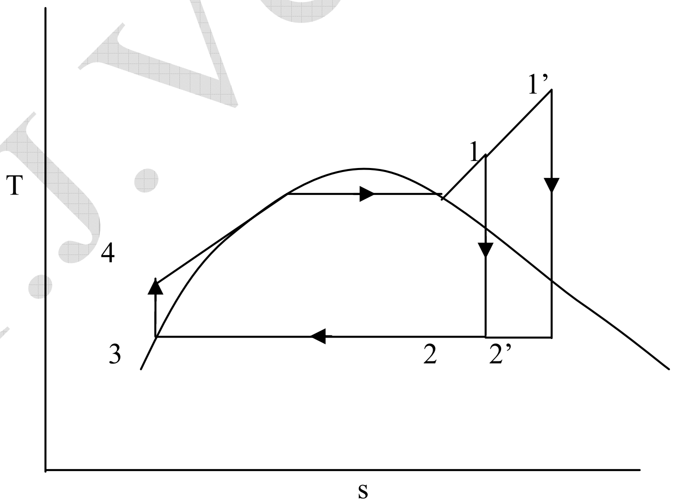
**Work Ratio**

$$WR = \frac{w_T - w_P}{w_T}$$

**Power developed**

$$P = m\,(w_T - w_P)$$

**Effect of initial temperature ($T_1$)**



As the temperature increases, $h_{1'}$, $w_T$ & $x_2$ increases.

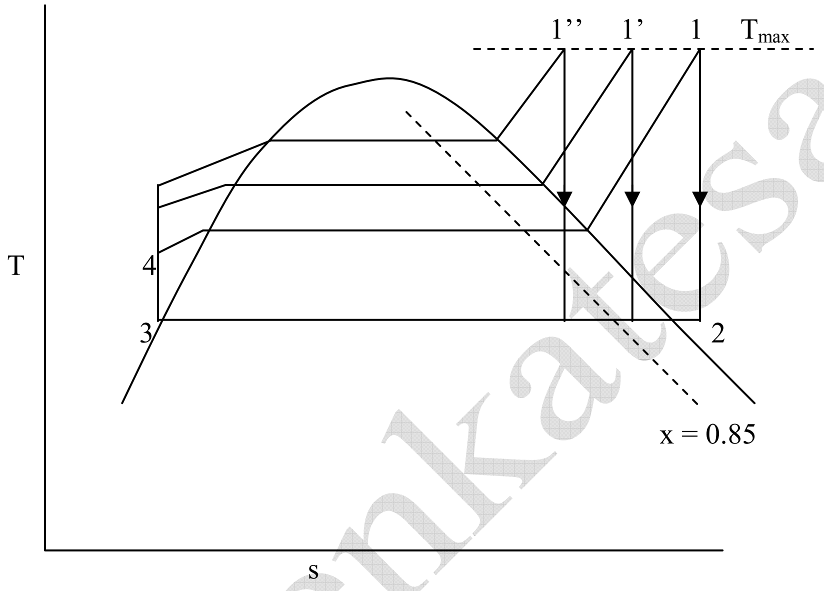
- At the outlet of turbine it is required to maintain quality of steam in order to prevent the damage of blades due to impingement of water particles.
- The maximum temperature of steam is fixed from metallurgical considerations.

**Effect of increase of pressure**

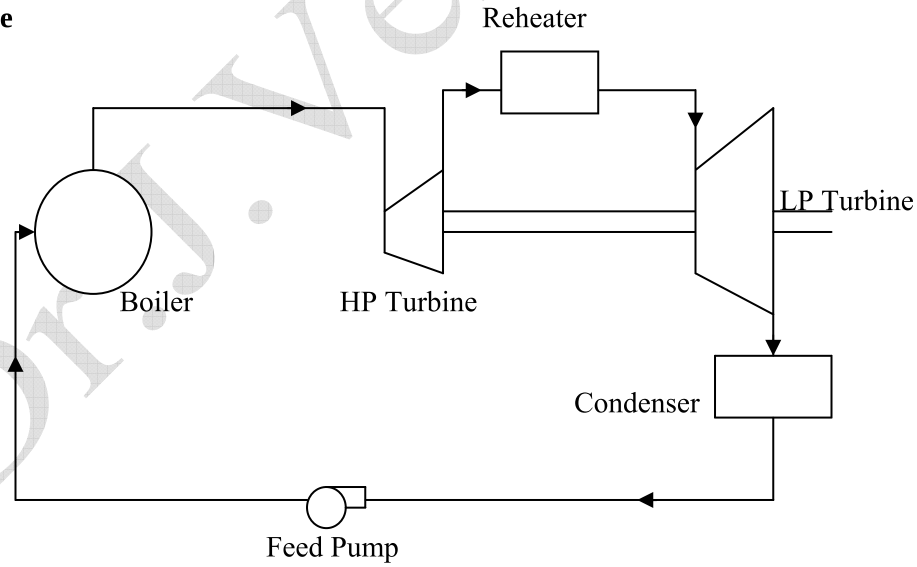
Increase in pressure for particular $T_{max}$,

- decreases the quality of exhaust steam
- decreases the heat supplied

If the moisture content of steam in the later stages of the turbine is higher, the entrained water particles along with the vapour coming out from the nozzles with high velocity strike the blades and erode their surfaces. From a consideration of the erosion of blades, in the later stages of the turbine, the maximum moisture content at the turbine exhaust is not allowed to exceed 15 % or the quality to fall below 85 %. Therefore like maximum temperature, maximum pressure at the turbine inlet also gets fixed.



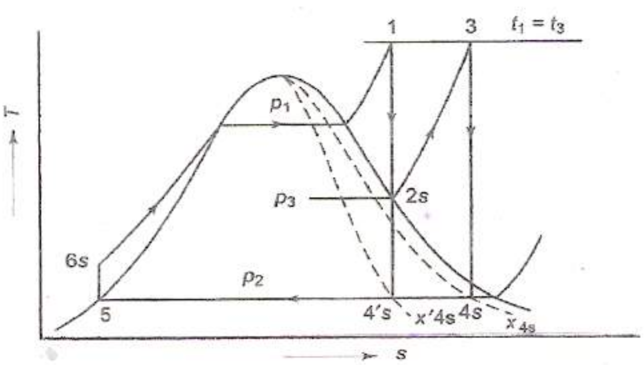
**Reheat Cycle**

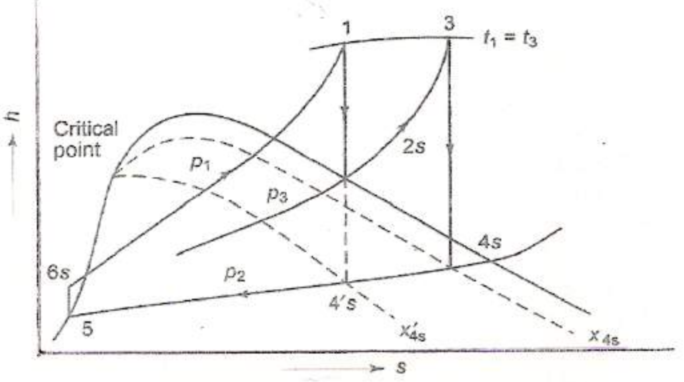


If a steam pressure higher than $p_{max}$ is used, in order to limit the quality to 0.85 at the turbine exhaust, reheat has to be adopted.

The steam from the HP turbine is reheated in the reheater using flue gases at constant pressure to initial temperature. Reheat pressure is an important factor. Low reheat pressure may bring down the $T_1$ and hence efficiency. A high reheat pressure increases the moisture content at turbine exhaust. Thus the reheat pressure is optimized.

For most of the modern power plants, the reheat pressure is 0.2 to 0.25 $p_1$.





Reheat -- increases the turbine work

-- reduces moisture content at turbine outlet

$$h_2 - h_{2'} < h_3 - h_4$$

$$\text{Total expansion} = (h_1 - h_2) + (h_3 - h_4)$$

The use of reheat only gives a marginal increase in cycle efficiency, but it increases the net work output by making possible the use of higher pressures, keeping the quality of steam at turbine exhaust within a permissible limit.

$$\text{Heat supplied} = (h_1 - h_6) + (h_3 - h_2)$$

$$\text{Pump work} = h_6 - h_5$$

$$\eta_{\text{reheat}} = \frac{w_T - w_P}{HS} = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}$$

$$\text{Steam rate} = \frac{3600}{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)} \text{ ------ kg/kWh}$$

Advantages of Reheating

- There is an increased output of the turbine.
- Erosion and corrosion problems in steam turbine are eliminated since the quality of steam is increased.
- There is an improvement in the thermal efficiency of the cycle.

- Final dryness fraction of steam is improved.
- There is an increase in the nozzle and blade efficiencies.

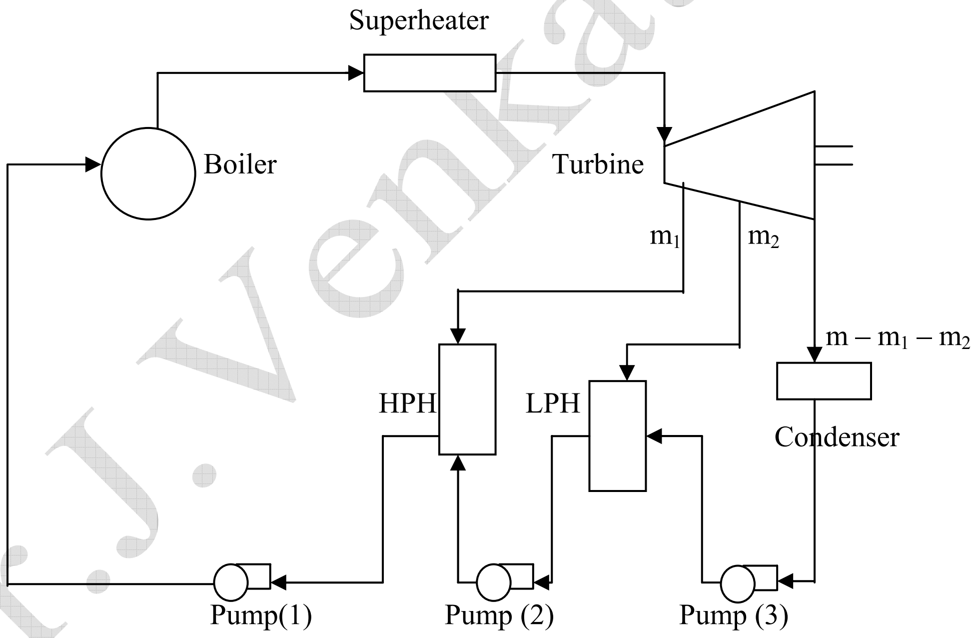
### Disadvantages

- Reheating requires more maintenance.
- The increase in thermal efficiency is depressed by the more cost of the heater.

### Regeneration

It may be observed in the Rankine cycle that the condensate which is at a fairly low temperature is pumped to the boiler. There is irreversible mixing of the cold condensate with hot boiler water. This results in loss of cycle efficiency. Methods are adopted to heat the feed water from the condenser reversibly by interchange of heat within the system and thus improving the cycle efficiency. This method of heating is called **regenerative feed heating** and the cycle is called **regenerative cycle**.

The principle of regeneration can be practically utilized by extracting steam from the turbine at several locations and supplying it to the regenerative heaters. The heating arrangement comprises of (i) for medium capacity turbines – not more than 3 heaters (ii) for high pressure high capacity turbines – not more than 7 heaters and (iii) for turbines of super critical parameters 8 to 9 heaters. The final condensate heating temperature is kept 50 to 60°C below the boiler saturated steam temperature so as to prevent evaporation of feed water.



Consider a cycle with 2 feed water heaters.

Let m = Mass of steam through the boiler.
$m_1$ = Mass of steam through heater-I
$m_2$ = Mass of steam through heater-II

∴ Mass of steam through the condenser = $m - m_1 - m_2$

5 – 6 → Pumping of $(m - m_1 - m_2)$ kg of feed water by pump (3)

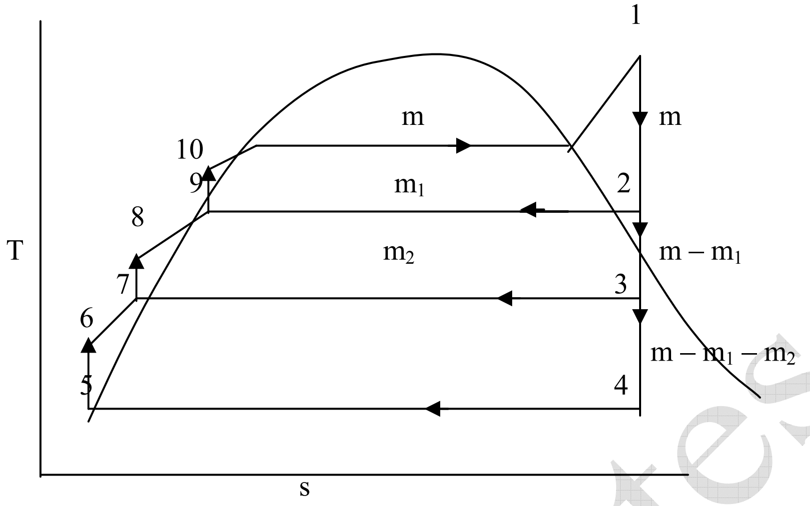
6 – 7 → Heating of $(m - m_1 - m_2)$ kg of feed water by $m_2$ kg of bled steam by LPH

7 – 8 → Pumping of $(m - m_1)$ kg of feed water by pump (2)

8 – 9 → Heating of $(m - m_1)$ kg of feed water by $m_1$ kg of bled steam by HPH

9 – 10 → Pumping of (m) kg of feed water by pump (1)

10 – 1 → Heating of (m) kg of feed water by supply of external heat in the boiler



$$\text{Efficiency} = \frac{\text{Workdone}}{\text{Heat supplied}}$$

$$\text{Workdone} = m (h_1 - h_2) + (m - m_1) (h_2 - h_3) + (m - m_1 - m_2) (h_3 - h_4)$$

Heat supplied = Heat supplied during process 10 – 1

$$= m (h_1 - h_{10})$$

**Advantages**

- The heating process in the boiler tends to become reversible.
- The thermal stress problems due to temperature difference between feed water from the condenser and boiler feed water are minimized.
- The thermal efficiency is improved.
- Heat supplied is reduced.
- Size of the condenser is reduced since it handles only ($m - m_1 - m_2$) kg of steam.

**Disadvantages**

- The plant becomes more complicated.
- Because of addition of heaters greater maintenance is required.
- Since the power output is slightly reduced due to extraction of steam, for a given power output, large capacity boiler is required.
- The heaters are costly. The gain in thermal efficiency is depressed by heavier costs.

**Steam Calorimeters**

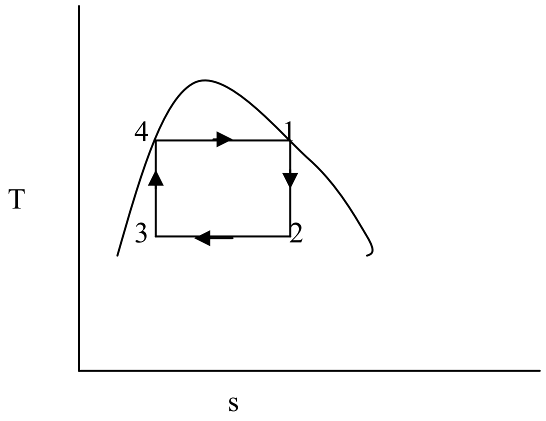
Steam calorimeters are used for measuring 'dryness fraction' of steam.

**Types**

- Tank or Bucket Calorimeter

- Throttling Calorimeter
- Separating Calorimeter
- Separating & Throttling Calorimeter
- Electrical Calorimeter

**Carnot cycle**



1-2 – Steam turbine

2-3 – Condenser

3-4 – Feed pump

4-1 – boiler

Carnot cycle is independent of the working fluid. It consists of two isothermal & two isentropic processes.

The vapour from wet region (3) is compressed isentropically from a lower temperature to the upper temperature of the cycle by means of a pump. At this higher temperature, heat is added at constant temperature in the boiler. The dry vapour is then expanded isentropically in a turbine. The wet vapour from the turbine outlet is condensed in a condenser at constant temperature, thereby completing a cycle.

As we know that the Carnot cycle efficiency is depending on temperature limits, the efficiency is given by,

$$\eta_C = \frac{T_1 - T_2}{T_1}$$

**Difficulties with Carnot cycle**

1. Carnot cycle requires heat addition and heat rejection at constant temperatures and a constant temperature heat transfer is not possible when the working medium is a gas or a superheated vapour.
2. The vapour compression process is difficult to accomplish.
3. Cycle efficiency is depending on temperature limits. The lower temperature is limited by the ambient conditions, The upper temperature is limited by the metallurgical considerations. Modern metals can withstand temperatures upto 650°C. The highest possible temperature at which a Carnot cycle can operate with saturated steam is below critical temperature (374.15°C) and the critical pressure being 221.2 bar. Thus present day metals can not be used advantageously in a Carnot cycle with steam as working fluid.

## PROBLEMS

1. **Calculate the dryness fraction of steam which has 1.5 kg of water in suspension with 50 kg of steam.**

Given

Mass of water ($m_f$) = 1.5 kg

Mass of steam ($m_g$) = 50 kg

Required : Dryness fraction (x)

Solution

$$\text{Dryness fraction } (x) = \frac{m_g}{m_g + m_f}$$

$$= \frac{50}{50 + 1.5} = \mathbf{0.971} \text{ --- } \textbf{Ans}$$

2. **Determine the amount of heat, which should be supplied to 2 kg of water at 25°C to convert it into steam at 5 bar and 0.9 dry.**

*Given:*

Mass of water ($m_f$) = 2 kg

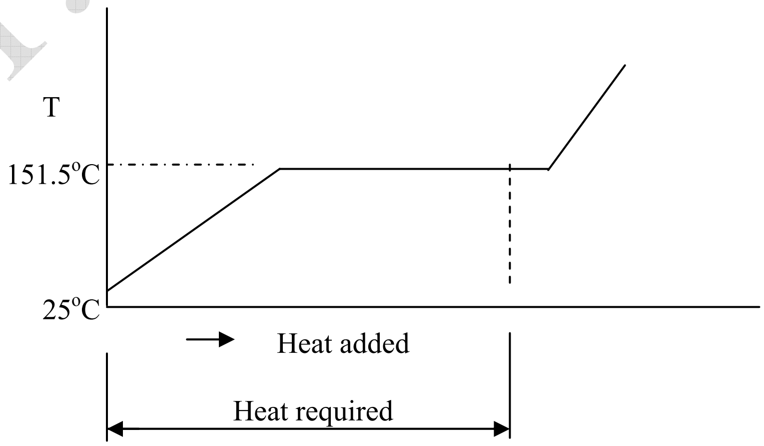
Initial temperature of water ($T_1$) = 25°C

Pressure of steam (p) = 5 bar

Dryness fraction of steam (x) = 0.9

*Required :* Heat required to convert water into steam.

*Solution:*



Heat required = Sensible heat addition + Latent heat addition

Sensible heat addition = m $C_{pw}$ $(t_s - T_1)$

$t_s$ = saturation temperature = 151.5°C at 5 bar from steam table

$C_{pw}$ = Specific heat at constant pressure = 4.186 kJ/kg

$\therefore$ Sensible Heat addition = 2 x 4.186 (151.5 – 25)

= 1059.058 kJ

Latent heat addition / kg = x $h_{fg}$

Latent heat ($h_{fg}$) = 2107.4 kJ/kg from steam table at 5 bar

Latent heat addition for 'm' kg = m x $h_{fg}$

= 2 x 0.9 x (2107.4)

= 3793.32 kJ

$\therefore$ Total heat required = 1059.058 + 3793.32

= **4852.372 kJ --- Ans**

**3. Steam is generated at 8 bar from water at 32°C. Determine the heat required to produce 1 kg of steam (a) when the dryness fraction is 0.85 (b) when steam is dry saturated and (c) when the steam is superheated to 305°C. The specific heat of superheated steam may be taken as 2.093 kJ/kg-K.**

*Given:*

Steam pressure (p) = 8 bar

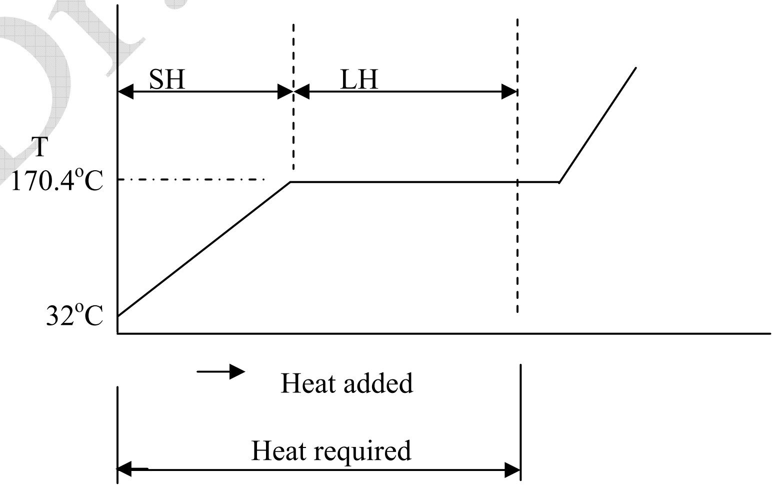
Initial temperature of water ($T_1$) = 32°C

Mass of steam (m) = 1 kg

*Required*: Heat required when (a) x = 0.85 (b) x = 1 (c) $T_{sup}$ = 305°C

*Solution:*

(a)



Heat required = Sensible heat addition + Latent heat addition

Sensible heat addition = m $C_{pw}$ $(t_s - T_1)$

$t_s$ = saturation temperature = 170.4°C at 8 bar from steam table

$C_{pw}$ = Specific heat at constant pressure = 4.186 kJ/kg (Taken)

∴ Sensible Heat addition = 1 x 4.186 x (170.4 – 32)

= 579.34 kJ/kg

Latent heat addition / kg = x $h_{fg}$

Latent heat ($h_{fg}$) = 2046.5 kJ/kg from steam table at 8 bar

Latent heat addition for 'm' kg = m x $h_{fg}$

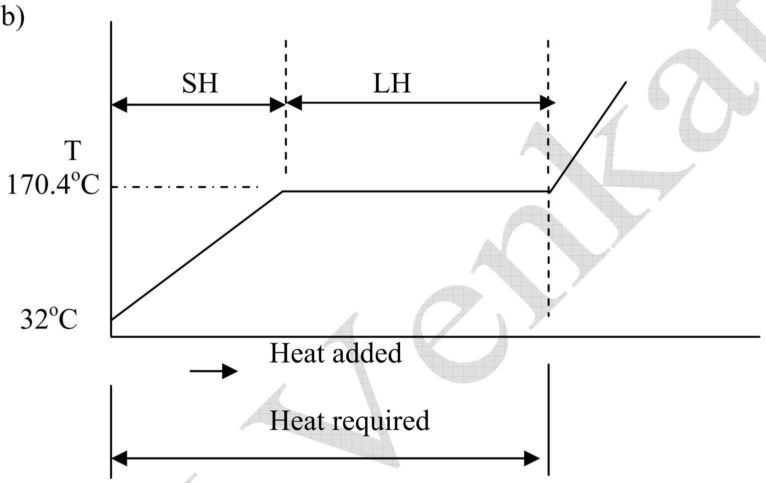
= 1 x 0.85 x (2046.5)

= 1739.525 kJ/kg

∴ Total heat required = 579.34 + 1739.525

= **2318.865 kJ/kg --- Ans**

(b)



Heat required = Sensible heat addition + Latent heat addition

Latent heat addition / kg = x $h_{fg}$

Latent heat ($h_{fg}$) = 2046.5 kJ/kg from steam table at 8 bar

Latent heat addition for 'm' kg = m x $h_{fg}$

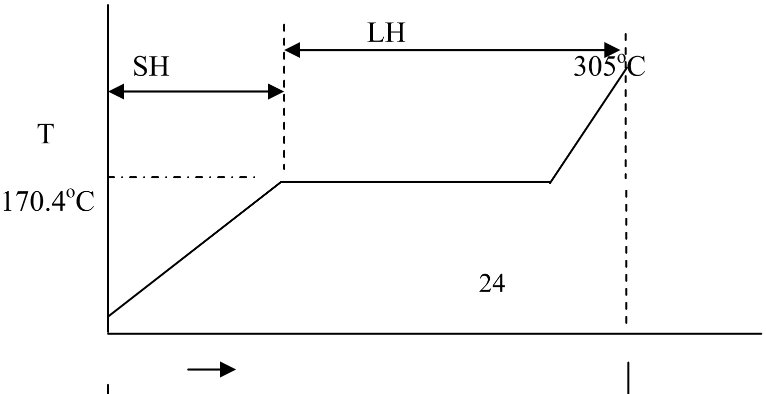
= 1 x 1 x (2046.5)

= 2046.5 kJ/kg

∴ Total heat required = 579.34 + 2046.5

= **2625.84 kJ/kg --- Ans**

(c)



32°C

Heat added

Heat required

Heat required = Sensible heat addition + Latent heat addition + Sensible Heat addition

Sensible heat addition to superheated steam = m $C_{pv}$ ($T_{sup}$ – $t_s$)

= 1 x 2.093 x (305 – 170.4)

= 281.72 kJ/kg

Latent heat addition / kg = $h_{fg}$

Latent heat ($h_{fg}$) = 2046.5 kJ/kg from steam table at 8 bar

$\therefore$ Total heat required = 579.34 + 2046.5 + 281.72

= **2907.56 kJ/kg --- Ans**

4. **Find specific volume, enthalpy and internal energy of wet steam at 18 bar, dryness fraction 0.9.**

*Given:*

Pressure of steam (p) = 8 bar

Dryness fraction of steam (x) = 0.9

*Required* : v, h, u

*Solution:*

Specific volume (v) = $v_f + x\, v_{fg}$

From steam table, at 18 bar,

$v_f$ = 0.001168 $m^3$/kg    $v_g$ = 0.11033 $m^3$/kg    $h_f$ = 884.5 kJ/kg

$h_{fg}$ = 1910.3 kJ/kg    $h_g$ = 2794.8 kJ/kg

$\therefore$ Specific volume (v) = 0.001168 + 0.9 (0.11033 – 0.001168)

**= 0.0994138 $m^3$/kg ---- Ans**

Specific enthalpy (h) = $h_f + x\, h_{fg}$

= 884.5 + 0.9 (1910.3)

**= 2603.77 kJ/kg ---- Ans**

Specific internal energy (u) = h – pv

= 2603.77 – (18 x $10^5$ x 0.0994138 / 1000)

**= 2424.825 kJ/kg ---- Ans**

Note : Divide the term ‘pv’ by 1000 to convert J/kg into kJ/kg.

5. **Find the dryness fraction, specific volume and internal energy of the steam at 7 bar and enthalpy 2600 kJ/kg.**

*Given:*

Pressure of steam (p) = 7 bar

Enthalpy of steam (h) = 2600 kJ/kg

*Required* : x, v, u

*Solution:*

We know , $h_{(wet)} = h_f + x\,h_{fg}$

At 7 bar, from steam table, $h_f = 697.1$ kJ/kg $h_{fg} = 2064.9$ kJ/kg

$v_f = 0.001108$ m$^3$/kg $v_g = 0.27268$ m$^3$/kg

$2600 = 697.7 + x\,(2064.9)$

**$x = 0.92$ ---- Ans**

Specific volume (v) $= v_f + x\,v_{fg}$

$= 0.001108 + 0.92\,(0.27268 - 0.001108)$

**$= 0.25095$ m$^3$/kg ---- Ans**

Specific internal energy (u) $= h - pv$

$= 2600 - (7 \times 10^5 \times 0.25095 / 1000)$

**$= 2424.335$ kJ/kg ---- Ans**

6. **A vessel of volume 0.04 m$^3$ contains a mixture of saturated water & saturated steam at a temperature of 250$^o$C. The mass of the liquid present is 9 kg. Find the pressure, mass, the specific volume, the enthalpy, the entropy and the internal energy.**

*Given:*

Total volume (V) $= 0.04$ m$^3$

Temperature of saturated steam ($t_s$) = 250$^o$C

Mass of the liquid ($m_f$) = 9 kg

*Required* : p, m, v, h, s, u

*Solution:*

From steam table at 250$^o$C, **$p_s = 39.73$ bar --- Ans**

From steam table at 250$^o$C,

$v_f = 0.0012512$ m$^3$/kg $v_g = 0.05013$ m$^3$/kg $h_f = 1085.36$ kJ/kg

$h_{fg} = 1716.2$ kJ/kg $s_f = 2.7927$ kJ/kg-K $s_{fg} = 3.2802$ kJ/kg-K

Mass of steam (m) $= m_g + m_f$

Mass of dry steam ($m_g$) $= V_g / v_g$

$V_g$ = Volume of dry steam $= V - V_f$

$= V - m_f\,(v_f)$

$= 0.04 - 9 \times 0.0012512$

$= 0.02874$ m$^3$

$\therefore$ Mass of dry steam ($m_g$) $= 0.02874 / 0.05013 = 0.5733$ kg

$\therefore$ Mass of steam (m) $= 0.5733 + 9 =$ **9.5733 ---- Ans**

Specific volume (v) $= v_f + x\,v_{fg}$

But, $$x = \frac{m_g}{m_g + m_f} = \frac{0.5733}{0.5733 + 9} = 0.06$$

$\therefore$ $v = 0.0012512 + 0.06\ (0.5013 - 0.0012512)$

$= \mathbf{0.0041839\ m^3/kg}$ **---- Ans**

Specific enthalpy (h) $= h_f + x\ h_{fg}$

$= 1085.36 + 0.06\ (1716.2)$

**= 1188.332 kJ/kg ---- Ans**

Specific entropy (s) $= s_f + x\ s_{fg}$

$= 2.7927 = 0.06\ (3.2802)$

**= 2.9895 kJ/kg ---- Ans**

Specific internal energy (u) $= h - pv$

$= 1188.332 - (39.73 \times 10^5 \times 0.0041839 / 1000)$

**= 1171.709 kJ/kg ---- Ans**

7. **Steam has a pressure of 1 bar and temperature of 150°C after throttling. The pressure before throttling is 14 bar. Find the dryness fraction of steam before throttling. Also find the change in entropy.**

*Given:*

Pressure after throttling ($p_2$) = 1 bar

Temperature after throttling ($T_2$) = 150°C

Pressure before throttling ($p_1$) = 14 bar

*Required* : Dryness fraction (x) & ($s_2 - s_1$)

*Solution:*

The condition of steam will be superheated after throttling.

For throttling process, $h_1 = h_2$

We know, for wet steam, $h_1 = h_{f1} + x\ h_{fg1}$

At 14 bar, from steam table, $h_f = 830$ kJ/kg

$h_{fg} = 1960$ kJ/kg

For super heated steam, at 1 bar & 150°C, from steam table,

$h_2 = 2776.3$ kJ/kg

$\therefore$ $830 + x\ (1960) = 2776.3$

**x = 0.993 --- Ans**

Change in entropy = ($s_2 - s_1$)

At 14 bar, from steam table, $s_f = 2.284$ kJ/kg-K

$s_{fg} = 4.185$ kJ/kg-K

$\therefore$ $s_1 = 2.284 + 0.993\ (4.185)$

$= 6.439705$ kJ/kg-K

At 1 bar & 150°C , $s_2 = 7.614$ kJ/kg-K

$\therefore \quad (s_2 - s_1) = 7.614 - 6.439705 = \mathbf{1.174295}$ **--- Ans**

8. **Calculate the state of steam, i.e., whether it is wet, dry or superheated for the following cases.**
   - **i) Steam has a pressure of 15 bar and specific volume of 0.12 m$^3$/kg**
   - **ii) Steam has a pressure of 10 bar and temperature of 250$^o$C**

*Given:*

i) $p = 15$ bar & $v = 0.12$ m$^3$/kg

ii) $p = 10$ bar & $T = 250^oC$

*Required :* To identify whether the steam is wet, dry or superheated.

*Solution*:

i) Assume that the steam is wet.

We know, for wet steam, $v = v_f + x(v_g - v_f)$

At 15 bar, from steam table, $v_f = 0.00115$ m$^3$/kg

$v_g = 0.13177$ m$^3$/kg

$\therefore \quad 0.12 = 0.00115 + x(0.13177 - 0.00115)$

$x = 0.91$

**$x < 1$, $\therefore$ The steam is wet ---- Ans**

ii) From steam table, at 10 bar, $t_s = 179.91^oC$.

But the given steam temperature is greater than saturation temperature.

**$\therefore$ the steam is superheated. ---- Ans**

9. **Steam at 10 bar, 0.95 dry from a boiler is sent to a superheater, where it is heated to 400$^o$C at constant pressure. Using Mollier chart only, find heat supplied and change in internal energy.**

*Given:*

Pressure of steam (p) = 10 bar

Dryness fraction of steam (x) = 0.95

Final temperature ($T_2$) = 400$^o$C

*Required* : Heat supplied & Change in internal energy

*Solution:*

Heat supplied = $h_2 - h_1$

Change in internal energy = $u_2 - u_1$

$= (h_2 - p_2v_2) - (h_1 - p_1v_1)$

From Mollier chart, at 10 bar & 0.95 dry,

$h_1 = 2675$ kJ/kg $v_1 = 0.195$ m$^3$/kg

At 400$^o$C & 10 bar, $h_2 = 3250$ kJ/kg $v_2 = 0.35$ m$^3$/kg

$\therefore (h_2 - h_1) = 3250 - 2675 =$ **575 kJ/kg ---- Ans**

$(u_2 - u_1) = [3250 - (10 \times 10^5 \times 0.35/1000)] -$

$[2900 - (10 \times 10^5 \times 0.195/1000)]$

**= 420 kJ/kg ---- Ans**

**10. A 0.25 $m^3$ vessel contains steam at 10 bar, 200$^oC$. Find i) the pressure, if the contents are cooled to dry saturated state, ii) final state such that thermometer fixed on the vessel shows 130$^oC$.**

*Given:*

Total volume (V) = 0.25 $m^3$

Initial pressure ($p_1$) = 10 bar

Initial temperature ($T_1$) = 200$^oC$

*Required* : i) $p_2$ if cooled to dry ii) 'x' if final temperature is 130$^oC$

*Solution:*

i) Constant volume cooling, since the vessel contains vapour.

$$\therefore\ v_1 = v_2$$

For solving this part, it is better to use Mollier chart. On Mollier chart locate point (1) at 10 bar & 200$^oC$. Follow the constant volume line upto saturation line, since the steam is dry after cooling, to locate point (2).

$$\therefore\ \mathbf{p_2 = 9.7\ bar} \text{ ---- } \textbf{Ans}$$

ii) For solving this part we can use steam table.

We know, for wet steam, $v_1 = v_{f2} + x\, v_{fg2}$

$v_1 = 0.2\ m^3/kg$ at 10 bar & 200$^oC$

At 130$^oC$, $v_f = 0.00107\ m^3/kg$

$v_g = 0.66814\ m^3/kg$

$$\therefore\quad 0.2 = 0.00107 + x\,(0.66814 - 0.00107)$$

$$\mathbf{x = 0.2982} \text{ ---- } \textbf{Ans}$$

**11. Find the volume, enthalpy and internal energy of steam when the condition of steam is i) 500 kPa and 0.75 dry ii) 1 MPa and 425$^oC$**

Given:

i) Pressure of steam (p) = 500 kPa = 5 bar

Dryness fraction (x) = 0.75

ii) Pressure of steam (p) = 10 bar

Temperature of steam (T) = 425$^oC$

*Required* : v, h & u

*Solution:*

i)

We know that, for wet steam, $v = v_f + x(v_g - v_f)$

$$h = h_f + x(h_{fg})$$

$$u = h - pv$$

At 5 bar, from steam table,

$v_f = 0.001093\ m^3/kg$ $v_g = 0.37466\ m^3/kg$ $h_f = 640.1\ kJ/kg$ $h_{fg} = 2107.4\ kJ/kg$

$$\therefore\quad v = 0.001093 + 0.75\,(0.37466 - 0.001093)$$

$$= \mathbf{0.281268\ m^3/kg} \text{ ---- } \textbf{Ans}$$

$h = 640.1 + 0.75\ (2107.4)$

$= \mathbf{2220.65\ kJ/kg}$ **---- Ans**

$u = 2220.65 - [5 \times 10^5 \times 0.281268 / 1000]$

$= \mathbf{2080.016\ kJ/kg}$ **----- Ans**

ii) the steam is superheated, since the saturation temperature is 179.91°C.

$v_{sup} = v_g\ [T_{sup}/t_s]$

At 10 bar, $v_g = 0.19444\ m^3/kg$

$v_{sup} = 0.19444 \times [(425 + 273) / (179.91 + 273)]$

$= \mathbf{0.29966\ m^3/kg}$ **----- Ans**

$h_{sup} = h_g + C_{p(sup)}\ (T_{sup} - t_s)$

or From steam table, at 10 bar & 425°C,

$\mathbf{h_{sup} = 3317.55\ kJ/kg}$ **---- Ans**

$u_{sup} = h_{sup} - p\ v_{sup}$

$= 3317.55 - (10 \times 10^5 \times 0.29966)/1000$

$= \mathbf{3017.89\ kJ/kg}$ **----- Ans**

**12. Steam at the rate of 400 kg/min enters the steam turbine of a simple Rankine cycle at 3 MPa and 500°C. The condenser pressure is 8 kPa. Find (a) Turbine work and Pump work (b) Power developed (c) Cycle efficiency (d) Specific steaming rate (e) Specific heat rate (f) Work ratio.**

*Given:*

Mass flow rate of steam ($m_s$) = 400 kg/min = 6.667 kg/s

Boiler pressure ($p_b$) = 3 MPa = 30 bar

Temperature of steam ($T_1$) = 500°C

Condenser pressure ($p_c$) = 8 kPa = 0.08 bar

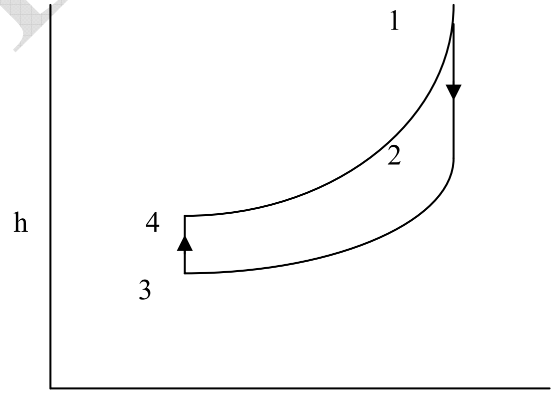
*Required* : (a) $w_T$ & $w_P$ (b) P (c) $\eta_R$ (d) SSC (e) SHR (f) WR

*Solution:*

***Using steam table***

(a)

Turbine work = $w_T = h_1 - h_2$



s

1-2 → Turbine 2-3 → Condenser 3-4 → Feed pump 4-1 → Boiler

$h_1$ = specific enthalpy at turbine inlet (i.e., at 30 bar & 500°C)

From super heated steam table, $h_1 = 3456.2$ kJ/kg

$h_2$ = specific enthalpy at turbine outlet (i.e., at condenser pressure of 0.08 bar)

$$h_2 = h_{f2} + x\, h_{fg2}$$

*From steam table at 0.08 bar*

$h_f = 173.9$ kg/kg $h_{fg} = 2403.2$ kJ/kg $s_f = 0.598$ kJ/kg-K

$s_{fg} = 7.637$ kJ/kg-K $v_f = 0.001008\ m^3/kg$

To find 'x'

In the turbine Isentropic expansion is taking place.

i.e., $s_1 = s_2$

$s_1 = 7.235$ kJ/kg-K at 30 bar & 500°C

$$s_2 = s_{f2} + x\, s_{fg2}$$

$$7.235 = 0.598 + x\,(7.637)$$

$$\therefore \quad x = 0.869$$

$$\therefore \quad h_2 = 173.9 + (0.869)\,(2403.2)$$

$$= 2262.28 \text{ kJ/kg}$$

$\therefore$ $w_T = 3456.2 - 2262.28$ **= 1193.92 kJ/kg ---- Ans**

Pump work = $w_P = v_{f3}\,(p_b - p_c) = h_4 - h_3$

$v_{f3}$ = specific volume of liquid at condenser pressure

$= 0.001008\ m^3/kg$

$$w_P = 0.001008\,(30 - 0.08) \times 10^5$$

= 3015.936 J/kg = **3.016 kJ/kg ---- Ans**

(b) Power developed (P) $= m_s\,(w_T - w_P)$

$= 6.667 \times (1193.92 - 3.016)$

**= 7939.75 kW --- Ans**

(c) Cycle efficiency ($\eta_R$) $= \dfrac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}$ → Considering pump work

$$= \frac{w_T - w_P}{h_1 - h_4}$$

$h_4 \approx h_3 = h_{f3}$ at condenser pressure = 173.9 kJ/kg

$$\therefore \quad \eta_R = \frac{1193.92 - 3.016}{3456.2 - 173.9}$$

**= 0.363 --- Ans**

(d) Specific steaming rate (SSC)

$$= \frac{3600}{w_T - w_P} = \frac{3600}{1193.92 - 3.016} = \mathbf{3.0229\ kg/kWh} \quad \text{---} \quad \mathbf{Ans}$$

(e) Specific heat rate (SHR) $= \dfrac{3600}{\eta_R} = 3600/0.363 = \mathbf{9917.35\ kJ/kWh}$ ---- **Ans**

(f) Work ratio $= \dfrac{w_T - w_P}{w_T} = \dfrac{1193.92 - 3.016}{1193.92} = \mathbf{0.9974}$ --- **Ans**

**Using Mollier chart**

- Locate point (1) at 30 bar & 500°C.

  At point (1) $h_1 = 3465$ kJ/kg
- Draw a straight vertical line to meet at 0.08 bar to locate point (2).

  At point (2) $h_2 = 2260$ kJ/kg
- But point (3) can't be located on the chart. For taking the values at point (3) refer steam table.

  $\therefore \quad h_3 = h_{f3} = 173.9$ kJ/kg

(a) Turbine work($w_T$) $= h_1 - h_2 = 3465 - 2260 = \mathbf{1205\ kJ/kg}$ ---- **Ans**

Pump work ($w_P$) $= h_4 - h_3 = v_{f3}\,(p_b - p_c)$

$= 0.001008 \times (30 - 0.08) \times 10^5$

$= \mathbf{3.016\ kJ/kg}$ --- **Ans**

(b) Power developed (P) $= m_s\,(w_T - w_P)$

$= 6.667\,(1205 - 3.016)$

$= \mathbf{8013.63\ kW}$ ---- **Ans**

(c) Cycle efficiency ($\eta_R$) $= \dfrac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}$ → Considering pump work

$$= \frac{w_T - w_P}{h_1 - h_4}$$

$$= \frac{1205 - 3.016}{3465 - 173.9} = \mathbf{0.365} \quad \text{---} \quad \mathbf{Ans}$$

d) Specific steaming rate (SSC)

$$= \frac{3600}{w_T - w_P} = \frac{3600}{1205 - 3.016} = \mathbf{2.995\ kg/kWh} \quad \text{---} \quad \mathbf{Ans}$$

(e) Specific heat rate (SHR) $= \dfrac{3600}{\eta_R} = 3600/0.365 = \mathbf{9863.01\ kJ/kWh}$ ---- **Ans**

(f) Work ratio $= \dfrac{w_T - w_P}{w_T} = \dfrac{1205 - 3.016}{1205}$ = **0.9975 --- Ans**

**13. Determine the Rankine cycle efficiency working between 6 bar and 0.4 bar when supplied with saturated steam. By what percentage is the efficiency increased by superheating to 300°C.**

*Given:*

Boiler pressure ($p_b$) = 6 bar

Condenser pressure ($p_c$) = 0.4 bar

Inlet condition = Dry steam

*Required* : $\eta_R$ & % increase in $\eta_R$ if inlet temperature is 300°C

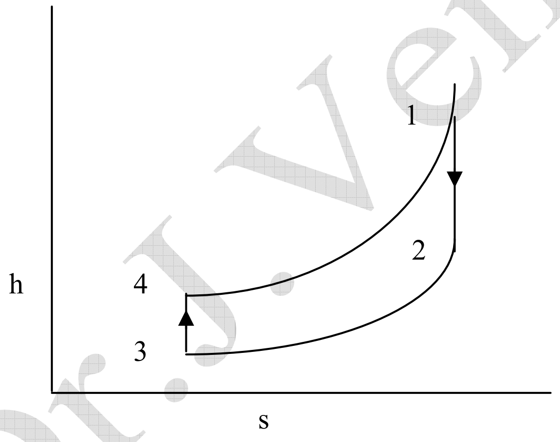
*Solution:*

Cycle efficiency ($\eta_R$) $= \dfrac{h_1 - h_2}{h_1 - h_4}$ → Neglecting pump work

$h_1 = h_g$ at 6 bar = 2755.5 kJ/kg

$h_2$ = specific enthalpy at turbine outlet (i.e., at condenser pressure of 0.4 bar)

$$h_2 = h_{f2} + x\, h_{fg2}$$



1-2 → Turbine 2-3 → Condenser 3-4 → Feed pump 4-1 → Boiler

*From steam table at 0.4 bar*

$h_f$ = 317.7 kg/kg $h_{fg}$ = 2319.2 kJ/kg $s_f$ = 1.026 kJ/kg-K $s_{fg}$ = 6.645 kJ/kg-K

<u>To find 'x'</u>

In the turbine Isentropic expansion is taking place.

i.e., $s_1 = s_2$

$s_1 = s_g$ = 6.758 kJ/kg-K at 6 bar

$$s_2 = s_{f2} + x\, s_{fg2}$$

$$6.758 = 1.026 + x\,(6.645)$$

$$\therefore \quad x = 0.862$$

$\therefore \quad h_2 = 317.7 + (0.862)(2319.2)$

$= 2316.85$ kJ/kg

$h_4 = h_{f3} = 317.7$ kJ/kg at 0.4 bar

$$\therefore \quad \eta_R = \frac{2755.5 - 2316.85}{2755.5 - 317.7} = \mathbf{0.18}$$ **---- Ans**

(ii) Inlet condition → 6 bar & 300°C

Outlet condition → 0.4 bar

From superheated steam table, $h_1 = 3062.3$ kJ/kg

$s_1 = 7.374$ kJ/kg-K

$h_2$ = specific enthalpy at turbine outlet (i.e., at condenser pressure of 0.4 bar)

$h_2 = h_{f2} + x\, h_{fg2}$

*To find 'x'*

In the turbine Isentropic expansion is taking place.

i.e., $s_1 = s_2$

$s_2 = s_{f2} + x\, s_{fg2}$

$7.374 = 1.026 + x(6.645)$

$\therefore \quad x = 0.955$

$\therefore \quad h_2 = 317.7 + (0.955)(2319.2)$

$= 2532.5$ kJ/kg

$h_4 = h_{f3} = 317.7$ kJ/kg at 0.4 bar

$$\therefore \quad \eta_R = \frac{3062.3 - 2532.5}{3062.3 - 317.7} = \mathbf{0.193}$$

$\therefore \quad$ % increase = $(0.193 - 0.18) \times 100/0.18$

= **7.22 % --- Ans**

**14. In a single heater regenerator cycle, the steam enters the turbine at 30 bar, 400°C and the exhaust pressure is 0.1 bar. The feed water heater is a direct contact type which operates at 5 bar. Find the efficiency and the steam rate of the cycle with and without regeneration. Pump work may be neglected.**

*Given:*

Pressure at turbine inlet ($p_1$) = 30 bar

Temperature at turbine inlet ($T_1$) = 400°C

Exhaust steam pressure ($p_3$) = 0.1 bar

Feed heater pressure ($p_2$) = 5 bar

*Required:* $\eta$ & Steam rate with and without regeneration

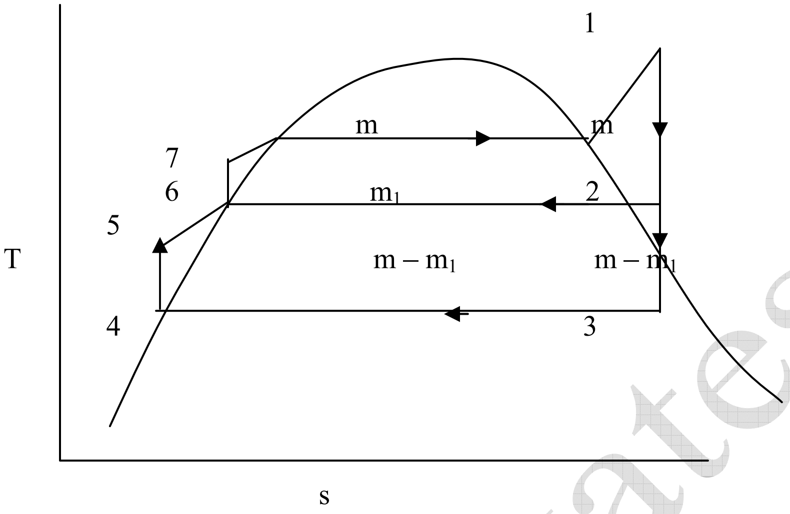
*Solution:*

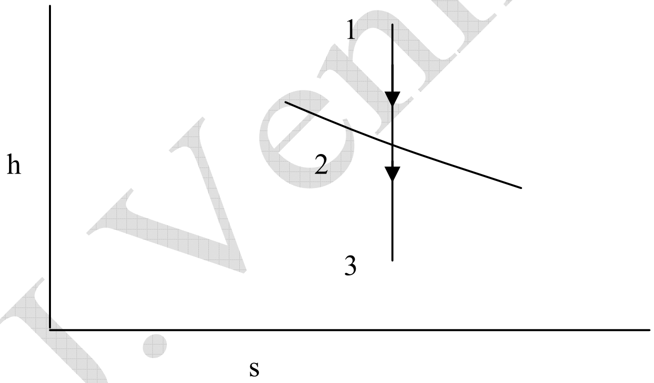
***With regeneration***

$$\text{Efficiency } (\eta) = \frac{\text{Workdone}}{\text{Heat supplied}}$$

Workdone = m $(h_1 - h_2) + (m - m_1)(h_2 - h_3)$

Take m = 1 kg





From Mollier Chart

- Locate the point (1) at 30 bar & 400°C
- Draw the vertical straight line to get point (2) at 5 bar and point (3) at 0.1 bar

$\therefore$ $h_1 = 3240$ kJ/kg $h_2 = 2800$ kJ/kg $h_3 = 2200$ kJ/kg

To find $m_1$

Heat lost by $m_1$ kg of bled steam = Heat gain by $(m - m_1)$ kg of feed water from pump (1)

i.e., $m_1 (h_2 - h_6) = (1 - m_1)(h_6 - h_5)$

$h_5 \approx h_4 = h_f$ at 0.1 bar = 191.8 kJ/kg from steam table

**Note:** Always use steam table for finding out $h_f$ values.

$h_6 = h_f$ at 5 bar = 640.1 kJ/kg from steam table

$\therefore$ $m_1 (2800 - 640.1) = (1 - m_1)(640.1 - 191.8)$

$m_1 = 0.1788$ kg

$$\text{Workdone} = 1 \times (3240 - 2800) + (1 - 0.1788)(2800 - 2200)$$

$$= 932.72 \text{ kJ/kg}$$

$$\text{Heat supplied} = m\,(h_1 - h_7)$$

$h_7 \approx h_6 = h_f$ at 5 bar = 640.1 kJ/kg

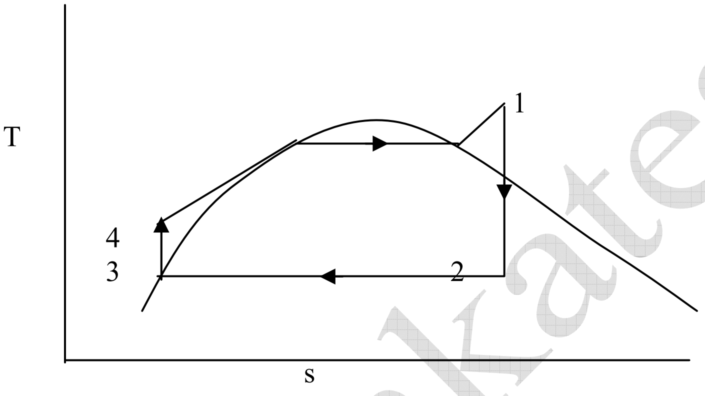
$$\therefore \text{ Heat supplied} = 1 \times (3200 - 640.1) = 2599.9 \text{ kJ/kg}$$

Efficiency = 932.72 / 2599.9 **= 0.359 --- Ans**

$$\text{Steam rate} = 3600 / (w_T - w_P)$$

= 3600 / 932.72 = **3.86 kg/kWh ---- Ans**

Without regeneration



$h_1 = 3240$ kJ/kg

$h_2 = 2200$ kJ/kg

$$\text{Workdone} = m\,(h_1 - h_2) = 1 \times (3240 - 2200) = 1040 \text{ kJ/kg}$$

$$\text{Heat supplied} = m\,(h_1 - h_4)$$

$h_4 \approx h_3 = h_f$ at 0.1 bar = 191.8 kJ/kg

$$\therefore \quad \text{Heat supplied} = 1 \times (3240 - 191.8) = 3008.2 \text{ kJ/kg}$$

Efficiency = 1040 / 3008.2 = **0.341 ---- Ans**

Steam rate = 3600 / $w_{net}$ = 3600 / 1040 **= 3.46 kg/kWh --- Ans**

15. **Steam is supplied to a turbine at a pressure of 30 bar and 400°C and is expanded isentropically to a pressure of 0.1 bar. The steam is trapped from turbine at two different location one at 10 bar and another at 3 bar, to supply feed water heaters. Find the thermal efficiency and steam rate of the cycle. Neglect pump work.**

*Given:*

Cycle with two feed water heaters

Pressure at turbine inlet ($p_1$) = 30 bar

Temperature at turbine inlet ($T_1$) = 400°C

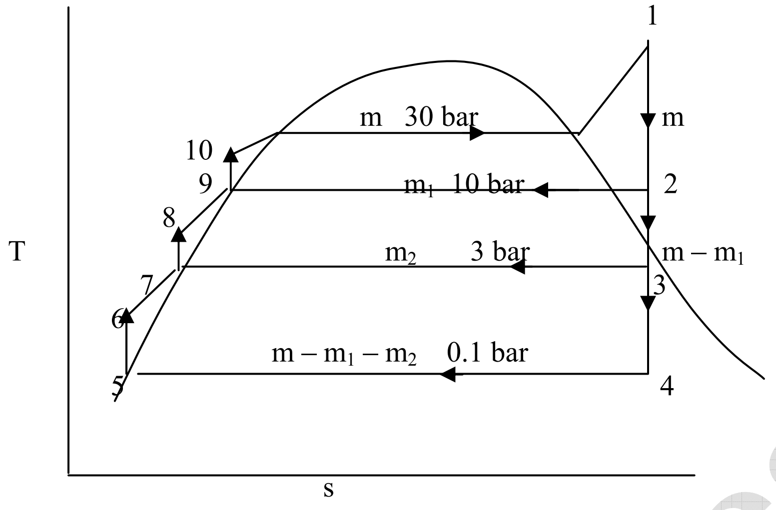
Exhaust steam pressure ($p_4$) = 0.1 bar

I-Feed heater pressure ($p_2$) = 10 bar

II-Feed heater pressure ($p_3$) = 3 bar

*Required:* $\eta$ & Steam rate

*Solution:*



$$\text{Efficiency} = \frac{\text{Workdone}}{\text{Heat supplied}}$$

Workdone = $m (h_1 - h_2) + (m - m_1)(h_2 - h_3) + (m - m_1 - m_2)(h_3 - h_4)$

Heat supplied = Heat supplied during process 10 – 1

$= m (h_1 - h_{10})$

Take m = 1 kg

From Mollier chart,

$h_1 = 3240$ kJ/kg $h_2 = 2950$ kJ/kg $h_3 = 2700$ kJ/kg $h_4 = 2200$ kJ/kg

To find $m_1$ & $m_2$

Heat lost by $m_1$ kg of bled steam = Heat gain by $(m - m_1)$ kg of feed water from pump (2)

i.e., $m_1 (h_2 - h_9) = (1 - m_1)(h_9 - h_8)$

Heat lost by $m_2$ kg of bled steam = Heat gained by $(m - m_1 - m_2)$ kg of feed water from pump (1)

$m_2 (h_3 - h_7) = (1 - m_1 - m_2)(h_7 - h_6)$

$h_9 = h_f$ at 10 bar = 762.6 kJ/kg from steam table

$h_8 \approx h_7 = h_f$ at 3 bar = 561.5 kJ/kg

$h_6 \approx h_5 = h_f$ at 0.1 bar = 191.8 kJ/kg

**Note:** Always use steam table for finding out $h_f$ values.

$m_1 (2950 - 762.6) = (1 - m_1)(762.6 - 561.5)$

$m_1 = 0.0842$ kg

and $m_2 (2700 - 561.5) = (1 - 0.0842 - m_2)(561.5 - 191.8)$

$m_2 = 0.135$ kg

Workdone = $1 \times (3240 - 2950) + (1 - 0.0842)(2950 - 2700)$

$+ (1 - 0.0842 - 0.135)(2700 - 2200)$

$= 909.35$ kJ/kg

Heat supplied = m $(h_1 – h_{10})$

$H_{10} \approx h_9 = h_f$ at 10 bar = 762.6 kJ/kg

$\therefore$ Heat supplied = 1 x (3240 – 762.6) = 2477.4 kJ/kg

Efficiency = 909.35 / 2477.4 **= 0.367 --- Ans**

Steam rate = 3600 / $(w_T – w_P)$ = 3600 / 2477.4 = **1.4531 kg/kWh ---- Ans**

**16. In a steam turbine steam at 20 bar, 360°C is expanded to 0.08 bar. It is then enters a condenser, where it is condensed to saturated liquid. The pump feeds back the water into the boiler. Assume ideal processes, find per kg of steam, the net work and the cycle efficiency.**

*Given:*

Turbine inlet pressure $(p_1)$ = 20 bar

Turbine exhaust pressure $(p_2)$ = 0.08 bar

Turbine inlet temperature $(T_2)$ = 360°C

*Required:* $w_T – w_P$ & $\eta$

*Solution:*

For solving this problem we can use either steam table or Mollier chart.

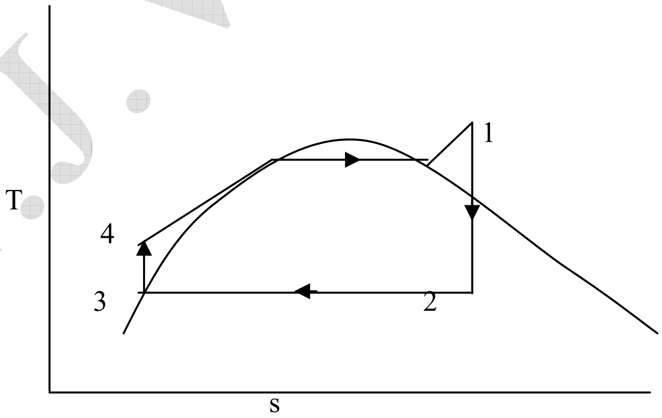
**Using steam table**

$$\text{Efficiency } (\eta) = \frac{\text{Net Work}}{\text{Heat supplied}}$$

Net work = $w_T - w_P$

Turbine work = $w_T = h_1 – h_2$

Pump work = $w_P = (p_1 – p_2)\, v_{f2}$



$h_1$ = 3160.74 kJ/kg from superheated steam table at 360°C & 20 bar

$h_2 = h_{f2} + x_2\, h_{fg2}$

$h_{f2}$ = 173.9 kJ/kg from steam table at 0.08 bar

$h_{fg2}$ = 2403.2 kJ/kg

<u>To find $x_2$</u>

$s_1 = s_2$

$s_1 = 6.994$ kJ/kg from steam table at $360^oC$ & 20 bar

$$s_2 = s_{f2} + x_2\, s_{fg2}$$

$s_{f2} = 0.593$ kJ/kg-K from steam table at 0.08 bar

$s_{fg2} = 7.637$ kJ/kg-K from table

$$\therefore \quad 6.994 = 0.593 + x_2\,(7.637)$$

$$x_2 = 0.838$$

$$h_2 = 173.9 + 0.838 \times 2403.2 = 2187.8 \text{ kJ/kg}$$

$$\therefore \quad w_T = 3160.74 - 2187.8 = 972.94 \text{ kJ/kg}$$

$v_{f2} = 0.001008\ m^3/kg$ from table at 0.08 bar

$$\therefore \quad w_P = 0.001008 \times (20 - 0.08) \times 10^5$$

$$= 2008 \text{ J/kg} = 2.008 \text{ kJ/kg}$$

$$\therefore \quad w_{net} = 972.94 - 2.008 = \mathbf{970.932\ kJ/kg}$$ **--- Ans**

Heat supplied = $h_1 - h_4$

To find $h_4$

$$w_P = h_4 - h_3 = 2.008$$

$$\therefore \quad h_4 = h_3 + 2.008$$

$h_3 = h_f$ at 0.08 bar = 173.9 kJ/kg

$$\therefore \quad h_4 = 2.008 + 173.9 = 175.908 \text{ kJ/kg}$$

$$\therefore \quad HS = 3160.74 - 175.908 = 2984.832 \text{ kJ/kg}$$

$$\eta = 972.94 / 2984.832 = \mathbf{0.326}$$ **--- Ans**

**17. In a Rankine cycle, the steam at inlet to turbine is saturated at a pressure of 35 bar and the exhaust pressure is 0.2 bar. Determine, the pump work, the turbine work, the Rankine efficiency, the condenser heat flow and dryness at the end of expansion. Assume flow rate of 9.5 kg/s.**

*Given:*

Turbine inlet pressure ($p_1$) = 35 bar

Turbine exhaust pressure ($p_2$) = 0.2 bar

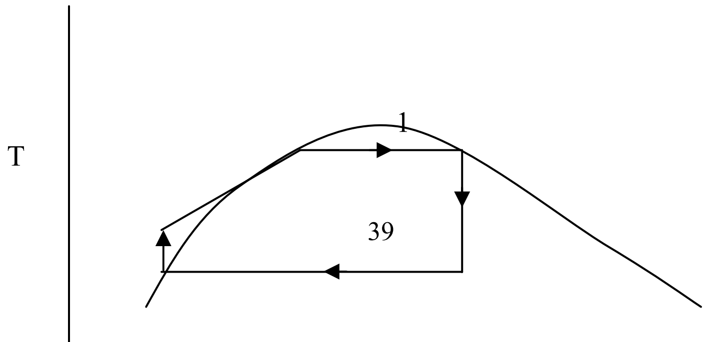
Dry steam at inlet

Mass flow rate of steam (m) = 9.5 kg/s

*Required:* $w_T$, $w_P$, $\eta$, $Q_{2-3}$ & $x_2$

*Solution:*

**Using steam table**



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s

$$\text{Efficiency } (\eta) = \frac{\text{Net Work}}{\text{Heat supplied}}$$

Net work = $w_T - w_P$

Turbine work = $w_T = h_1 - h_2$

Pump work = $w_P = (p_1 - p_2)\, v_{f2}$

Heat flow through the condenser = $Q_{2\text{-}3} = m\,(h_2 - h_3)$

$h_1 = h_g$ at 35 bar from steam table = 2802 kJ/kg

$h_2 = h_{f2} + x_2\, h_{fg2}$

$h_{f2}$ = 251.5 kJ/kg from steam table at 0.2 bar

$h_{fg2}$ = 2358.4 kJ/kg

To find $x_2$

$s_1 = s_2$

$s_1 = s_g$ at 35 bar from steam table = 6.123 kJ/kg-K

$s_2 = s_{f2} + x_2\, s_{fg2}$

$s_{f2}$ = 0.832 kJ/kg-K from steam table at 0.2 bar

$s_{fg2}$ = 7.077 kJ/kg-K from table

$\therefore \quad 6.123 = 0.832 + x_2\,(7.077)$

$\mathbf{x_2 = 0.7476}$ **--- Ans**

$h_2 = 251.5 + 0.7476 \times 2358.4 = 2014.64$ kJ/kg

$\therefore \quad w_T = 2802 - 2014.64 = 787.36$ kJ/kg

Turbine work = 9.5 x 787.36 = **7479.92 kW ---- Ans**

$v_{f2} = 0.001017\ \text{m}^3/\text{kg}$ from table at 0.2 bar

$\therefore \quad w_P = 0.001017 \times (35 - 0.2) \times 10^5$

$= 3539.16$ J/kg = 3.53916 kJ/kg

Pump work = 9.5 x 3.53916 = **33.622 kW ---- Ans**

$\therefore \quad w_{net} = 787.36 - 3.53916 = 783.82084$ kJ/kg

Heat supplied = $h_1 - h_4$

To find $h_4$

$w_P = h_4 - h_3 = 3.53916$

$\therefore \quad h_4 = h_3 + 3.53916$

$h_3 = h_f$ at 0.2 bar = 251.5 kJ/kg

$\therefore \quad h_4 = 3.53916 + 251.5 = 255.04$ kJ/kg

∴ HS = 2802 – 255.04 = 2546.96 kJ/kg

$\eta$ = 783.82084 / 2546.96 **= 0.308 --- Ans**

Condenser heat flow = 9.5 x (2014.64 – 251.5) **= 16749.83 kW ---- Ans**

**18. In a steam power plant operating on an reheat Rankine cycle, the steam enters the high pressure turbine at 3 MPa and 400°C. After expansion to 0.6 MPa, the steam is reheated to 400°C and then expanded in the low pressure turbine to the condenser pressure of 10 kPa. Determine the thermal efficiency of the cycle and the quality of the steam at the outlet of the low pressure turbine.**

*Given:*

Reheat cycle

Pressure at inlet of HP turbine ($p_1$) = 3 MPa = 30 bar

Temperature at inlet of HP turbine ($T_1$) = 400°C = 673 K

Pressure at the inlet of reheater ($p_2$) = 0.6 MPa = 6 bar

Temperature at the exit of reheater ($T_3$) = 400°C = 673 K

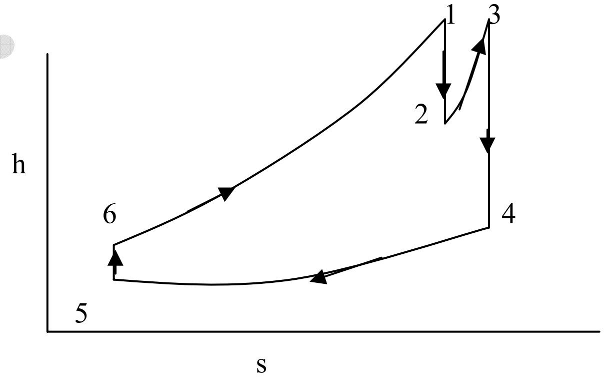
Pressure at the exit of the LP turbine ($p_4$) = 10 kPa = 0.1 bar

*Required:* Thermal efficiency and Dryness fraction at exit of LP turbine

*Solution:*

**Using Mollier diagram**:

- Locate the point (1) at 400°C and 30 bar in the Mollier chart
- 1-2 → Isentropic expansion in HP turbine → Draw the vertical line to meet 6 bar line and locate point (2)
- 2-3 → Constant pressure heat addition → Follow the constant pressure line (6 bar) and reach 400°C and mark point (3)
- 3-4 → Isentropic expansion in LP turbine → Draw the vertical line to meet 0.1 bar line and mark point (4)



$h_1$ = 3230 kJ/kg  $h_2$ = 2830 kJ/kg  $h_3$ = 3275 kJ/kg  $h_4$ = 2445 kJ/kg

$x_4$ = 0.94 **--- Ans**

$$\eta = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}$$

$(h_6 – h_5) = v_{f5} (p_6 – p_5)$

From steam table,

$v_{f5} = 0.00101\ m^3/kg$

$h_{f5} = 191.8$ kJ/kg $= h_5$

$(h_6 - h_5) = 0.00101 \times (30 - 0.1) \times 10^5/10^3 = 3.0199$ kJ/kg

$h_6 = 3.0199 + 191.8 = 194.82$ kJ/kg

$$\eta = \frac{(3230 - 2830) + (3275 - 2445) - (3.0199)}{(3230 - 194.82) + (3275 - 2830)} = 0.3525 \quad ----Ans$$

**Using Steam table:**

From superheated steam table at 30 bar and 400°C,

$h_1 = 3232.5$ kJ/kg $\qquad s_1 = 6.925$ kJ/kg-K

To check type of steam at (2)

$s_1 = s_2 = s_{f2} + x_2\, s_{fg2}$

From steam table at 6 bar,

$t_{s2} = 158.8^oC \quad s_{f2} = 1.931$ kJ/kg-K $\quad s_{fg2} = 4.827$ kJ/kg-K $\quad h_{g2} = 2755.5$ kJ/kg

Therefore, $6.925 = 1.93 + x_2\,(4.827)$

$x_2 = 1.0348$

Therefore the steam is superheated at HP turbine outlet.

$h_2 = h_{g2} + Cp_{sup}\,(T_2 - t_{s2})$

$t_{s2} = 158.8^oC$

There are two unknowns, $T_2$ and $Cp_{sup}$

At $s_2 = 6.925$ kJ/kg-K and 6 bar, from superheated steam table, the temperature, $T_2$ is obtained.

$s = s_{g2} = 6.758$ kJ/kg-K, at $158.8^oC$,

$= 6.966$ kJ/kg-K, at $200^oC$

$= 6.925$ kJ/kg-K, at $T_2$

Therefore, $$T_2 = \frac{(158.8 - 200)}{(6.758 - 6.966)}(6.925 - 6.966) + 200$$

$= 191.9^oC = 464.9$ K

To find $Cp_{sup}$

$s_1 = s_2 = s_{g2} + Cp_{sup} \ln (T_2/t_{s2})$

$6.925 = 6.758 + Cp_{sup} \ln (464.9/431.8)$

$Cp_{sup} = 2.261$ kJ/kg-K

Therefore, $h_2 = 2755.5 + 2.261 \times (191.9 - 158.8) = 2830.3$ kJ/kg

From superheated steam table, at 6 bar and 400°C,

$h_3 = 3270.6$ kJ/kg $\qquad s_3 = 7.709$ kJ/kg-K

From saturated table, at 0.1 bar

$h_{f4} = 191.8$ kJ/kg

$h_{fg4} = 2392.9$ kJ/kg

$s_{f4} = 0.649$ kJ/kg-K

$s_{fg4}$ = 7.502 kJ/kg-K

<u>To find $x_4$</u>

$s_4 = s_{f4} + x_4\, s_{fg4}$

Process 1-4 is isentropic, $s_4 = s_3$

$7.709 = 0.649 + x_4 (7.502)$

$x_4 = 0.941$ ----- **Ans**

Therefore, $h_4 = h_{f4} + x_4\, h_{fg4}$

= 191.8 + 0.941 (2392.9) = 2443.52 kJ/kg

$(h_6 - h_5) = v_{f5} (p_6 - p_5)$

From steam table,

$v_{f5} = 0.00101\ m^3/kg$

$h_{f5}$ = 191.8 kJ/kg = $h_5$

$(h_6 - h_5) = 0.00101 \times (30 - 0.1) \times 10^5/10^3 = 3.0199$ kJ/kg

$h_6$ = 3.0199 + 191.8 = 194.82 kJ/kg

$$\eta = \frac{(3232.5 - 2830.3) + (3270.6 - 2443.52) - (3.0199)}{(3232.5 - 194.82) + (3270.6 - 2830.3)} = 0.3526 \quad ----Ans$$

# UNIT-IV

# IDEAL AND REAL GASES, THERMO DYNAMIC RELATIONS

## Ideal gas

A perfect gas or an ideal gas is defined as a gas having no intermolecular forces. A gas which follows the gas laws at all ranges of pressures and temperatures can be considered as an ideal gas, but no such gas exists in nature.

## Boyle's law

Boyle's law states that if the temperature of a gas in a closed system is maintained constant during a process, the volume of the gas will vary increasingly with absolute pressure during the change of state.

$$V \alpha\ 1/p \quad \text{or} \quad pV = \text{Constant} \quad \text{when T is constant}$$

## Charle's law

If the pressure of the gas in a closed system is maintained constant during a process, the volume of gas will vary directly with the absolute temperature.

$$V \alpha\ T \quad \text{or} \quad V/T = C$$

## Avogadro's law

A mole of a substance has a mass numerically equal to the molecular weight of the substance.

Example: 1 kg mole of $O_2$ has a mass of 32 kg

1 kg mole of $N_2$ has a mass of 28 kg

Avogadro's law states that the volume of a 1 kg mole of all gases at STP ($0^oC$ and 1.01325 bar) is the same and is equal to 22.4 $m^3$.

Example: 1 kg mole of $O_2$ has a mass of 32 kg and volume of 22.4 $m^3$

1 kg mole of $N_2$ has a mass of 28 kg and volume of 22.4 $m^3$

At p = 1.01325 bar and T = 273.15 K, for 1 kg mole of gas V = 22.4 $m^3$

$$v_m = 22.4\ m^3/\text{kg mole}$$

$$p\ v_m = R_u\ T$$

$$1.01325 \times 10^5 \times 22.4 = R_u \times 273.15$$

$$R_u = 8314.3\ \text{J/kg mol K}$$

$R_u$ → Universal gas constant; $v_m$ → Molar volume

## Joule's law

This law states that there is no change of temperature when a gas expands without doing external work and without receiving or rejecting heat.

$$u = f(T) \quad \text{only for ideal gas}$$

## Equation of state

The functional relationship among the properties, pressure p, molar or specific volume v and temperature T, is known as an equation of state, which may be expressed in the form,

$$f\,(p, v, T) = 0$$

If two properties of a gas are known, the third can be evaluated from the equation of state.

$$p\ v = R\ T$$

## Real gas

Most of the real gases obey Boyle's and Charle's law at low pressures and temperatures. But the actual behaviour of real gases at elevated pressures and at low temperatures deviates considerably.

The ideal gas equation p v = R T can be derived analytically using the kinetic theory of gases by making the following assumptions:

i. A finite volume of gas contains large number of molecules.

ii. The collision of molecules with one another and with the walls of the container is perfectly elastic.

iii. The molecules are separated by large distances compared to their own dimensions.

iv. The molecules do not exert forces on one another except when they collide.

As long as the above assumptions are valid the behaviour of a real gas approaches closely that of an ideal gas.

### Van der Waals equation

The ideal gas equation p v = R T is being used with two important assumptions that there is little or no attraction between the two molecules of the gas and that the volume occupied by the molecules themselves is negligibly small compared to the volume of the gas. When the pressure is very small or temperature is very large, the intermolecular attraction and the volume of the molecules compared to the total volume of the gas are not of much importance, and the real gas obeys very closely the ideal gas equation. But as pressure increases, the intermolecular forces of attraction and repulsion increase, and also the volume of the molecules becomes appreciable compared to the total volume. Then the real gases deviate considerably from the ideal gas equation. Van der Waals by applying the laws of mechanics to the individual molecules, introduced two corrections terms in the ideal gas equation.

The Van der Waals equation for a real gas may be written as

$$\left(p+\frac{a}{v^2}\right)(v-b)=RT$$

a → a constant to account for the existence of mutual attraction between the molecules.

$a/v^2$ → the force of attraction

b → the coefficient introduced to account for the volumes of molecules, and is known as co-volume

$$a=\frac{27\,R^2\,T_c^2}{64\,p_c} \qquad b=\frac{R\,T_c}{8\,p_c}$$

$T_c$ → Critical temperature

$p_c$ → Critical pressure

Unique feature

- Van der Waals equation qualitatively accounts for the heating effect observed at ordinary temperatures.

Limitations

- The values of a and b (which are assumed to be constant) are found to vary with temperature. Thus the results obtained from the equation are incorrect when the variation of a and b is large with respect to temperature.
- The equation is not accurate enough in the critical region.

### Virial expansions

The virial expansion, also called the virial equation of state, is the most interesting and versatile of the equations of state for gases. The virial expansion is a power series in powers of the variable, B/V, and has the form,

$$\frac{pv}{RT} = A_o + A_1 p + A_2 p^2 + A_3 p^3 + ------$$

$$\frac{pv}{RT} = B_o + \frac{B_1}{v} + \frac{B_2}{v^2} + \frac{B_3}{v^3} + ------$$

The coefficient, $A_o$, $A_1$, $A_2$, ……. and $B_o$, $B_1$, $B_2$, …….. are called the virial coefficients and are functions of temperature. $A_o$, $B_o$ are first virial coefficient and $A_1$, $B_1$ are second virial coefficients and so on.

## Law of corresponding states

For each substance there is a compressibility factor. It would be very convenient if one chart could be used for all substances. The general shapes of the vapour dome and of the constant temperature lines on the p-v plane are similar for all substances, although the scales may be different. This similarity can be exploited by using dimensionless properties called 'reduced properties'.

Reduced pressure, $p_r = \frac{p}{p_c}$

Reduced temperature, $T_r = \frac{T}{T_c}$

Reduced volume, $v_r = \frac{v}{v_c}$

Generally for all substances, $v_r = f(p_r, T_r)$ and $Z = f(p_r, T_r)$

The relation among the reduced properties, $p_r$, $T_r$ and $v_r$ is known as the law of corresponding states.

## Compressibility

In thermodynamics and fluid mechanics, compressibility is a measure of the relative volume change of a fluid or solid as a response to a pressure (or mean stress) change.

$$K = \frac{1}{v}\frac{\partial v}{\partial p} \qquad ----- (1)$$

where v is specific volume and p is pressure. The above statement is incomplete, because for any object or system the magnitude of the compressibility depends strongly on whether the process is adiabatic or isothermal. Accordingly we define the isothermal compressibility as:

$$K_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T \qquad ----- (2)$$

where the subscript T indicates that the partial differential is to be taken at constant temperature. The adiabatic compressibility as:

$$K_S = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_S \qquad ----- (3)$$

where S is entropy. For a solid, the distinction between the two is usually negligible.

Coefficient of volume expansion is defined as,

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p \qquad \text{----- (4)}$$

The inverse of the compressibility is called the bulk modulus.

We know that, $\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V = -1$ ----- (5)

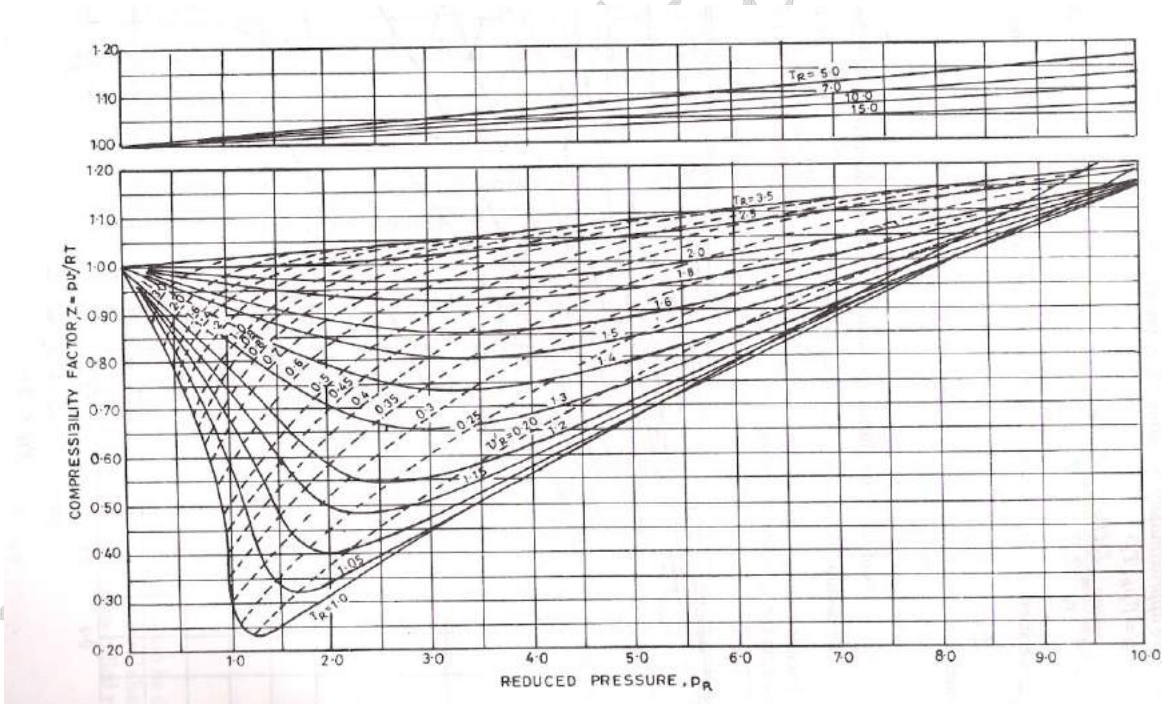
$$\beta v = \left(\frac{\partial v}{\partial T}\right)_p \text{ from (4)}$$

$$-K_T v = \left(\frac{\partial v}{\partial p}\right)_T \text{ from (2)}$$

Substituting in (5), $\left(\frac{-1}{K_T v}\right)_T (\beta v)\left(\frac{\partial T}{\partial p}\right)_V = -1$

$$\frac{\beta}{K_T} = \left(\frac{\partial p}{\partial T}\right)_V \qquad \text{----- (6)}$$

**Compressibility Chart**



The compressibility factor (Z) is used to alter the ideal gas equation to account for the real gas behaviour. The compressibility factor is usually obtained from the compressibility chart. Mathematically, it is defined as,

$$Z = \frac{pv}{RT} = \frac{v_{actual}}{v_{ideal}}$$

p → the pressure,

v → the specific volume of the gas,

T → the temperature, and

R → the gas constant.

Z = 1 for ideal gas

Critical compressibility factor, $Z_c = \dfrac{p_c\, v_c}{R T_c}$

Z can, in general, be either greater or less than unity for a real gas. The deviation from ideal gas behavior tends to become particularly significant (or, equivalently, the compressibility factor strays far from unity) near the critical point, or in the case of high pressure or low temperature. In these cases, a generalized Compressibility chart or an alternative equation of state better suited to the problem must be utilized to produce accurate results.

**Rules on partial derivatives**

**Theorem 1 (Exact differential)**

If a relation exists among the variables x, y and z, then z may be expressed as function of x and y.

$$z = f\,(x, y)$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

Let, $M = \left(\dfrac{\partial z}{\partial x}\right)_y$ and $N = \left(\dfrac{\partial z}{\partial y}\right)_x$

$$dz = M\,dx + N\,dy$$

Differentiating M partially with respect to y, and N with respect to x,

$$\left(\frac{\partial M}{\partial y}\right)_x = \frac{\partial^2 z}{\partial x\,\partial y}$$

$$\left(\frac{\partial N}{\partial x}\right)_y = \frac{\partial^2 z}{\partial y\,\partial x}$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

This is the condition of exact or perfect differential.

**Theorem 2**

If a quantity f is a function of x, y and z and a relation exists among x, y and z, then f is a function of any two of x, y and z.

If f = f(x, y, z)

$$\left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f \left(\frac{\partial z}{\partial x}\right)_f = 1$$

**Theorem 3**

Among the variables x, y and z, any one variable may be considered as a function of the other two.

If x = x (y, z) y = y (x, z) z = z(x, y)

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial y}{\partial z}\right)_z = -1$$

Among the thermodynamics variables, p, v and T, the following relation holds good.

$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V = -1$$

**Maxwell's equations**

A pure substance existing in a single phase has only two independent variables. Of the eight quantities, p, v, T, s, u, h, F (Helmholts function) and G (Gibbs function), any one may be expressed as a function of any two others.

(i) Consider a non flow process.

$$q = w + \Delta u$$

$$dq = dw + du$$

$$du = dq - dw$$

$$= T\,ds - p\,dv \qquad \text{----- (1)}$$

Therefore, u = f (s, v)

$$du = \left(\frac{\partial u}{\partial s}\right)_V ds + \left(\frac{\partial u}{\partial v}\right)_S dv \qquad \text{----- (2)}$$

Comparing (1) and (2)

$$T = \left(\frac{\partial u}{\partial s}\right)_V \quad \text{and} \quad -p = \left(\frac{\partial u}{\partial v}\right)_S$$

$$\left(\frac{\partial T}{\partial v}\right)_S = \frac{\partial^2 u}{\partial v\,\partial s} \qquad \text{and} \qquad \left(-\frac{\partial p}{\partial S}\right)_S = \frac{\partial^2 u}{\partial s\,\partial v}$$

But, $\left(\frac{\partial^2 u}{\partial v\,\partial s}\right) = \left(\frac{\partial^2 u}{\partial s\,\partial v}\right)$

Therefore, $\left(\frac{\partial T}{\partial v}\right)_S = -\left(\frac{\partial p}{\partial s}\right)_V$ → **I – Maxwell equation**

The rate of increase in temperature with respect to volume during isentropic process is equal to the rate of decrease in pressure with respect to entropy during constant volume process.

(ii) Consider a flow process.

$$q = w + \Delta h$$

$$dq = dw + dh$$

$$dh = dq - dw$$

$$= T\,ds + v\,dp \qquad \text{----- (1)}$$

Therefore, h = f (s, p)

$$dh = \left(\frac{\partial h}{\partial s}\right)_p ds + \left(\frac{\partial h}{\partial p}\right)_S dp \qquad \text{----- (2)}$$

Comparing (1) and (2)

$$T = \left(\frac{\partial h}{\partial s}\right)_p \quad \text{and} \quad v = \left(\frac{\partial h}{\partial p}\right)_S$$

$$\left(\frac{\partial T}{\partial p}\right)_S = \frac{\partial^2 h}{\partial p\,\partial s} \quad \text{and} \quad \left(\frac{\partial v}{\partial s}\right)_p = \frac{\partial^2 h}{\partial s\,\partial p}$$

But, $\left(\frac{\partial^2 h}{\partial p\,\partial s}\right) = \left(\frac{\partial^2 h}{\partial s\,\partial p}\right)$

Therefore, $\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial v}{\partial s}\right)_p$ → **II – Maxwell equation**

The rate of increase in temperature with respect to pressure during isentropic process is equal to the rate of increase in volume with respect to entropy during constant pressure process.

(iii) F = u – Ts

dF = du – (T ds + s dT)

= du – T ds – s dT

But, du = T ds – p dv

Therefore, dF = T ds – p dv – T ds – s dT

= - p dv – s dT ----- (1)

Therefore, F = f (v, T)

$$dF = \left(\frac{\partial F}{\partial T}\right)_V dT + \left(\frac{\partial F}{\partial v}\right)_T dv \qquad \text{----- (2)}$$

Comparing (1) and (2)

$$-s = \left(\frac{\partial F}{\partial T}\right)_V \quad \text{and} \quad -p = \left(\frac{\partial F}{\partial v}\right)_T$$

$$-\left(\frac{\partial s}{\partial v}\right)_T = \frac{\partial^2 F}{\partial v\,\partial T} \quad \text{and} \quad -\left(\frac{\partial p}{\partial T}\right)_V = \frac{\partial^2 F}{\partial T\,\partial v}$$

But, $\left(\frac{\partial^2 F}{\partial v\,\partial T}\right) = \left(\frac{\partial^2 F}{\partial T\,\partial v}\right)$

Therefore, $\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$ → **III – Maxwell equation**

The rate of increase in entropy with respect to volume during isothermal process is equal to the rate of increase in pressure with respect to temperature during constant volume process.

(iv) g = h – Ts

dg = dh – (T ds + s dT)

= dh – T ds – s dT

But, dh = T ds + v dp

Therefore, dg = T ds + v dp – T ds – s dT

= v dp – s dT ----- (1)

Therefore, g = f (p, T)

$$dg = \left(\frac{\partial g}{\partial T}\right)_p dT + \left(\frac{\partial g}{\partial p}\right)_T dp \qquad \text{----- (2)}$$

Comparing (1) and (2)

$$-s = \left(\frac{\partial g}{\partial T}\right)_p \quad \text{and} \quad v = \left(\frac{\partial g}{\partial p}\right)_T$$

$$-\left(\frac{\partial s}{\partial p}\right)_T = \frac{\partial^2 g}{\partial p\,\partial T} \quad \text{and} \quad \left(\frac{\partial v}{\partial T}\right)_p = \frac{\partial^2 g}{\partial T\,\partial p}$$

But, $\left(\frac{\partial^2 g}{\partial p\,\partial T}\right) = \left(\frac{\partial^2 g}{\partial T\,\partial p}\right)$

Therefore, $\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$ → **IV – Maxwell equation**

The rate of increase in entropy with respect to pressure during isothermal process is equal to the rate of decrease in volume with respect to temperature during constant pressure process.

**TdS equations**

(i) Let s = f (T, v) → T and v independent variables

$$ds = \left(\frac{\partial s}{\partial T}\right)_V dT + \left(\frac{\partial s}{\partial v}\right)_T dv \qquad \text{------ (1)}$$

Let u = f (T, v)

$$du = \left(\frac{\partial u}{\partial T}\right)_V dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

$$= C_V\, dT + \left(\frac{\partial u}{\partial v}\right)_T dv \qquad \text{------ (2)}$$

Also, du = T ds – p dv ------ (3)

Equating (2) and (3),

$$T\,ds - p\,dv = C_V\, dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

$$ds = \frac{C_V}{T} dT + \frac{1}{T}\left[\left(\frac{\partial u}{\partial v}\right)_T + p\right] dv \qquad \text{------ (4)}$$

Comparing (1) and (4), $\left(\frac{\partial s}{\partial T}\right)_V = \frac{C_V}{T}$ ------ (5)

From Maxwell III-equation, $\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$ ------ (6)

Substituting (5) and (6) in (1),

$$ds = \frac{C_V}{T}\,dT + \left(\frac{\partial p}{\partial T}\right)_V dv$$

$$T\,ds = C_V\,dT + T\left(\frac{\partial p}{\partial T}\right)_V dv$$ → **First T ds equation**

We know that, $\frac{\beta}{K_T} = \left(\frac{\partial p}{\partial T}\right)_V$

$$T\,ds = C_V\,dT + T\left(\frac{\beta}{K_T}\right) dv$$ → **First T ds equation**

(ii) Let s = f (T, p) → T and p are independent variables

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp$$ ------ (1)

Let h = f (T, p)

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$

$$= C_p\,dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$ ------ (2)

Also, dh = T ds + v dp ------ (3)

Equating (2) and (3),

$$T\,ds + v\,dp = C_p\,dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$

$$ds = \frac{C_p}{T}\,dT + \frac{1}{T}\left[\left(\frac{\partial h}{\partial p}\right)_T - v\right] dp$$ ------ (4)

Comparing (1) and (4), $\left(\frac{\partial s}{\partial T}\right)_p = \frac{C_p}{T}$ ------ (5)

From Maxwell IV-equation, $\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$ ------ (6)

Substituting (5) and (6) in (1),

$$ds = \frac{C_p}{T} dT - \left(\frac{\partial v}{\partial T}\right)_p dp$$

$$T\,ds = C_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp$$ → **Second-T ds equation**

We know that, $\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$

$$\beta v = \left(\frac{\partial v}{\partial T}\right)_p$$

$$T\,ds = C_p dT - T\,\beta v\;dp$$ → **Second T ds equation**

(iii) Let u = f (p, v) → p and v are independent variables

$$du = \left(\frac{\partial u}{\partial p}\right)_V dp + \left(\frac{\partial u}{\partial v}\right)_p dv \qquad \text{------ (1)}$$

$$\left(\frac{\partial u}{\partial p}\right)_v = \left(\frac{\partial u}{\partial T}\right)_V \left(\frac{\partial T}{\partial p}\right)_V$$

$$= C_v \frac{K_T}{\beta} \qquad \text{------ (2)}$$

Also, u = h – pv

$$\left(\frac{\partial u}{\partial v}\right)_p = \left(\frac{\partial h}{\partial v}\right)_p - p$$

$$= \left(\frac{\partial h}{\partial T}\right)_p \left(\frac{\partial T}{\partial v}\right)_p - p$$

$$= C_p \frac{1}{v\beta} - p \qquad \text{------ (3)}$$

Substituting (2) and (3) in (1),

$$du = C_v \frac{K_T}{\beta} dp + \left[\frac{C_p}{v\beta} - p\right] dv \qquad \text{------ (4)}$$

We know that T ds = du + p dv

$$T\,ds = C_v \frac{K_T}{\beta} dp + \left[\frac{C_p}{v\beta} - p\right] dv + p\,dv$$

$$= C_v \frac{K_T}{\beta} dp + \frac{C_p}{v\beta} dv \qquad \text{------ (5)}$$

$$T\,ds = C_V \frac{K_T}{\beta} dp + C_p \frac{1}{\beta v} dv$$ → **Third T ds equation**

**Change in internal energy**

du = T ds – p dv

$$T\,ds = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dv$$ → First T ds equation

$$du = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dv - p\,dv$$

$$du = C_V dT + \left[T\left(\frac{\partial p}{\partial T}\right)_V - p\right]dv$$

**Internal energy of an ideal gas is function of temperature only**

$$du = C_V dT + \left[T\left(\frac{\partial p}{\partial T}\right)_V - p\right]dv$$

For an ideal gas, p v = R T

$$p = \frac{RT}{v}$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v} = \frac{p}{T}$$

$$du = C_V dT + \left[T\left(\frac{p}{T}\right)_V - p\right]dV = C_v dT$$

Therefore the internal energy is depending only on temperature.

**Change in internal energy when a gas obeys Van der Waals equation**

We know that, $du = C_V dT + \left[T\left(\frac{\partial p}{\partial T}\right)_V - p\right]dv$

Van der Waals equation is, $\left(p + \frac{a}{v^2}\right)(v - b) = RT$

$$p = \frac{RT}{v - b} - \frac{a}{v^2}$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v - b}$$

Therefore, $$du = C_V\,dT + \left[T\left(\frac{R}{v-b}\right) - \frac{RT}{v-b} + \frac{a}{v^2}\right]dv$$

$$du = C_V\,dT + \left[\frac{a}{v^2}\right]dv$$

$$\int_1^2 du = C_V\int_1^2 dT + a\int_1^2 \frac{dv}{v^2}$$

$$u_2 - u_1 = C_V\,(T_2 - T_1) + a\left[\frac{1}{v_1} - \frac{1}{v_2}\right]$$

**Change in entropy when a gas obeys Van der Waals equation**

$$T\,ds = C_V\,dT + T\left(\frac{\partial p}{\partial T}\right)_V dv$$ → First T ds equation

$$ds = C_V\,\frac{dT}{T} + \left(\frac{\partial p}{\partial T}\right)_V dv$$

Van der Waals equation is, $\left(p + \frac{a}{v^2}\right)(v-b) = RT$

$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

Therefore, $$ds = C_V\,\frac{dT}{T} + \left(\frac{R}{v-b}\right) dv$$

$$\int_1^2 ds = C_V\int_1^2 \frac{dT}{T} + R\int_1^2 \frac{dv}{v-b}$$

$$s_2 - s_1 = C_v\,\ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2 - b}{v_1 - b}\right)$$

**Change in enthalpy**

dh = T ds + v dp

$$T\,ds = C_p\,dT - T\left(\frac{\partial v}{\partial T}\right)_p dp$$ → Second T ds equation

$$dh = C_p\,dT - T\left(\frac{\partial v}{\partial T}\right)_p dp + v\,dp$$

$$dh = C_p\,dT - \left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]dp$$

$$dh = C_p dT + \left[ v - T\left(\frac{\partial v}{\partial T}\right)_p \right] dp$$

**Enthalpy of an ideal gas is function of temperature only**

$$dh = C_p dT + \left[ v - T\left(\frac{\partial v}{\partial T}\right)_p \right] dp$$

For an ideal gas, $\quad p\,v = R\,T$

$$v = \frac{RT}{p}$$

$$\left(\frac{\partial v}{\partial T}\right)_v = \frac{R}{p} = \frac{v}{T}$$

$$dh = C_p dT + \left[ v - T\left(\frac{v}{T}\right) \right] dp = C_p dT$$

Therefore the enthalpy is depending only on temperature.

**Difference in heat capacities ($C_p - C_v$)**

To derive $C_p - C_v = -T\left(\frac{\partial v}{\partial T}\right)_p^2 \left(\frac{\partial p}{\partial v}\right)_T$ or $(C_p - C_v) = R$ or $C_p - C_v = \frac{T\,v\,\beta^2}{K_T}$

Note: Derive First and Second T ds equations and proceed as follows:

$$T\,ds = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dv \rightarrow \text{First T ds equation}$$

$$T\,ds = C_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp \rightarrow \text{Second T ds equation}$$

Equating the above equations,

$$C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dv = C_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp$$

$$\left(C_p - C_v\right) dT = T\left(\frac{\partial p}{\partial T}\right)_v dv + T\left(\frac{\partial v}{\partial T}\right)_p dp$$

$$dT = \left(\frac{T}{C_p - C_v}\right)\left(\frac{\partial p}{\partial T}\right)_v dv + \left(\frac{T}{C_p - C_v}\right)\left(\frac{\partial v}{\partial T}\right)_p dp \quad \text{----- (1)}$$

We can write, T = f (p, v)

$$dT = \left(\frac{\partial T}{\partial p}\right)_V dp + \left(\frac{\partial T}{\partial v}\right)_p dv \quad \text{----- (2)}$$

Comparing (1) and (2),

$$\left(\frac{\partial T}{\partial v}\right)_p = \frac{T}{C_p - C_v}\left(\frac{\partial p}{\partial T}\right)_v \qquad \text{----- (3)}$$

From (3), $C_p - C_v = T\left(\frac{\partial p}{\partial T}\right)_v\left(\frac{\partial v}{\partial T}\right)_p$ ----- (4)

But, $\left(\frac{\partial p}{\partial T}\right)_v\left(\frac{\partial T}{\partial v}\right)_p\left(\frac{\partial v}{\partial p}\right)_T = -1$

$$\left(\frac{\partial p}{\partial T}\right)_v = -\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial v}\right)_T \qquad \text{----- (5)}$$

Substituting (5) in (4),

$$C_p - C_v = -T\left(\frac{\partial v}{\partial T}\right)_p^2\left(\frac{\partial p}{\partial v}\right)_T \qquad \text{----- (6)}$$

p v = R T

$$v = \frac{RT}{p}$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} \qquad \text{----- (7)}$$

$$p = \frac{RT}{v}$$

$$\left(\frac{\partial p}{\partial v}\right)_T = \frac{-RT}{v^2} \qquad \text{----- (8)}$$

Substituting (7) and (8) in (6),

$$C_p - C_v = -T\frac{R^2}{p^2}\left(\frac{-RT}{v^2}\right) = \frac{R^3T^2}{p^2v^2}$$

p v = R T

$$C_p - C_v = \frac{R^3T^2}{R^2T^2} = R$$

$\mathbf{C_p - C_v = R}$ **----- (9)**

We know that, $\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p \rightarrow \left(\frac{\partial v}{\partial T}\right)_p = \beta v$

$$K_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T \rightarrow \left(\frac{\partial v}{\partial p}\right)_T = -vK_T$$

Therefore equation (6) can be written as,

$$C_p - C_v = -T\beta^2v^2\,\frac{-1}{vK_T}$$

$$C_p - C_v = \frac{T\,v\,\beta^2}{K_T} \qquad \text{----- (10)}$$

**Ratio of heat capacities**

Note: Derive First and Second T ds equations and proceed as follows:

$$T\,ds = C_V\,dT + T\left(\frac{\partial p}{\partial T}\right)_V dv \rightarrow \text{First T ds equation}$$

$$T\,ds = C_p\,dT - T\left(\frac{\partial v}{\partial T}\right)_p dp \rightarrow \text{Second T ds equation}$$

Consider isentropic process, ds = 0

$$C_p\,dT = T\left(\frac{\partial v}{\partial T}\right)_p dp$$

$$C_p = T\left(\frac{\partial v}{\partial T}\right)_p \frac{dp}{dT}$$

$$C_V\,dT = -T\left(\frac{\partial p}{\partial T}\right)_V dv$$

$$C_V = -T\left(\frac{\partial p}{\partial T}\right)_V \frac{dv}{dT}$$

$$\frac{C_p}{C_v} = -\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v \frac{dp}{dT}\frac{dT}{dv}$$

$$\frac{C_p}{C_v} = -\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v \frac{dp}{dv}$$

But, $\dfrac{dp}{dv} = \left(\dfrac{\partial p}{\partial v}\right)_s$ for isentropic process

Therefore, $\dfrac{C_p}{C_v} = -\left(\dfrac{\partial v}{\partial T}\right)_p \left(\dfrac{\partial T}{\partial p}\right)_v \left(\dfrac{\partial p}{\partial v}\right)_s$ ----- (1)

We can write, $\left(\dfrac{\partial p}{\partial T}\right)_v \left(\dfrac{\partial T}{\partial v}\right)_p \left(\dfrac{\partial v}{\partial p}\right)_T = -1$

$$\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial T}{\partial v}\right)_p = -\left(\frac{\partial v}{\partial p}\right)_T$$

Substituting in (1), $\dfrac{C_p}{C_v} = \left(\dfrac{\partial v}{\partial p}\right)_T \left(\dfrac{\partial p}{\partial v}\right)_s$ ----- (2)

$$K_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T \rightarrow \left(\frac{\partial v}{\partial p}\right)_T = -v\,K_T$$

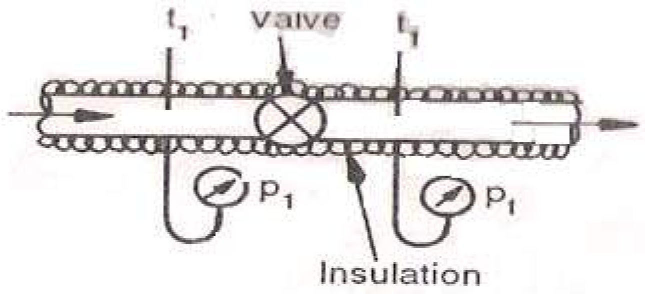
$$K_S = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_S \rightarrow \left(\frac{\partial v}{\partial p}\right)_S = -v\,K_S$$

Substituting in (2), $\frac{C_p}{C_v} = \frac{K_T}{K_S}$ ----- (3)

**Joule-Thomson coefficient**

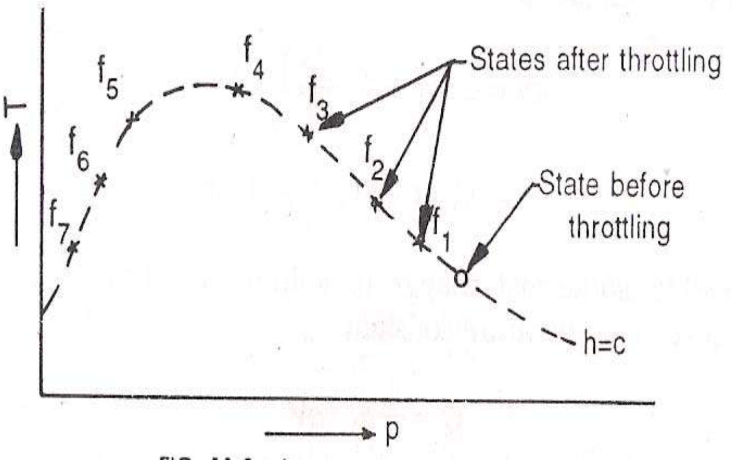
A gas is made to undergo continuous throttling process by a valve, as shown. The pressures and temperatures of the gas in the insulated pipe upstream and downstream of the valve are measured with suitable manometers and thermometers.

$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h$$

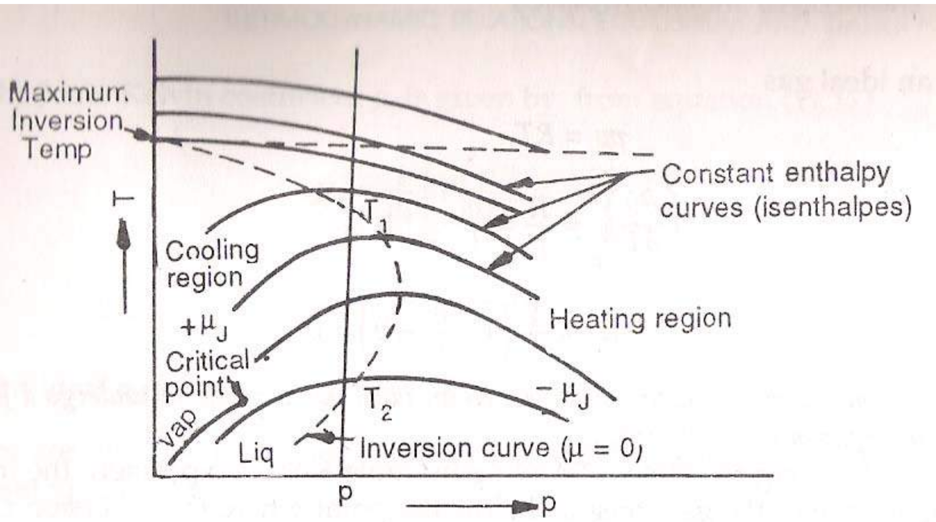


Joule-Thomson expansion

Let $p_1$ and $T_1$ be the arbitrarily chosen pressure and temperature before throttling and let them be kept constant. By operating the valve manually the gas is throttled successively to different pressures and temperatures. These are then plotted on the T-p coordinates. The curve passing through all these points is an isenthalpic curve.



Isenthalpic state of a gas



Isenthalpic curves and the inversion curve

The initial pressure and temperature of the gas (before throttling) are then set to new values, and by throttling to different states, a family of isenthalpes is obtained for the gas. The curve passing through the maxima of these isenthalpes is called the inversion curve.

The numerical value of the slope of an isenthalpe on a T-p diagram at any point is called the 'Joule-Thomson coefficient' and is denoted by $\mu_J$.

At inversion curve, $\mu_J = 0$

At right side of the inversion curve, dT is positive, dp is negative.

$\mu_J = -$ ve (Heating region)

At left side of the inversion curve, dT is negative, dp is negative.

$\mu_J = +$ ve (Cooling region)

$$dh = T\,ds + v\,dp \quad \text{----- (1)}$$

Second T ds equation is given by,

$$T\,ds = C_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp$$

Substituting in (1),

$$dh = C_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp + v\,dp$$

$$= C_p dT - \left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] dp$$

The second term in the above equation stands only for a real gas, because for an ideal gas,

$$dh = C_p\,dT$$

dh = 0 for isenthalpic process

Therefore, $$C_p dT - \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right] dp = 0$$

$$C_p dT = \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right] dp$$

$$\frac{dT}{dp} = \frac{1}{C_p} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right]$$

$$\left( \frac{\partial T}{\partial p} \right)_h = \frac{1}{C_p} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right]$$

Therefore, $$\mu_J = \left( \frac{\partial T}{\partial p} \right)_h = \frac{1}{C_p} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right]$$

<u>For an ideal gas</u>

$$p\,v = R\,T$$

$$v = \frac{RT}{v}$$

$$\left( \frac{\partial v}{\partial T} \right)_p = \frac{R}{p} = \frac{v}{T}$$

$$\mu_J = \left( \frac{\partial T}{\partial p} \right)_h = \frac{1}{C_p} \left[ T \left( \frac{v}{T} \right) - v \right] = 0$$

There is no change in temperature when an ideal gas is made to undergo a throttling.

<u>Another form</u>

$$\mu_J = \left( \frac{\partial T}{\partial p} \right)_h = \frac{1}{C_p} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right]$$

But, $\beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$

$$\mu_J = \frac{1}{C_p} \left[ T \beta v - v \right] = \frac{v}{C_p} \left[ T \beta - 1 \right]$$

$\beta = \frac{1}{T}$ for an ideal gas since $\mu_J = 0$

**Clausius-Clapeyron equation**

Clausius-Clapeyron equation is a relationship between the saturation pressure, temperature, the enthalpy of evaporation and the specific volume of the two phases involved. This equation provides a basis for calculations of properties in a two phase region. It gives the slope of a curve separating the two phases in the p-T diagram.

Let us consider then change of state from saturated liquid to saturated vapour of a pure substance which takes place at constant temperature. During the evaporation, the pressure and temperature are independent of volume.

III-Maxwell equation, $\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$

$$\left(\frac{dp}{dT}\right) = \left(\frac{ds}{dv}\right) = \frac{s_g - s_f}{v_g - v_f} = \frac{s_{fg}}{v_{fg}}$$

$s_g$ = specific entropy of saturated vapour

$s_f$ = specific entropy of saturated liquid

$v_g$ = specific volume of saturated vapour

$v_f$ = specific volume of saturated liquid

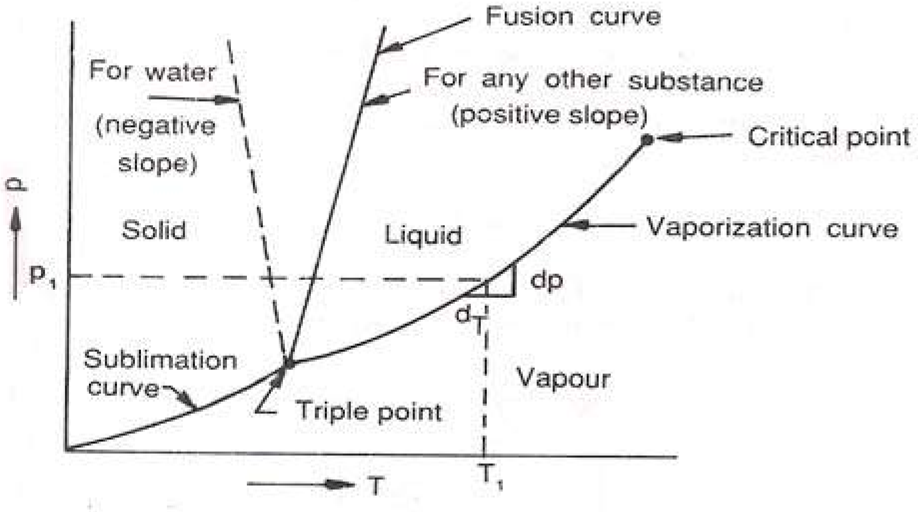
$h_{fg}$ = increase in specific entropy

$v_{fg}$ = increase in specific volume

Also, $$s_g - s_f = \frac{h_{fg}}{T}$$

$$\left(\frac{dp}{dT}\right) = \frac{s_g - s_f}{v_g - v_f} = \frac{h_{fg}}{v_{fg}} = \frac{h_{fg}}{T\,v_{fg}}$$

This equation is known as Clausius-Clapeyron equation for evaporation of liquids. The derivative dp/dT is the slope of vapour pressure versus temperature curve. Knowing this slope and the specific volume $v_g$ and $v_f$ from experimental data, we can determine the enthalpy of evaporation, $h_{fg}$.



# UNIT-V

## GAS MIXTURES AND PSYCHROMETRY

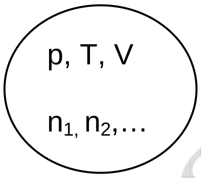
### Gas Mixtures

A pure substance is defined as a substance having a constant and uniform chemical composition. A homogeneous mixture of gases which do not react with one another is considered to a pure substance. Air is a homogeneous mixture of nitrogen, oxygen, and traces of other substances like, argon, helium, carbon dioxide, etc, and as they do not react with one another, air is regarded a pure substance. The properties of the mixture are determined from the properties of the constituent gases.

### Dalton's law of partial pressures

Let us consider a homogeneous mixture of ideal gases at a temperature T, a pressure p and a volume V.

Suppose there are $n_1$ moles of gas $A_1$, $n_2$ moles of gas $A_2$, ……… $n_c$ moles of gas $A_c$.



$$pV = (n_1 + n_2 + \ldots\ldots\ldots + n_c) R_u T$$

$$p = \frac{n_1 R_u T}{V} + \frac{n_2 R_u T}{V} + \ldots\ldots\ldots\ldots + \frac{n_c R_u T}{V}$$

$$= p_1 + p_2 + \ldots\ldots\ldots\ldots\ldots + p_c$$

$p_1$, $p_2$, ………. $p_3$ are partial pressure of individual components

p = Total pressure

$R_u$ = Universal gas constant = 8314 J/kmol-K

Dalton's law of partial pressures states that the total pressure of a mixture of ideal gases is equal to the sum of the partial pressures.

$$p = p_1 + p_2 + \ldots\ldots\ldots + p_c$$

### Properties of gas mixture

$$\sum n = n_1 + n_2 + \ldots\ldots\ldots\ldots + n_c$$

$x_1 = \frac{n_1}{\sum n}$ → Mole fraction or volume fraction of component-1

$x_2 = \frac{n_2}{\sum n}$ → Mole fraction or volume fraction of component-2, ……

Partial pressure of component-1, $p_1 = \frac{m_1 R_1 T}{V} = \frac{n_1 R_u T}{V}$

Partial volume of component-1, $V_1 = \frac{m_1 R_1 T}{p} = \frac{n_1 R_u T}{p}$

Mass of component-1, $m_1 = M_1 n_1 = m_{f1} m$

Gas constant of component-1, $R = R_u/M_1$

$R_1$ = Characteristic gas constant (J/kg-K)

$M_1$ = Molar mass (Molecular weight) of component-1

$n_1$ = Number of moles of component-1

$m_{f1}$ = Mass fraction of component-1

Mass of gas mixture, $m = m_1 + m_2 + \ldots\ldots\ldots\ldots + m_c$

Volume of gas mixture, $V = V_1 + V_2 + \ldots\ldots\ldots\ldots + V_c$

Internal energy of mixture,

$$u = \frac{m_1u_1 + m_2u_2 + \ldots\ldots\ldots\ldots + m_cu_c}{m_1 + m_2 + \ldots\ldots\ldots + m_c}$$

Enthalpy of mixture,

$$h = \frac{m_1h_1 + m_2h_2 + \ldots\ldots\ldots\ldots + m_ch_c}{m_1 + m_2 + \ldots\ldots\ldots + m_c}$$

Specific heat of mixture,

$$C_p = \frac{m_1C_{p1} + m_2C_{p2} + \ldots\ldots\ldots\ldots + m_cC_{pc}}{m_1 + m_2 + \ldots\ldots\ldots + m_c}$$

$$C_v = \frac{m_1C_{v1} + m_2C_{v2} + \ldots\ldots\ldots\ldots + m_cC_{vc}}{m_1 + m_2 + \ldots\ldots\ldots + m_c}$$

Molar mass of mixture,

$$M = \frac{n_1M_1 + n_2M_2 + \ldots\ldots\ldots\ldots + n_cM_c}{n_1 + n_2 + \ldots\ldots\ldots + n_c}$$

$$= x_xM_1 + x_2M_2 + \ldots\ldots\ldots\ldots + x_cM_c$$

Gas constant of mixture,

$$R = \frac{m_1R_1 + m_2R_2 + \ldots\ldots\ldots\ldots + m_cR_c}{m_1 + m_2 + \ldots\ldots\ldots + m_c} = \frac{R_u}{M}$$

Density of mixture,

$$\rho = \frac{p}{RT}$$

Specific heat ratio of mixture, $\gamma = x_1\gamma_1 + x_2\gamma_2 + \ldots\ldots\ldots\ldots + x_c\gamma_c$

Note: Molecular weight of the mixture is also called as apparent molecular weight.

## PROBLEMS

1. **Air contains 21% $O_2$ and 79% $N_2$ by volume. Determine the molecular weight, gas constant and density of air at STP.**

   *Given*

   Mole fraction of $O_2$ ($x_{O2}$) = 0.21

   Mole fraction of $N_2$ ($x_{N2}$) = 0.79

   Temperature of mixture (T) = 273 K (Std temperature)

   Pressure of mixture (p) = 1 atm = 1.01325 bar (Std pressure)

   *Required*: M, R, ρ

   *Solution*

Molecular weight, $M = x_{CO2} M_{CO2} + x_{N2} M_{N2}$

$$M = 0.21(32) + 0.79(28) = 28.84 \text{ ----- } \textbf{Ans}$$

Gas constant, $R = \dfrac{m_{O2} R_{O2} + m_{N2} R_{N2}}{m_{O2} + m_{N2}} = \dfrac{R_u}{M}$

= 8314 / 28.84

= 288.3 J/kg-K ----- **Ans**

Density of mixture, $\rho = \dfrac{p}{RT} = \dfrac{1.01325 x 10^5}{288.3 x 273} = 1..287\ kg/m^3$ ----- **Ans**

2. **0,45 kg of CO and 1 kg of air is contained in a vessel of volume 0.4 $m^3$ at 15$^o$C. Air has 23.3% of $O_2$ and 76.7% of $N_2$ by mass. Calculate the partial pressure of each constituent and total pressure in the vessel. Molar masses of CO, $O_2$ and $N_2$ are 28, 32 and 28 kg/kmol.**

*Given:*

Mass of CO ($m_{CO}$) = 0.45 kg
Mass of air ($m_a$) = 1 kg
Volume of mixture (V) = 0.4 $m^3$
Temperature of mixture (T) = 15$^o$C = 288 K
Mass fraction of $O_2$ ($m_{fO2}$) = 0.233
Mass fraction of $N_2$ ($m_{fN2}$) = 0.767
Molar mass of CO ($M_{CO}$) = 28
Molar mass of $O_2$ ($M_{O2}$) = 32
Molar mass of $N_2$ ($M_{N2}$) = 28

*Required*: $p_{CO}$, $p_{O2}$, $p_{N2}$ and p

*Solution:*

Mass of $O_2$ ($m_{O2}$) = $m_{fO2}$ $m_a$ = 0.233 x 1 = 0.233 kg
Mass of $N_2$ ($m_{N2}$) = $m_{fN2}$ $m_a$ = 0.767 x 1 = 0.767 kg
Mass of mixture (m) = $m_{CO}$ + $m_{O2}$ + $m_{N2}$
= 0.45 + 0.233 + 0.767 = 1.45 kg
Gas constant of CO ($R_{CO}$) = $R_u$ / $M_{CO}$ = 8324 / 28 = 296.93 J/kg-K
Gas constant of $O_2$ ($R_{O2}$) = $R_u$ / $M_{O2}$ = 8314 / 32 = 259.81 J/kg-K
Gas constant of $N_2$ ($R_{N2}$) = $R_u$ / $M_{N2}$ = 8314 / 28 = 296.93 J/kg-K

Partial pressure of CO, $p_{CO} = \dfrac{m_{CO} R_{CO} T}{V} = \dfrac{0.45 x 296.93 x 288}{0.4}$

= 96205.3 Pa ---- **Ans**

Partial pressure of $O_2$, $p_{O2} = \dfrac{m_{O2} R_{O2} T}{V} = \dfrac{0.233 x 259.81 x 288}{0..4}$

= 43585.7 Pa ---- **Ans**

Partial pressure of $N_2$, $p_{N2} = \dfrac{m_{N2} R_{N2} T}{V} = \dfrac{0.767\,x\,296.93\,x\,288}{0.4}$

$= 163976.6$ Pa ---- **Ans**

Total pressure, $p = p_{CO} + p_{O2} + p_{N2}$

$= 96205.3 + 43585.7 + 163976.6$

$= 303767.6$ Pa $= 3.04$ bar ----- **Ans**

**3. A vessel of 0.35 $m^3$ capacity contains 0.4 kg of carbon monoxide (Molecular weight = 28) and 1 kg of air at $20^oC$. Calculate the partial pressure of each component and the total pressure in the vessel. The gravimetric analysis of air is to be taken as 23.3% $O_2$ and 76.7% $N_2$.**

*Given:*

Mass of CO ($m_{CO}$) = 0.4 kg

Mass of air ($m_a$) = 1 kg

Volume of mixture (V) = 0.35 $m^3$

Temperature of mixture (T) = $20^oC$ = 293 K

Mass fraction of $O_2$ ($m_{fO2}$) = 0.233

Mass fraction of $N_2$ ($m_{fN2}$) = 0.767

Molar mass of CO ($M_{CO}$) = 28

*Required***:** $p_{CO}$, $p_{O2}$, $p_{N2}$ and p

*Solution:*

Mass of $O_2$ ($m_{O2}$) = $m_{fO2}\, m_a$ = 0.233 x 1 = 0.233 kg

Mass of $N_2$ ($m_{N2}$) = $m_{fN2}\, m_a$ = 0.767 x 1 = 0.767 kg

Mass of mixture (m) = $m_{CO} + m_{O2} + m_{N2}$

= 0.4 + 0.233 + 0.767 = 1.4 kg

Gas constant of CO ($R_{CO}$) = $R_u$ / $M_{CO}$ = 8324 / 28 = 296.93 J/kg-K

Gas constant of $O_2$ ($R_{O2}$) = $R_u$ / $M_{O2}$ = 8314 / 32 = 259.81 J/kg-K

Gas constant of $N_2$ ($R_{N2}$) = $R_u$ / $M_{N2}$ = 8314 / 28 = 296.93 J/kg-K

Partial pressure of CO, $p_{CO} = \dfrac{m_{CO} R_{CO} T}{V} = \dfrac{0.4\,x\,296.93\,x\,293}{0.35}$

$= 99429.1$ Pa ---- **Ans**

Partial pressure of $O_2$, $p_{O2} = \dfrac{m_{O2} R_{O2} T}{V} = \dfrac{0.233\,x\,259.81\,x\,293}{0..35}$

$= 50677$ Pa ---- **Ans**

Partial pressure of $N_2$, $p_{N2} = \dfrac{m_{N2} R_{N2} T}{V} = \dfrac{0.767\,x\,296.93\,x\,293}{0.35}$

$= 190655.6$ Pa ---- **Ans**

Total pressure, $p = p_{CO} + p_{O2} + p_{N2}$

$= 99429.1 + 50677 + 190655.6$

= 340761.7 Pa = 3.407 bar ----- **Ans**

**4. A mixture of ideal gases consists of 3 kg of nitrogen and 5 kg of carbon dioxide at a pressure of 300 kPa and a temperature of 20°C. Find (i) mole fraction of each constituent, (ii) the equivalent molecular weight of the mixture, (iii) the equivalent gas constant of the mixture, (iv) the partial pressures and partial volumes, and (v) the volume and density of the mixture.**

*Given:*

Mass of nitrogen ($m_{N2}$) = 3 kg

Mass of Carbon dioxide ($m_{CO2}$) = 5 kg

Mixture pressure (p) = 300 kPa

Mixture temperature (T) = 20°C = 293 K

*Required:* (i) $x_{N2}$, $x_{CO2}$ (ii) M (iii) R (iv) $p_{N2}$, $p_{CO2}$, $V_{N2}$, $V_{CO2}$ (v) V, ρ

*Solution:*

(i) Number of moles of $N_2$, $n_{N2} = \dfrac{m_{N2}}{M_{N2}} = \dfrac{3}{28} = 0.1071$

Number of moles of $CO_2$, $n_{CO2} = \dfrac{m_{CO2}}{M_{CO2}} = \dfrac{5}{44} = 0.1136$

Mole fraction of $N_2$ $x_{N2} = \dfrac{n_{N2}}{n_{N2} + n_{CO2}} = \dfrac{0.1071}{0.1071 + 0.1136} = 0.4853$ ----- **Ans**

Mole fraction of $N_2$ $x_{CO2} = \dfrac{n_{CO2}}{n_{N2} + n_{CO2}} = \dfrac{0.1136}{0.1071 + 0.1136} = 0.5147$ ----- **Ans**

(ii) Equivalent molecular weight, $M = x_{CO2} M_{CO2} + x_{N2} M_{N2}$

$= 0.5147\,(44) + 0.4853\,(28) = 36.23$ ----- **Ans**

(iii) Equivalent gas constant, $R = \dfrac{m_{CO2} R_{CO2} + m_{N2} R_{N2}}{m_{CO2} + m_{N2}} = \dfrac{R_u}{M}$

= 8314 / 36.23

= 229.48 J/kg-K ----- **Ans**

(iv) Partial pressure of $N_2$, $p_{N2} = x_{N2}\, p = 0.4853\ (300) = 145.59$ kPa ----- **Ans**

Partial pressure of $CO_2$, $p_{CO2} = x_{CO2}\, p = 0.5147\ (300) = 154.41$ kPa ----- **Ans**

Partial volume of $N_2$, $V_{N2} = \dfrac{m_{N2} R_{N2} T}{p}$

$R_{N2} = R_u / M_{N2} = 8314 / 28 = 296.93$ J/kg-K

$V_{N2} = \dfrac{3\,(296.93)\,(293)}{300\, x 10^3} = 0.87\, m^3$ ----- **Ans**

Partial volume of $CO_2$, $V_{CO2} = \dfrac{m_{CO2} R_{CO2} T}{p}$

$R_{CO2} = R_u / M_{CO2} = 8314 / 44 = 188.95$ J/kg-K

$$V_{CO2} = \frac{5(188.95)(293)}{300\,x10^3} = 0.923\,m^3$$ ----- **Ans**

(v) Total volume, $V = V_{N2} + V_{CO2} = 0.87 + 0.923 = 1.793\ m^3$ ----- **Ans**

Density of the mixture, $\rho = \frac{p}{RT} = \frac{300\,x10^3}{229.48\,x\,293} = 4.4618\ kg/m^3$ ----- **Ans**

## PSYCHROMETRY

Atmospheric air always contains water vapour. The water vapour content in air plays an important role in comfort air conditioning. The partial pressure of water vapour in atmospheric air is very low and the vapour exists either in superheated or saturated.

The science which deals with the study of the behaviour of air and water vapour mixture is known as Psychrometry. The properties of air and water vapour mixture are known as psychrometric properties.

**Dry air** : Air with no water vapour is called 'dry air'.

$O_2 = 21\ \%$ ; $N_2 = 78.1\ \%$ ; $CO_2 = 0.03\ \%$

Since dry air is never found, it always may contain some water vapour.

**Moisture :** The water vapour present in the air is known as moisture.

**Moist air :** It is a mixture of dry air and water vapour.

**Unsaturated and Saturated air :** The moist air in which the water vapour exists in superheated state, is known as Unsaturated air and such air will be invisible

If the water vapour is added to dry air or unsaturated air, a limit will be reached when the air will be saturated and can hold no more water vapour. Such air will also be invisible and the air is called 'saturated air'. But if more water is added, the drops of water may remain in suspension and make air misty or foggy. Thus such drops are the condensed particles of water vapour. This will happen only beyond saturation limit.

According to the Dalton's law of partial pressures,

$$p_b = p_a + p_v$$

$p_b$ = Barometric pressure

$p_a$ = Partial pressure of dry air

$p_v$ = Partial pressure of water vapour

**Dry Bulb Temperature (DBT) :** The temperature of the air measured by ordinary thermometer whose bulb is dry, is known as DBT of the air.

**Wet Bulb Temperature (WBT) :** The temperature of the air measured by a thermometer when its bulb is covered with wet cloth and is exposed to a current of air is known as WBT of air.

**Wet bulb depression** = DBT – WBT

**Dew Point Temperature (DPT) :** It is the temperature of air at which water vapour in the air starts condensing when the is cooled. Thus this temperature will corresponds to the saturation temperature at partial pressure of water vapour. DPT can be obtained from steam table at $p_v$.

**Dew point depression =** DBT – DPT

Note:

For saturated air, DBT = WBT = DPT

### Specific humidity (W)

It is the mass of water vapour present with one kg of dry air.

$$W = 0.622 \frac{p_v}{p_b - p_v} \text{ kg / kg dry air}$$

Also, $$W = \frac{C_{pa}(T_w - T_d) + W_w\, h_{fgW}}{h_{gd} - h_{fw}} = \frac{\rho_v}{\rho_a} = \frac{m_v}{m_a}$$

$$W_w = 0.622 \frac{p_{sw}}{p_b - p_{sw}}$$

$p_{sw}$ = Saturation pressure at WBT

$h_{fgw}$ = $h_{fg}$ at WBT in kJ/kg

$h_{gd}$ = $h_g$ at DBT in kJ/kg

$h_{fw}$ = $h_f$ at WBT in kJ/kg

$C_{pa}$ = Specific heat of air = 1.005 kJ/kg-K

$\rho_v$ = Density of water vapour in the mixture

$\rho_a$ = Density of dry air in the mixture

$m_v$ = Mass of water vapour

$m_a$ = Mass of dry air

### Density of dry air and water vapour

Density of dry air can be calculated as,

$\rho_a = p_a / (R_a T_d)$

$T_d$ = Dry bulb temperature in K

$R_a$ = 287 J/kg-K

$p_a$ = Partial pressure of dry air in Pa

Density of water vapour can be calculated from,

$\rho_v = W \rho_a$

### Mass of dry air and water vapour

Mass of dry air ($m_a$) is calculated from,

$$p_a V = m_a R_a T_d$$

Mass of water vapour ($m_v$) is calculated from,

$$p_v V = m_v R_v T_d$$

If gas constant ($R_v$) for water vapour is not available, $m_v$ can be calculated from,

$$m_v = W m_a$$

### Specific humidity of saturated air ($W_s$)

It is the mass of water vapour present with 1 kg of dry air when the air is saturated.

$$W_s = 0.622 \frac{p_s}{p_b - p_s} \text{ kg / kg dry air}$$

$p_s \rightarrow$ Saturation pressure at DBT

### Degree of saturation (or) Saturation ratio (μ)

$$\mu = \frac{W}{W_s} = \frac{p_v(p_b - p_s)}{p_s(p_b - p_v)}$$

If $\Phi = 0$, $p_v = 0$, $W = 0$, $\mu = 0$

If $\Phi = 100\%$, $p_v = p_s$, $W = W_s$, $\mu = 1$

Therefore μ varies between 0 and 1.

### Relative humidity (ϕ)

It is the ratio of actual mass of water vapour in a given volume of moist air to the mass of water vapour in the same volume of saturated air at the same temperature & pressure.

$$\phi = \frac{p_v}{p_s} = \frac{\mu}{1-(1-\mu)\dfrac{p_s}{p_b}}$$

$p_v$ can be calculated from, Carrier's equation,

$$p_v = p_{sw} - \frac{(p_b - p_{sw})(T_d - T_w)}{1527.4 - 1.3T_w}$$

Note: $T_w \rightarrow$ WBT in °C

$p_{sw}$ = Saturation pressure at WBT $\quad$ $p_b$ = Barometric pressure

$p_v$ also can be determined from, $W = 0.622\dfrac{p_v}{p_b - p_v}$ knowing W.

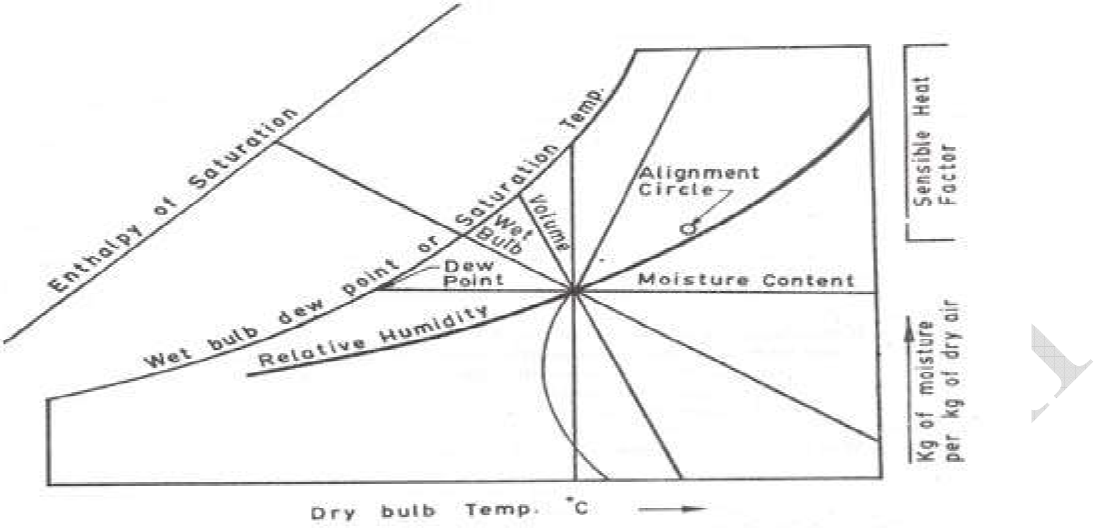
### Enthalpy of moist air

$$h = C_{pa}T_d + W[h_{gd} + 1.88(T_d - T_{dp})]$$

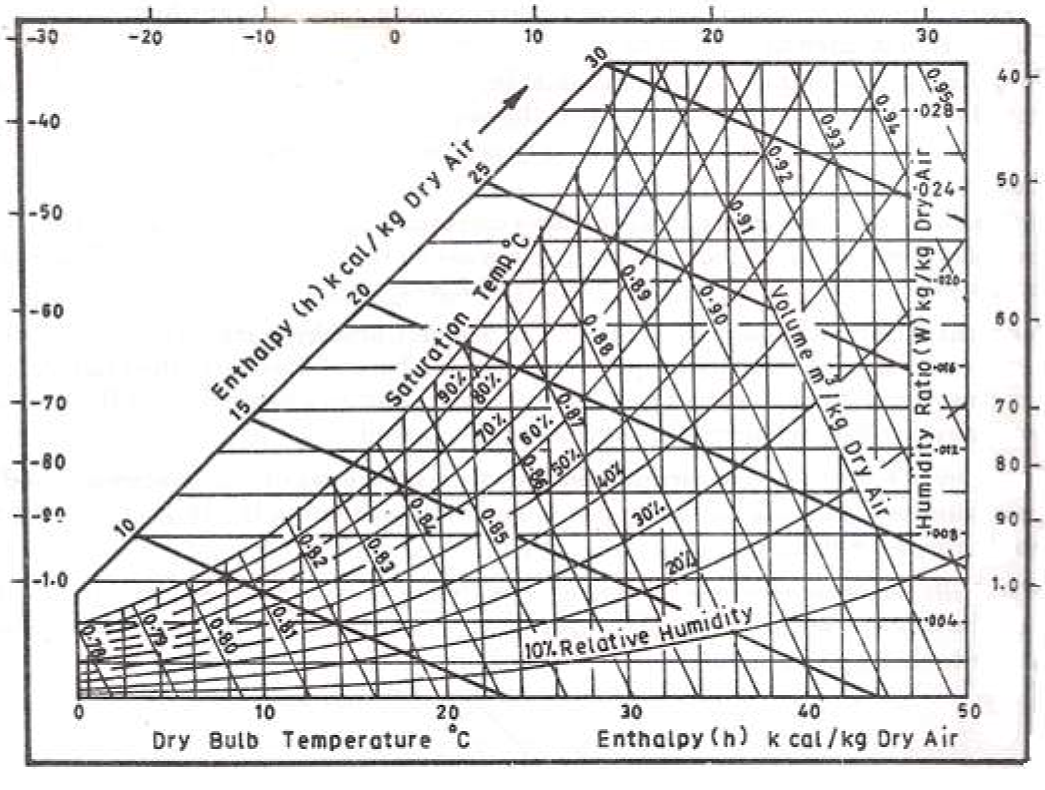
$T_d$ = DBT in °C $\quad$ $C_{pa}$ = 1.005 kJ/kg-K $\quad$ W = Specific humidity in kg/kg dry air

$T_{dp}$ = DPT in °C

### Psychrometric Chart

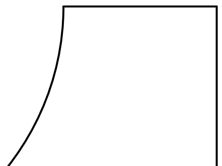


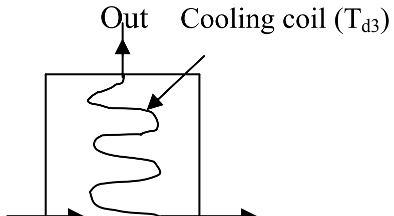
DBT - lines ➔ Uniformly spaced vertical lines.
W - lines ➔ Uniformly spaced horizontal lines.
DPT – lines ➔ Non-uniformly spaced horizontal lines.
WBT – lines ➔ Non-uniformly spaced inclined lines.
$\phi$ - lines ➔ Curved lines.



## Psychrometric processes

### Sensible cooling





Air out

Air in ($T_{d1}$) ($T_{d2}$)

3 2 1 $W_1 = W_2$

Refrigerant in

$T_{d3}$ $T_{d2}$ $T_{d1}$

The cooling of air without change in its specific humidity is known as ‘Sensible cooling’.

$$By\ pass\ factor\,(BPF) = \frac{T_{d2} - T_{d3}}{T_{d1} - T_{d3}}$$

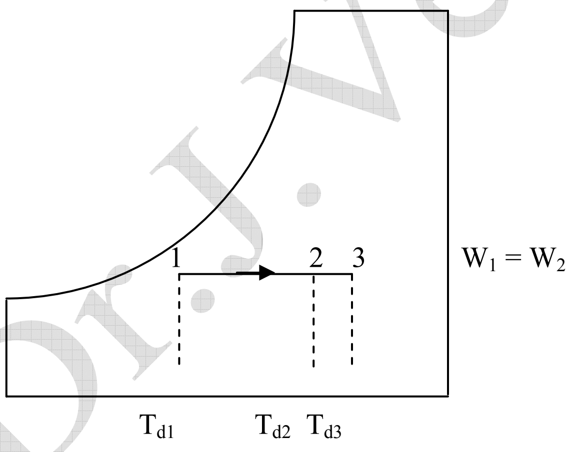
$T_{d3}$ = Coil temperature

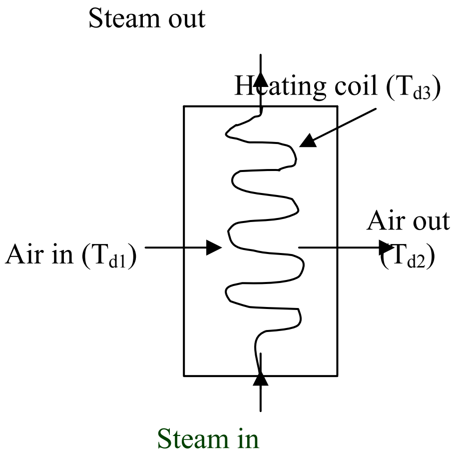
Coil efficiency ($\eta_c$) = 1 – BPF

## Sensible heating

The heating of air without change in its specific humidity is known as ‘Sensible heating’.

$$By\ pass\ factor\,(BPF) = \frac{T_{d3} - T_{d2}}{T_{d3} - T_{d1}}$$



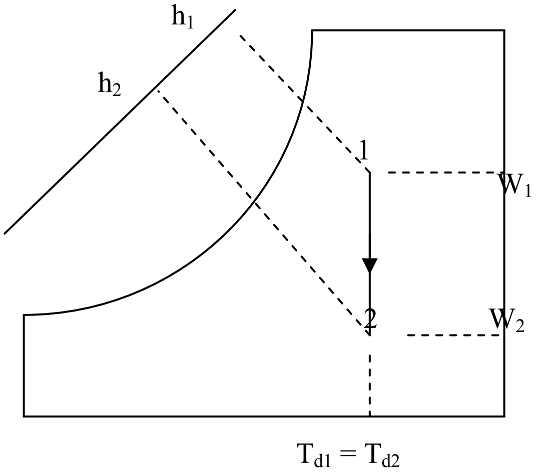


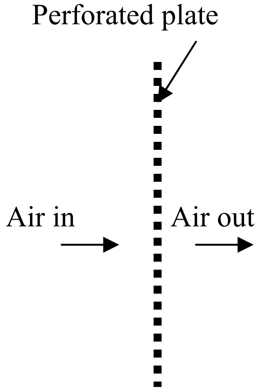
$T_{d3}$ = Coil temperature

Coil efficiency ($\eta_h$) = 1 – BPF

## Dehumidification

The removal of moisture from the air, without change in its dry bulb temperature is known as ‘dehumidification’. Perforated plate is used as ‘dehumidifier’ which removes water particles from the air.

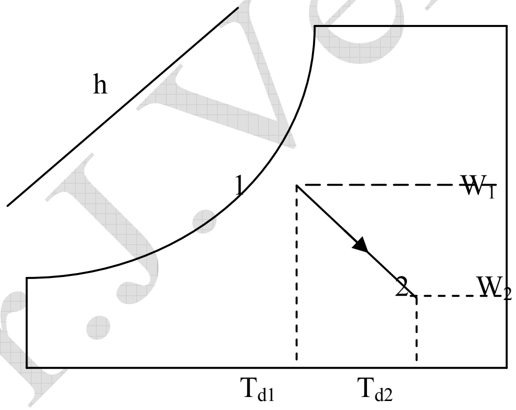




## Chemical dehumidification

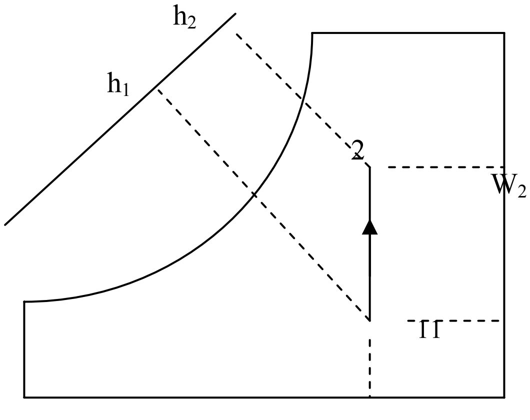
Some substances like silica gel and activated alumina have great affinity with water vapour. They are called absorbents. When air passes through a bed of silica gel, water vapour molecules get absorbed on its surface. Latent heat of condensation is released.

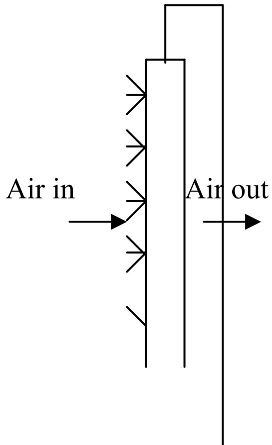
For vapour to condense, it has to lose its heat to surrounding air. So, the DBT of air increases. This process is called 'Chemical dehumidification'.

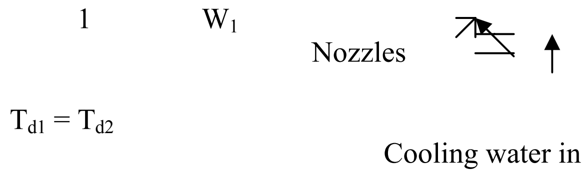


## Humidification

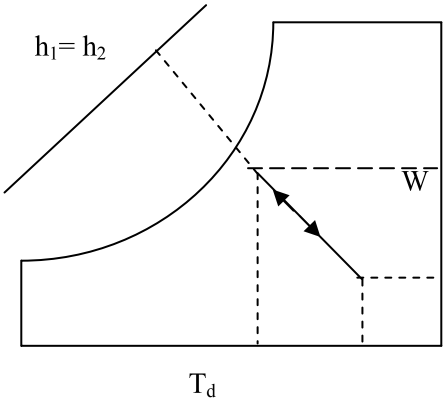
Addition of moisture to the air, without change in its dry bulb temperature is known as 'Humidification'.







## Adiabatic dehumidification and humidification



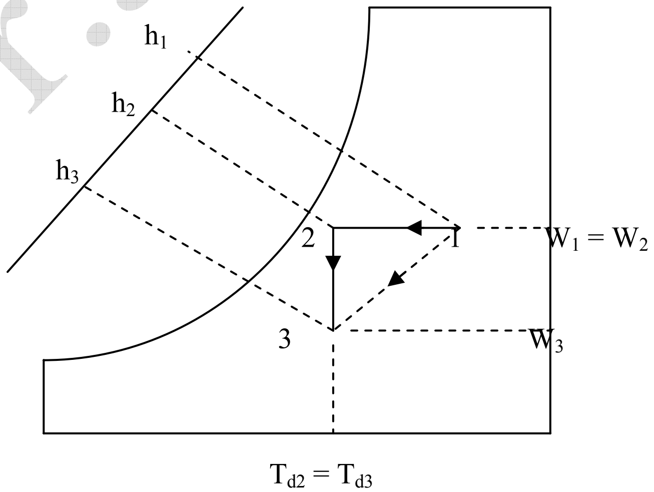
Dehumidification or humidification of air at constant enthalpy with no heat transfer is called ‘adiabatic dehumidification’ or ‘adiabatic humidification’.

## Cooling & Dehumidification

This process is generally used in Summer air conditioning to cool and dehumidify the air.

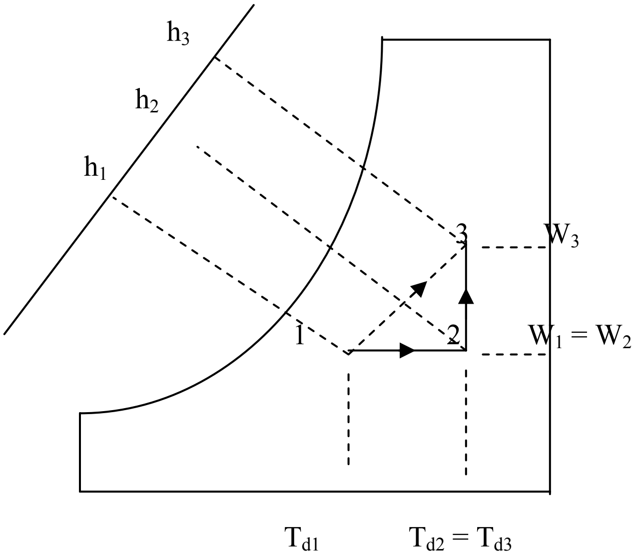
$$\text{Sensible heat factor (SHF)} = \frac{\text{Sensible heat}}{\text{Total heat}}$$

$$= \frac{h_1 - h_2}{h_1 - h_3}$$



## Heating & Humidification

This process is generally used in Winter air conditioning to warm & humidify the air.



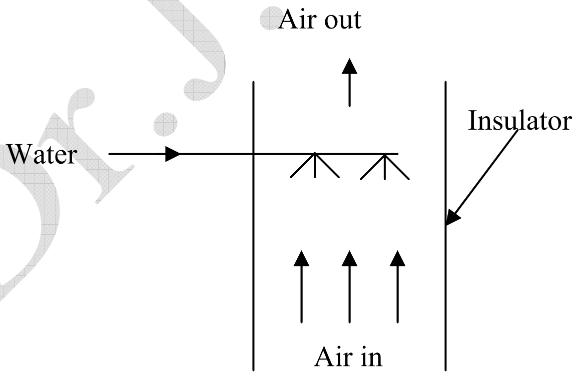
$$\text{Sensible heat factor (SHF)} = \frac{\text{Sensible heat}}{\text{Total heat}}$$

$$= \frac{h_2 - h_1}{h_3 - h_1}$$

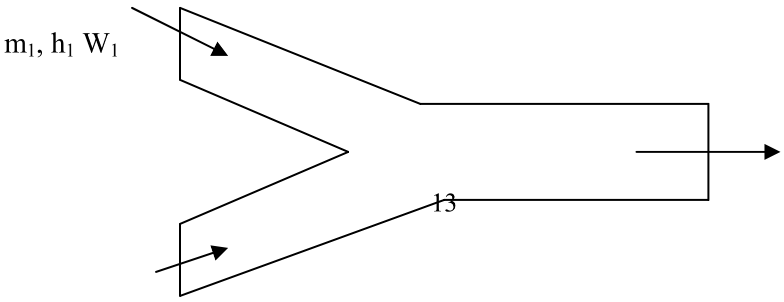
## Adiabatic evaporative cooling

A large quantity of water is constantly circulated through a spray chamber. The moist air is passed through this spray water. Some water evaporates into moist air. During this evaporation the water absorbs latent heat from the air reducing air temperature and increasing air specific humidity. This process is called ‘Evaporative cooling’ of air. If the air is cooled in an insulated chamber, then the cooling is known as ‘adiabatic evaporative cooling’.

Cooling tower utilizes the phenomenon of evaporative cooling to cool warm water below the DBT of the air.



## Adiabatic mixing of two streams



$m_3, h_3, W_3$

$m_2, h_2\ W_2$

$m_1$ = Mass flow rate of dry air of First stream

$h_1$ = Specific enthalpy of moist air/kg of dry air of first stream

$W_1$ = Specific humidity of moist air of first stream

$m_2, h_2, W_2$ ➔ Corresponding parameters of second stream

$m_3, h_3, W_3$ ➔ Corresponding parameters of mixed stream

We can write,

$$m_1 + m_2 = m_3 \qquad \text{------ (1)}$$

$$m_1 h_1 + m_2 h_2 = m_3 h_3 \qquad \text{------ (2)}$$

$$m_1 W_1 + m_2 W_2 = m_3 W_3 \qquad \text{-------(3)}$$

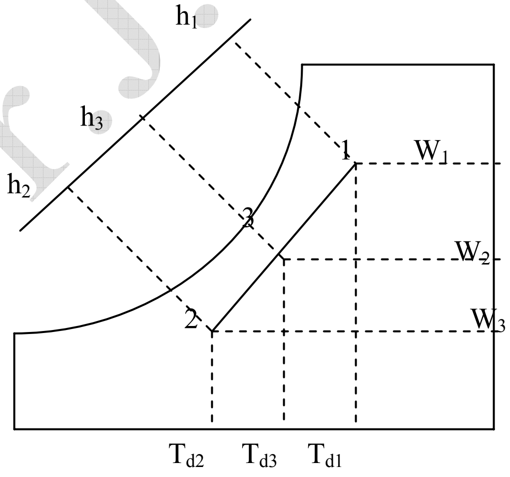
From (1) & (2),
$$m_1 W_1 + m_2 W_2 = (m_1 + m_2) W_3$$

$$m_1(W_1 - W_3) = m_2 (W_3 - W_2)$$

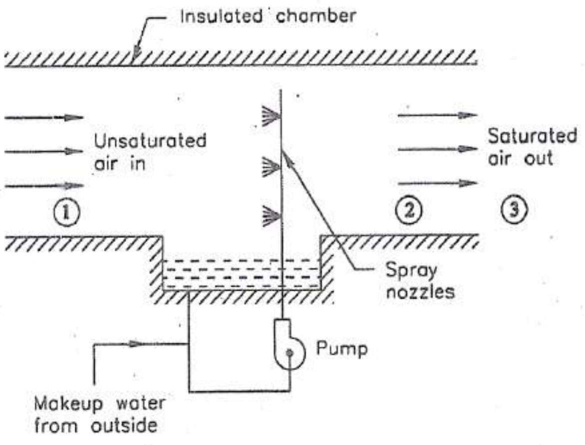
$$\frac{m_1}{m_2} = \frac{W_3 - W_2}{W_1 - W_3}$$

And similarly we can write,

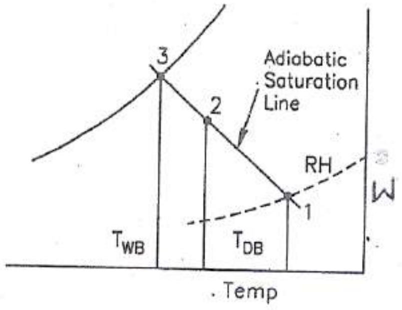
$$\frac{m_1}{m_2} = \frac{h_3 - h_2}{h_1 - h_3}$$



## Adiabatic saturation



(*a*) Adiabatic saturation arrangement



(*b*) Representation of adiabatic saturation on psychrometric chart

When unsaturated air flows over a long sheet of water in an insulated chamber, the water evaporates, and the specific humidity of the air increases. Both the air and water are cooled as evaporation takes places. The process continues until the energy transferred from the air to the water is equal to the energy required to evaporate the water. When this point is reached, thermal equilibrium exists with respect to the water, air and water vapour, and correspondingly the air is saturated. This process is called adiabatic saturation. And the enthalpy is constant as there is no heat transfer during the process.

## PROBLEMS

**5. 5 gm of water vapour per kg of atmospheric air at 35°C, 60% RH and 1.013 bar is removed and temperature of air after removing the water vapour is 25°C DBT. Determine, RH and DPT.**

*Given:*

Mass of water vapour ($m_v$) = 5 gm

Initial DBT ($T_{d1}$) = 35°C

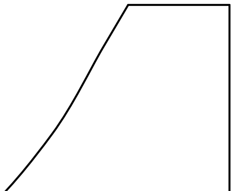
Initial RH ($\Phi_1$) = 60%

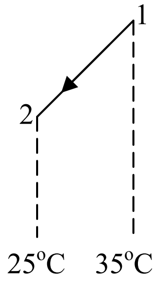
Total pressure of air ($p_b$) = 1.013 bar

Final DBT ($T_{d2}$) = 25°C

*Required***:** $\Phi_2$ and $T_{dp2}$

*Solution:*





$$\phi_1 = \frac{p_{v1}}{p_{s1}}$$

$p_{s1} = 0.05628$ bar at $T_{d1} = 35^oC$ from steam table

Therefore, $$0.6 = \frac{p_{v1}}{0.05628}$$

$p_{v1} = 0.03377$ bar

$$W_1 = 0.622 \frac{p_{v1}}{p_b - p_{v1}}$$

$$W_1 = 0.622 \frac{0.03377}{1.013 - 0.03377} = 0.02145\, kg / kg\, dry\, air$$

= 21.45 gm/kg dry air

Therefore, $W_2 = 21.45 – 5 = 16.45$ gm/kg dry air

$p_{s2} = 0.03169$ bar at $T_{d2} = 25^oC$ from steam table

$$W_2 = 0.622 \frac{p_{v2}}{p_b - p_{v2}}$$

$$0.01645 = 0.622\, x\, \frac{p_{v2}}{1.013 - p_{v2}}$$

$0.02679 – 0.026447\, p_{v2} = p_{v2}$

$p_{v2} = 0.0261$ bar

$$\phi_1 = \frac{p_{v2}}{p_{s2}} = \frac{0.0261}{0.03169} = 0.823 = 82.3\%$$ **---- Ans**

From steam table, $T_{dp2} = 22^oC$ at $p_{v2} = 0.0261$ bar **----- Ans**

6. **The sling psychrometer reads 40°C DBT and 28°C WBT. Calculate, (i) specific humidity, (ii) relative humidity (iii) vapour density of air, (iv) DPT and (v) enthalpy of mixture. Assume atmospheric pressure to be 1.013 bar.**

*Given:*

Atmospheric pressure ($p_b$) = 1.013 bar

Dry bulb temperature ($T_d$) = 40°C

Wet bulb temperature ($T_w$) = 28°C

*Required:* (i) W (ii) Φ (iii) $\rho_v$ (iv) $T_{dp}$ (v) h

*Solution:*

(i) $$W = 0.622 \frac{p_v}{p_b - p_v}$$

Also, $$W = \frac{C_{pa}(T_w - T_d) + W_w\, h_{fgW}}{h_{gd} - h_{fw}}$$

$$W_w = 0.622 \frac{p_{sw}}{p_b - p_{sw}}$$

$p_{sw}$ = Saturation pressure at WBT = 0.03778 bar from steam table

$$W_w = 0.622 \frac{0.03778}{1.013 - 0.03778} = 0.024096\ kg/kg\, dry\, air$$

$h_{fgw} = h_{fg}$ at WBT = 2435.4 kJ/kg from steam table at 28°C

$h_{gd} = h_g$ at DBT = 2574.4 kJ/kg from steam table at 40°C

$h_{fw} = h_f$ at WBT = 117.3 kJ/kg from steam table at 28°C

$C_{pa}$ = Specific heat of air = 1.005 kJ/kg-K

$$W = \frac{1.005(28-40) + 0.024096(2435.4)}{2574.4 - 117.3} = 0.018975\ kg/kg\, dry\, air$$

(ii) $$\phi = \frac{p_v}{p_s}$$

$$W = 0.622 \frac{p_v}{p_b - p_v}$$

$$0.018975 = 0.622\, x\, \frac{p_v}{1.013 - p_v}$$

$p_v$ = 0.02998 bar

$p_s$ = 0.07375 bar at 40°C from steam table

Therefore, Φ = 0.02998/0.07375 = 0.4065 = 40 % ----- **Ans**

(iii) Density of water vapour can be found out from, $W = \rho_v / \rho_a$

We know that, $\rho_a = \dfrac{p_a}{R_a T_a}$

$R_a$ = 287 J/kg-K

$T_a$ = DBT

$p_a$ = Partial pressure of dry air = $p_b - p_v$

$p_a$ = 1.013 – 0.02998 = 0.98302 bar

$$\rho_a = \frac{0.98302\,x10^5}{287\,x(40+273)} = 1.0943\ kg/m^3$$

Therefore, $\rho_v$ = 0.018975 x 1.0943 = **0.020764 kg/m$^3$ ----- Ans**

(iv) At $p_v$ = 0.020764 bar from steam table, $T_{dp}$ = **24$^o$C ----- Ans**

(v) $$h = C_{pa}T_d + W[h_{gd} + 1.88(T_d - T_{dp})]$$

$$= 1.005(40) + 0.018975[2574.4 + 1.88(40-24)]$$

= **89.62 kJ/kg dry air ---- Ans**

Note: Substitute $T_d$ in $^o$C

7. **Atmospheric air at 1.0132 bar has a DBT of 32$^o$C and WBT of 26$^o$C. Compute (a) the partial pressure of water vapour, (b) the specific humidity, (c) the DPT, (d) the RH, (e) the degree of saturation, (f) the density of air in the mixture, (g) the density of vapour in the mixture and (h) the enthalpy of the mixture.**

*Given:*

Atmospheric pressure ($p_b$) = 1.0132 bar

Dry bulb temperature ($T_d$) = 32$^o$C

Wet bulb temperature ($T_w$) = 26$^o$C

*Required***:** (a) $p_v$ (b) W (c) $T_{dp}$ (d) $\Phi$ (e) $\mu$ (f) $\rho_a$ (g) $\rho_v$ (h) h

*Solution:*

(a)

I-Method (Using Carrier's equation)

$$p_v = p_{sw} - \frac{(p_b - p_{sw})(T_d - T_w)}{1527.4 - 1.3T_w}$$

Note: $T_w$ → WBT in $^o$C

$p_{sw}$ = Saturation pressure at WBT = 0.03360 bar from steam table

$$p_v = 0.03360 - \frac{(1.0132 - 0.03360)(32-26)}{1527.4 - 1.3(26)}$$

= **0.0296 bar ----- Ans**

II-Method

$$W = \frac{C_{pa}(T_w - T_d) + W_w\,h_{fgW}}{h_{gd} - h_{fw}}$$

$$W_w = 0.622\frac{p_{sw}}{p_b - p_{sw}}$$

$p_{sw}$ = Saturation pressure at WBT = 0.03360 bar from steam table

$$W_w = 0.622\frac{0.03360}{1.0132 - 0.03360} = 0.021334\ kg/kg\,dry\,air$$

$h_{fgw}$ = $h_{fg}$ at WBT = 2440.2 kJ/kg from steam table at 26$^o$C

$h_{gd}$ = $h_g$ at DBT = 2560 kJ/kg from steam table at 32$^o$C

$h_{fw} = h_f$ at WBT = 108.9 kJ/kg from steam table at 26°C

$C_{pa}$ = Specific heat of air = 1.005 kJ/kg-K

$$W = \frac{1.005(26-32)+0.021334(2440.2)}{2560-108.9} = 0.01878\ kg/kg\,dry\,air$$

Also, $$W = 0.622\frac{p_v}{p_b - p_v}$$

$$0.01878 = 0.622\,x\,\frac{p_v}{1.0132 - p_v}$$

$p_v$ = **0.0297 bar** ---- **Ans**

(b) $$W = 0.622\frac{p_v}{p_b - p_v}$$

$$= 0.622\frac{0.0296}{1.0132-0.0296} = 0.01872\ kg/kg\,dry\,air$$ ---- **Ans**

(c) $T_{dp}$ = 24°C at $p_v$ from steam table ----- **Ans**

(d) $$\phi = \frac{p_v}{p_s}$$

$p_s$ = 0.04753 bar at 32°C from steam table

Therefore, Φ = 0.0296/0.04753 = 0.623 = 62.3 % ----- **Ans**

(e) $$\mu = \frac{W}{W_s} = \frac{p_v(p_b - p_s)}{p_s(p_b - p_v)}$$

$$= \frac{0.0296\,(1.0132-0.04753)}{0.04753\,(1.0132-0.0296)} = 0.6114$$ ----- **Ans**

(f) We know that, $$\rho_a = \frac{p_a}{R_a T_a}$$

$R_a$ = 287 J/kg-K

$T_a$ = DBT

$p_a$ = Partial pressure of dry air = $p_b - p_v$

$p_a$ = 1.0132 – 0.0296 = 0.9836 bar

$$\rho_a = \frac{0.9836\,x\,10^5}{287\,x\,(32+273)} = 1.1236\ kg/m^3$$ ----- **Ans**

(g) Density of water vapour can be found out from, $W = \rho_v / \rho_a$

Therefore, $\rho_v$ = 0.01872 x 1.1236 = **0.021 kg/m$^3$** ----- **Ans**

(h) $$h = C_{pa}T_d + W[h_{gd} + 1.88(T_d - T_{dp})]$$

At $p_v$ = 0.0296 bar from steam table, $T_{dp}$ = **24°C** ----- **Ans**

$$h = 1.005\,(32) + 0.01872\,[2560 + 1.88(32-24)]$$

**= 80.365 kJ/kg dry air ---- Ans**

8. **Air at 20$^o$C, 40 % RH is mixed adiabatically with air at 40$^o$C, 40 % RH in the ratio of 1 kg of the former with 2 kg of latter (on dry basis). Find the final condition of air. Draw the process in chart also as diagram.**

*Given:*

DBT of I-stream ($T_{d1}$) = 20$^o$C

RH of I-stream ($\Phi_1$) = 40 %

DBT of II-stream ($T_{d2}$) = 40$^o$C

RH of II-stream ($\Phi_2$) = 40 %

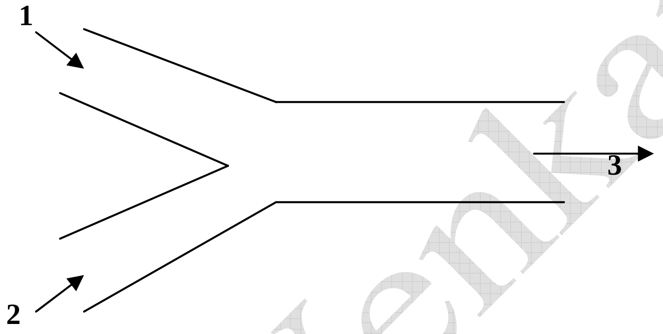
$m_{a1}/m_{a2}$ = 1/2

Mass of dry air in the I-stream ($m_{a1}$) = 1 kg

Mass of dry air in the II-stream ($m_{a2}$) = 2 kg

*Required*: Final condition of air ($h_3$, $W_3$)

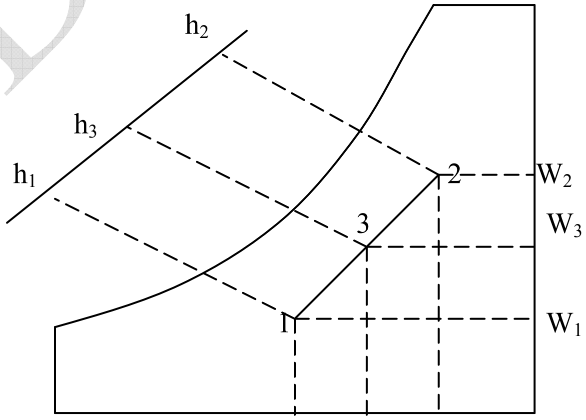
*Solution:*



We can write,

$$m_{a1}\, h_1 + m_{a2}\, h_2 = (m_{a1} + m_{a2})\, h_3$$

$$m_{a1}\, W_1 + m_{a2}\, W_2 = (m_{a1} + m_{a2})\, W_3$$



$T_{d1}$ $T_{d3}$ $T_{d2}$

Using Steam table and equations

$$\phi_1 = \frac{p_{v1}}{p_{s1}}$$

$p_{s1}$ = 0.02337 bar at $T_{d1}$ = 20°C from steam table

$$0.4 = \frac{p_{v1}}{0.02337}$$

$p_{v1}$ = 0.009348 bar

$$W_1 = 0.622 \frac{p_{v1}}{p_b - p_{v1}}$$

$$= 0.622\,x\,\frac{0.009348}{1.0132 - 0.009348} = 0.0058\ kg/kg\ dry\ air$$

$$\phi_2 = \frac{p_{v2}}{p_{s2}}$$

$p_{s2}$ = 0.07375 bar at $T_{d2}$ = 40°C from steam table

$$0.4 = \frac{p_{v2}}{0.07375}$$

$p_{v2}$ = 0.0295 bar

$$W_2 = 0.622 \frac{p_{v2}}{p_b - p_{v2}}$$

$$= 0.622\,x\,\frac{0.0295}{1.0132 - 0.0295} = 0.01865\ kg/kg\ dry\ air$$

$$h_1 = C_{pa}T_{d1} + W_1[h_{gd1} + 1.88(T_{d1} - T_{dp1})]$$

At $p_{v1}$ = 0.09348 bar from steam table, $T_{dp1}$ = 6°C

$h_{gd1}$ = 2538.2 kJ/kg

$$h_1 = 1.005(20) + 0.0058[2538.2 + 1.88(20 - 6)]$$

= 34.97 kJ/kg dry air

$$h_2 = C_{pa}T_{d2} + W_2[h_{gd2} + 1.88(T_{d2} - T_{dp2})]$$

At $p_{v2}$ = 0.0295 bar from steam table, $T_{dp2}$ = 24°C

$h_{gd2}$ = 2574.4 kJ/kg

$$h_2 = 1.005(40) + 0.01865[2574.4 + 1.88(40 - 24)]$$

= 88.8 kJ/kg dry air

1 (34.97) + 2 (88.8) = (1 + 2) $h_3$

$h_3$ = 70.86 kJ/kg dry air ----- **Ans**

1 (0.0058) + 2 (0.01865) = (1 + 2) $W_3$

$W_3 = 0.01436$ kg/kg dry air ----- **Ans**

Using chart

- Locate the point (1) on the chart at $\Phi_1 = 40\%$ and $T_{d1} = 20^oC$
- Locate the point (2) at $\Phi_2 = 40\%$ and $T_{d2} = 40^oC$
- Get $h_1$, $h_2$, $W_1$ and $W_2$ from chart

| | |
|---|---|
| $h_1 = 35$ kJ/kg dry air | $W_1 = 0.0058$ kg/kg dry air |
| $h_2 = 90$ kJ/kg dry air | $W_2 = 0.0187$ kg/kg dry air |

$1\,(35) + 2\,(90) = (1 + 2)\,h_3$

$h_3 = 71.67$ kJ/kg dry air ----- **Ans**

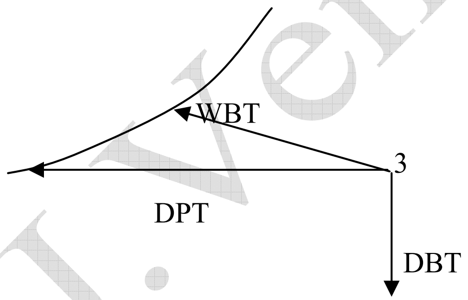
$1\,(0.0058) + 2\,(0.0187) = (1 + 2)\,W_3$

$W_3 = 0.0144$ kg/kg dry air ----- **Ans**

We can also get other properties from chart.

- Locate point (3) on the chart at $W_3$ or $h_3$
- Point (3) lies between (1) and (2) and on the line joining (1) and (2)
- Get WBT and DPT on the saturation curve
- Get DBT on the x-axis
- Get RH and specific volume at point (3)

$T_{w3} = 23.8^oC$, $T_{d3} = 33.1^oC$, $T_{dp3} = 19.6^oC$, $v_3 = 0.887$ m$^3$/kg dry air, $\Phi_3 = 46\%$



**9. Air-water vapour mixture at 0.1 MPa, 30$^o$C, 80% RH has a volume of 50 m$^3$. Calculate the specific humidity, DPT, WBT, mass of dry air and mass of water vapour.**

*Given:*

Total pressure of mixture ($p_b$) = 0.1 MPa = 1 bar

DBT of mixture ($T_d$) = 30$^o$C

RH of mixture ($\Phi$) = 80%

Volume of mixture (V) = 50 m$^3$

*Required*: W, DPT, WBT, $m_a$ and $m_w$

*Solution:*

$$W = 0.622\frac{p_v}{p_b - p_v}$$

$$\phi = \frac{p_v}{p_s}$$

$p_s = 0.04242$ bar at $T_d = 30^oC$ from steam table

$p_v = 0.8\ (0.04242) = 0.03394$ bar

$$W = 0.622\, x\, \frac{0.03394}{1-0.03394} = 0.02185\ kg/kg\ dry\ air$$ ---- **Ans**

$T_{dp} = 26^oC$ at $p_v = 0.03394$ bar from steam table ---- **Ans**

- Using Carrier's equation, only by trial and error we can find WBT. Therefore it is advisable to refer chart for obtaining the WBT.
- Locate the point on the chart at 30°C and 80% RH
- Get WBT on saturation curve

$T_w = 27^oC$ ---- **Ans**

To find mass of dry air

$p_a = p_b - p_v = 1 - 0.03394 = 0.96606$ bar

$p_a\ V = m_a\ R_a\ T_a$

$0.96606 \times 10^5 \times 50 = m_a \times 287 \times (30 + 273)$

$m_a = 55.54$ kg ----- **Ans**

To find mass of water vapour

$$W = \frac{\rho_v}{\rho_a} = \frac{m_v}{m_a}$$

$m_v = 0.02185\ (55.54) = 1.2136$ kg ---- **Ans**

Note: If we know the value of $R_w$, $m_w$ can be found out from, $p_w\ V = m_w\ R_w\ T_v$

$T_a = T_v = T_d$

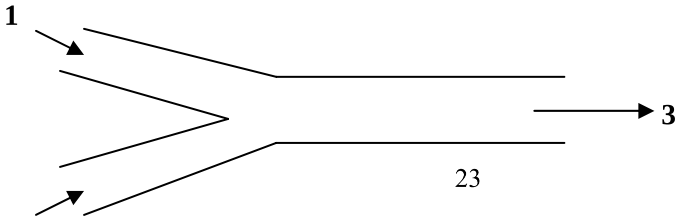
**10. Saturated air at 20°C at a rate of 1.16 $m^3/s$ is mixed adiabatically with the outside air at 35°C and 50% RH at a rate of 0.5 $m^3/s$. Assuming adiabatic mixing condition at 1 atm, determine specific humidity, relative humidity, dry bulb temperature and volume flow rate of the mixture.**

*Given:*

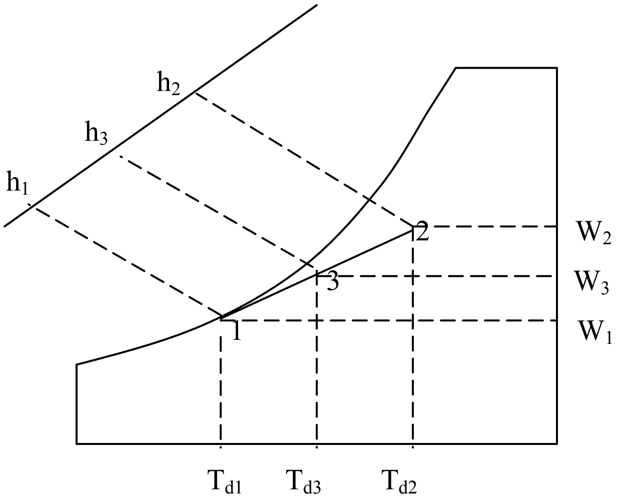
| | |
|---|---|
| Saturated air, i.e., $\Phi_1$ | = 100% |
| DBT of I-stream ($T_{d1}$) | = 20°C |
| Volume flow rate of I-stream ($V_1$) | = 1.16 $m^3/s$ |
| DBT of II-stream ($T_{d2}$) | = 35°C |
| RH of II-stream ($\Phi_2$) | = 50% |
| Volume flow rate of II-stream ($V_2$) | = 0.5 $m^3/s$ |
| Total pressure ($p_b$) | = 1 atm = 1.01325 bar |

*Required*: $W_3$, $\Phi_3$ $T_{d3}$ and $V_3$

*Solution:*



**2**



We can write,

$$m_{a1}\, h_1 + m_{a2}\, h_2 = (m_{a1} + m_{a2})\, h_3$$

$$m_{a1}\, W_1 + m_{a2}\, W_2 = (m_{a1} + m_{a2})\, W_3$$

Using chart

- Locate the point (1) on the chart at $\Phi_1 = 100\%$ and $T_{d1} = 20^oC$
- Locate the point (2) at $\Phi_2 = 50\%$ and $T_{d2} = 35^oC$
- Get $h_1$, $h_2$, $W_1$ and $W_2$ from chart

$h_1 = 58$ kJ/kg dry air $W_1 = 0.015$ kg/kg dry air $v_1 = 0.851\ m^3$/kg dry air

$h_2 = 81.5$ kJ/kg dry air $W_2 = 0.018$ kg/kg dry air $v_2 = 0.898\ m^3$/kg dry air

$$m_1 = \frac{V_1}{v_1} = \frac{1.16}{0.851} = 1.3631\ kg/s$$

$$m_2 = \frac{V_2}{v_2} = \frac{0.5}{0.898} = 0.5568\ kg/s$$

$1.3631\ (0.015) + 0.5568\ (0.018) = (1.3631 + 0.5568)\ W_3$

$W_3 = 0.0159$ kg/kg dry air ----- **Ans**

- Locate point (3) on the chart at $W_3$
- Point (3) lies between (1) and (2) and on the line joining (1) and (2)
- Get DBT on the x-axis
- Get RH and specific volume at point (3)

From chart, $\Phi_3 = 80\%$ ----- **Ans**

$T_{d3} = 25^oC$ ------ **Ans**

$v_3 = 0.866\ m^3$/kg dry air

$V_3 = m_3\, v_3 = (1.3631 + 0.5568)\ (0.866)$

$= 1.6626\ m^3/s$ ----- **Ans**

**11. Air at 16°C and 25% RH passes through a heater and then through a humidifier to reach final DBT of 30°C and 50% RH. Calculate the heat and moisture added to the air. What is the sensible heat factor?**

*Given:*

Initial DBT of air ($T_{d1}$) = 16°C

Initial RH of air ($\Phi_1$) = 25%

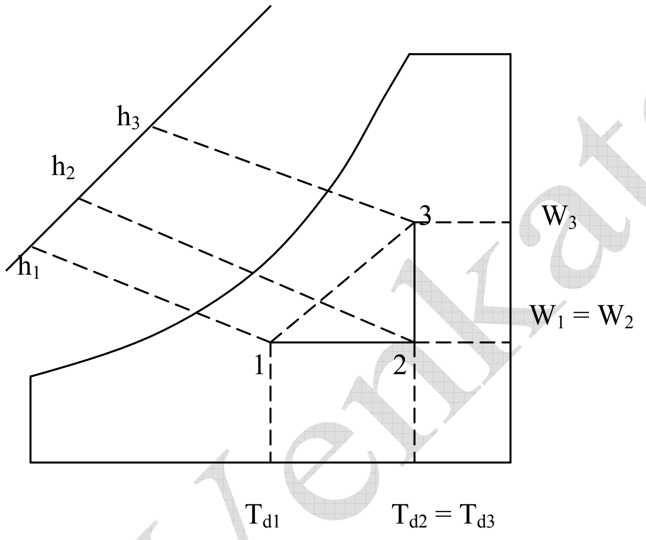
Final DBT of air ($T_{d3}$) = 30°C

Final RH of air ($\Phi_3$) = 50%

Process → Heating and humidification

*Required:* Heat added, Moisture added and SHF

*Solution:*



**Using chart**

- Locate the point (1) at $T_{d1}$ and $\Phi_1$
- Locate the point (3) at $T_{d3}$ and $\Phi_3$
- Draw the horizontal line (heating) from (1) and Vertical line (humidification) from (3) and get intersection point (2)
- Get $h_1$, $h_2$, $h_3$, $W_1$, $W_2$ and $W_3$

$h_1$ = 23 kJ/kg dry air  $h_2$ = 38 kJ/kg dry air  $h_3$ = 64 kJ/kg dry air

$W_1 = W_2$ = 0.0033 kg/kg dry air  $W_3$ = 0.0134 kg/kg dry air

Heat added = SH + LH = $(h_2 - h_1) + (h_3 - h_2)$

$= h_3 - h_1$

= 64 – 23 = 41 kJ/kg dry air ---- **Ans**

Moisture added = $W_3 - W_2$ = 0.0134 – 0.0033

= 0.0101 kg/kg dry air ----- **Ans**

$$\text{Sensible heat factor (SHF)} = \frac{SH}{SH + LH} = \frac{h_2 - h_1}{(h_2 - h_1) + (h_3 = h_2)} = \frac{h_2 - h_1}{h_3 - h_1}$$

$$= \frac{38-23}{64-23} = 0.366 \text{ ----- } \textbf{Ans}$$

**12. For a hall to be air conditioned, the following conditions are given:**

**Outdoor condition → 40$^{o}$C DBT, 20$^{o}$C WBT**

**Required comfort condition → 20$^{o}$C DBT, 60% RH**

**Seating capacity of hall = 1500**

**Amount of air supplied = 0.3 m$^{3}$/min per person**

**If the required condition is achieved first by adiabatic humidification and then by cooling, estimate (a) capacity of the cooling coil in tones, and (b) the capacity of the humidifier.**

*Given:*

Process → Adiabatic humidification and Cooling

<u>Before adiabatic humidification</u>

DBT of air ($T_{d1}$) = 40$^{o}$C

WBT of air ($T_{w1}$) = 20$^{o}$C

<u>After adiabatic humidification</u>

RH of air ($\Phi_2$) = 60%

<u>After cooling (sensible)</u>

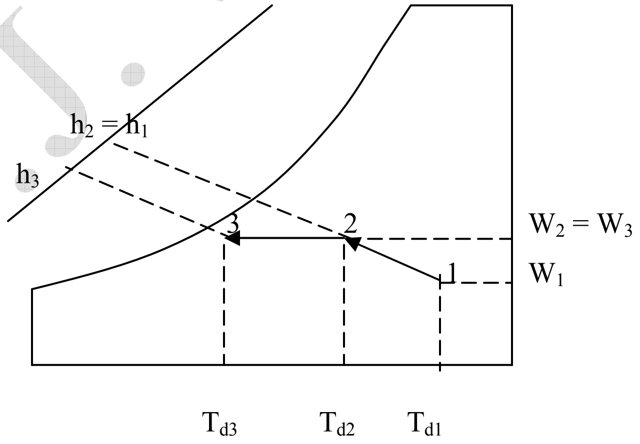
DBT of air ($T_{d3}$) = 20$^{o}$C

Amount of air supplied ($V_{a1}$) = 0.3 m$^{3}$/min per person

Seating capacity = 1500

*Required*: (a) Capacity of cooling coil (b) Capacity of humidifier

*Solution:*



- Locate the point (1) at $T_{d1}$ = 40$^{o}$C and $T_{w1}$ = 20$^{o}$C
- Locate the point (3) at $T_{d3}$ = 20$^{o}$C and RH = 60 %
- During adiabatic humidification, h = constant. During cooling, W = constant. Therefore follow the constant WBT line from point (1) and constant W line from point (3) to get point (2).

From chart, $h_1 = h_2 = 57$ kJ/kg dry air

$h_3 = 42$ kJ/kg dry air

$W_1 = 0.0065$ kg/kg dry air

$W_2 = W_3 = 0.0088$ kg/kg dry air

$v_1 = 0.896$ $m^3$/kg dry air

(a) Capacity of cooling coil $= m_a (h_2 - h_3)$

Volume flow rate of air ($V_{a1}$) = 0.3 x 1500/60 = 7.5 $m^3$/s

Mass flow rate of air ($m_a$) $= V_{a1}/v_1 = 7.5 / 0.896 = 8.37$ kg/s

Capacity of cooling coil = 8.37 (57 – 42) = 125.55 kJ/s

= 125.55 / 3.89 = 32.27 TR ---- **Ans**

Capacity of humidifier $= m_a (W_2 - W_1)$ = 8.37 (0.0088 – 0.0065)

= 0.019251 kg/s

= 69.3 kg/h ----- **Ans**

# ME18301 ENGINEERING THERMODYNAMICS
## SOLUTIONS TO THE PROBLEMS- UNIT-I

**2. 1 Kg of air at a pressure of 1bar and 25 $^{o}$C is heated at constant volume till the pressure is doubled. It is then expanded isothermally to the original pressure and then cooled to the initial condition at constant pressure. Show the process on P-V and T-S diagrams and calculate the work and heat interactions during the cycle. (AU May 2006)**

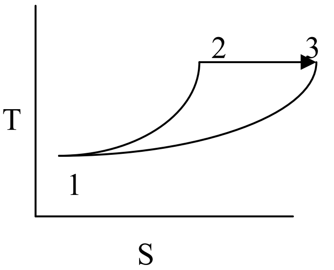
**Data Given**

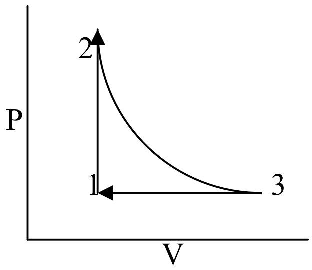
m=1kg

P1=1bar=1x10$^5$ N/m$^2$

$T_1$=25 **$^{o}$C=298K**

$\left(\frac{P_2}{P_1}\right)=2$





**To Find**: work and Heat transfer

**Solution:**

**PROCESS 1-2 – CONSTANT VOLUME PROCESS**

**During this process work done is zero, since dV=0**

**Heat transfer=change in Internal Energy**

**ΔU=** m $C_v$ ($T_2$ –$T_1$)

To Find $T_2$:

For constant Volume process, $\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)$

$T_2$ =$(2)$ x298 =**596K**

**ΔU=1x712x(596-298)=212.176kJ**

**δQ=212.176kJ**

**PROCESS 2-3 – CONSTANT TEMPERATURE PROCESS**

**In Iso- thermal process change in Internal Energy is zero. Hence Heat transfer=work transfer**

**Work transfer** $W_{2-3}$= $Q_{2-3}$ = mR $T_2$ ln ($V_3$/ $V_2$) = mR $T_2$ ln ($P_2$/ $P_3$)

$P_3$= $P_1$ , Since $\left(\frac{P_2}{P_1}\right)=2$ & $P_1$= 1bar , P2=2bar

Therefore $W_{2-3}$= $Q_{2-3}$=1x287x596 ln(2)=**118.564kJ**

**PROCESS 3-1 – CONSTANT PRESSURE PROCESS**

**Work Transfer=** $W_{3-1}$= $P_3$($V_1$ –$V_3$)=mR($T_1$ –$T_3$)

$T_1$ is given . Since 2-3 is isothermal process $T_2$=$T_3$

$W_{3-1}$=1x287x(298-596)= -85.526kJ

Heat transfer **δQ=mx $C_p$x**($T_1$ –$T_3$)=1x1005x(298-596)= **-299.490kJ**

Net Heat Transfer During The Cycle=§Q=**δ$Q_{1-2}$ +δ$Q_{2-3}$ +δ$Q_{3-1}$**
=212.17+118.56+(-299.49)

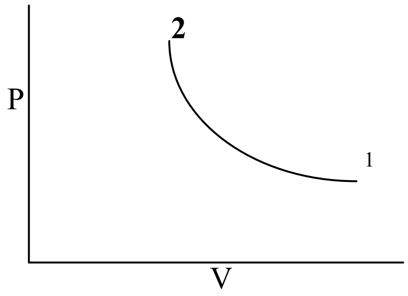
**§Q =31.24kJ**

According to First law of thermodynamics For a Cyclic process

**Net Heat Transfer=Net work Transfer**

**4. Air at 1.02 bar, 22°C, initially occupying a cylinder volume of 0.015 m$^3$, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate: (a) The final temperature (b) The final volume (c) The work done on the mass of air in the cylinder**.

**<u>Data Given :</u>**



$P_1$=1.02 bar=1.02x10$^5$ N/m$^2$
$V_1$=0.015m$^3$
$T_1$=22°C=22+273=295K
$P_2$=6.8 bar=6.8x10$^5$ N/m$^2$

**<u>To Find:</u>**

$T_2$ , $V_2$ &W

**<u>SOLUTION:</u>**

$$\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\left(\nu-1/\nu\right)}=\left(\frac{6.8}{1.02}\right)^{\left(.4/1.4\right)}$$

$$T_2=\left(\frac{6.8}{1.02}\right)^{(.286)}x295=\mathbf{507.53K}$$

$$\left(\frac{P_1}{P_2}\right)=\left(\frac{V_2}{V_1}\right)^{\gamma}$$

$$V_2=\left(\frac{P_1}{P_2}\right)^{\left(1/\gamma\right)}\text{x } V_1=\left(\frac{1.02}{6.8}\right)^{1/1.4}x.015\ \ =\mathbf{3.871x10^{-3}\ m^3}$$

To find mass of Air:

$P_1$ $V_1$=m$RT_1$

$$m=\frac{P_1\,V_1}{T_1}=\frac{1.02x10^5\ x\ 0.015}{287\ x295}=0.0181kg$$

Workdone during reversible Adaiabatic process = m $C_v$ ($T_2$ –$T_1$)
=0.0181x718x(295-507.5)

**W = -2761.61kJ**

**Work done on the system is 2761.61kJ**

**6. 1 kg of gas at 1.1 bar,27 °C is compressed to 6.6bar as per the law $PV^{1.3}$=constant. Calculate work and heat transfer, if**

**(1) When the gas is ethane ($C_2H_6$) with molar mass 30kg/k mol and $C_p$ of 2.1kJ/kgK**

**(2) When the gas is argon (Ar) with molar mass 40kg/k mol and $C_p$ of 0.52kJ/kgK ( AU DEC2005)**

**Data Given:**

**m=1kg**
**$P_1$=**$1.1\text{bar}=1.1\text{x}10^5\ \text{N/m}^2$
**$T_1$=27 °C=300K**
**$P_2$=6.6bar=6.6**$\text{x}10^5\ \text{N/m}^2$
**$PV^{1.3}$=constant,n=1.3**

**To Find:** Heat and Work Transfer For Ethane and argon

**Solution:**

For Polytropic Process, Work Transfer is

$$W_{1-2} = \frac{m\,R\,(T_1 - T_2)}{n-1}$$

Heat transfer during Polytropic process: $Q_{1-2} = W_{1-2} \text{ x } (\gamma\text{-}n/\gamma\text{-}1)$

Where γ=adiabatic Index

n=Index of Compression

To find Final Temperature after Polytropic Compression:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(n-1/n)} = \left(\frac{6.6}{1.1}\right)^{(.3/1.3)}$$

$$T_2 = \left(\frac{6.6}{1.1}\right)^{(.3/1.3)} \text{x}300 = 453.81\text{K}$$

Gas Constant R=Universal Gas Constant/ Molecular weight

**<u>For the gas Methane</u>:**

For Methane M=30;$C_p$=**2.1kJ/kgK** (Given)

R=8314/30=277.13J/kg.moleK

$C_v$= $C_p$-R=2100-277.13=1822.87**J/kgK**

$\gamma$= $C_p$/ $C_v$=2100/1822.87=1.15

$$W_{1-2} = \frac{1x277.13x(300-453.81)}{0.3} = \mathbf{-142.084kJ}$$

$Q_{1-2}$= (-142.084) x (1.15-1.3)/0.15

**$Q_{1-2}$ = 702kJ**

**<u>For the gas Argon:</u>**

For argon M=40;$C_p$=0**.52kJ/kgK** (Given)

R=8314/40=207.85J/kg.moleK

$C_v$= $C_p$-R=520-207.85=312.15**J/kgK**

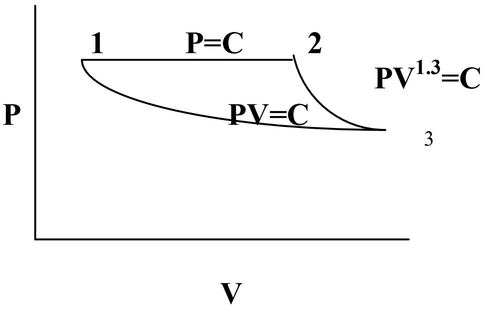
$\gamma = C_p / C_v = 520/312.15 = 1.67$

$$W_{1\text{-}2} = \frac{1x207.8513x(300-453.81)}{0.3} = \mathbf{-106.565kJ}$$

$Q_{1\text{-}2} = (-106.565) \quad x(1.67-1.3)/0.67$

$\mathbf{Q_{1\text{-}2} = -58.84kJ}$

**8. A certain quantity of air initially at state 1 has a pressure of 5bar,volume of 0.43 $m^3$ and temperature of 227 $^oC$ . Heat is added at constant pressure till the air reaches state 2 where its volume is three times that at state 1.It is now expanded to state 3 according to $PV^{1..3}$=constant. An isothermal process brings the air from state 3 to state 1 completing a cycle. (i) Sketch the cycle on p-v diagram (ii) Find heat rejected /received during each of the three processes (iii) determine the network done during the cycle**



Data Given

$P_1 = 5 \text{ bar} = 5x10^5 \text{ N/m}^2$
$V_1 = 0.43m^3$
$T_1 = 227^oC = 227+273 = 500K$
$V_2 = 3V_1$

**To Find**

$Q_{1\text{-}2}, Q_{2\text{-}3}, \quad Q_{3\text{-}1}$ & Net Work done

**Solution**

To find Mass of Air

$$P_1 V_1 = mRT_1$$

$$m = \frac{P_1 V_1}{RT_1} \qquad \frac{5x10^5 \text{ x } 0.43}{287 \text{ x}500} = \mathbf{1.50kg}$$

**To find $Q_{1\text{-}2}$:**

Heat transfer during Constant Pressure Process=$mC_p(T_2 - T_1)$

For the process 1-2

$$(T_2/T_1) = (V_2/V_1)$$
$$T_2 = (V_2/V_1) \text{ x } T_1$$
$$= 3x\ 500 = 1500K$$

$Q_{1\text{-}2}$ =1.5 x1005 x (1500-500)
=1507.5 kJ

**Heat received during the process 1-2 is 1507.5 kJ**

**To find $Q_{2\text{-}3}$:**

Heat transfer during Polytropic process $PV^{1.3}$=C: $Q_{2\text{-}3} = W_{2\text{-}3} \text{ x } (\nu\text{-}n/\nu\text{-}1)$

$$\text{Work done during the process } W_{2\text{-}3} = \frac{m R (T_2 - T_3)}{n\text{-}1} = \frac{1.5 \text{ x } 287 \text{ x } (1500-500)}{0.3} = 1435.005 \text{ kJ}$$

$Q_{2-3}$ = 1435.005 x (1.4-1.3/0.4)
=358.775kJ

**Heat received during the process 2-3 is 358.775 kJ**

**<u>To find $Q_{3-1}$ :</u>**

Heat transfer during Isothermal process = Work done during that process

$Q_{3-1} = W_{3-1} = mR\ T_3 \ln (V_1/ V_3) = mR\ T_3 \ln (P_3/ P_1)$

$[V_1/ V_3 = P_3/ P_1]$

=1.5 x 287 x 500 x ln (0.043/5)

**$Q_{3-1}$ = - 1023.735 kJ**

**To find $P_3$:**

In the process 2-3

$(T_3/T_2) = (P_3/ P_2)^{(n-1/n)}$

$P_3 = (T_3/T_2)^{(n/n-1)} \times P_2$

$= (500/1500)^{(1.3/0.3)} \times 5$
=0.043 bar

**Heat rejected during the process 3-1 is 1023.735 kJ**

**<u>THE NETWORK DONE DURING THE CYCLE :</u>**

According to First law of Thermodynamics

$$\oint \partial W = \oint \partial Q$$

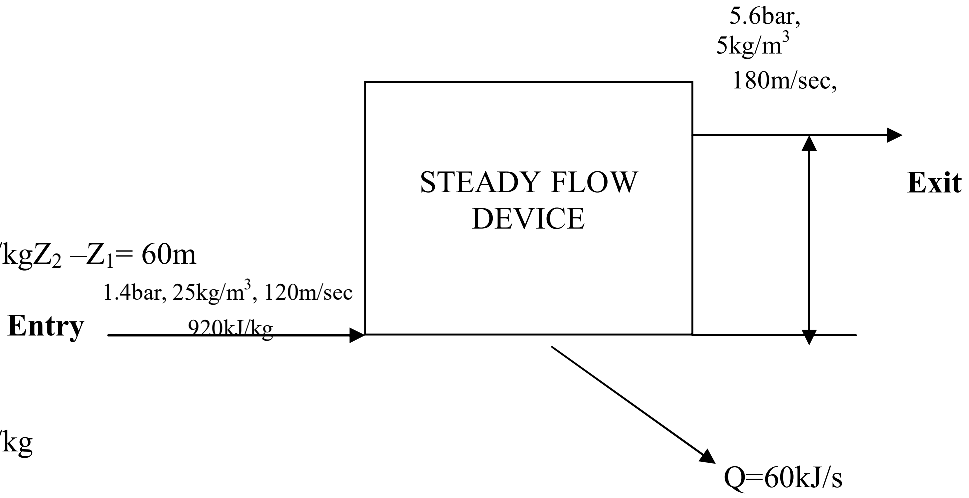
**The Network done during the cycle = The Heat transfer during the cycle**
=1507.5 kJ + 358.775kJ - 1023.735 kJ
**=842.54 kJ**

**Net work done during the cycle 1-2-3-1 is 842.54 kJ**

**In a certain steady flow process, 12kg of fluid per minute enters at a pressure of 1.4bar,density25kg/m$^3$ ,velocity 120m/sec and internal energy 920kJ/kg. The fluid properties at exit are pressure 5.6bar,density5kg/m$^3$ , velocity 180m/sec and internal energy 720kJ/kg. During the process, the fluid rejects 60kJ/sec of heat and rises through 60m. Determine work done during the process in kW.**

**<u>Data Given</u>:**

m= 12kg/min=0.2kg/sec



At Entry:

720kJ/kg

$P_1$=1.4bar
$\rho_1$=25kg/m$^3$:$v_1$=1/$\rho_1$
$V_1$=120m/sec
$U_1$=920kJ/kg=920 x10$^3$J/kg$Z_2$ –$Z_1$= 60m

At Exit:

$P_2$=5.6bar
$\rho_2$=5kg/m$^3$:$v_2$ =1/$\rho_2$
$V_2$=180m/sec
$U_2$=720kJ/kg=920 x10$^3$J/kg

Q= -60kJ/sec= -60 x10$^3$J/sec
$Z_2$ –$Z_1$= 60m

**<u>To Find</u>:** W

**<u>Solution:</u>**

Applying steady flow energy equation

$$Q + m (U_1+ P_1 v_1 +V_1^2/2 +gZ_1) = m (U_2+ P_2 v_2 +V_2^2/2 +gZ_2) +W$$

$$W= m\,[(U_1 - U_2)+(P_1v_1 - P_2v_2)+\left(\frac{V_1^2 - V_2^2}{2}\right)+g(Z_1 - Z_2)] + Q$$

$$=0.2\,[(920-720)\times10^3 + \left(\frac{1.4}{25}-\frac{5.6}{5}\right)\times10^5 + \frac{120^2-180^2}{2} + 9.81(-60)] -60\text{x}10^3$$ **($v_1$=1/$\rho_1$: $v_2$ =1/$\rho_2$)**

$$=0.2[200\text{x}10^3 + (-1.064\text{x}10^5) +(-9000) +(-588.6)]- 60\text{x}10^3$$

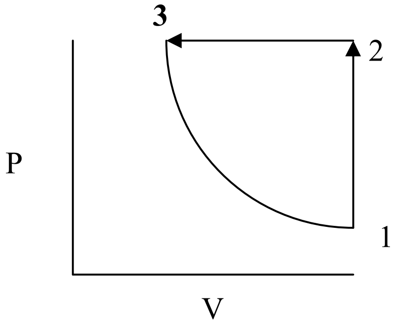
$$=0.2[200\text{x}10^3 -115988.6] - 60\text{x}10^3$$

**W = -43.198 kW**

**WORK INPUT TO THE STEADY FLOW DEVICE IS 43.198kW**

## ADDITIONAL PROBLEMS

**1. A system receives 200kJ of energy as heat at constant volume. Then it is cooled at constant pressure when 50kJ of work was done on the system while it rejects 70kJ of heat. Supposing the system is restored to the initial state by an adiabatic process, how much of work and heat will be done by the system.** (AU May 2006)



Data Given:

**$\delta Q_{1-2}$= 200kJ**
**$\delta W_{2-3}$= -50kJ**
**$\delta Q_{2-3}$= -70kJ**

**To Find:**

Heat and work interactions in the process 3-1

**Solution:**

Since 3-1 is an adiabatic process heat transferred in the process is zero.

Net Heat Transfer During The Cycle=§Q=$\delta Q_{1-2} + \delta Q_{2-3} + \delta Q_{3-1}$
=200-70-0
=130kJ

According to First law of thermodynamics For a Cyclic process
Net Heat Transfer=Net work Transfer

130kJ=$\delta W_{1-2} + \delta W_{2-3} + \delta W_{3-1}$

Since 1-2 is a constant Volume process $\delta W_{1-2}=0$

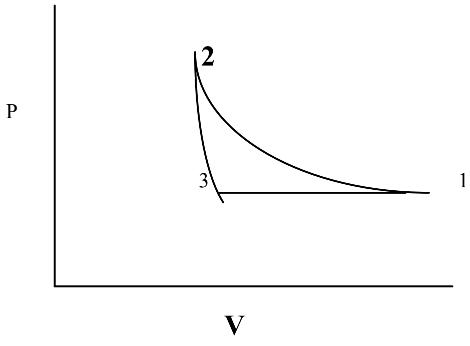
130=0 +(-50) +$\delta W_{3-1}$

$\delta W_{3-1}$=180kJ

**180kJ of work will be done by the system to restore the initial position**

**2. Air undergoes a cyclic process in a cylinder and piston arrangement. Atmospheric air at 1bar and 27 °C is compressed adiabatically to 10bar,expanded isothermally to initial pressure and bought to initial condition at constant pressure. Find the change in internal energy; enthalpy change heat transfer and work transfer for each process and for the cycle. (AU may 2003)**

**Data Given:**



**m=1kg**
**$P_1$=1bar=1x$10^5$ N/m$^2$**
**$T_1$=27°C=300K**
**$P_2$=10bar=10x$10^5$ N/m$^2$**

**To Find:**

Change in Internal energy& Enthalpy ,Heat transfer & work transfer for the process and cycle.

**Solution:**

### Process 1-2: Reversible adiabatic Compression

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\left(\nu-1/_\nu\right)} = \left(\frac{10}{1}\right)^{\left(.4/_{1.4}\right)}$$

$T_2 = (10)^{(.286)}$x300 =**579.6K**

**ΔU=** m $C_v$ ($T_2$ –$T_1$)= 1x 718x ( 579.6-300) = 200.745kJ/kg

In Reversible adiabatic process, Q=0 & W= **-ΔU, Hence**

**W= -200.745kJ/kg**

**Δh=** m $C_p$ ($T_2$ –$T_1$)=1x1000x(579.36-300)=**279.6kJ/kg**

### Process 2-3: Isothermal expansion

**$T_3$=579.6K ,$P_3$=1bar**

**In isothermal Process ΔU=0 & Δh=0**

**Work transfer** $W_{2\text{-}3}$= $Q_{2\text{-}3}$ = mR $T_2$ ln ($V_3$/ $V_2$) = mR $T_2$ ln ($P_2$/ $P_3$)

**=1x287x579.6 ln(10/1)= 383.024kJ**

### Process 3-1: constant pressure process

**W**$_{3\text{-}1}$= $P_3$ ($V_1$ –$V_3$)=mxR x($T_1$-$T_3$)=1x287x(300-579.6)= **-80.245kJ**

**$Q_{3\text{-}1}$=m Cp** ($T_1$-$T_3$)=1x 1000x (300-579.6)= **-279.6kJ**

In Constant pressure process **Δh =Q**

**Δh= -279.6kJ**

**ΔU=** m $C_v$ ($T_1$ –$T_3$)=1x718x(300-579.6)= **-200.745kJ/kg**

**FOR THE CYCLE**

§**Δh=Δh$_{1\text{-}2}$ +Δh$_{2\text{-}3}$ +Δh$_{3\text{-}1}$**
**=279.6 +0 + (-279.6)**
**=0**

§**ΔU=ΔU$_{1\text{-}2}$ +ΔU$_{2\text{-}3}$ +ΔU$_{3\text{-}1}$**
**= 200.745 +0 +(-220.745)**
**=0**

Since U and H are property of the system its cyclic integral will be always zero.

§W= -200.745 + 383.024 + (-80.245)

**§W= 102.034kJ**

**3. A fluid undergoes a reversible adiabatic compression from 0.5Mpa, 0.2m3 to 0.05m$^3$ according to the law PV$^{1.3}$ =constant. Determine the change in enthalpy, Internal Energy and entropy.** **(AU DEC2005)**

**Data Given**

$V_1$=0.2m$^3$

$P_1$=0.5MPa =0.5x10$^6$ N/m$^2$

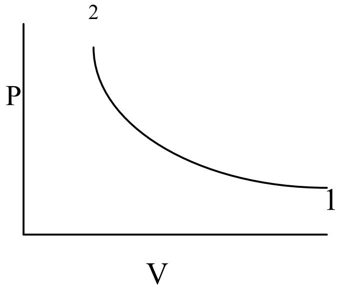
$V_2$=0.05 m$^3$

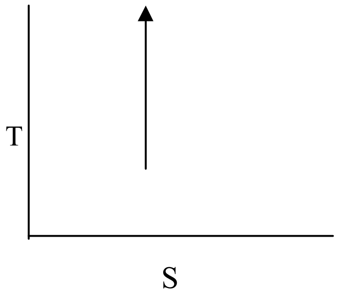
**PV$^{1.3}$ =constant**

**To Find:**

Δh, ΔU, ΔS

**Solution:**





Work done during adiabatic process is given by

$$W=\frac{P_1V_1-P_2V_2}{\gamma-1}$$

To find $P_2$:

$$\left(\frac{P_2}{P_1}\right)=\left(\frac{V_1}{V_2}\right)^{\gamma}=\left(\frac{0.2}{0.05}\right)^{1.3}$$

$$P_2=\left(\frac{0.2}{0.05}\right)^{1.3}x0.5x10^6=3.03x10^6\ N/m^2$$

$$W=\frac{(0.5x0.2-3.03x0.05)x10^6}{0.3}=\frac{-51500}{0.3}=\ -171.67kJ$$

Change in Enthalpy $\Delta h=h_2-h_1$

$=(U_2+P_2V_2)-(U_1+P_1V_1)$

$=(U_2-U_1)-(P_1V_1-P_2V_2)$

In Reversible adiabatic Process ΔU= -W

There fore **$\Delta U=(U_2-U_1)$= 171.67kJ**

Then Δh=171.67-(-51500)

**Δh =223.17kJ**

For Reversible adiabatic Process **ΔS=0**