## UNIT - I

## GAS POWER CYCLES

Air Standard cycles: Cycles using a perfect gas, having the properties of air useful in the study of the I. C. Engine because they represent a limit to which actual cycle may approach and they are subjected to simple mathematical and explanatory treatment.

## Assumptions made for analysis

- The properties of the working medium can be calculated by the application of the perfect gas equation.
i.e., $\mathrm{pV}=\mathrm{mRT}$
- The Specific heat of the substance remains same during all the processes in the cycle.
i.e., $C_{p} \& C_{v}$ are unchanged.
- The cycles are composed of reversible processes.
- The gas does not undergo any chemical changes.


## Various cycles

1. Carnot cycle.
2. Otto cycle.
3. Diesel cycle.
4. Stirling cycle.
5. Brayton cycle.
6. Erricson cycle.
7. Dual cycle.

## Air standard efficiency of a cycle

The thermal efficiency of an ideal air standard cycle is called the "Air standard efficiency".

In an ideal air standard cycle, the working fluid is air. The petrol and diesel engines working on Otto cycle and diesel cycle use petrol and diesel oil with air. This air fuel mixture behaves like air before the combustion takes place. The properties of combustion products are also not different from those of air. Therefore the efficiencies of petrol and diesel engines are calculated assuming them working on air standard cycles.

The efficiency of a cycle is given by,

$$
\begin{aligned}
& \text { Heat supplied - Heat rejected } \\
& \text { = ---------------------------------------- }
\end{aligned}
$$

## Carnot cycle

It is an ideal cycle and has maximum efficiency.
Carnot cycle efficiency is depending only on the temperature limits.

$$
\eta=\frac{W}{H S}=\frac{T_{1}-T_{2}}{T_{1}}
$$

Otto cycle: (Constant volume cycle)
Operation : Let the air is filled in the cylinder and the condition of air initially at point (1) are $p_{1,}$, $\mathrm{V}_{1}, \& \mathrm{~T}_{1}$. The piston compresses the air adiabatically from $\mathrm{V}_{1}$ to $\mathrm{V}_{2}$. At the end of the compression at point (2), the conditions of air are $\mathrm{p}_{2}, \mathrm{~V}_{2}, \& \mathrm{~T}_{2}$. Here the air occupies the clearance volume of the cylinder. Now the air is heated at constant volume by bringing the hot body is contact with the cylinder. This causes to rise the pressure from $p_{2}$ to $p_{3}$. At point (3), the conditions of air are $p_{3}, V_{3}, \& T_{3}$. Now the hot body is removed and the air expands adiabatically from (3) to (4). At point (4) the conditions of air are $\mathrm{p}_{4}, \mathrm{~V}_{4}, \& \mathrm{~T}_{4}$. Now air is cooled at constant volume by bringing a cold body in contact with the cylinder. This causes to drop the pressure from point (4) to (1). At point (1), the air finally returns to its original conditions $p_{1}, V_{1}, \& T_{1}$. Thus the cycle is completed.


Otto cycle consists of four processes:
$1-2 \rightarrow$ Isentropic compression.
$2-3 \rightarrow$ Constant volume heat addition.
3-4 $\rightarrow$ Isentropic expansion.
4-1 $\rightarrow$ Constant volume heat rejection.

## Efficiency of cycle

Process - 1 - $2 \rightarrow$ Isentropic compression
$\therefore$ Heat transferred $=Q_{1-2}=0$
Process - $2-3 \rightarrow$ Constant volume heat addition
$\therefore$ Heat supplied to air $=Q_{2-3}=\mathrm{m} \mathrm{C}_{\mathrm{v}}\left(\mathrm{T}_{3}-\mathrm{T}_{2}\right)$
Process - $3-4 \rightarrow$ Isentropic expansion
$\therefore$ Heat transferred $=Q_{3-4}=0$
Process - 4-1 $\rightarrow$ Constant volume heat rejection
$\therefore$ Heat rejected from air $=\mathrm{Q}_{4-1}=\mathrm{m} \mathrm{C}_{\mathrm{v}}\left(\mathrm{T}_{4}-\mathrm{T}_{1}\right)$
$\therefore$ Cycle efficiency $(\eta)=[$ Heat supplied - Heat rejected $] /$ Heat supplied

$$
=\frac{m C_{v}\left(T_{3}-T_{2}\right)-\mathrm{mC}_{\mathrm{v}}\left(\mathrm{~T}_{4}-\mathrm{T}_{1}\right)}{\mathrm{m} \mathrm{C}_{\mathrm{v}}\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right)}
$$

$$
\begin{aligned}
& =1-\frac{\left(T_{4}-T_{1}\right)}{\left(T_{3}--T_{2}\right)}
\end{aligned}
$$

$$
\text { Let } r=\text { compression ratio }=V_{1} / V_{2}=\text { Total Volume } / \text { Clearance Volume }
$$

Note:
But for Otto cycle, Compression ratio $=$ Expansion ratio $\left(\mathrm{V}_{4} / \mathrm{V}_{3}\right)$

## $\underline{I}_{1}$ in terms of $T_{2}$

1-2 $\rightarrow$ Isentropic process.
$\therefore$ we can write $T_{1} / T_{2}=\left(\mathrm{V}_{2} / \mathrm{V}_{1}\right)^{\gamma-1}=(1 / \mathrm{r})^{\gamma-1}$

$$
\therefore \mathrm{T}_{1}=\mathrm{T}_{2}(1 / \mathrm{r})^{\gamma-1}
$$

$\underline{I}_{4}$ in terms of $T_{3}$
3-4 $\rightarrow$ Isentropic process.
$\therefore$ we can write $\mathrm{T}_{4} / \mathrm{T}_{3}=\left(\mathrm{V}_{3} / \mathrm{V}_{4}\right)^{\gamma-1}$

$$
\begin{aligned}
& =(1 / r)^{\gamma-1} \\
& \therefore \mathrm{~T}_{4}=\mathrm{T}_{3}(1 / \mathrm{r})^{\gamma-1} \\
& =1-\frac{\mathrm{T}_{3}(1 / \mathrm{r})^{\gamma-1}-\mathrm{T}_{2}(\mathrm{l} / \mathrm{r})^{\gamma-1}}{\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& \eta=1-\frac{1}{(r)^{\gamma-1}}
\end{aligned}
$$

## Characteristics of Otto cycle

Cycle efficiency depends on,

- The compression ratio (r)
- The ratio of specific heats $(\gamma)$

Application: Used in petrol engines.
Diesel cycle : (Constant pressure cycle)

## Operation

It consists of four processes

1-2 $\boldsymbol{\rightarrow}$ Isentropic compression.
$2-3 \rightarrow$ Constant pressure heat addition.
3-4 $\rightarrow$ Isentropic expansion.
4-1 $\rightarrow$ Constant volume heat rejection.


Let the air is filled in the cylinder and the condition of air at point (1) are $p_{1}, V_{2}, \& T_{1}$. The piston compresses the air isentropically from $\mathrm{V}_{1}$ to $\mathrm{V}_{2}$. At the end of compression at point (2), the conditions of air are $p_{2}, V_{2}, \& T_{2}$ Here the air occupies the clearance volume of the cylinder. Now the air is heated at constant pressure by bringing the hot body in contact with the cylinder. This causes the volume to rise from $\mathrm{V}_{2}$ to $\mathrm{V}_{3}$. At point (3), the conditions of air are $\mathrm{p}_{3}$, $V_{3}, \& T_{3}$. Now the hot body is removed and the air expands adiabatically from point (3) to (4). At point (4) the conditions of air are $\mathrm{p}_{4}, \mathrm{~V}_{4}, \& \mathrm{~T}_{4}$. Now air is cooled at constant volume by bringing the cold body in contact with the cylinder. This causes to drop the pressure from point (4) to (1). At point (1), the air finally returns to its original conditions $p_{1}, V_{1}, \& T_{1}$. Thus the cycle is completed.

## Efficiency of cycle

Process - 1-2 $\rightarrow$ Isentropic compression
$\therefore$ Heat transferred $=Q_{1-2}=0$
Process - $2-3 \rightarrow$ Constant pressure heat addition
$\therefore$ Heat supplied to air $=Q_{2-3}=\mathrm{m} \mathrm{Cr}_{\mathrm{P}}\left(\mathrm{T}_{3}-\mathrm{T}_{2}\right)$
Process - 3-4 $\rightarrow$ Isentropic expansion
$\therefore$ Heat transferred $=Q_{3-4}=0$
Process - 4-1 $\rightarrow$ Constant volume heat rejection
$\therefore$ Heat rejected from air $=Q_{4-1}=\mathrm{m}_{\mathrm{v}}\left(\mathrm{T}_{4}-\mathrm{T}_{1}\right)$
$\therefore$ Cycle efficiency $(\eta)=[$ Heat supplied - Heat rejected $] /$ Heat supplied

$$
\begin{aligned}
& =\frac{m C_{p}\left(T_{3}-T_{2}\right)-\mathrm{mC}_{\mathrm{v}}\left(\mathrm{~T}_{4}-\mathrm{T}_{1}\right)}{m \mathrm{C}_{\mathrm{p}}\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right)} \\
& =1-\frac{\left(T_{4}-\mathrm{T}_{1}\right)}{\gamma\left(\mathrm{T}_{3}----\mathrm{T}_{2}\right)}
\end{aligned}
$$

Let $\mathrm{r}=$ compression ratio $=\mathrm{V}_{1} / \mathrm{V}_{2}$
$\rho=$ Cut off ratio $=V_{3} / V_{2}$

## $\underline{I}_{4}{\underline{i n ~ t e r m s ~ o f ~} T_{3}}^{3}$

Process - 3-4 $\rightarrow$ Isentropic process.

$$
\begin{aligned}
\therefore T_{4} / T_{3} & =\left(V_{3} / V_{4}\right)^{\gamma-1} \\
& =\left[\left(V_{3} / V_{2}\right) \times\left(V_{2} / V_{4}\right)\right]^{\gamma-1}=(\rho / r)^{\gamma-1} \\
\therefore T_{4} & =T_{3}(\rho / r)^{\gamma-1}
\end{aligned}
$$

$\underline{T}_{2} \underline{\text { in terms of }} T_{3}$
Process - $2-3 \rightarrow$ Constant pressure process.

$$
\begin{aligned}
\therefore \mathrm{V}_{2} / \mathrm{T}_{2} & =\mathrm{V}_{3} / \mathrm{T}_{3} \\
\quad \therefore \mathrm{~T}_{2} & =\mathrm{T}_{3}\left(\mathrm{~V}_{2} / \mathrm{V}_{3}\right)=\mathrm{T}_{3}(1 / \rho)
\end{aligned}
$$

$\underline{I}_{1} \underline{\text { in terms of }} T_{3}$
Process - $1-2 \rightarrow$ Isentropic process .

$$
\begin{aligned}
& \therefore \quad \mathrm{T}_{1} / \mathrm{T}_{2}=\left(\mathrm{V}_{2} / \mathrm{V}_{1}\right)^{\gamma-1}=(1 / \mathrm{r})^{\gamma-1} \\
& \therefore \quad \mathrm{~T}_{1}=\mathrm{T}_{2}(1 / \mathrm{r})^{\gamma-1} \\
& =T_{3}(1 / \rho)(1 / r)^{\gamma-1} \\
& =T_{3}\left[1 /\left(\rho r^{\gamma-1}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& \left.=1-\frac{1}{\gamma} \frac{(\rho / r)^{\gamma-1}-\left[1 /\left(\rho r^{\gamma-1}\right)\right]}{[------------------}\right]
\end{aligned}
$$

Note: Expansion ratio $\left(\mathrm{V}_{4} / \mathrm{V}_{3}\right) \neq$ Compression ratio $\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)$
Characteristics of Diesel cycle
Cycle efficiency depends on,

- Cut-off ratio ( $\rho$ )
- Compression ratio (r)
- Ratio of specific heat $(\gamma)$

Application: Used in Diesel engine.

## Dual Cycle (Limited Pressure or Mixed Cycle)

This cycle is a combination of Otto and Diesel cycles. In this cycle the heat is added partially at constant volume and partially at constant pressure. The advantage of this cycle is increased time to fuel for injection.



S
$1-2 \rightarrow$ Isentropic compression
$2-3 \rightarrow$ Constant volume heat addition
3-4 $\rightarrow$ Constant pressure heat addition
$4-5 \rightarrow$ Isentropic expansion
$5-1 \rightarrow$ Constant volume heat rejection

Mean effective pressure $=$ Workdone $/$ Stroke volume

$$
\begin{aligned}
& r_{p} \gamma(\rho-1)+\left(r_{p}-1\right)-r^{1-\gamma}\left(r_{p} \rho^{\gamma}-1\right) \\
& p_{m}=p_{1} r^{\gamma}[-------------------------------------------] \\
& (\gamma-1)(r-1)
\end{aligned}
$$

$r=$ Compression ratio $=V_{1} / V_{2}$
$\rho=$ Cut-off ratio $=V_{4} / V_{3}$
$r_{p}=$ Explosion ratio or Pressure ratio $=p_{3} / p_{2}$

## Brayton cycle (Joule cycle)



It consists of four processes:
$1-2 \rightarrow$ Isentropic compression.
$2-3 \rightarrow$ Constant pressure heat addition.
3-4 $\rightarrow$ Isentropic expansion.
$4-3 \rightarrow$ Constant pressure heat rejection.

## Flow diagram

Brayton cycle is used in Gas turbines.


## Operation

Let the air is filled in compressor and the conditions of air initially at point (1) are $p_{1}, V_{1, ~ \& ~}$ $T_{1}$. The compressor, compresses the air isentropically from $\mathrm{V}_{1}$ to $\mathrm{V}_{2}$. At the end of compression at point (2), the conditions of air are $p_{2}, V_{2}, \& T_{2}$. Now the air is heated in the heater at constant pressure. Therefore the volume of air is raised from $V_{2}$ to $V_{3}$. the conditions of air at point (3) is $p_{3}, V_{3}$ \& $T_{3}$. Now the air is in the gas turbine isentropically. The condition of air is denoted by $p_{4}, V_{4}, \& T_{4}$ at point (4). The air then allowed to pass through the cooler where heat is removed from air at constant pressure to bring the air to original condition $p_{1}, V_{1}, \& T_{1}$. Thus the cycle is completed.
Efficiency of cycle
Process - 1 - $2 \rightarrow$ Isentropic compression

$$
\therefore \quad \text { Heat transferred }=\mathrm{Q}_{1-2}=0
$$

Process - $2-3 \rightarrow$ Constant pressure heat addition

$$
\therefore \quad \text { Heat supplied to air }=\mathrm{Q}_{2-3}=\mathrm{m} \mathrm{C}_{\mathrm{p}}\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right)
$$

Process - 3-4 $\rightarrow$ Isentropic expansion
$\therefore \quad$ Heat rejected from the air $=Q_{4-1}=m C_{p}\left(T_{4}-T_{1}\right)$
$\therefore$ Cycle efficiency $(\eta)=[$ Heat supplied - Heat rejected $] /$ Heat supplied

$$
\begin{aligned}
& =\frac{m C_{p}\left(T_{3}-T_{2}\right)-\mathrm{m}_{\mathrm{p}}\left(\mathrm{~T}_{4}-\mathrm{T}_{1}\right)}{\mathrm{m} \mathrm{C}_{\mathrm{p}}\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right)} \\
& =\frac{\left(T_{3}-T_{2}\right)-\left(T_{4}-T_{1}\right)}{\left(T_{3}-T_{2}\right)}=1-\frac{\left(T_{4}-T_{1}\right)}{\left(T_{3}-T_{2}\right)}
\end{aligned}
$$

$$
\begin{aligned}
\text { Let } r & =\text { Compression ratio }=V_{1} / V_{2} \\
\qquad r_{p} & =\text { Pressure ratio }=p_{2} / p_{1}=p_{3} / p_{4}
\end{aligned}
$$

$\underline{I}_{\underline{1}} \underline{\text { in terms of }} T_{2}$
1-2 $\rightarrow$ Isentropic process.
$\therefore$ we can write $\mathrm{T}_{1} / \mathrm{T}_{2}=\left(\mathrm{V}_{2} / \mathrm{V}_{1}\right)^{\gamma-1}=(1 / \mathrm{r})^{\gamma-1}$

$$
\therefore \mathrm{T}_{1}=\mathrm{T}_{2}(1 / \mathrm{r})^{\gamma-1}
$$

$\underline{T}_{4}$ in terms of $T_{3}$
3-4 $\rightarrow$ Isentropic process.
$\therefore$ we can write $\left.\mathrm{T}_{4} / \mathrm{T}_{3}=\left(\mathrm{V}_{3} / \mathrm{V}_{4}\right)\right)^{\gamma-1}=(1 / r)^{\gamma-1}$

$$
\begin{aligned}
\therefore \mathrm{T}_{4}= & \mathrm{T}_{3}(1 / r)^{\gamma-1} \\
& =1-\mathrm{T}_{3}(1 / \mathrm{r})^{\gamma-1} \mathrm{~T}_{2}(1 / \mathrm{r})^{\gamma-1} \\
& \left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right)
\end{aligned}
$$

$$
\begin{gathered}
1 \\
\eta=1--------1 \\
(r)^{\gamma-1} \\
\hline
\end{gathered}
$$

But, $\quad p_{1} V_{1}^{\gamma}=p_{2} V^{\gamma}$

$$
\begin{aligned}
& \therefore \mathrm{p}_{2} / \mathrm{V}_{1}=\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{\gamma}=(\mathrm{r})^{\gamma} \\
& \therefore \mathrm{r}_{\mathrm{p}}=(\mathrm{r})^{\gamma} \\
& \text { (or) } \quad r=\left(\mathrm{r}_{\mathrm{p}}\right)^{1 / \gamma} \\
& \eta=1-\cdots-1 \\
&\left(r_{p}\right)^{(\gamma-1) / \gamma}
\end{aligned}
$$

## Characteristics of Brayton cycle

Cycle efficiency depends on,

- Specific heat ratio $(\gamma)$
- Pressure ratio ( $r_{p}$ )/Compression ratio ( $r$ )

Application: Used in Gas turbines.

## Effect of pressure ratio on the efficiency of Brayton cycle

The efficiency of the ideal cycle increases with the pressure ratio. The limit of maximum pressure is determined by the limiting temperature of the material of the turbine.


## Pressure ratio for the maximum work

We know that the net work is given by,

$$
\begin{aligned}
& W=W_{T}-W_{C}=H S-H R \\
& =m C_{p}\left(T_{3}-T_{2}\right)-m C_{p}\left(T_{4}-T_{1}\right) \\
& =m C_{p}\left(T_{3}-T_{4}\right)-m C_{p}\left(T_{2}-T_{1}\right) \\
& =m C_{p} T_{3}\left(1-\frac{T_{4}}{T_{3}}\right)-m C_{p} T_{1}\left(\frac{T_{2}}{T_{1}}-1\right)
\end{aligned}
$$

For gas turbines, the minimum temperature, $\mathrm{T}_{1}$ and the maximum temperature, $\mathrm{T}_{3}$ are fixed, i.e, $\mathrm{T}_{1} \& \mathrm{~T}_{3}$ are constants.

We can write, $\frac{T_{3}}{T_{4}}=\left(r_{p}\right)^{(\gamma-1) / \gamma}=\frac{T_{2}}{T_{1}}$

$$
\begin{aligned}
& \text { Let } z=\frac{\gamma-1}{\gamma} \\
& \begin{aligned}
\therefore W & =m C_{p}\left[T_{3}\left(1-\frac{1}{r_{p}^{z}}\right)-T_{1}\left(r_{p}^{z}-1\right)\right] \\
& =m C_{p}\left[T_{3}\left(1-r_{p}^{-z}\right)-T_{1}\left(r_{p}^{z}-1\right)\right]
\end{aligned}
\end{aligned}
$$

The only variable in the above equation is $r_{p}$. Therefore differentiating the above equation with respect to $r_{p}$ and equating to zero, to obtain the condition for maximum work,

$$
\begin{aligned}
& \frac{d W}{d r_{p}}=m C_{p}\left[T_{3}\left(0-(-z)\left(r_{p}^{-z-1}\right)\right)-T_{1}\left(z r_{p}^{z-1}-0\right)\right]=0 \\
& m C_{p}\left[T_{3} \frac{z}{r_{p}^{z+1}}-T_{1} z r_{p}^{z-1}\right]=0
\end{aligned}
$$

The product $m \mathrm{C}_{\mathrm{p}}$ is constant and cannot be zero.

$$
\begin{aligned}
T_{3} \frac{z}{r_{p}^{z+1}} & =T_{1} z r_{p}^{z-1} \\
\frac{T_{3}}{T_{1}} & =r_{p}^{2 z} \\
r_{p} & =\left(\frac{T_{3}}{T_{1}}\right)^{1 / 2 z}=\left(\frac{T_{3}}{T_{1}}\right)^{\gamma / 2(\gamma-1)}
\end{aligned}
$$

## Compression of Otto \& Diesel Cycle

| Otto Cycle | Diesel cycle |
| :---: | :---: |
| 1. Heat is added at constant volume | Heat is added at constant pressure. |
| 2. At the time of heat addition the piston is at TDC (or) IDC. | When the piston is at TDC, the heat addition begins and ends at a portion of piston backward stroke. |
| 3. Isentropic expansion takes place during the complete backward stroke of the piston. | Expansion starts from cut-off point in the backward stroke. |
| 4. Compression ratio is less (6:1 to 10 : 1) | Compression ratio is more (12: 1 to $22: 1$ ) |
| 5. Heat rejection takes place at constant volume. | Heat rejection takes place at constant volume. |
| 6. Petrol engines work on this cy | Diesel engines work on this cycle. |
| 7. Efficiency is more for some compression ratio. | Efficiency is less for some compression ratio. |

## Mean effective pressure ( MEP )

It is defined as the constant pressure acting on the piston that will produce the same amount of work as that produced by the actual varying pressure, during the cycle.


$$
\therefore \text { MEP }=\mathrm{p}_{\mathrm{m}}=\frac{\text { Workdone }}{\text { S-------------- }}
$$

Stroke volume $=\mathrm{V}_{1}-\mathrm{V}_{2}$

$$
\begin{aligned}
& =(\pi / 4) D^{2} L---m^{3} \\
& =(\pi / 4) D^{2} L(N / 60)---m^{3} / \mathrm{s}
\end{aligned}
$$

Significance: Mean effective pressure is usually preferred to compare air standard cycles of reciprocating engines. A cycle with a higher MEP will produce a large work output per unit swept volume and the engine size will be small for a given work output.

## Mean Effective Pressure of Otto Cycle

$$
p_{m}=\frac{W}{V_{1}-V_{2}}=\frac{W_{e}-W_{c}}{V_{1}-V_{2}}
$$

Work of expansion

$$
W_{e}=\frac{p_{3} V_{3}-p_{4} V_{4}}{\gamma-1}=\frac{p_{3} V_{c}-p_{4} V_{1}}{\gamma-1}
$$

Work of compression $\quad W_{c}=\frac{p_{2} V_{2}-p_{1} V_{1}}{\gamma-1}=\frac{p_{2} V_{c}-p_{1} V_{1}}{\gamma-1}$

$$
\begin{aligned}
W=W_{e}-W_{c} & =\frac{p_{3} V_{c}-p_{4} V_{1}-p_{2} V_{c}+p_{1} V_{1}}{\gamma-1} \\
& =\frac{\left(p_{3}-p_{2}\right) V_{c}-\left(p_{4}-p_{1}\right) V_{1}}{\gamma-1} \\
V_{1}-V_{2} & =\left(\frac{V_{1}}{V_{2}}-1\right) V_{2}=(r-1) V_{2} \\
p_{m} & =\frac{\left(p_{3}-p_{2}\right) V_{c}-\left(p_{4}-p_{1}\right) V_{1}}{(\gamma-1)(r-1) V_{2}} \\
& =\frac{\left(p_{3}-p_{2}\right)-\left(p_{4}-p_{1}\right) r}{(\gamma-1)(r-1)} \\
& =\frac{\left(\frac{p_{3}}{p_{2}}-1\right) p_{2}-\left(\frac{p_{4}}{p_{1}}-1\right) p_{1} r}{(\gamma-1)(r-1)} \\
& =\frac{\left(r_{p}-1\right) p_{2}-\left(r_{p}-1\right) p_{1} r}{(\gamma-1)(r-1)} \\
& =\frac{\left(r_{p}-1\right)\left(p_{2}-p_{1} r\right)}{(\gamma-1)(r-1)}
\end{aligned}
$$

$$
\begin{aligned}
= & \frac{\left(r_{p}-1\right) p_{1} r\left(\frac{p_{2}}{p_{1} r}-1\right)}{(\gamma-1)(r-1)} \\
= & \frac{\left(r_{p}-1\right) p_{1} r\left(\left(\frac{V_{1}}{V_{2}}\right)^{\gamma} \frac{1}{r}-1\right)}{(\gamma-1)(r-1)} \\
= & \frac{\left(r_{p}-1\right) p_{1} r\left(\frac{r^{\gamma}}{r}-1\right)}{(\gamma-1)(r-1)} \\
p_{m}= & p_{1} r \frac{\left(r_{p}-1\right)\left(r^{\gamma-1}-1\right)}{(\gamma-1)(r-1)}
\end{aligned}
$$

$r_{p}=$ pressure ratio $=p_{3} / p_{2}$

$$
=p_{4} / p_{1}
$$

## Mean Effective Pressure of Diesel Cycle

$$
p_{m}=p_{1} r^{\gamma} \frac{\gamma(\rho-1)-r^{1-\gamma}\left(\rho^{\gamma}-1\right)}{(\gamma-1)(r-1)}
$$

$\rho=$ Cut-off ratio.
$r=$ Compression ratio.

## Mean Effective Pressure of Dual Cycle

$$
p_{m}=p_{1} r^{\gamma} \frac{\gamma\left[\beta(\rho-1)+(\beta-1)-r^{1-\gamma}\left(\beta \rho^{\gamma}-1\right)\right]}{(\gamma-1)(r-1)}
$$

$\beta=$ Pressure or Explosion ratio $=p_{3} / p_{2}$

## Comparison of Otto, Diesel and dual Cycles

For same compression ratio


Fig shows the comparison for the air standard efficiency of Otto, Diesel and Dual cycles at various compression ratios with given cut-off ratio. It can be observed that the efficiency increases with increase in the compression ratio. For a given compression ratio,

$$
\eta_{\text {otto }}>\eta_{\text {Dual }}>\eta_{\text {Diesel }}
$$

## For the same compression ratio and the same heat input

We know that, $\eta=1$ - Heat Rejected / Heat Supplied
If heat supplied is constant, the efficiency is dependent of heat rejection. p-V and T-s diagrams are shown for Otto, Diesel and Dual cycles for the same compression ratio and the same heat input.


From, T-s diagram, it can be observed that the heat rejection is least in Otto cycle and highest in Diesel cycle. Thus, Otto cycle is the most efficient cycle and the Diesel cycle is least efficient.

$$
\eta_{\text {Otto }}>\eta_{\text {Dual }}>\eta_{\text {Diesel }}
$$

## Comparison of Otto and Diesel cycles for same constant maximum pressure and heat

 supplied
(a)

(b)

Figure shows the Otto and Diesel cycles on p-v and T-s diagrams for constant maximum pressure and heat input. For the maximum pressure the points 3 and 3 ' must lie on a constant pressure line. On T-s diagram the heat rejected from the diesel cycle is represented by the area under the line 4 to 1 and this area is less than the Otto cycle area under the curve 4 ' to 1 , hence the Diesel cycle is more efficient than the Otto cycle.

## PROBLEMS

1. The efficiency of an Otto cycle is $60 \%$ and $\gamma=1.5$, what is the compression ratio? Given:

Cycle efficiency $(\eta)=0.6$

$$
\gamma=1.5
$$

Required : r

Solution:

$$
\text { Compression ratio }(r)=V_{1} / V_{2}
$$

we know that, $\eta=1-1 / r^{\gamma-1}$

$$
\begin{aligned}
& 0.6=1-1 / r^{1.5-1} \\
& r=6.25-- \text { Ans }
\end{aligned}
$$

2. An engine of $\mathbf{2 5 0 ~ m m}$ bore and 375 mm stroke works on Otto cycle. The clearance volume is $0.00263 \mathrm{~m}^{3}$. The initial pressure and temperature are 1 bar and $50^{\circ} \mathrm{C}$. If the maximum pressure is limited to 25 bar, find (i) air standard efficiency of the cycle (ii) the mean effective pressure of the cycle.

Given:

$$
\begin{array}{ll}
\text { Bore }(\mathrm{d}) & =0.25 \mathrm{~m} \\
\text { Stroke }(\mathrm{L}) & =0.375 \mathrm{~m} \\
\text { Clearance volume }\left(\mathrm{V}_{2}\right)=0.00263 \mathrm{~m}^{3} \\
\text { Initial pressure } \quad\left(\mathrm{p}_{1}\right)=1 \mathrm{bar} \\
\text { Initial temperature }\left(\mathrm{T}_{1}\right)=50^{\circ} \mathrm{C}=323 \mathrm{~K} \\
\text { Maximum pressure }\left(\mathrm{p}_{3}\right)=25 \mathrm{bar}
\end{array}
$$

Required: (i) $\eta$ (ii) $p_{m}$


Solution:
(i) Cycle efficiency $(\eta)=1-1 / r^{\gamma-1}$

$$
\begin{aligned}
r & =\text { compression ratio }=\mathrm{V}_{1} / \mathrm{V}_{2} \\
\mathrm{~V}_{1} & =\text { stroke volume }+ \text { clearance volume } \\
& =\left(\mathrm{V}_{1}-\mathrm{V}_{2}\right)+\mathrm{V}_{2} \\
& =(\pi / 4) \mathrm{D}^{2} \mathrm{~L}+\mathrm{V}_{2} \\
& =(\pi / 4) \times 0.25^{2}+0.375+0.00263 \\
& =0.021038 \mathrm{~m}^{3} \\
\therefore \quad & r
\end{aligned}
$$

(ii) Mean effective pressure $\left(\mathrm{p}_{\mathrm{m}}\right)$

$$
\begin{aligned}
& p_{m}=p_{1} r \underset{(\gamma-1)(r-1)}{[-------------1)} \quad \rightarrow \text { for Otto cycle } \\
& \mathrm{r}_{\mathrm{p}}=\mathrm{p}_{3} / \mathrm{p}_{2} \\
& p_{2} V_{2}^{\gamma}=p_{1} V_{1}^{\gamma} \\
& p_{2}=\left(V_{1} / V_{2}\right)^{\gamma} p_{1}=(8)^{1.4} \times 1=18.38 \text { bar } \\
& \therefore r_{p}=25 / 18.38=1.36 \\
& \left(8^{1.4-1}-1\right)(1.36-1) \\
& \mathrm{p}_{\mathrm{m}}=1 \times 8[-----------------------\mathrm{-}) \\
& =1.334 \text { bar }--- \text { Ans }
\end{aligned}
$$

3. Find the air standard efficiency of a diesel cycle when the compression ratio and cut-off ratio are 15 \& 1.84 respectively. Assume $\gamma=1.4$.
Given:
Compression ratio (r) $=15$

$$
\begin{aligned}
\text { Cut }- \text { off ratio } \quad(\rho) & =1.84 \\
\gamma & =1.4
\end{aligned}
$$

Required: $\eta$
Solution:

4. In a diesel engine the pressure at the beginning of compression is 1 bar. Compression ratio is 14: 1 and cut-off takes place at $10 \%$ of the stroke. Calculate the air standard efficiency and ideal mep of the cycle ( $\gamma=1.4$ for air ).
Given:
Initial pressure $\left(\mathrm{p}_{1}\right)=1$ bar.
Compression ratio (r) $=14$.
Cut - off takes place at $10 \%$ of stroke,

$$
\text { i.e., } \begin{aligned}
V_{3}-V_{2} & =0.1\left(V_{1}-V_{2}\right) \\
\gamma & =1.4
\end{aligned}
$$

Required: $\eta$ \& $\mathrm{p}_{\mathrm{m}}$
Solution:

$$
\begin{gathered}
\rho=\text { cut }- \text { off ratio }=\mathrm{V}_{3} / \mathrm{V}_{2} \\
\mathrm{r}=\mathrm{V}_{1} / \mathrm{V}_{2}=14 \\
\therefore \mathrm{~V}_{1}=14 \mathrm{~V}_{2} \\
\therefore \mathrm{~V}_{3}-\mathrm{V}_{2}=0.1\left(14 \mathrm{~V}_{2}-\mathrm{V}_{2}\right) \\
\mathrm{V}_{3}=2.3 \mathrm{~V}_{2} . \\
\mathrm{V}_{3} / \mathrm{V}_{2}=\rho=2.3
\end{gathered}
$$

$$
=0.577 \text {----- Ans }
$$

Mean effective pressure $\left(p_{m}\right)$

$$
\begin{aligned}
& \gamma(\rho-1)-r^{1-\gamma}\left(\rho^{\gamma}-1\right) \\
& \mathrm{p}_{\mathrm{m}}=\mathrm{p}_{1} \mathrm{r}^{\gamma}[------------------------\mathrm{-}] \quad \rightarrow \text { for Diesel cycle } \\
& 1.4(2.3-1)-14^{1-1.4}\left(2.3^{1.4}-1\right)
\end{aligned}
$$

$$
\begin{aligned}
& =0.612 \text {----- Ans }
\end{aligned}
$$

5. Calculate air standard efficiency of a gas turbine power plant working between 1 bar and 6 bar. If the minimum and maximum temperatures are $27^{\circ} \mathrm{C}$ and $527^{\circ} \mathrm{C}$ respectively, determine the temperature after isentropic compression and after isentropic expansion.
Given:
Initial pressure ( $\mathrm{p}_{1}$ )
Pressure after compression ( $p_{1}$ )

$$
\begin{aligned}
& =1 \mathrm{bar} \\
& =6 \mathrm{bar} \\
& =27^{\circ} \mathrm{C}=300 \mathrm{~K} \\
& =527^{\circ} \mathrm{C}=800 \mathrm{~K}
\end{aligned}
$$

Required: $\eta, T_{2}$ \& $T_{4}$
Solution:



Cycle efficiency $(\eta) \quad=1-1 /\left(r_{p}\right)^{(\gamma-1) / \gamma}$
$r_{p}=$ pressure ratio $=p_{2} / p_{1}=6 / 1=6$
$\therefore \quad \eta=1-1 /(6)\left(^{1.4-1) / 1.4}=\mathbf{0 . 4}\right.$----- Ans
Process 1-2 $\rightarrow$ isentropic

$$
\begin{aligned}
T_{2} / T_{1} & =\left(p_{2} / p_{1}\right)^{(\gamma-1) / \gamma} \\
T_{2} / 300 & =(6 / 1)^{1.4-1 / 1.4}=479.5 \mathrm{~K} \text {---- Ans }
\end{aligned}
$$

Process 3-4 $\rightarrow$ Isentropic

$$
\begin{aligned}
T_{4} / T_{3} & =\left(p_{4} / p_{3}\right)^{(\gamma-1) / \gamma}=\left(p_{2} / p_{1}\right)^{(\gamma-1) / \gamma} \\
T_{4} & =800(6 / 1)^{(1.4-1) / 1.4}=479.5 \mathrm{~K}-\cdots-- \text { Ans }
\end{aligned}
$$

6. A diesel engine, operating on air standard cycle, has 20 cm bore and 30 cm stroke. The clearance volume is $420 \mathrm{~cm}^{3}$. The fuel is injected at constant pressure for $5 \%$ of the stroke. Calculate air standard efficiency.
Given:
Bore
(D) $=0.2 \mathrm{~m}$
Stroke
$(\mathrm{L})=0.3 \mathrm{~m}$

Clearance volume $\left(V_{2}\right)=420 \mathrm{~cm}^{3}=420 \times 10^{-6} \mathrm{~m}^{3}$
Cut-off volume $\left(\mathrm{V}_{3}-\mathrm{V}_{2}\right)=0.05\left(\mathrm{~V}_{1}-\mathrm{V}_{2}\right)$
Required: $\eta$

## Solution:

$$
\begin{aligned}
& \eta=1-\underset{\gamma r^{\gamma-1}}{\substack{1 \\
\rho-----------]}} \\
& V_{1}-V_{2}=(\pi / 4) D^{2} L=(\pi / 4) \times 0.2^{2} \times 0.3=0.009424 \mathrm{~m}^{3} \\
& \therefore \mathrm{~V}_{3}-\left(420 \times 10^{-6}\right)=0.05(0.009424) \\
& V_{3}=0.0008912 \mathrm{~m}^{3} \\
& \mathrm{~V}_{1}=\left(\mathrm{V}_{1}-\mathrm{V}_{2}\right)+\mathrm{V}_{2} \\
& =0.009424+\left(420 \times 10^{-6}\right) \\
& =0.009844 \mathrm{~m}^{3} \\
& \therefore r=V_{1} / V_{2}=0.009844 / 420 \times 10^{-6}=23.43 \\
& \rho=V_{3} / V_{2}=0.0008912 / 420 \times 10^{-6}=2.12
\end{aligned}
$$

$$
\begin{aligned}
& =0.663 \text {----- Ans }
\end{aligned}
$$

7. Air enters an air standard Otto cycles at $100 \mathrm{kN} / \mathrm{m}^{2}$ and 290 K . The ratio of heat rejection to heat supplied is 0.4 . The maximum temperature in the cycle is 1500 K . Find (a) efficiency, (b) net work, (c) mep, \& (d) compression ratio (r).
Given:
$\begin{array}{ll}\text { Initial pressure }\left(\mathrm{p}_{1}\right) & =100 \mathrm{kN} / \mathrm{m}^{2}=1 \mathrm{bar} \\ \text { Initial temperature }\left(\mathrm{T}_{1}\right) & =290 \mathrm{~K}\end{array}$

Heat rejection $/$ Heat supplied $=0.4$
Maximum temperature $\left(\mathrm{T}_{3}\right)=1500 \mathrm{~K}$
Required: (a) $\eta$ (b) $W_{\text {net }}$ (c) mep (d) $r$

## Solution:

(a) Cycle efficiency $(\eta)=1-1 / /^{\gamma-1}$

$$
\begin{aligned}
& =\text { (heat supplied }- \text { heat rejection) } / \text { heat supplied } \\
& =1-(\mathrm{HR} / \mathrm{HS})=1-0.4=0.6--- \text { Ans }
\end{aligned}
$$

(b) Net work $\left(W_{\text {net }}\right) \quad=$ Heat supplied - Heat rejected

$$
\text { Heat supplied }=m C_{v}\left(T_{3}-T_{2}\right)
$$

Take $\mathrm{m}=1 \mathrm{~kg} \quad \& \mathrm{C}_{\mathrm{v}}=0.717 \mathrm{~kJ} / \mathrm{kgK}$



To find $T_{2}$

$$
\begin{aligned}
\mathrm{T}_{2} / \mathrm{T}_{1} & =\left(\mathrm{p}_{2} / \mathrm{p}_{1}\right)^{(\gamma-1) / \gamma}=\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{(\gamma-1)} \\
& =\mathrm{r}^{\gamma-1}
\end{aligned}
$$

From, $\quad \eta=1-\left(1 / r^{\gamma-1}\right)$
i.e., $\quad 0.6=1-\left(1 / r^{1.4-1}\right)$

$$
r=9.88
$$

$$
\therefore \mathrm{T}_{2}=290(9.88)^{1.4-1}=725 \mathrm{~K}
$$

$\therefore \quad$ Heat supplied $=1 \times 0.717 \times(1500-725)=555.675 \mathrm{~kJ}$

$$
\text { Heat rejected }=0.4(555.675)=222.27 \mathrm{~kJ}
$$

$$
\therefore \mathrm{W}_{\text {net }}=555.675-222.27=333.405 \mathrm{~kJ} \quad---\mathrm{Ans}
$$

(c) mep

$$
2-3 \rightarrow \text { Constant volume process }
$$

$$
\begin{aligned}
& \mathrm{p}_{3} / \mathrm{T}_{3}=\mathrm{p}_{2} / \mathrm{T}_{2} \\
& \therefore \mathrm{p}_{3} / \mathrm{p}_{2}=\mathrm{T}_{3} / \mathrm{T}_{2}
\end{aligned}
$$

To find $T_{2}$

$$
\mathrm{T}_{2} / \mathrm{T}_{1}=\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{\gamma-1}
$$

$$
\begin{aligned}
& \mathrm{p}_{\mathrm{m}}=\mathrm{p}_{1} \mathrm{r} \underset{(\gamma-1)(\mathrm{r}-1)}{\left.\stackrel{\left(r^{\gamma-1}-1\right)\left(r_{p}-1\right)}{[-------1}\right]} \rightarrow \text { for Otto cycle } \\
& \mathrm{r}_{\mathrm{p}}=\mathrm{p}_{3} / \mathrm{p}_{2}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{T}_{2}=290(9.88)^{1.4-1}=724.94 \mathrm{~K} \\
& r_{p}=p_{3} / p_{2}=1500 / 724.94=2.069 \\
& \left(9.88^{1.4-1}-1\right)(2.069-1) \\
& \mathrm{p}_{\mathrm{m}}=1 \times 9.88 \\
& (1.4-1)(9.88-1) \\
& =4.459 \text { bar }--- \text { Ans }
\end{aligned}
$$

(d) Compression ratio ( r ) $=9.88 \quad---$ Ans
8. Air enters a Brayton cycle at $100 \mathrm{kPa}, \& 300 \mathrm{~K}$. The compression ratio is $8: 1$. The maximum temperature in the cycle is 1300 K . Find (i) air standard efficiency, (ii) compressor and turbine work and (iii) work ratio.
Given:
Initial pressure $\left(\mathrm{p}_{1}\right) \quad=1$ bar
Initial temperature $\left(\mathrm{T}_{1}\right)=300 \mathrm{~K}$
Compression ratio (r) $=8$
Maximum temperature $\left(\mathrm{T}_{3}\right)=1300 \mathrm{~K}$
Required: (i) $\eta$ (ii) $W_{C} \& W_{T}$ (iii) $\left(W_{T}-W_{C}\right) / W_{T}$
Solution:
(i) Cycle efficiency $(\eta) \quad=1-1 /\left(r_{p}\right)^{\gamma-1 / \gamma}$

$$
=1-1 /(r)^{\gamma-1}=1-1 /(8)^{1.4-1}=0.565---- \text { Ans }
$$

(ii) Compressor work $\left(\mathrm{W}_{\mathrm{C}}\right)=\gamma\left(\mathrm{p}_{2} \mathrm{~V}_{2}-\mathrm{p}_{1} \mathrm{~V}_{1}\right) /(\gamma-1)$

$$
=\gamma \mathrm{mR}\left(\mathrm{~T}_{2}-\mathrm{T}_{1}\right) /(\gamma-1)
$$

Take $m=1 \mathrm{~kg}$ \& $R=287 \mathrm{~J} / \mathrm{kgK}$


To find $T_{2}$

$$
\begin{array}{rlrl} 
& \mathrm{T}_{2} / \mathrm{T}_{1} & =\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{\gamma-1} \\
\mathrm{~T}_{2} & =300(8)^{1.4-1}=689.2 \mathrm{~K} \\
\therefore \quad & & \mathrm{~W}_{\mathrm{C}} & =1.4 \times 1 \times 287 \times(689.2-300) /(1.4-1)
\end{array}
$$

= 390951.4 J/kg ---- Ans

Turbine network $\left(W_{T}\right)=\gamma \mathrm{mR}\left(\mathrm{T}_{3}-\mathrm{T}_{4}\right) /(\gamma-1)$
To find $T_{4}$
3-4 $\rightarrow$ Isentropic process.

$$
\begin{aligned}
\mathrm{T}_{4} / \mathrm{T}_{3} & =\left(\mathrm{p}_{4} / \mathrm{p}_{3}\right)^{\gamma-1 / \gamma}=\left(\mathrm{p}_{1} / \mathrm{p}_{2}\right)^{\gamma-1 / \gamma}=\left(\mathrm{V}_{2} / \mathrm{V}_{1}\right)^{\gamma-1} \\
\mathrm{~T}_{4} / 1300 & =(1 / 8)^{1.4-1} \\
\therefore \quad & \mathrm{~T}_{4}
\end{aligned}=565.86 \mathrm{~K} .
$$

(iii) Work ratio
9. 1 kg of air is taken through a diesel cycle. Initially the air is at $15^{\circ} \mathrm{C}$ and 1 atm . The compression ratio is 15 and the heat added is 1850 kJ . Calculate the ideal cycle efficiency.
Given:
Mass of air (m)

$$
=1 \mathrm{~kg}
$$

Initial temperature of air $\left(p_{1}\right)=15^{\circ} \mathrm{C}=288 \mathrm{~K}$
Initial pressure ( $\mathrm{p}_{1}$ )

$$
=1 \mathrm{~atm}=1.013 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}
$$

Compression ratio (r)

$$
=15
$$

Heat added ( $\mathrm{Q}_{2-3}$ )
$=m C_{P}\left(T_{3}-T_{2}\right)=1850 \mathrm{~kJ}$
Required: $\eta$
Solution:


$$
\eta=1-\begin{gathered}
1 \\
\gamma r^{\gamma-1}
\end{gathered} \frac{\rho^{\gamma-1}-1}{\rho----------]}
$$

Cut - off ratio $(\rho)=V_{3} / V_{2}=\left(T_{3} / T_{2}\right)$
To find $\mathrm{T}_{2}$

$$
\begin{aligned}
\therefore \quad \mathrm{T}_{2} / \mathrm{T}_{1} & =\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{\gamma-1} \\
\mathrm{~T}_{2} & =288(15)^{1.4-1} \\
& =850.8 \mathrm{~K}
\end{aligned}
$$

To find $T_{3}$

$$
\begin{array}{rlrl} 
& 1850 & =1 \times 1.005\left(\mathrm{~T}_{3}-850.8\right) \\
\mathrm{T}_{3} & =2690.8 \mathrm{~K} \\
\therefore \quad & \rho & =2690.8 / 850.8=3.16
\end{array}
$$

$$
\eta=1-\frac{1}{1.4(15)^{1.4-1}} \frac{3.16^{1.4-1}-1}{3 .--------------------16-1}
$$

$$
=0.5514 \text {---- Ans }
$$

10. An engine working on the Otto cycle is supplied with air at $0.1 \mathrm{MPa}, 35^{\circ} \mathrm{C}$. The compression ratio is 8 . Heat supplied is $2100 \mathrm{~kJ} / \mathrm{kg}$. Calculate the maximum pressure and temperature of the cycle, the cycle efficiency and the mean effective pressure.
Given:

| Initial pressure $\left(\mathrm{p}_{1}\right)$ | $=1 \mathrm{bar}$ |
| :--- | :--- |
| Initial temperature $\left(\mathrm{T}_{1}\right)$ | $=35^{\circ} \mathrm{C}=308 \mathrm{~K}$ |
| Compression ratio $(\mathrm{r})$ | $=8$ |

Heat supplied $=\mathrm{C}_{\mathrm{v}}\left(\mathrm{T}_{3}-\mathrm{T}_{2}\right)=2100 \mathrm{~kJ} / \mathrm{kg}$
Required: $\mathrm{p}_{3}, \mathrm{~T}_{3}, \eta \& \mathrm{p}_{\mathrm{m}}$


Solution:

To find $p_{2}$

$$
\begin{aligned}
\mathrm{p}_{3} / \mathrm{T}_{3} & =\mathrm{p}_{2} / \mathrm{T}_{2} \\
\mathrm{p}_{3} & =\mathrm{p}_{2}\left(\mathrm{~T}_{3} / \mathrm{T}_{2}\right)
\end{aligned}
$$

$$
\mathrm{p}_{1} \mathrm{~V}_{1}^{\gamma}=\mathrm{p}_{2} \mathrm{~V}_{2}^{\gamma}
$$

$$
\begin{array}{ll} 
& \mathrm{p}_{2} / \mathrm{p}_{1}=\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{\gamma}=(8)^{1.4} \\
\therefore & \mathrm{p}_{2}=1 \times(8)^{1.4}=18.38 \mathrm{bar}
\end{array}
$$

To find $T_{2}$

$$
\left.\begin{array}{rl} 
& \left(\mathrm{p}_{2} / \mathrm{p}_{1}\right)^{(\gamma-1) / \gamma}
\end{array}=\mathrm{T}_{2} / \mathrm{T}_{1}\right)
$$

To find T 3

$$
\begin{aligned}
& 2100=0.717\left(\mathrm{~T}_{3}-707.6\right) \\
& T_{3}=3636.5 \cdots \text { Ans } \\
& \therefore \quad \mathrm{p}_{3}=18.38(3636.5 / 707.6)=94.46 \text { bar }-- \text { Ans } \\
& \eta=1-1 /(r)^{\gamma-1}=1-1 /(8)^{1.4-1}=0.5647 \text {---- Ans } \\
& \left(r^{\gamma-1}-1\right)\left(r_{p}-1\right) \\
& p_{m}=p_{1} r\left[\begin{array}{ll}
(\gamma-1)(r-1)
\end{array}\right] \rightarrow \text { for Otto cycle } \\
& r_{p}=p_{3} / p_{2}=94.46 / 18.38=5.4 \\
& \left(8^{1.4-1}-1\right)(5.4-1)
\end{aligned}
$$

$$
\begin{aligned}
& \text { = } 16.31 \text { bar ---- Ans }
\end{aligned}
$$

11. Air enters an Otto cycle at $27^{\circ} \mathrm{C}$ and 1 bar. The compression ratio is 7.5. The maximum temperature in the cycle is 1000 K . Find mep and efficiency of the cycle.
Given:
Initial temperature $\left(\mathrm{T}_{1}\right)=27^{\circ} \mathrm{C}=300 \mathrm{~K}$
Initial pressure $\left(p_{1}\right)=1$ bar
Compression ratio (r) $\quad=7.5$
Maximum temperature $\left(\mathrm{T}_{3}\right)=1000 \mathrm{~K}$
Required: $\mathrm{p}_{\mathrm{m}} \& \eta$
Solution:

$$
p_{m}=p_{1} r \frac{\left(r^{\gamma-1}-1\right)\left(r_{p}-1\right)}{(\gamma-------------1)(r-1)} \rightarrow \text { for Otto cycle }
$$

$$
r_{p}=\text { pressure ratio }=p_{3} / p_{2}
$$

$2-3 \rightarrow$ Constant volume process

$$
\begin{aligned}
& \mathrm{p}_{3} / \mathrm{T}_{3}=\mathrm{p}_{2} / \mathrm{T}_{2} \\
& \mathrm{p}_{3} / \mathrm{p}_{2}=\mathrm{T}_{3} / \mathrm{T}_{2}
\end{aligned}
$$




To find $T_{2}$

$$
\begin{aligned}
& \mathrm{T}_{2} / \mathrm{T}_{1}=\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{\gamma-1} \\
& \mathrm{~T}_{2}=300 \times(7.5)^{1.4-1}=671.6 \mathrm{~K} \\
& \therefore \quad r_{p}=p_{3} / p_{2}=1000 / 671.6=1.49 \\
& \left(7.5^{1.4-1}-1\right)(1.49-1) \\
& \mathrm{p}_{\mathrm{m}}=1 \times 7.5[---------------------------\mathrm{-}] \\
& \text { = } 1.751 \text { bar ---- Ans } \\
& \eta=1-\left[1 / r^{\gamma-1}\right] \\
& =1-\left[1 / 7.5^{1.4-1}\right]=0.553 \text {--- Ans }
\end{aligned}
$$

12. In an engine working on diesel cycle inlet pressure \& temperature are 1 bar and $17^{\circ} \mathrm{C}$ respectively. Pressure at the end of adiabatic compression is 35 bar. The ratio of expansion, i.e., after constant pressure heat addition is 5 . Calculate the heat addition, heat rejection and the efficiency of the cycle. Assume $\gamma=1.4, \mathrm{C}_{\mathrm{p}}=$ $1.004 \mathrm{~kJ} / \mathrm{kgK}$ and $\mathrm{C}_{\mathrm{v}}=0.717 \mathrm{~kJ} / \mathrm{kgK}$.
Given:
Initial pressure ( $\mathrm{p}_{1}$ )

$$
=1 \mathrm{bar}
$$

Initial temperature ( $\mathrm{T}_{1}$ )

$$
=17^{\circ} \mathrm{C}=290 \mathrm{~K}
$$

Pressure after compression $\left(p_{2}\right)=35$ bar
Expansion ratio $\left(\mathrm{V}_{4} / \mathrm{V}_{3}\right)=5$
Required: Heat addition, Heat rejection \& Cycle efficiency
Solution:

$$
\text { Heat addition }=m C_{P}\left(T_{3}-T_{2}\right)
$$

To find $T_{2}$

$$
\begin{aligned}
\mathrm{T}_{2} / \mathrm{T}_{1} & =\left(\mathrm{p}_{2} / \mathrm{p}_{1}\right)^{(\gamma-1) / \gamma} \\
\mathrm{T}_{2} & =290 \times(35 / 1)^{(1.4-1) / 1.4} \\
& =800.86 \mathrm{~K}
\end{aligned}
$$



s

To find $T_{3}$

$$
\begin{aligned}
\mathrm{V}_{2} / \mathrm{T}_{2} & =\mathrm{V}_{3} / \mathrm{T}_{3} \\
\mathrm{~T}_{3} & =\mathrm{T}_{2}\left(\mathrm{~V}_{3} / \mathrm{V}_{2}\right) \\
\mathrm{p}_{2} / \mathrm{p}_{1} & =\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{\gamma} \\
\therefore \mathrm{V}_{1} / \mathrm{V}_{2} & =(35 / 1)^{1 / 1.4} \\
& =12.67 \\
\therefore \mathrm{~V}_{3} / \mathrm{V}_{2} & =\left(\mathrm{V}_{3} / \mathrm{V}_{4}\right) \times\left(\mathrm{V}_{4} / \mathrm{V}_{2}\right)=\left(\mathrm{V}_{3} / \mathrm{V}_{4}\right) \times\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right) \\
& =1 / 5 \times 12.67=2.534
\end{aligned}
$$

$$
\mathrm{T}_{3}=800.86 \times 2.534=2029.4 \mathrm{~K}
$$

$\therefore \quad$ Heat addition $=1.004 \times(2029.4-800.86)$

$$
=1233.45 \mathrm{~kJ} / \mathrm{kg} \quad---- \text { Ans }
$$

Heat rejection $=\mathrm{mCv}\left(\mathrm{T}_{4}-\mathrm{T}_{1}\right)$
To find $\mathrm{T}_{4}$

$$
\begin{aligned}
\mathrm{T}_{4} / \mathrm{T}_{3} & =\left(\mathrm{V}_{3} / \mathrm{V}_{4}\right)^{\gamma-1} \\
\therefore \quad \mathrm{~T}_{4} & =2029.4(1 / 5)^{1.4-1} \\
& =1066.05 \mathrm{~K}
\end{aligned}
$$

$\therefore \quad$ Heat rejection $=0.717 \times(1066.05-290)$

$$
=556.43 \mathrm{~kJ} / \mathrm{kg} \quad--- \text { Ans }
$$

13. In an Otto cycle air at $17^{\circ} \mathrm{C}$ and 1 bar is compressed adiabatically until the pressure is 15 bar. Heat is added at constant volume until the pressure rises to 40 bar. The swept volume is $0.711 \mathrm{~m}^{3}$. Calculate the air standard efficiency, the compression ratio and the mean effective pressure.
Given:
Initial pressure $\left(\mathrm{p}_{1}\right) \quad=1$ bar
Initial temperature $\left(T_{1}\right) \quad=17^{\circ} \mathrm{C}=290 \mathrm{~K}$
Pressure after compression $\left(p_{2}\right)=15$ bar
Pressure after heat supplied $\left(p_{3}\right)=40$ bar
Swept volume $\left(\mathrm{V}_{1}-\mathrm{V}_{2}\right)$
$=0.711 \mathrm{~m}^{3}$
Required : Cycle efficiency, Compression ratio, MEP

## Solution:

p



Compression ratio (r) $=\mathrm{V}_{1} / \mathrm{V}_{2}$

$$
=\left[p_{2} / p_{1}\right]^{1 / \gamma}=[15]^{1 / 1.4}=6.92 \text {---- Ans }
$$

Cycle efficiency $(\eta)=1-1 / r^{r-1}=1-1 / 6.92^{1.4-1}=0.5387$--- Ans
Mean effective pressure ( $\mathrm{p}_{\mathrm{m}}$ )

$$
r_{p}=\text { Pressure ratio }=p_{3} / p_{2}=40 / 15=2.666
$$

$$
\left(6.92^{1.4-1}-1\right)(2.666-1)
$$

$$
\begin{aligned}
& =(\mathrm{HS}-\mathrm{HR}) / \mathrm{HS} \\
& =1233.45-556.43 / 1233.45 \\
& \text { = } 0.55 \text {---- Ans }
\end{aligned}
$$

## = 5.686 bar ---- Ans

14. The compression ratio in an air standard Otto cycle is 8. At the beginning of compression process the pressure is 1 bar and temperature is 300 K . The heat transfer to the air per cycle is $1900 \mathrm{~kJ} / \mathrm{kg}$ of air. Calculate (i) the pressure and temperature at the end of each process of the cycle (ii) thermal efficiency and (iii) the mean effective pressure.
Given:

| Initial pressure $\left(p_{1}\right)$ | $=1 \mathrm{bar}$ |
| :--- | :--- |
| Initial temperature $\left(\mathrm{T}_{1}\right)$ | $=300 \mathrm{~K}$ |
| Compression ratio $(\mathrm{r})$ | $=8$ |

Heat supplied $=C_{v}\left(T_{3}-T_{2}\right)=1900 \mathrm{~kJ} / \mathrm{kg}$
Required: (i) $\mathrm{p}_{2}, \mathrm{~T}_{2}, \mathrm{p}_{3}, \mathrm{~T}_{3}, \mathrm{p}_{4}, \mathrm{~T}_{4}$, (ii) $\eta$ (iii) $\mathrm{p}_{\mathrm{m}}$


Solution:
(i)

To find $\underline{p}_{2}$

$$
\begin{aligned}
& \mathrm{p}_{1} \mathrm{~V}_{1}^{\gamma}=\mathrm{p}_{2} \mathrm{~V}_{2}^{\gamma} \\
\mathrm{p}_{2} / \mathrm{p}_{1} & =\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{\gamma}=(8)^{1.4} \\
\therefore \quad & \mathrm{p}_{2}=1 \times(8)^{1.4}=18.38 \text { bar }-- \text { Ans }
\end{aligned}
$$

To find $T_{2}$

$$
\begin{aligned}
& \left(\mathrm{p}_{2} / \mathrm{p}_{1}\right)^{(\gamma-1) / \gamma} & =\mathrm{T}_{2} / \mathrm{T}_{1} \\
\therefore & \mathrm{~T}_{2} / 300 & =(18.38 / 1)^{(1.4-1) / 1.4} \\
& \therefore \quad \mathrm{~T}_{2} & =\mathbf{6 8 9 . 2 2} \mathrm{K}--- \text { Ans }
\end{aligned}
$$

To find $T_{3}$

$$
\begin{aligned}
1900 & =0.717\left(\mathrm{~T}_{3}-689.22\right) \\
\mathrm{T}_{3} & =3339.2 \mathrm{~K}---\mathrm{Ans}
\end{aligned}
$$

To find $\underline{p}_{3}$

$$
\mathrm{p}_{3} / \mathrm{T}_{3}=\mathrm{p}_{2} / \mathrm{T}_{2}
$$

$$
\begin{array}{ll} 
& \mathrm{p}_{3}=\mathrm{p}_{2}\left(\mathrm{~T}_{3} / \mathrm{T}_{2}\right) \\
\therefore \quad & \mathrm{p}_{3}=18.38(3339.2 / 689.22)=89.05 \text { bar --- Ans }
\end{array}
$$

To find ( $\mathrm{T}_{4}$ )

$$
\begin{aligned}
\mathrm{T}_{4} / \mathrm{T}_{3} & =\left[\mathrm{V}_{3} / \mathrm{V}_{4}\right]^{\gamma-1} \\
\mathrm{~T}_{4} / 3339.2 & =[1 / 8]^{1.4-1} \\
\mathrm{~T}_{4} & =1453.5 \mathrm{~K} \text {---- Ans }
\end{aligned}
$$

To find ( $\mathrm{p}_{4}$ )
(ii)

$$
\begin{aligned}
& \mathrm{p}_{4} / \mathrm{p}_{3}=\left[\mathrm{V}_{3} / \mathrm{V}_{4}\right]^{\gamma} \\
& \mathrm{p}_{4} / 89.05=[1 / 8]^{1.4} \\
& \mathrm{p}_{4}=4.845 \text { bar }--- \text { Ans } \\
& \eta=1-1 /(r)^{\gamma-1}=1-1 /(8)^{1.4-1} \\
& =0.5647 \text {---- Ans } \\
& p_{m}=p_{1} r\left[\begin{array}{c}
\left(r^{\gamma-1}-1\right)\left(r_{p}-1\right) \\
(\gamma-1)(r---------1)
\end{array} \rightarrow\right. \text { for Otto cycle } \\
& r_{p}=p_{3} / p_{2}=89.05 / 18.38=4.845
\end{aligned}
$$

$$
\begin{aligned}
& \text { = } 14.2528 \text { bar ---- Ans }
\end{aligned}
$$

15. In an SI engine working on the ideal Otto cycle, the compression ratio is 5.5. The pressure and temperature at the beginning of the compression are 1 bar and 300 $K$ respectively. The peak pressure is 30 bar. Determine the pressure and temperature at salient points, the air standard efficiency and mep. Assume ratio of specific heats to be 1.4 for air.
Given:

| Initial pressure $\left(p_{1}\right)$ | $=1 \mathrm{bar}$ |
| :--- | :--- |
| Initial temperature $\left(\mathrm{T}_{1}\right)$ | $=300 \mathrm{~K}$ |
| Compression ratio $(\mathrm{r})$ | $=5.5$ |
| Peak pressure $\left(\mathrm{p}_{3}\right)$ | $=30 \mathrm{bar}$ |

Required: $\mathrm{p}_{2}, \mathrm{~T}_{2}, \mathrm{~T}_{3}, \mathrm{p}_{4}, \mathrm{~T}_{4}, \eta, \mathrm{p}_{\mathrm{m}}$

## Solution:

To find $p_{2}$

$$
\mathrm{p}_{1} \mathrm{~V}_{1}^{\gamma}=\mathrm{p}_{2} \mathrm{~V}_{2}^{\gamma}
$$

$$
\begin{array}{ll} 
& \mathrm{p}_{2} / \mathrm{p}_{1}=\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{\gamma}=(5.5)^{1.4} \\
\therefore & \mathrm{p}_{2}=1 \times(5.5)^{1.4}=\mathbf{1 0 . 8 7 7} \text { bar }-- \text { Ans }
\end{array}
$$

To find $\mathrm{T}_{2}$

$$
\begin{array}{rlrl} 
& \left(\mathrm{T}_{2} / \mathrm{T}_{1}\right) & =\left[\mathrm{V}_{1} / \mathrm{V}_{2}\right]^{\gamma-1} \\
\therefore \quad \mathrm{~T}_{2} / 300 & =(5.5)^{(1.4-1)} \\
\therefore \quad & \mathrm{T}_{2} & =593.3 \mathrm{~K} \text {--- Ans }
\end{array}
$$




To find $\mathrm{T}_{3}$

$$
\begin{aligned}
\mathrm{p}_{3} / \mathrm{T}_{3} & =\mathrm{p}_{2} / \mathrm{T}_{2} \\
\mathrm{~T}_{3} & =\mathrm{T}_{2}\left(\mathrm{p}_{3} / \mathrm{p}_{2}\right) \\
\therefore \quad \mathrm{T}_{3} & =593.3 \times(30 / 10.877)=\mathbf{1 6 3 6 . 4} \mathrm{K} \text {--- Ans }
\end{aligned}
$$

To find ( $\mathrm{T}_{4}$ )

$$
\begin{aligned}
\mathrm{T}_{4} / \mathrm{T}_{3} & =\left[\mathrm{V}_{3} / \mathrm{V}_{4}\right]^{\gamma-1} \\
\mathrm{~T}_{4} / 1636.4 & =[1 / 5.5]^{1.4-1} \\
\mathrm{~T}_{4} & =\mathbf{8 2 7 . 4} \mathbf{K} \cdots \text {--- Ans }
\end{aligned}
$$

To find ( $\mathrm{p}_{4}$ )

$$
\begin{aligned}
& \mathrm{p}_{4} / \mathrm{p}_{3}=\left[\mathrm{V}_{3} / \mathrm{V}_{4}\right]^{\gamma} \\
& \mathrm{p}_{4} / 30=[1 / 5.5]^{1.4} \\
& \mathrm{p}_{4}=2.758 \text { bar ---- Ans } \\
& \eta=1-1 /(r)^{\gamma-1}=1-1 /(5.5)^{1.4-1} \\
& =0.494 \text {---- Ans }
\end{aligned}
$$

$$
\begin{aligned}
& r_{p}=p_{3} / p_{2}=30 / 10.877=2.758 \\
& p_{m}=1 \times 5.5[-\cdots-\cdots--\cdots-\cdots-1)(2.758-1) \\
&(1.4-1)(5.5-1) \\
&=5.2515 \text { bar }--- \text { Ans }
\end{aligned}
$$

16. A gas engine operating on the ideal Otto cycle has a compression ratio of 6:1. The pressure and temperature at the commencement of compression are 1 bar and 300 K . The heat added during the constant volume combustion process is 1170 $\mathrm{kJ} / \mathrm{kg}$. Determine the pressure and temperatures at the salient points, work output per kg of air and air standard efficiency. Assume $\mathrm{C}_{\mathrm{v}}=0.717 \mathrm{~kJ} / \mathrm{kgK}$ and ratio of specific heats to be 1.4 for air. (Oct 2002)
Given:

| Initial pressure $\left(p_{1}\right)$ | $=1 \mathrm{bar}$ |
| :--- | :--- |
| Initial temperature $\left(\mathrm{T}_{1}\right)$ | $=300 \mathrm{~K}$ |
| Compression ratio $(\mathrm{r})$ | $=6$ |

Heat supplied $=\mathrm{C}_{\mathrm{v}}\left(\mathrm{T}_{3}-\mathrm{T}_{2}\right)=1170 \mathrm{~kJ} / \mathrm{kg}$
Required: $\mathrm{p}_{2}, \mathrm{~T}_{2}, \mathrm{p}_{3}, \mathrm{~T}_{3}, \mathrm{p}_{4}, \mathrm{~T}_{4}, \mathrm{~W}, \eta$


Solution:
To find $\mathrm{p}_{2}$

$$
\begin{aligned}
\mathrm{p}_{1} \mathrm{~V}_{1}^{\gamma} & =\mathrm{p}_{2} \mathrm{~V}_{2}^{\gamma} \\
\mathrm{p}_{2} / \mathrm{p}_{1} & =\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{\gamma}=(6)^{1.4} \\
\therefore \quad & \mathrm{p}_{2}
\end{aligned}=1 \times(6)^{1.4}=\mathbf{1 2 . 2 8 6} \text { bar --- Ans }
$$

To find $T_{2}$

$$
\begin{array}{rlrl} 
& & \left(\mathrm{T}_{2} / \mathrm{T}_{1}\right) & =\left[\mathrm{V}_{1} / \mathrm{V}_{2}\right]^{\gamma-1} \\
\therefore \quad \mathrm{~T}_{2} / 300 & =(6)^{(1.4-1)} \\
\therefore \quad \mathrm{T}_{2} & =\mathbf{6 1 4 . 3} \mathrm{K} \text {--- Ans } \\
1170 & =0.717\left(\mathrm{~T}_{3}-614.3\right)
\end{array}
$$

To find $T_{3}$

To find $\mathrm{p}_{3}$

$$
\mathrm{T}_{3}=2246.1 \mathrm{~K}--- \text { Ans }
$$

$$
\begin{aligned}
\mathrm{p}_{3} / \mathrm{T}_{3} & =\mathrm{p}_{2} / \mathrm{T}_{2} \\
\mathrm{p}_{3} & =\mathrm{p}_{2}\left(\mathrm{~T}_{3} / \mathrm{T}_{2}\right) \\
\therefore \quad \mathrm{p}_{3} & =12.286 \times(2246.1 / 614.3)=44.922 \text { bar }- \text { Ans }
\end{aligned}
$$

To find ( $T_{4}$ )

$$
\begin{aligned}
\mathrm{T}_{4} / \mathrm{T}_{3} & =\left[\mathrm{V}_{3} / \mathrm{V}_{4}\right]^{\gamma-1} \\
\mathrm{~T}_{4} / 2246.1 & =[1 / 6]^{1.4-1} \\
\mathrm{~T}_{4} & =1096.9 \mathrm{~K}--- \text { Ans }
\end{aligned}
$$

To find $\left(p_{4}\right)$

$$
\begin{aligned}
\mathrm{p}_{4} / \mathrm{p}_{3} & =\left[\mathrm{V}_{3} / \mathrm{V}_{4}\right]^{\gamma} \\
\mathrm{p}_{4} / 44.922 & =[1 / 6]^{1.4} \\
\mathrm{p}_{4} & =3.656 \text { bar }-- \text { Ans }
\end{aligned}
$$

$$
W=H S-H R
$$

$$
\begin{aligned}
\mathrm{HS} & =m \mathrm{C}_{\mathrm{v}}\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right) \\
& =1 \times 717 \times(2246.1-614.3)=1170000.6 \mathrm{~J} / \mathrm{kg} \\
\mathrm{HR} & =m \mathrm{C}_{\mathrm{v}}\left(\mathrm{~T}_{4}-\mathrm{T}_{1}\right) \\
& =1 \times 717 \times(1096.9-300)=571377.3 \mathrm{~J} / \mathrm{kg}
\end{aligned}
$$

$$
W=1170000.6-571377.3=598623.3 \mathrm{~J} / \mathrm{kg} \quad-- \text { Ans }
$$

$$
\eta=W / H S=598623.3 / 1170000.6=0.5116 \text {--- Ans }
$$

$$
\text { Or } \quad \eta=1-1 /(r)^{\gamma-1}=1-1 /(6)^{1.4-1}=0.5116--- \text { Ans }
$$

17. A certain quantity of air at a pressure of 1 bar and temperature of $70^{\circ} \mathrm{C}$ is compressed reversibly and adiabatically until the pressure is 7 bar in an Otto cycle engine. 460 kJ of heat per kg of air is now added at constant volume. Determine (i) compression ratio of the engine (ii) temperature at the end of compression and (iii) temperature at the end of heat addition. Take for air $C_{p}=1$ $\mathrm{kJ} / \mathrm{kgK}$ and $\mathrm{C}_{\mathrm{v}}=0.707 \mathrm{~kJ} / \mathrm{kgK}$.

Given:
Initial pressure $\left(p_{1}\right)=1$ bar
Initial temperature $\left(\mathrm{T}_{1}\right) \quad=70^{\circ} \mathrm{C}=343 \mathrm{~K}$
Final pressure $\left(\mathrm{p}_{2}\right)=7$ bar
Heat supplied $=C_{v}\left(T_{3}-T_{2}\right)=460 \mathrm{~kJ} / \mathrm{kg}$
Required: (i) r (ii) $\mathrm{T}_{2}$ (iii) $\mathrm{T}_{3}$

Solution:
$\gamma=C_{p} / C_{v}=1 / 0.707=1.414$
(i) Compression ratio (r) $=\mathrm{V}_{1} / \mathrm{V}_{2}=\left[\mathrm{p}_{2} / \mathrm{p}_{1}\right]^{1 / \gamma}=[7 / 1]^{1 / 1.414}=3.96$--- Ans
(ii) $\underline{T o}^{\text {find }} T_{2}$

$$
\begin{array}{rlrl} 
& & \left(\mathrm{T}_{2} / \mathrm{T}_{1}\right) & =\left[\mathrm{V}_{1} / \mathrm{V}_{2}\right]^{\gamma-1} \\
& \therefore \quad \mathrm{~T}_{2} / 343 & =(3.96)^{(1.414-1)} \\
& \therefore \quad \mathrm{T}_{2} & =606.4 \mathrm{~K} \text {--- Ans }
\end{array}
$$


(iii) To find $T_{3}$

$$
\begin{aligned}
460 & =0.707 \times\left(T_{3}-606.4\right) \\
T_{3} & =1257.03 \mathrm{~K}--- \text { Ans }
\end{aligned}
$$

18. An engine working on Otto cycle has a volume of $0.45 \mathrm{~m}^{3}$, pressure of 1 bar and temperature of $30^{\circ} \mathrm{C}$ at the beginning of compression stroke. At the end of compression stroke the pressure is 11 bar. 210 kJ of heat is added at constant volume. Determine (i) pressure, temperature and volume at salient points (ii) percentage clearance (iii) efficiency (iv) net work per cycle (v) mep (vi) ideal power developed by the engine if the number of working cycles per min is 210.

Given:

| Initial pressure $\left(p_{1}\right)$ | $=1 \mathrm{bar}$ |
| :--- | :--- |
| Initial temperature $\left(\mathrm{T}_{1}\right)$ | $=30^{\circ} \mathrm{C}=313 \mathrm{~K}$ |
| Initial volume $\left(\mathrm{V}_{1}\right)$ | $=0.45 \mathrm{~m}^{3}=\mathrm{V}_{4}$ |
| Final pressure $\left(\mathrm{p}_{2}\right)$ | $=11 \mathrm{bar}$ |

Heat supplied $=\mathrm{mC}_{\mathrm{v}}\left(\mathrm{T}_{3}-\mathrm{T}_{2}\right)=210 \mathrm{~kJ}$
Working cycle per min $=210$
Required: (i) $\mathrm{V}_{2}, \mathrm{~T}_{2}, \mathrm{p}_{3}, \mathrm{~T}_{3}, \mathrm{p}_{4}, \mathrm{~T}_{4}$ (ii) k (iii) $\eta$ (iv) W (v) mep (vi) P

## Solution:

(i)

To find $T_{2}$

$$
\left(T_{2} / T_{1}\right)=\left[p_{2} / p_{1}\right]^{(\gamma-1) / \gamma}
$$

$$
\begin{aligned}
& \therefore & \mathrm{T}_{2} / 313 & =(11 / 1)^{(1.4-1) / 1.4} \\
& \therefore & \mathrm{~T}_{2} & =621 \mathrm{~K}--\mathrm{Ans}
\end{aligned}
$$

To find $\mathrm{V}_{2}$

$$
\begin{array}{rlrl} 
& \left(V_{2} / V_{1}\right) & =\left[p_{1} / p_{2}\right]^{1 / \gamma} \\
\therefore \quad V_{2} / 0.45 & =(1 / 11)^{1 / 1.4} \\
\therefore \quad & V_{2} & =0.081 \mathrm{~m}^{3}-- \text { Ans }=V_{3}
\end{array}
$$



To find $T_{3}$

$$
\mathrm{p}_{1} \mathrm{~V}_{1}=\mathrm{mR} \mathrm{~T}_{1}
$$

Note: The value of $m$ can not be assumed since $p_{1}, V_{1}$ and $T_{1}$ are given.

$$
\begin{aligned}
1 \times 10^{5} \times 0.45 & =\mathrm{m} \times 287 \times 313 \\
\mathrm{~m} & =0.5009 \mathrm{~kg} \\
210 & =0.5009 \times 0.717 \times\left(\mathrm{T}_{3}-621\right) \\
\mathrm{T}_{3} & =\mathbf{1 2 0 5 . 7} \mathrm{K} \text {---. Ans }
\end{aligned}
$$

To find $p_{3}$

$$
\begin{array}{rlrl}
\mathrm{p}_{3} / \mathrm{T}_{3} & =\mathrm{p}_{2} / \mathrm{T}_{2} \\
\mathrm{p}_{3} & =\mathrm{p}_{2}\left(\mathrm{~T}_{3} / \mathrm{T}_{2}\right) \\
\therefore \quad & \mathrm{p}_{3} & =11 \times(1205.7 / 621)=\mathbf{2 1 . 3 5} \text { bar }-- \text { Ans }
\end{array}
$$

To find ( $\mathrm{T}_{4}$ )

To find ( $p_{4}$ )

$$
\begin{aligned}
\mathrm{T}_{4} / \mathrm{T}_{3} & =\left[\mathrm{V}_{3} / \mathrm{V}_{4}\right]^{\gamma-1} \\
\mathrm{~T}_{4} / 1205.7 & =[0.081 / 0.45]^{1.4-1} \\
\mathrm{~T}_{4} & =607.2 \mathrm{~K} \cdots-\mathrm{Ans} \\
\mathrm{p}_{4} / \mathrm{p}_{3} & =\left[\mathrm{V}_{3} / \mathrm{V}_{4}\right]^{\gamma} \\
\mathrm{p}_{4} / 21.35 & =[0.081 / 0.45]^{1.4} \\
\mathrm{p}_{4} & =1.935 \text { bar }-\cdots-\text { Ans }
\end{aligned}
$$

(ii) $\%$ Clearance $=V_{2} /\left(V_{1}-V_{2}\right) \times 100=0.081 /(0.45-0.081) \times 100$

$$
=21.95 \% \text {--- Ans }
$$

(iii)

$$
\eta=(H S-H R) / H R
$$

$$
H S=m C_{v}\left(T_{3}-T_{2}\right)
$$

$$
=0.5009 \times 717 \times(1205.7-621)=209992.2 \mathrm{~J}
$$

$$
\mathrm{HR}=\mathrm{m} \mathrm{C}_{\mathrm{v}}\left(\mathrm{~T}_{4}-\mathrm{T}_{1}\right)
$$

$$
=0.5009 \times 717 \times(607.2-313)=105660.5 \mathrm{~J}
$$

$$
W=209992.2-105660.5=104331.7 \mathrm{~J}
$$

$$
\eta=W / H S=104331.7 / 209992.2=0.497 \text {--- Ans }
$$

Or

$$
\begin{align*}
\eta & =1-1 /(\mathrm{r})^{\gamma-1}=1-1 /(0.45 / 0.081)^{1.4-1} \\
& =0.4964--- \text { Ans } \\
\mathrm{W} & =104331.7 \mathrm{~J}-- \text { Ans }  \tag{iv}\\
\mathrm{mep} & =\mathrm{W} /\left(\mathrm{V}_{1}-\mathrm{V}_{2}\right)  \tag{v}\\
& =104331.7 /(0.45-0.081)=282741.7 \mathrm{~N} / \mathrm{m}^{2} \\
& =2.827 \text { bar }-- \text { Ans }
\end{align*}
$$

$$
\begin{equation*}
\text { Power }=\mathrm{W} \times \mathrm{N} / 60=104331.7 \times 210 / 60=365161 \mathrm{~W}--- \text { Ans } \tag{vi}
\end{equation*}
$$

19. An oil engine works on air standard dual cycle with compression ratio of 10. The pressure and temperature at the beginning of compression are 1 bar and $30^{\circ} \mathrm{C}$. The maximum pressure reached is 40 bar and maximum temperature is $1400^{\circ} \mathrm{C}$. Determine (i) temperature at the end of constant volume heat addition and (ii) cutoff ratio. Take $C_{p}=1.004 \mathrm{~kJ} / \mathrm{kgK}$ and $\mathrm{C}_{\mathrm{v}}=0.717 \mathrm{~kJ} / \mathrm{kgK}$.
Given:
Dual cycle
Compression ratio (r) $=10$
Initial pressure $\left(p_{1}\right)=1$ bar
Initial temperature $\left(\mathrm{T}_{1}\right) \quad=30^{\circ} \mathrm{C}=303 \mathrm{~K}$
Maximum pressure $\left(\mathrm{p}_{3}\right)=40$ bar
Maximum temperature $\left(\mathrm{T}_{4}\right) \quad=1400^{\circ} \mathrm{C}=1673 \mathrm{~K}$
Required: (i) $\mathrm{T}_{3}$ (ii) $\mathrm{V}_{4} / \mathrm{V}_{3}$

## Solution:

(i) 2-3 $\rightarrow$ Constant volume process

$$
p_{3} / T_{3}=p_{2} / T_{2}
$$

To find $p_{2}$

$$
\gamma=1.004 / 0.717=1.4
$$

$$
p_{1} V_{1}^{\gamma}=p_{2} V_{2}^{\gamma}
$$



To find $T_{2}$

$$
\begin{array}{rlrl} 
& \left(\mathrm{T}_{2} / \mathrm{T}_{1}\right) & =\left[\mathrm{V}_{1} / \mathrm{V}_{2}\right]^{\gamma-1} \\
& \therefore \quad \mathrm{~T}_{2} / 303 & =(10)^{(1.4-1)} \\
& \therefore \quad \mathrm{T}_{2} & =761.1 \mathrm{~K} \\
\therefore \quad & 40 / \mathrm{T}_{3} & =25.119 / 761.1 \\
& \mathrm{~T}_{3} & =1212 \mathrm{~K} & \cdots-{ }^{(1)} \text { Ans }
\end{array}
$$

(ii) Cut-off ratio $(\rho)=V_{4} / V_{3}$

3-4 $\rightarrow$ Constant pressure process

$$
\begin{aligned}
& V_{3} / T_{3}=V_{4} / T_{4} \\
& V_{4} / V_{3}=T_{4} / T_{3}=1673 / 1212=1.38 \text {--. Ans }
\end{aligned}
$$

20. An air standard dual cycle has a compression ratio of 9.5:1. The pressure and temperature at the beginning of compression are 1 bar and $25^{\circ} \mathrm{C}$. Maximum pressure reached is 38 bar and the maximum temperature is $1300^{\circ} \mathrm{C}$. Determine workdone per kg of air and cycle efficiency. Take $\mathrm{C}_{\mathrm{p}}=1.004 \mathrm{~kJ} / \mathrm{kgK}$ and $\mathrm{C}_{\mathrm{v}}=0.717$ kJ/kgK.
Given:
Compression ratio (r) $\quad=9.5$
Initial pressure $\left(p_{1}\right)=1$ bar
Initial temperature $\left(T_{1}\right) \quad=25^{\circ} \mathrm{C}=298 \mathrm{~K}$
Maximum pressure $\left(\mathrm{p}_{3}\right)=38$ bar
Maximum temperature $\left(\mathrm{T}_{4}\right) \quad=1300^{\circ} \mathrm{C}=1573 \mathrm{~K}$
Required: W, $\eta$
Solution:
W = HS - HR

$$
\begin{aligned}
& H S=m C_{v}\left(T_{3}-T_{2}\right)+m C_{p}\left(T_{4}-T_{3}\right) \\
& H R=m C_{v}\left(T_{5}-T_{1}\right)
\end{aligned}
$$

To find $p_{2}$

$$
\begin{gathered}
\gamma=1.004 / 0.717=1.4 \\
p_{1} \mathrm{~V}_{1}^{\gamma}=\mathrm{p}_{2} \mathrm{~V}_{2}^{\gamma} \\
\mathrm{p}_{2} / \mathrm{p}_{1}=\left(\mathrm{V}_{1} / \mathrm{V}_{2}\right)^{\gamma}=(9.5)^{1.4} \\
\mathrm{p}_{2}=1 \times(9.5)^{1.4}=23.378 \mathrm{bar} \\
\mathrm{p}
\end{gathered}
$$

To find $T_{2}$

$$
\begin{array}{rlrl} 
& & \left(\mathrm{T}_{2} / \mathrm{T}_{1}\right) & =\left[\mathrm{V}_{1} / \mathrm{V}_{2}\right]^{\gamma-1} \\
\therefore \quad \mathrm{~T}_{2} / 298 & =(9.5)^{(1.4-1)} \\
& \therefore \quad \mathrm{T}_{2} & =733.5 \mathrm{~K}
\end{array}
$$

To find $\mathrm{T}_{3}$

$$
\begin{aligned}
& & \mathrm{p}_{3} / \mathrm{T}_{3} & =\mathrm{p}_{2} / \mathrm{T}_{2} \\
& & 38 / \mathrm{T}_{3} & =23.378 / 733.5 ; \quad \mathrm{T}_{3}=1191.95 \mathrm{~K}
\end{aligned}
$$

To find $\mathrm{T}_{5}$

$$
\begin{aligned}
\mathrm{T}_{5} / \mathrm{T}_{4} & =\left[\mathrm{V}_{4} / \mathrm{V}_{5}\right]^{\gamma-1}=\left[\mathrm{V}_{4} / \mathrm{V}_{1}\right]^{\gamma-1} \\
\mathrm{p}_{2} \mathrm{~V}_{2} & =\mathrm{mR} \mathrm{~T}_{2} \\
23.378 \times 10^{5} \times \mathrm{V}_{2} & =1 \times 287 \times 733.3 \\
\mathrm{~V}_{2} & =0.090023 \mathrm{~m}^{3}=\mathrm{V}_{3} \\
\mathrm{~V}_{1} & =9.5 \mathrm{~V}_{2}=9.5 \times 0.090023=0.8552185 \mathrm{~m}^{3}
\end{aligned}
$$

3-4 $\rightarrow$ Constant pressure process

$$
\begin{aligned}
\mathrm{V}_{3} / \mathrm{T}_{3} & =\mathrm{V}_{4} / \mathrm{T}_{4} \\
0.090023 / 1191.95 & =\mathrm{V}_{4} / 1573 \\
\mathrm{~V}_{4} & =0.1188 \mathrm{~m}^{3} \\
\therefore \quad \mathrm{~T}_{5} / 1573 & =[0.1188 / 0.8552185
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{T}_{5}=714.2 \mathrm{~K} \\
& \mathrm{HS}=1 \times 0.717 \times(1191.95-733.3)+1 \times 1.004 \times(1573-1191.5) \\
&=711.426 \mathrm{~kJ} / \mathrm{kg} \\
& \mathrm{HR}=1 \times 0.717 \times(714.2-298)=298.4154 \mathrm{~kJ} / \mathrm{kg} \\
& \mathrm{~W}=711.426-298.415=413.011 \mathrm{~kJ} / \mathrm{kg}-- \text { Ans } \\
& \eta=\mathrm{W} / \mathrm{HS}=413.011 / 711.426=0.5805 \cdots \text { Ans }
\end{aligned}
$$

## UNIT-II

## INTERNAL COMBUSTION ENGINES

## Heat Engines

A heat engine or a thermal engine is a machine which converts heat energy into mechanical work. Heat is produced by burning any combustible material.

Heat engines are classified as
(i) Internal combustion engines (I.C engines)
(ii) External combustion engines (E.C engines)

## I.C engines

In an I.C engine, the chemical energy of fuel is released as heat by way of combustion inside the engine cylinder where power is produced. Both combustion and power developed take place inside the cylinder.

## E.C engines

E.C engines are steam engines and steam turbines. In these units, heat energy is produced during combustion of fuel in a boiler furnace. This energy is used to generate steam under pressure in the boiler. The steam expands in an engine and thereby does work. In this case, power is produced in a unit other than the one where heat is generated.

## Nomenclature of an I.C engine

In any engine, during its working, cycle of operation take place again and again. In an I.C engine, the cycle consists of four operations. They occur one after other in the order given below:
(i) Suction
(ii) Compression
(iii) Ignition and expansion
(iv) Exhaust

In order to perform the four operations, a piston moves within the cylinder up and down, or to and fro depending on the type of engine. The piston, cylinder and other details of an engine is shown in fig.
TDC $\rightarrow$ Top dead centre $\rightarrow$ Top most position of the piston in the cylinder of vertical engines.
$B D C \rightarrow$ Bottom dead centre $\rightarrow$ Bottom most position of the piston in the cylinder of vertical engines.
IDC $\rightarrow$ Inner dead centre $\rightarrow$ Inner most position of the piston in the cylinder of horizontal engines.
ODC $\rightarrow$ Outer dead centre $\rightarrow$ Outer most position of the piston in the cylinder of horizontal engines.
Stroke $\rightarrow$ The distance (L) between two dead centres. The piston completes one stroke in $1 / 2$ revolution of the crank shaft $\left(180^{\circ}\right)$.
Bore $\rightarrow$ Diameter of the cylinder (D).
Clearance volume $\rightarrow$ The space above the piston when the piston is at TDC.
Cylinder volume $\rightarrow$ The volume above the piston when the piston is at BDC.

Stroke volume $\rightarrow$ Also called swept volume. The volume displaced by the piston during a stroke is called swept volume.

Square engine $\rightarrow \mathrm{D}=\mathrm{L}$
Under square engine $\rightarrow \mathrm{D}<\mathrm{L} \rightarrow$ Low speed engines. Large industrial engines and tractor engines are under square engines.

Over square engine $\rightarrow \mathrm{D}>\mathrm{L} \rightarrow$ High speed engines. This is the most common engine design.
Cubic capacity $\rightarrow$ The cubic capacity of an engine or engine displacement or engine size is the product of stroke volume in one cylinder and the number of cylinders in the engine.
Compression ratio $\rightarrow$ The ratio of cylinder volume to the clearance volume is called compression ratio.

Cylinder head

Crank case


## Components of I.C engines

(i) Cylinder block
(ii) Cylinder head
(iii) Piston assembly
(iv) Connecting rod
(v) Crank shaft
(vi) Crank case
(vii) Valves and Valve operating mechanism
(viii) Fuel supply system
(ix) Lubrication system
(x) Cooling system
(xi) Inlet and Exhaust system

The cylinder block is the main structure for the various components. The cylinder head is mounted on the cylinder block. The cylinder head and cylinder block are provided with water jackets in the case of water cooled engines or with cooling fins in the case of air cooled engines.

The cylinder head is held tight to the cylinder block by number of bolts and studs. The bottom portion of the cylinder block is called crank case. The piston reciprocates inside the cylinder and the motion of the piston is transmitted to the crank shaft by connecting rod and crank assembly. Inlet and exhaust valves are provided for suction of charge and removal of exhaust gases.

Fuel is supplied to the engine from the fuel tank for combustion through fuel filter, fuel pump and fuel injector in case of C.I engines and carburetor in case of S.I engines.

Lubricating system supplies lubricating oil to the various parts of the engine where there is relative motion. This reduces friction between the parts and thereby increases engine life.

Cooling system abstracts excess heat from various engine parts which are heated due to combustion. This prevents the failure of the components due to overheating and increases the engine life. The coolant may be either liquid or air.

Inlet manifold is provided on suction side which allows the charge entering the cylinder during suction process. Exhaust manifold is provided on exhaust side which allows the exhaust gases letting to atmosphere during exhaust process.

## Classification of I.C engines

Based on working cycle
(i) Two stroke engines
(ii) Four stroke engines

Based on method of ignition
(i) Compression ignition engines (C.I engines)
(ii) Spark ignition engines (S.I engines)

Based on Fuel used
(i) Light fuel oil engines (Petrol engines)
(ii) Diesel engines
(iii) Gas engines

Based on applications
(i) Stationary engines
(ii) Portable engines
(iii) Automobile engines
(iv) Marine engines
(v) Aero engines

Based on arrangement of the cylinder
(i) Horizontal engines
(ii) Vertical engines
(iii) Radial engines
(iv) V-type engines

Based on speed of the engine
(i) Slow speed engines
(ii) Medium speed engines
(iii) High speed engines

Based on number of cylinders
(i) Single cylinder engines
(ii) Multi-cylinder engines

Based on method of cooling
(i) Water cooled engines
(ii) Air cooled engines

## Indicator Diagram ( $p-V$ diagram) and Diagram factor

The indicator diagram is a graph between pressure and volume. This is obtained by an instrument known as "Indicator". The indicator diagrams are of two types: (1) Theoretical or Hypothetical and (2) Actual. The theoretical indicator diagram is always longer in size as compared to the actual, since in the former losses are neglected.

Diagram Factor $=\frac{\text { Area of the Actual indicator diagram }}{\text { Area of the theoretical indicator diagram }}$

## Working of Four Stroke S.I Engines (Petrol engines and Gas engines)

In four stroke engine all the operations are completed in four strokes or in two revolutions of the crank shaft.

I.V = Intel valve, E.V. = Exhaust valve, E.C. $=$ Engine cylinder, C.R. $=$ Connecting rod $\mathrm{C}=$ Crank, S.P. $=$ Spark plug.

Suction Stroke: During this stroke (also called induction stroke) the piston moves from TDC to BDC. The inlet valve opens and proportionate fuel-air mixture is sucked in the engine cylinder. The operation is represented by the line $5-1$. The exhaust valve remains closed through out the stroke.

Compression Stroke: In this stroke the piston moves towards TDC and compresses the enclosed fuel air mixture drawn in the cylinder during suction stroke. The pressure of the
mixture rises in the cylinder to a value of about 8 bar (1-2). Just before end the stroke, a high intensity spark is applied to ignite the mixture and combustion takes place at constant volume ( 2 -3 ). Both inlet and exhaust valves remain closed during this stroke.

Expansion or Power stroke: The high pressure and high temperature hot gases obtained by the combustion, throws the piston from TDC to BDC. Thus the work is obtained in this stroke (34). Both the valves remain closed during this stroke. When the piston just reaches the BDC, the exhaust valve opens and there will be sudden pressure drop and heat rejection at constant volume (4-1).
Exhaust Stroke: This is last stroke of the cycle. During this stroke, the used gases are allowed to escape through exhaust valve to atmosphere. The piston moves from BDC to TDC and the


## Working of Four Stroke C.I Engines (Diesel engines)

Suction Stroke: During this stroke (also called induction stroke) the piston moves from TDC to BDC. The inlet valve opens and proportionate air is sucked in the engine cylinder. The operation is represented by the line $5-1$. The exhaust valve remains closed through out the stroke.

Compression Stroke: In this stroke the piston moves towards TDC and compresses the enclosed air drawn in the cylinder during suction stroke. The pressure of the mixture rises in the cylinder to a value of about 35 bar ( $1-2$ ). The temperature will be around $600^{\circ} \mathrm{C}$. Just before end the stroke, fuel is injected. Both inlet and exhaust valves remain closed during this stroke.

Expansion or Power stroke: As the piston starts moving from TDC, the injected fuel into the hot compressed air burn the fuel. The burning is taking place at constant pressure ( $2-3$ ). The point 2 represents the beginning of fuel injection and point 3 represents the end of fuel injection. The expansion starts at point 3 and continues upto BDC. Thus the work is obtained in this stroke from $(3-4)$. Both the valves remain closed during this stroke. When the piston just reaches the $B D C$, the exhaust valve opens and there will be sudden pressure drop and heat rejection at constant volume (4-1).

Exhaust Stroke: This is last stroke of the cycle. During this stroke, the used gases are allowed to escape through exhaust valve to atmosphere. The piston moves from BDC to TDC and the exhaust gases are driven out of the cylinder. This operation is represented by $1-5$.


Theoretical p-V diagram


## Working of Two Stroke S.I engine

In two stroke engines all the operations are completed in two strokes or in single revolution of the crank rotation. Two stroke engines have ports instead of valves. The three ports are (i) Transfer port (TP) (ii) Exhaust port (EP) and (iii) Suction port (SP) or Valve (V).

$\mathrm{L}=$ Cylinder E.P. $=$ Exhaust port T.P. $=$ Transfer port $\mathrm{V}=$ valve, $\mathrm{C} . \mathrm{C}=$ Crank chamber


Theoretical and actual p-V diagrams of Two Stroke Otto cycle

Suction stage: In this stage, while going down towards BDC, uncovers both transfer port and exhaust port. The fresh fuel-air mixture enters the cylinder through transfer port.

Compression stage: In this stage, the piston, while moving up, first covers the transfer port and then exhaust port. The air-fuel mixture is compressed. In this stage the inlet port or a valve opens and fuel air mixture enters the crank case.

Expansion stage: Shortly before the piston reaches the TDC (during compression stroke), the charge is ignited with the help of spark plug. It suddenly increases the pressure and temperature at constant volume. The piston is pushed downwards and thus the work is obtained.

Exhaust stage: In this stage, the exhaust port is opened as the piston moves downwards. The exhaust gases are sent to the atmosphere through exhaust manifold.

## Working of Two Stroke C.I engine

Except that fuel is admitted near the TDC, the working of two stroke cycle diesel engine is similar to that of two stroke cycle petrol engine.
Suction stage: In this stage, while going down towards BDC, uncovers both transfer port and exhaust port. The air enters the cylinder through transfer port.

Compression stage: In this stage, the piston, while moving up, first covers the transfer port and then exhaust port. The air is compressed. In this stage the inlet port or a valve opens and air enters the crank case.

Expansion stage: Shortly before the piston reaches the TDC (during compression stroke), the fuel is injected in the form of fine spray into the cylinder through the injector. At this moment the temperature of the air is sufficiently high to ignite the fuel. It suddenly increases the pressure and temperature. The fuel is continuously injected for a fraction of the crank revolution. The fuel is assumed to be burnt at constant pressure The piston is pushed downwards and thus the work is obtained.

Exhaust stage: In this stage, the exhaust port is opened as the piston moves downwards. The exhaust gases are sent to the atmosphere through exhaust manifold.


A valve timing diagram is a graphical representation of the exact moments, in the sequence of operations, at which the two valves (inlet and exhaust valves) open and close as well as firing of the fuel. It is generally expressed in terms of angular positions of the crank shaft.

## Theoretical Valve Timing Diagram of Four stroke engine



The theoretical valve timing diagram for the four stroke cycle engine is shown in fig. The inlet valve opens at TDC and suction takes place from TDC to BDC. At BDC the inlet valve closes and the compression takes place from BDC to TDC. At TDC the fuel is fired and the expansion takes place from TDC to BDC. At the end of expansion (BDC) the exhaust valve opens and exhaust takes place from BDC to TDC.

## Valve Timing Diagram of Four stroke petrol engine



In actual practice it is difficult to open and close the valve instantaneously. The inlet valve is opened $10^{\circ}$ to $30^{\circ}$ in advance of the TDC position to enable the fresh charge to enter the cylinder and to help the burnt gases at the same time, to escape to the atmosphere. The suction of the mixture continues upto $30^{\circ}$ to $40^{\circ}$ or even $60^{\circ}$ after $B D C$ position. The inlet valve closes and the compression of the entrapped mixture starts.

The spark plug produces a spark $30^{\circ}$ to $40^{\circ}$ before the TDC position, thus fuel gets more time to burn. The pressure becomes maximum nearly $10^{\circ}$ past the TDC position. The exhaust valve opens $30^{\circ}$ to $60^{\circ}$ before BDC position and the exhaust gases are driven out of the cylinder by piston during its upward movement. The exhaust valve closes when piston is nearly $10^{\circ}$ past TDC position.

## Valve Timing Diagram of Four stroke diesel engine

The inlet valve opens $10^{\circ}$ to $25^{\circ}$ in advance of TDC position and closes $25^{\circ}$ to $50^{\circ}$ after the BDC position. Exhaust valve opens $30^{\circ}$ to $50^{\circ}$ in advance of BDC position and closes $10^{\circ}$ to $15^{\circ}$ after the TDC position. The fuel injection takes place $5^{\circ}$ to $10^{\circ}$ before TDC position and continues upto $15^{\circ}$ to $25^{\circ}$ near TDC position.


## Port Timing Diagram of Two stroke engine

The expansion of the charge after ignition starts as the piston moves from TDC towards BDC. First the exhaust port opens before the piston reaches BDC and the burnt gases start leaving the cylinder. After a small fraction of the crank revolution, the transfer port also opens and the fresh charge enters into the engine cylinder. This is done as the fresh incoming charge helps in pushing out the burnt gases. Now the piston reaches the BDC and then starts moving upwards. As the piston moves little beyond BDC, first the transfer port closes and then exhaust port also closes. This is done to suck fresh charge through the transfer port and to exhaust the burnt gases through the exhaust port simultaneously. Now the charge is compressed with both the ports closed and then ignited with the help of spark plug (petrol engine) or injector (diesel engine) before the end of the compression stroke. This is done as the charge requires some time to ignite. By the time the piston reaches TDC, the burnt gases push the piston downwards with full force and expansion of the burnt gases takes place.


Comparison of Four stroke and Two stroke engines

| SI.No | Aspect | Four stroke | Two stroke |
| :---: | :--- | :--- | :--- |
| 1 | Completion of <br> cycle | In four strokes of the piston <br> or in two revolution of the <br> crankshaft. | In two strokes of the piston or <br> in one revolution of the crank <br> shaft. |
| 2 | Flywheel <br> required | Heavier flywheel <br> required. | Lighter flywheel is needed. |
| 3 | Power <br> produced | One power stroke for two <br> revolution. | One power stroke in one <br> revolution. Double the power <br> as that developed by four <br> stroke engine (theoretically). |
| 4 | Cooling and <br> lubrication <br> requirements | Because of one power <br> stroke in two revolution, <br> lesser cooling and <br> lubrication requirements. <br> Lesser rate of wear and <br> tear. | Because of one power stroke <br> in one revolution greater <br> cooling and lubrication <br> requirements. Great rate of <br> wear and tear. |
| 5 | Valve <br> mechanism | Contains valves and <br> maintenance required. | Contains ports. No valves. <br> Less maintenance problems. |
| 6 | Initial cost | Because of heavy weight <br> and complication of valve <br> mechanism, initial cost is <br> high. | Because of light weight and <br> simplicity due to absence of <br> valves, initial cost is less. |


| 7 | Volumetric <br> efficiency | More due to more time of <br> induction. | Less due to lesser time of <br> induction. |
| :---: | :--- | :--- | :--- |
| 8 | Thermal <br> efficiency | Higher | Lower |
| 9 | Part load <br> efficiency | Higher | Lower |
| 10 | Applications | Used where efficiency is <br> important. In cars, buses, <br> trucks, industrial engines, <br> power generators, etc. | Used where low cost, <br> compactness and light weight <br> is required. In scooters, <br> ships, motor cycles, etc. |

Comparison of S.I and C.I engines

| SI.No | Aspect | S.l engines | C.l engines |
| :---: | :---: | :---: | :---: |
| 1 | Fuel used | Petrol | Diesel |
| 2 | Air-Fuel ratio | 10: 1 to $20: 1$ | 18: 1 to $100: 1$ |
| 3 | Compression ratio | 7 to 11 | 12 to 24 |
| 4 | Combustion | Spark ignition | Compression ignition |
| 5 | Fuel supply | By carburetor - cheap. | By injector - expensive. |
| 6 | Cycle of operation | Otto cycle | Diesel cycle for slow speed engines. <br> Dual cycle for high speed engines. |
| 7 | Power developed | Less | More |
| 8 | Control of power | Quantity governing | Quality governing |
| 9 | Running cost | Higher | Lower |
| 10 | Applications | Used where low cost, compactness and light weight is required. In scooters, ships, motor cycles, air crafts, etc. | Used where efficiency is important. In cars, buses, trucks, industrial engines, power generators, etc. |

## Applications of I.C engines

The I.C engines are generally used for
(i) Road vehicles (Scooter, motor cycle, buses, etc)
(ii) Air craft
(iii) Locomotives
(iv) Civil engineering equipment (bull-dozer, scraper, power shovels, etc)
(v) Pumping sets
(vi) Cinemas
(vii) Hospitals
(viii) Several industrial applications

## Function of components

Crank shaft: The crank shaft runs under the action of piston through the connecting rod and crank pin and transmits the work from the piston to the driven shaft.

Piston: Transmits the power developed by the combustion of fuels to the crank shaft.
Piston rings: The piston rings lubricated with engine oil produces gas-tight seal between the piston and the cylinder liners. The three main functions of piston rings in reciprocating engines are: 1. Sealing the combustion/expansion chamber, 2. Supporting heat transfer from the piston to the cylinder wall and 3.Regulating engine oil consumption.

Compression rings $\rightarrow$ Prevents leakage of high pressure air to crank case during compression process.

Oil rings $\rightarrow$ Scraps the lubricating oil from the cylinder wall and allow it to return to the oil sump.
Connecting rod: The connecting rod changes and transmits the reciprocating motion of the piston to the continuously rotating crank pin during the working stroke and vice versa during other strokes.

Push rod and Rocker arm: The motion of the cam is transmitted to the valve through the pushrod and rocker arm. A push rod is the component of the valve system of piston engines. Its function is essentially to push the valve open. The push rod is driven by the rocker arm which is actuated by the cam.
Cam shaft: It is driven from the crank shaft by a timing gear. It operates the intake valve and the exhaust valve through the cams, followers, push rods and rocker arms.
Crank case: The main body of the engine to which the cylinders are attached and which contains crankshaft and camshaft bearing. It protects the parts from dirt, etc.

Flywheel: A flywheel (Steel or cast iron disc) secured on the crankshaft takes care of the fluctuations in speed and stores energy during the power stroke and releases during the other strokes. It also makes crankshaft rotation more uniform.

Governor: When the speed decreases due to increase in load the supply valve is opened by the mechanism operated by the governor and therefore speeds up again to the original speed. If the speed increases due to a decrease in load the governor mechanism closes the supply valve sufficiently to slow the engine to its original speed. Thus the function of a governor is to control the fluctuations of engine sped due to changes of load.
Spark plug: The function of the spark plug is to ignite the mixture after completing the compression in the petrol engine.
Carburettor: To supply uniform air-fuel mixture in the cylinder of petrol engines through the intake manifold.

Fuel pump: It forces the fuel oil at high pressure through fuel nozzle into the cylinder at the end of compression stroke in diesel engine.

## Spark Plug

The spark plug consists of a metal shell having two electrodes which are insulated from each other with an air gap. High tension current jumping from the supply electrode produces the necessary spark. The correct type of plug with correct width of gap between the electrodes is important factor. The spark plug gap can be easily checked by means of a feeler gauge and set as per manufacturer's specifications.


The spark plug entails the following requirements:
(i) It must withstand peak pressures up to atleast 55 bar.
(ii) It must provide suitable insulation between the electrodes to prevent short circuiting.
(iii) It must be capable of withstanding high temperatures $\left(2000^{\circ} \mathrm{C}\right.$ to $\left.2500^{\circ} \mathrm{C}\right)$ over long period of operation.
(iv) It must offer maximum resistance to erosion.
(v) It must possess high heat resistance.
(vi) The insulating material must withstand the chemical reaction effects of the fuel and hot products of combustion.

Gas tight joints between the insulator and metal parts are essential under all operating conditions.

## Carburettor

The carburettor is a device for atomising and vapourising the fuel and mixing it with air in varying proportions to suit the charging operation conditions in the engine. This process of breaking up and mixing the petrol with air is called carburetion.

Vapourisation $\rightarrow$ Change of phase from liquid phase to vapour phase
Atomisation $\rightarrow$ Reduction of fuel into fine particles by mechanical breaking up process

The parts of a simple carburettor are shown in fig.
The main function of the carburettor is to vapourise and mix the petrol and air by means of engine suction and to supply the required quantity of mixture in proper proportion.

As the engine is started, suction is created inside the cylinder and the air flows from atmosphere into the cylinder. As the air passes through the venturi, the pressure of the air falls below the atmosphere. The pressure at the nozzle tip also is less than atmosphere. But the pressure in the fuel tank is atmosphere. This pressure difference causes the flow of the fuel through the fuel jet into the air stream. As the fuel and air pass ahead, the fuel gets vapourised and uniform mixture is supplied to the engine. The quantity of mixture supplied to the engine depends upon the opening of the throttle valve. In case of stationary engine the opening of throttle valve is controlled by the governor according to the load on the engine. In case of automobile engines on road, the opening of the throttle valve is controlled by the driver through the accelerator pedal.


## Air-Fuel ratio

Air is needed for complete combustion. The theoretical air fuel ratio is about $15: 1$. The correct air fuel ratio ensures the burning without leaving excess of air or fuel. It is difficult to get correct air fuel ratio in practice.

Too little air $\rightarrow$ Rich mixture $\rightarrow$ some of the fuel goes unburnt or simply charred to carbon.

Too much air $\rightarrow$ Lean mixture $\rightarrow$ burns slowly and erratically and power is less.
Air fuel ratio is depending on fuel characteristics, shape of the combustion chamber, temperature and pressure in the combustion chamber.

## Desired mixture:

For average speeds : 15:1
During starting : 12:1
For maximum economy : $16: 1$ to $17: 1$

## Calculation of Air-Fuel ratio

Mass flow rate of air considering air to be incompressible


Air in

$$
\dot{m}_{a}=\rho_{1} A_{1} C_{1}=\rho_{2} A_{2} C_{2}
$$

$\rho_{1}=$ Density of air at inlet
$\mathrm{A}_{1}=$ Flow area at inlet
$\mathrm{C}_{1}=$ Velocity of air at inlet
$\rho_{2}, A_{2}$ and $C_{2}=$ Corresponding values at section 2
Assuming the air to be incompressible and applying Bernoulli's equation between $1 \& 2$,

$$
\frac{p_{1}}{\rho_{1}}+\frac{C_{1}^{2}}{2}+g Z_{1}=\frac{p_{2}}{\rho_{2}}+\frac{C_{2}^{2}}{2}+g Z_{2}
$$

But $\rho_{1}=\rho_{2}=\rho_{\mathrm{a}}$ and $Z_{2}-Z_{1} \approx 0, C_{1} \approx 0$

$$
\begin{aligned}
& \frac{p_{1}}{\rho_{a}}=\frac{p_{2}}{\rho_{a}}+\frac{C_{2}^{2}}{2} \\
& C_{2}=\sqrt{\frac{2\left(p_{1}-p_{2}\right)}{\rho_{a}}}
\end{aligned}
$$

Actual velocity of air,

$$
C_{2}=C_{d a} \sqrt{\frac{2\left(p_{1}-p_{2}\right)}{\rho_{a}}}
$$

$\mathrm{C}_{\mathrm{da}}=$ Coefficient of discharge of venturi

Therefore,

$$
\begin{aligned}
\dot{m}_{a} & =C_{d a} \rho_{a} A_{2} \sqrt{\frac{2\left(p_{1}-p_{2}\right)}{\rho_{a}}} \\
& =C_{d a} A_{a} \sqrt{2 \rho_{a}\left(p_{1}-p_{2}\right)}
\end{aligned}
$$

Mass flow rate of air considering air to be compressible
Steady flow energy equation is given by,

$$
h_{1}+\frac{C_{1}^{2}}{2}+g Z_{1}+q=w+h_{2}+\frac{C_{2}^{2}}{2}+g Z_{2}
$$

But, for a flow through Venturi, $\mathrm{C}_{1}=0, \mathrm{w}=0, \mathrm{q}=0$ (assuming adiabatic expansion), $\mathrm{Z}_{2}-\mathrm{Z}_{1}=0$
Therefore,

$$
\begin{aligned}
C_{2} & =\sqrt{2\left(h_{1}-h_{2}\right)}=\sqrt{2 C_{p}\left(T_{1}-T_{2}\right)} \\
& =\sqrt{2 C_{p} T_{1}\left(1-\frac{T_{2}}{T_{1}}\right)} \\
& =\sqrt{2 C_{p} T_{1}\left(1-\left(\frac{p_{2}}{p_{1}}\right)^{(\gamma-1) / \gamma}\right)}
\end{aligned}
$$

Mass flow rate of air,
$\dot{m}_{a}=\rho_{1} A_{1} C_{1}=\rho_{2} A_{2} C_{2}$

We can write,

$$
\begin{aligned}
& p_{1} v_{1}^{\gamma}=p_{2} v_{2}^{\gamma} \text { or } \frac{v_{1}}{v_{2}}=\left(\frac{p_{2}}{p_{1}}\right)^{1 / \gamma} \text { or } \frac{\rho_{2}}{\rho_{1}}=\left(\frac{p_{2}}{p_{1}}\right)^{1 / \gamma} \\
& \rho_{2}=\rho_{1}\left(\frac{p_{2}}{p_{1}}\right)^{1 / \gamma} \\
& \dot{m}_{a}=\rho_{1}\left(\frac{p_{2}}{p_{1}}\right)^{1 / \gamma} A_{2} \sqrt{2 C_{p} T_{1}\left(1-\left(\frac{p_{2}}{p_{1}}\right)^{(\gamma-1) / \gamma}\right)} \\
& \dot{m}_{a}=C_{d a} \rho_{1}\left(\frac{p_{2}}{p_{1}}\right)^{1 / \gamma} A_{2} \sqrt{2 C_{p} T_{1}\left(1-\left(\frac{p_{2}}{p_{1}}\right)^{(\gamma-1) / \gamma}\right)}
\end{aligned}
$$

Actual mass flow rate,

Mass flow rate of fuel


Let $\mathrm{z}=$ Distance between tip of fuel nozzle and fuel level in the float chamber

$$
\dot{m}_{f}=\rho_{1} A_{1} C_{1}=\rho_{2} A_{2} C_{2}=\rho_{\mathrm{f}} \mathrm{~A}_{\mathrm{f}} \mathrm{C}_{\mathrm{f}}
$$

$\rho_{1}=$ Density of fuel at float chamber
$\mathrm{A}_{1}=$ Flow area at float chamber
$\mathrm{C}_{1}=$ Velocity of fuel at float chamber
$\rho_{2}\left(\rho_{f}\right), A_{2}\left(A_{f}\right)$ and $C_{2}\left(C_{f}\right)=$ Corresponding values at exit of fuel nozzle (section 2)
Fuel is incompressible and applying Bernoulli's equation between $1 \& 2$,

$$
\frac{p_{1}}{\rho_{1}}+\frac{C_{1}^{2}}{2}+g Z_{1}=\frac{p_{2}}{\rho_{2}}+\frac{C_{2}^{2}}{2}+g Z_{2}
$$

But $\rho_{1}=\rho_{2}=\rho_{f}$ and $Z_{2}-Z_{1}=z, C_{1} \approx 0$

$$
\begin{gathered}
\frac{p_{1}}{\rho_{f}}=\frac{p_{2}}{\rho_{f}}+\frac{C_{f}^{2}}{2}+g\left(Z_{2}-Z_{1}\right) \\
C_{f}=\sqrt{2\left[\frac{\left(p_{1}-p_{2}\right)}{\rho_{f}}-g z\right]}
\end{gathered}
$$

Actual velocity of fuel,

$$
C_{f}=C_{d f} \sqrt{2\left[\frac{\left(p_{1}-p_{2}\right)}{\rho_{f}}-g z\right]}
$$

$C_{d a}=$ Coefficient of discharge of fuel nozzle

Therefore,

$$
\begin{aligned}
\dot{m}_{f} & =C_{d f} \rho_{f} A_{f} \sqrt{2\left[\frac{\left(p_{1}-p_{2}\right)}{\rho_{f}}-g z\right]} \\
& =C_{d f} A_{f} \sqrt{2 \rho_{f}\left[\left(p_{1}-p_{2}\right)-g z \rho_{f}\right]}
\end{aligned}
$$

$A_{f}=$ Cross sectional area of the fuel jet
Critical velocity of air $\left(\mathrm{C}_{\mathrm{a}}\right)_{\text {critical }}=C_{d a} \sqrt{\frac{2 g z \rho_{f}}{\rho_{a}}}$

## Fuel injection system

The main functions of fuel injection system are:

1. Filter the fuel
2. Measure the correct quantity of fuel to be injected
3. Time the fuel injection
4. Control the rate of fuel injection
5. Atomise or break up the fuel to the fine particles
6. Properly distribute the fuel in the combustion chamber

## Air injection

In this method of fuel injection air is compressed in the compressor to a very high pressure (much higher than that developed in the engine cylinder at the end of the compression stroke) and then injected through the fuel nozzle into the engine cylinder. The rate of fuel admission can be controlled by varying the pressure of injection air. The high pressure air is obtained from the compressor driven by the engine. This method is absolute these days.

## Solid or Airless injection

It is also termed as mechanical injection. Here a fuel pump is used which supplies a measured quantity of fuel to the atomiser or injector which injects the fuel at a very high velocity into the engine cylinder in the form of sprays. The injection pressure varies from 100 to 180 bar. This pressure is produced by fuel pump.

This system is consisting of

1. Fuel pump
2. Fuel injector

## Fuel Pump



This is widely used to supply fuel under high pressure in diesel engines. In this pump the plunger stroke remains constant but the effective stroke is reduced by changing the position of helix on the plunger with respect to fuel inlet port.

It has a cylindrical plunger which is closely fitted into the barrel by lapping. The plunger has a constant stroke and reciprocating motion of the plunger is given by the cam. A vertical
hole extends from the top of the plunger to another helical groove. When the inlet and spill port or the by pass ports are closed by the upper edge of the plunger during its ascending motion, the oil above the plunger gets compressed and the delivery valve gets lifted off its seat due to the high pressure developed. Then the fuel is forced through the pipe to the injector. The supply of fuel to the injector is continued until the helical groove reaches the inlet port. The plunger top in this position of the plunger communicates with the spill port through the spiral slot and the vertical groove is thus connected to the atmosphere. The pressure on the spring side is released. The delivery valve falls back to its seat and the supply of fuel to the atomizer is stopped. At this time, the fuel in the barrel escapes from the spill port or the by pass port.

During the downward stroke of the plunger, suction is created and fuel is drawn into the barrel through the supply port.

The quantity of fuel being supplied to the injector depends upon the position of helical groove. The plunger can be rotated by means of a rack. By turning the plunger to the right, the effective pumping stroke can be increased, i.e. the instant, at which the space communication with the spill port can be delayed and so, more fuel can be supplied to the engine at higher loads. The rack is operated by the governor.

## Fuel Injector (Atomiser)



It consists of a nozzle valve fitted in the nozzle body. The nozzle valve is held on its seat by a spring which exerts pressure through the spindle. The nozzle valve lift can be adjusted by adjusting screw. Usually the nozzle valve is set to lift at 135 to 170 bar pressure. The feeling pin indicates whether valve is working properly or not. The oil under pressure from fuel pump enters the injector through the passage $B$ and $C$ and lifts the nozzle valve. The fuel travels down the
nozzle and injected into the engine cylinder in the form of fine sprays. When the pressure of the oil falls, the nozzle valve occupies its seat under the spring force and fuel supply is cut off. Any leakage of fuel accumulated above the valve is led to the fuel tank through the passage $A$. The leakage occurs when the nozzle valve is worn out.

## Ignition System

The ignition system is one of the most important systems of the petrol and gas engines. Spark ignition systems are universally and successfully used in all cars and automobiles using petrol as fuel while compression ignition is used in diesel engines.

## Requirement of spark Ignition systems

1. The voltage across the spark plug electrodes should be sufficiently large to produce an arc required to initiate the combustion. The voltage necessary to overcome the resistance of the spark gap and to release enough energy to initiate the self propagating flame front in the combustible mixture is about 10000 to 20000 Volts.
2. The intensity of spark should lie in a specified limit because too high intensity may burn the electrodes and too low intensity may not ignite the mixture properly.
3. The volume of the mixture (clearance volume) at the end of compression should not be too large, otherwise the spark produced may not be sufficient to ignite the whole charge. There is definite relation between the size of the spark and clearance volume.
4. There should be no missing cycle due to failure of spark.

## Battery or Coil ignition system



Most of the modern spark ignition engines use battery ignition system. This system consists of the following components.

1. Battery ( 6 to 12 volts)
2. Ignition switch
3. Induction coil
4. Circuit/Contact breaker
5. Condenser
6. Distributor

One terminal of the battery is ground to the frame of the engine and other is connected through the ignition switch to one primary terminal of the ignition coil (consisting of a few turns of thick wire). The other primary terminal is connected to one end of the contact points of the circuit breaker and through closed points to ground. The primary circuit of the ignition coil thus gets completed when contact points of the circuit breaker are together and switch is closed. The secondary terminal of the coil is connected to the central contact of the distributor and hence to distributor rotor. The secondary circuit consists of secondary winding (consisting of large number of turns of fine wire) of the coil, distributor and four spark plugs. The contact breaker is driven by a cam whose speed if half the engine speed (for four stroke engines) and breaks the primary circuit one for each cylinder during one complete cycle of the engine.

To start with, the ignition switch is made on and the engine is cranked the contacts touch, the current flows from battery through the switch, primary winding of the induction coil to circuit breaker points and the circuit is completed through the ground. A condenser connected across the terminals of the contact breaker points prevent the sparking at these points. The rotating cam breaks open the contacts immediately and breaking of this primary circuit brings about a change of magnetic field, due to which a very high voltage to the tune of 8000 V to 12000 V is produced across the secondary terminals. Due to high voltage the spark jumps across the gap in the spark plug and air-fuel mixture is ignited in the cylinder.

On account of its combined cheapness, convenience of maintenance, attention and general suitability, it has been adopted universally on automobiles.

## Magneto-Ignition system



The magneto ignition system has the same principle of working as that of coil ignition system, except that no battery is required, as the magneto acts as its own generator.

It consists of either rotating magnets in fixed coils or rotating coils in fixed magnets. The current produced by the magneto is made to flow to the induction coil which works in the same way as that of coil ignition system. The high voltage current is then made to flow to the distributor which connects the sparking plugs in rotation depending upon the firing order of the engine.

This type of ignition system is generally employed in small spark ignition engines such as scooters, motor cycles and small motor boat engines.

## Cooling System

In an I.C engine, the temperature of the gases inside the engine cylinder may vary from $35^{\circ} \mathrm{C}$ or less to as high as $2750^{\circ} \mathrm{C}$ during the cycle. If an engine is allowed to run without
external cooling, the cylinder walls, cylinder and piston will tend to assume the average temperatures of the gases to which they are exposed, which may be of the order of 1000 to $1500^{\circ} \mathrm{C}$. Obviously at such high temperatures, the metals will loose their characteristics and piston will expand considerably and seize the liner. If the cylinder wall temperature is allowed to rise above a certain limit, about $65^{\circ} \mathrm{C}$, the lubricating oil will begin to evaporate rapidly and both cylinder and piston may be damaged. In view of this, part of the heat generated inside the engine cylinder is allowed to be carried away by the cooling system.

## Air Cooling

In this method, heat is carried away by the air flowing over and around the engine cylinder. It is used in scooters, motor cycles, etc. Here fins are cast on the cylinder head and cylinder barrel which provide additional surface for heat transfer. The fins are arranged such a way that they are at right angles to the cylinder axis.


## Advantages

1. Simple in design and cheap
2. Absence of cooling pipes, radiator, etc. makes the cooling system simpler
3. No damage of coolant leakage
4. The engine is not subjected to freezing troubles, etc. usually encountered in water cooled engines
5. The weight per unit power output is less than that of water cooled engines
6. Easy installation

## Disadvantages

1. Their movement is noisy
2. Non-uniform cooling
3. The output of air cooled engine is less than that of water cooled engines
4. Maintenance is not easy
5. Smaller useful compression ratio

## Liquid Cooling

In this method of cooling system, the cylinder walls and heads are provided with jackets through which the cooling liquid can circulate. The heat is transferred from the cylinder walls to the liquid by convection and conduction. The liquid becomes heated in its passage through the jackets and is itself cooled by means of an air-cooled radiator system. The heat from liquid in turn is transferred to air.

## Thermostat water cooling system

Too lower cylinder temperature may result in severe corrosion damage due to condensation of acids in cylinder walls. To avoid such situation, it is customary to use a thermostat (a temperature controlling device) to stop flow of coolant below a pre-set cylinder wall temperature. Most modern cooling systems employ a thermostat device which prevents the water in the engine jacket from circulating through the radiator for cooling until its temperature has reached to a value suitable engine operation.

The water cooling system as shown in fig is used in the engines of cars, busses, trucks, etc. In this system, the water is circulated through water jackets around each of the combustion chambers, cylinders, valve seats and valve stems. The water is kept continuously in motion by a centrifugal pump which is driven by a V-belt from the pulley on the engine crankshaft. After passing through the engine jackets in the cylinder block and heads, the water is passed through the radiator. In the radiator the water is cooled by air drawn through the radiator by a fan. Usually fan and water pump are mounted and driven on a common shaft. After passing through the radiator, the water is drained and delivered to the water pump through a cylinder inlet passage. The water is again circulated through the engine jackets.


## Advantages liquid cooling

1. Compact design of engine is possible
2. The fuel consumption of high compression liquid cooled engine is lower than air-cooled engine
3. Uniform cooling of cylinder barrels (walls) and heads
4. Installation is not necessary at the front of vehicles as in the case of air-cooled engines

## Disadvantages

1. This is dependent system in which supply of water for circulation in the jacket is required
2. Power absorbed by the pump for water circulation is considerably higher than that for cooling fans
3. In the event of failure of cooling system serious damage may be caused to the engine
4. Cost of the system is considerably high
5. Requires more maintenance

## Lubrication System

## Purpose of lubrication

1. Reduce the friction and wear between the parts having relative motion
2. Cool the surface by carrying away heat generated due to friction
3. Seal a space adjoining the surfaces such as piston rings and cylinder liner
4. Clean the surface by carrying away the carbon and metal particles caused by wear
5. Absorb shock between bearings and other parts and consequently reduce noise

## Properties of lubricants

1. Viscosity
2. Flash point and Fire point
3. Cloud point
4. Pour point
5. Oiliness
6. Corrosion
7. Emulsification
8. Physical and Chemical stability
9. Neutralisation number
10. Adhesiveness
11. Film strength
12. Specific gravity

Main Parts of an engine to be lubricated

1. Main crankshaft bearing
2. Big end bearing
3. Small or gudgeon pin bearing
4. Piston rings and cylinder walls
5. Timing gears
6. Camshaft and camshaft bearings
7. Valve mechanism
8. Valve guides, valve tappets and rocker arms

## Wet sump lubrication

These systems employ a large capacity oil sump at the base of crank chamber, from which the oil is drawn by low pressure oil pump and delivered to various parts. Oil there gradually returns back to the sump after serving the purpose.

The general arrangement of wet sump lubrication system is shown in fig. In this case oil is always contained in the sump which is drawn by the pump through a strainer.


## Dry sump lubrication system



In this system, the oil from the sump is carried to a separate storage tank outside the engine cylinder block. The oil from sump is pumped by means of a sump pump through filters to the storage tank. Oil from storage tank is pumped to the engine cylinder through oil cooler. Oil pressure may vary from 3 to 8 bar. Dry sump lubrication system is generally adopted for high capacity engines.

## Mist lubrication system

This system is used for two stroke cycle engines. Most of these engines are crank charged, i.e. they employ crankcase compression and thus, are not suitable for crankcase lubrication. These engines are lubricated by adding 2 to $3 \%$ lubricating oil in the fuel tank. The oil and fuel mixture is induced through the carburettor. The gasoline is vapourised, and the oil in the form of mist, goes via crankcase into the cylinder. The oil which impinges on the crankcase walls lubricates the main and connecting rod bearings and rest of the oil which passes on the cylinder during charging and scavenging periods, lubricates the piston, piston rings and the cylinder.

## Advantages

1. System is simple
2. Low cost (because no oil pump, filter, etc. are required)

## Disadvantages

1. A portion of lubrication oil burns in the combustion chamber leading to increasing exhaust emission and formation of deposits on the piston
2. Lubricating oil loses its anti-corrosion properties as it comes in contact with the acidic vapours produced during combustion.
3. There should be thorough mixing of lubricants and fuels for effective lubrication. This requires separate mixing prior to use or some additives to give the oil good mixing characteristics.
4. Because of burning of some lubricating oil in the combustion chamber, there will be excess consumption of 5 to $15 \%$ lubricant.
5. Since there is no control over the lubricating oil, once introduced with fuel, most of the two stroke engines are over-oiled most of the time.

## Fuels

The majority of fuels for I.C engines are obtained by the distillation of crude petroleum. This may be followed by pressure and heat treatment to reform some of the products coupled with mixing of some additives.

Petrol may be defined as a distillate with a boiling point that does not exceed $200^{\circ} \mathrm{C}$ and diesel fuel as a mixture of gas oils and fuel oils.

## Rating of SI engine fuels - Octane number

The hydrocarbon fuels used in spark ignition engines have a tendency to cause engine knock when the engine operating conditions become severe. The knocking tendency of a fuel in SI engines is generally expressed by its octane number. The percentage by volume of isooctane in a mixture of iso-octane and normal heptane, which exactly matches the knocking intensity of a given fuel, in a standard engine, under given standard operating conditions, is termed as the octane number rating of the fuel. Thus if a mixture of $50 \%$ iso-octane and $50 \%$ of normal heptane matches the fuel under test, then the fuel is assigned an octane number of 50. This octane number rating is an expression which indicates the ability of a fuel to resist knock in a SI engine.

Sine iso-octane is a very good anti-knock fuel, therefore it is assigned a rating of 100 octane number. On the other hand, normal heptane has a very poor anti-knock qualities, therefore it is given a rating of 0 (zero) octane number.

## Rating of Cl engine fuels - Cetane number

The knocking tendency is also found in compression ignition engines with an effect similar to that of SI engines, but it is due to different phenomenon. The knock in Cl engine is due to sudden ignition and abnormally rapid combustion of accumulated fuel in the combustion chamber. Such a situation occurs because of an ignition lag in the combustion of fuel between the time of injection and the actual burning.

The property of ignition lag is generally measured in terms of Cetane number. It is defined as the percentage, by volume, of Cetane in a mixture of Cetane and alpha-methylnaphthalene that produces the same ignition lag as the fuel being tested, in the same engine and under the same operating conditions. For example, a fuel of Cetane number 60 has the same ignition quality as a mixture of $60 \%$ Cetane and $40 \%$ alpha-methyl-naphthalene.

The Cetane which is a straight chain paraffin with good ignition quality is assigned a Cetane number of 100 and alpha-methyl-naphthalene which is a hydrocarbon with poor ignition quality, is assigned 0 Cetane number.

## Detonation (SI engines)

The loud pulsating noise within the engine cylinder is known as detonation. It is caused due to the propagation of a high speed pressure wave created by the auto ignition of end portion of unburned fuel. The blow of this pressure wave may be of sufficient strength to break the piston. Thus the detonation is harmful to the engine and must be avoided. The following are the factors which cause the detonation:
5. The shape of the combustion chamber
6. The relative position of the spark plugs
7. The chemical nature of the fuel
8. The initial temperature and pressure of the fuel
9. The rate of combustion of that portion of the fuel which is the first to ignite

The detonation in petrol engines can be reduced or avoided by the addition of lead ethide or ethyl fluid. This is called doping.

## Knocking (Cl engines)

In Cl engines the fuel, which is in atomized form, is considerably colder than the hot compressed air in the cylinder. Although the actual ignition is almost instantaneous, an appreciable time elapses before the combustion is in full progress. This time occupied is called the delay period or ignition lag. It is the time immediately following injection of the fuel during which the ignition process is being initiated and the pressure does not rise beyond the value it would have due to compression of air. The delay period extends for about $13^{\circ}$ of crank rotation.

If the delay period in Cl engine is long a large amount of fuel will be injected and accumulated in the chamber. The auto ignition of this large amount of fuel may cause high rate of pressure rise and high maximum pressure which may cause knocking in diesel engines.

## Exhaust gas analysis

The combustion products are mainly gaseous. When a sample is taken for analysis it is usually cooled down to a temperature which is below saturation temperature of the steam present. The steam content is therefore not included in the analysis, which is then quoted as analysis of the dry products. Since the products are gaseous, it is usual to quote the analysis by volume. An analysis which includes the steam in the exhaust is called a wet analysis.

The most common means of analysis of the combustion products is the Orsat apparatus.

## Orsat apparatus

An Orsat apparatus consists of a burette, a gas cleaner and four absorption pipettes. The pipettes are interconnected by means of a manifold fitted with cocks $S_{1}, S_{2}, S_{3}$ and $S_{4}$ and contain different chemicals to absorb carbon dioxide $\left(\mathrm{CO}_{2}\right)$, carbon monoxide ( CO ) and oxygen $\left(\mathrm{O}_{2}\right)$. Each pipette is also fitted with a number of small glass tubes which provide a greater amount of surface. These tubes are wetted by the absorbing agents and are exposed to the gas under analysis. The measuring burette is surrounded by a water jacket to prevent changes in temperature and density of the gas. The pipette 1 contains KOH (caustic soda) to absorb $\mathrm{CO}_{2}$, pipette 2 contains an alkaline solution of pyrogallic acid to absorb $\mathrm{O}_{2}$, pipette $3 \& 4$ contain an acid solution of cuprous chloride to absorb CO. Furthermore the apparatus has a leveling bottle and a three way cock to connect the apparatus either to gases or to the atmosphere.

100 cc of gas whose analysis is to be made is drawn into the bottle by lowering the traveling bottle. The stop cock $S_{4}$ is then opened and the whole gas is forced to pipette 1 . The gas remains in the pipette 1 for sometime and most of the $\mathrm{CO}_{2}$ is absorbed. The leveling bottle is then lowered to allow the chemical to come to its original level. The volume of gas thus absorbed is read on the scale of the measuring bottle. The gas is then forced through pipette 1 number of times to ensure that the whole of the $\mathrm{CO}_{2}$ is absorbed. Further the remaining gas is
then forced to the pipette 2 which absorbs $\mathrm{O}_{2}$. The reading on the measuring bottle will be the sum of volume of $\mathrm{CO}_{2}$ and $\mathrm{O}_{2}$. The $\mathrm{O}_{2}$ content can be found out by subtraction. Finally, as before, the sample is forced through the pipette $3 \& 4$ to absorb CO completely.

The amount of nitrogen in the sample can be determined by subtracting from total volume of gas the sum of $\mathrm{CO}_{2}, \mathrm{O}_{2}$ and CO .


## Pollution control norms

The rising number of automobiles has had one negative fall out: air pollution, however one a positive note one must add that the emission in motor vehicles has seen a steady decrease over the years. According to SIAM (Society of automobile Manufacturers in India) the country has seen an $86 \%$ reduction in pollution levels over the last decade or so. It has to be noted that India has some of the stringent standards for two-wheeler prevalent anywhere in the world. With the introduction of Euro III (Bharat III in the Indian context) in select cities and the rest of the country moving to Euro II standards, Indians could breathe easy and breathe cleaner air. All car manufacturers have already started phasing out Euro I cars because they won't be allowed to be sold in the country because of these new emission norms.

The emission norms in India came into force from 1991 for Petrol vehicles and in the following year it was extended to Diesel vehicles that were plying in the country. From the year 1995 it was made mandatory for all petrol vehicles in the four metros to use catalytic converters. Unleaded petrol was also made available to these four cities then, which was later extended to other parts of the country by the year 2000.

The Central Pollution Control Board (CPCB) has taken some steps to reduce air pollution in the country.

- Establishment of Ambient air quality monitoring throughout the country.
- Notification of Ambient air quality standards under the Environment Protection Act.
- Notification of Air pollution norms
- Improving the fuel quality
- Introduction of cars that run on alternative fuel like CNG/LPG etc.
- Improvement of public transport system
- Phasing out old and polluting commercial vehicles
- Creating awareness and public campaigns

In the wake of making the country pollution free various car companies have taken steps to improve the situation. Maruti Suzuki which is one of the oldest car manufacturers in the country organizes free pollution check camps and is also pushing aggressively for CNG kits for its cars. Recently it has launched a Wagon R which comes fitted with LPG kit; it also has a LPG version of its popular car OMNI in its kitty.

Hyundai Motor Company globally has been in the forefront of innovating environmental friendly vehicles like Hybrid Electric Vehicle and Fuel Cell Electric Vehicles. Companies like Tata Motors and Mahindra and Mahindra are also working hard to meet the stringent Bharat III or the Euro III standards in their diesel engines. It has been able to meet the standard with a conventional diesel engine; however it is also working on a CRDI engine to stay in the competition. Mahindra on the other hand has straightway introduced a CRDI version of its flagship vehicle Scorpio. Not to be left behind the two-wheeler manufacturers have already taking steps to meet the strict emission standards that are prevalent in the country. Hero Honda which is one of the largest two wheeler manufacturer in the country has a philosophy of continuously innovating new products to improve environmental compatibility. Similarly automobile users have a key role to play to keep the environment pollution free, they should maintain their vehicles and drive responsibly to make the country a better place to drive about.

## Performance of I.C engines

Engine performance is an indication of the degree of success with which it does its assigned work, i.e., conversion of heat energy into useful mechanical work.

The basic performance parameters are
(i) Power and Mechanical efficiency
(ii) Brake thermal efficiency
(iii) Indicated thermal efficiency
(iv) Volumetric efficiency
(v) Specific fuel consumption
(vi) Mean effective pressure and Torque

## Indicated Power (IP)

The ideal power developed by the combustion of fuel in the cylinder is called IP.

$$
I P=\frac{z p_{m i} L A n}{60}--W
$$

Where, $\quad Z \rightarrow$ Number of cylinders
$\mathrm{N} \rightarrow$ Compressor speed in rpm
$\mathrm{n} \rightarrow$ Number of working cycles $=\mathrm{N}$ for two stroke engines
= N/2 for four stroke engines
$\mathrm{p}_{\mathrm{mi}} \rightarrow$ Indicated mean effective pressure in Pa
A $\rightarrow$ Area of the cylinder $=(\pi / 4) D^{2}$
$D \rightarrow$ Bore or diameter of the cylinder in $m$
$L \rightarrow$ Stroke in $m$

## Brake Power (BP)

Power available at the crank shaft is called BP.

$$
B P=\frac{z p_{m b} L A n}{60}=\frac{2 \pi N T}{60}
$$

Where, $\quad \mathrm{p}_{\mathrm{mb}} \rightarrow$ Brake mean effective pressure in Pa

$$
\mathrm{T} \rightarrow \text { Torque in Nm }
$$

## Friction Power (FP)

Power required to overcome the friction between moving parts.

$$
\mathrm{FP}=\mathrm{IP}-\mathrm{BP}
$$

## Mechanical Efficiency ( $\eta_{m}$ )

The ratio of BP and IP is called mechanical efficiency.

$$
\eta_{\mathrm{m}}=\mathrm{BP} / \mathrm{IP}
$$

## Mean Effective Pressure ( $\mathbf{p}_{\mathrm{m}}$ )

It is hypothetical pressure which is assumed to be acting on the piston throughout the power stroke.

Based on BP, $\quad \mathrm{p}_{\mathrm{mb}}=B P /$ Stroke volume $\rightarrow B M E P$
Based on IP, $\quad \mathrm{p}_{\mathrm{mi}}=\mathrm{IP} /$ Stroke volume $\rightarrow$ IMEP

## Volumetric Efficiency ( $\eta_{\mathrm{v}}$ )

It is the actual volume of charge drawn in during the suction stroke to the swept volume of the piston.

## Specific Fuel Consumption (SFC)

It is defined as the quantity of fuel required per hour to produce unit power output.
Based on BP, $\quad \mathrm{SFC}=\mathrm{m}_{\mathrm{f}} / \mathrm{BP}$
Based on IP, $\quad$ SFC $=m_{f} / I P$

## Brake Thermal Efficiency ( $\eta_{\mathrm{bt}}$ )

$$
\eta_{b t}=B P /\left(m_{f} \times C V\right)
$$

IP $\rightarrow$ Indicated power in W
$\mathrm{m}_{\mathrm{f}} \rightarrow$ Mass flow rate of fuel in $\mathrm{kg} / \mathrm{h}$
$\mathrm{CV} \rightarrow$ Calorific value of fuel in $\mathrm{J} / \mathrm{kg}$
Indicated Thermal Efficiency $\left(\eta_{i t}\right)$

$$
\eta_{\mathrm{it}}=\mathrm{IP} /\left(\mathrm{m}_{\mathrm{f}} \times \mathrm{CV}\right)
$$

## Relative Efficiency ( $\eta_{r}$ )

$\eta_{r}=$ Thermal efficiency / Air standard efficiency
Based on $\eta_{\mathrm{bt}}, \quad \eta_{\mathrm{r}}=$ Brake thermal efficiency / Air standard efficiency
Based on $\eta_{\mathrm{it}}, \quad \eta_{\mathrm{r}}=$ Indicated thermal efficiency / Air standard efficiency

## Measurement of fuel consumption

The fuel from the fuel tank is sent to the engine through " three way cock ". Three way cock consists of three individual values one for fuel tank, one for engine, and third for measuring gauge.

Let $t_{f}=$ time taken for ' $V$ ' cc of fuel consumption. Now the tank line is closed and fuel is taken from measuring burette. Now time taken for 10 cc of fuel consumption is noted.
$\therefore$ Total fuel consumption ( TFC ) $=\left(\mathrm{V} / \mathrm{t}_{\mathrm{f}}\right) \mathrm{x}$ density of fuel
Specific fuel consumption (SFC) = TFC / BP
SFC is defined as the fuel required for producing 1 kWh of shaft power output.


## Measurement of air consumption

The air consumption can be measured by
3. Air box method
4. Viscous-flow meter

Air Box Method


It consists of air tight chamber fitted with a sharp edged orifice. The orifice is located away from the suction connection to the engine. Due to the suction of the engine, there is a pressure depression in the air box which causes the flow through the orifice. For obtaining
steady flow the volume of chamber should be sufficiently large compared with the swept volume of the cylinder ( 500 to 600 times the swept volume).

A water manometer is used to measure the pressure difference across the orifice. The orifice should be designed such that the depression across the orifice should not exceed 100 to 150 mm of water.

Let, $\quad a=$ Area of the orifice
$d=$ Diameter of the orifice
$h_{w}=$ Pressure depression across the orifice in $m$ of water
$C_{d}=$ Coefficient of discharge of orifice
$h_{\mathrm{a}}=$ Pressure depression across the orifice in m of air
$\rho_{w}=$ Density of water in $\mathrm{kg} / \mathrm{m}^{3}$
$\rho_{\mathrm{a}}=$ Density of air in $\mathrm{kg} / \mathrm{m}^{3}$
$m_{a}=$ Mass flow rate of air in $\mathrm{kg} / \mathrm{s}$
$V_{a}=$ Velocity of air through the orifice $=\sqrt{2 g h_{a}} \mathrm{~m} / \mathrm{s}$
We can write, $\rho_{w} h_{w}=\rho_{a} h_{a}$

$$
h_{a}=\frac{\rho_{\mathrm{w}}}{\rho_{a}} h_{\mathrm{w}}
$$

$$
\dot{m}_{a}=C_{d} \rho_{a} a V_{a}
$$

Viscous-flow air meter


Alcock viscous-flow air meter is another design of air meter. It is not subjected to the errors of the simple type flow meters. With the air box method the flow is proportional to the square of the pressure difference across the orifice. With the Alcock meter the air flows through a form of honey comb so that the flow is viscous. The resistance of the element is directly proportional to the air velocity and is measured by means of an inclined manometer. The accuracy is improved by fitting a damping vessel between the meter and the engine to reduce the effect of pulsations

## Measurement of Brake Power (BP)

## Absorption dynamometers

Absorption dynamometers are those that absorb the power to be measured by friction. The power absorbed in friction is finally dissipated in the form of heat energy.

1. Prony brake dynamometer
2. Rope brake dynamometer
3. Hydraulic brake dynamometer
4. Fan brake dynamometer
5. Electrical brake dynamometer (Eddy current and swinging field dynamometers)

## Transmission dynamometers

These are also called Torque meters. These are very accurate and are used where continuous transmission of load is necessary. These are mainly used in automatic units.
Rope brake dynamometer


A rope is wound round the circumference of the brake wheel. To prevent the rope from slipping small wooden blocks are laced to rope. The spring balance is attached to one end of the rope and to the other end the load carrier (to carry dead weights). The speed of the engine is noted using tachometer.
Let, $\quad W=$ Weight on the load carrier in N
$\mathrm{S}=$ Spring balance reading in N
$R=$ Radius of the brake drum in $m$
$r=$ Radius of the rope in $m$
$R_{m}=$ Mean radius of the brake drum $=R+r$
$\mathrm{T}=$ Torque in Nm
$\mathrm{N}=$ Engine speed in rpm

$$
B P=\frac{2 \pi N T}{60}=\frac{2 \pi N(W-S) R_{m}}{60}
$$

## Prony brake dynamometer

The arrangement of braking system is shown in fig. It consists of brake shoes made of wood and these are clamped to the rim of the brake wheel by means of the bolts. The pressure on the rim is adjusted with the help of nut and springs. A load bar extends from top of the brake and a load carrier is attached to the end of the load bar. The load arm is kept horizontal to keep the arm length constant.

The energy supplied by the engine to the brake is eventually dissipated as heat. Therefore most of the brakes are provided with supply of cooling water to the inside rim of the brake drum.


$$
B P=\frac{2 \pi N T}{60}=\frac{2 \pi N W L}{60}
$$

W = Load on the carrier
$\mathrm{L}=$ Distance from the centre of the shaft to the point of load-meter
Prony brake is inexpensive, simple in operation and easy to construct. It is extensively used for testing of low speed engines.

Hydraulic dynamometers

$$
B P=\frac{2 \pi N T}{60}=\frac{2 \pi N W R}{60}
$$

R = Arm length.

## Measurement of mean effective pressure


(a) Equivalent rectangle method.

(b) Mid-ordinate method.

Mean effective pressure is defined as hypothetical constant pressure which is thought to be acting on the piston throughout the power stroke.

If it is based on IP, it is called indicated mean effective pressure (IMEP). If it is based on $B P$, it is called brake mean effective pressure (BMEP).

IMEP is obtained from the indicator diagram by any one of the following methods:

1. By drawing the diagram on a squared paper and then finding its area by counting the number of squares.
2. By finding the area of the indicator diagram with the help of a planimeter.
3. By mid-ordinates taken from one end to another.

In all the methods, the aim is to determine the height of a rectangle of an area equal to the area of the indicator diagram. The height of this rectangle gives the IMEP.

Fig (a) shows the indicator diagram and the equivalent rectangle having the same area of the indicator diagram, whose length is equal to the length of the indicator diagram or card.

$$
\begin{aligned}
& I M E P=\frac{\text { Workdone }}{\text { Stroke volume }} \rightarrow \text { Theoretical } \\
& I M E P=\frac{\text { Area of indicator card }(\mathrm{mm}) \times \text { scale of the indicator spring }(\mathrm{bar} / \mathrm{mm})}{\text { Length of the indicator card }(\mathrm{mm})} \rightarrow \text { Actual }
\end{aligned}
$$

In case of mid-ordinate method, the indicator diagram is divided into strips of equal width. At the centre of each strip, mid-ordinates are drawn. All these mid-ordinates are added and the total is divided by number of ordinates to get the mean height of the indicator diagram.

IMEP = Mean height $x$ Scale of the indicator spring

## Measurement of Indicated Power (IP)

The IP of the engine at a particular running condition is obtained from the indicator diagram. The indicator diagram is the $\mathrm{p}-\mathrm{V}$ diagram for one cycle at that load drawn with the help of indicator fitted on the engine. The area of the indicator diagram is IP and is measured using planimeter. Or from the IMEP the IP can be obtained.

## Measurement of Friction Power (FP)

FP of an engine can be determined by the following methods:

1. Willan's line method (only for Cl engines)
2. Morse test (Only for multi-cylinder engines)
3. Motoring test
4. Difference between IP and BP
5. Retardation test

## Motoring test

In this test the engine is first run upto the desired speed by its own power and allowed to remain under the given speed and load conditions for some time so that oil, water and engine component temperatures reach stable conditions. The power of the engine during this period is absorbed by a dynamometer (usually electrical type). The fuel supply is then cut-off and by suitable electrical switching devices the dynamometer is converted to run as a motor to drive or motor the engine at the same speed at which it was previously running. The power supply to the motor is measured which is a measure of FP of the engine.

## Willan's line method

At constant speed the load is varied and the corresponding BP and TFC (Total fuel consumption) readings are taken. A graph is drawn between BP and TFC as shown in fig. The graph drawn is called Willan's line and is extrapolated back to cut the BP axis at the point L . The reading OL is taken as the power loss (FP) of the engine at that speed.


## Morse test

The engine is run at required speed and the BP is measured. One cylinder is cut out, by shorting the spark plug if a SI engine is under test, or by disconnecting an injector if a Cl engine is under test. The speed falls because of the loss of power with one cylinder cut out, but is restored by reducing the load. The BP is measured again when the speed has reached its original value.

If the value of IP of the cylinders are denoted by $\mathrm{IP}_{1}, \mathrm{IP}_{2}, \mathrm{IP}_{3}$ and $\mathrm{IP}_{4}$ (considering four cylinder engine) and the power loses in the each cylinder are denoted by $\mathrm{FP}_{1}, \mathrm{FP}_{2}, \mathrm{FP}_{3}$ and $\mathrm{FP}_{4}$ then the values of BP at test speed with all cylinders firing is given by,

$$
\begin{align*}
\mathrm{BP}_{1234} & =\mathrm{BP}_{1}+\mathrm{BP}_{2}+\mathrm{BP}_{3}+\mathrm{BP}_{4} \\
& =\left(\mathrm{IP}_{1}-\mathrm{FP}_{1}\right)+\left(\mathrm{IP}_{2}-\mathrm{FP}_{2}\right)+\left(\mathrm{IP}_{3}-\mathrm{FP}_{3}\right)+\left(\mathrm{IP}_{4}-\mathrm{FP}_{4}\right) \tag{1}
\end{align*}
$$

If Cylinder-1 is cut out,

$$
\mathrm{IP}_{1}=0 \text {, but } \mathrm{FP}_{1} \text { remains same. }
$$

Therefore

$$
\begin{equation*}
\mathrm{BP}_{234}=\left(-\mathrm{FP}_{1}\right)+\left(\mathrm{IP}_{2}-\mathrm{FP}_{2}\right)+\left(\mathrm{IP}_{3}-\mathrm{FP}_{3}\right)+\left(\mathrm{IP}_{4}-\mathrm{FP}_{4}\right) \tag{2}
\end{equation*}
$$

(1) - (2) $\rightarrow \quad \mathrm{BP}_{1234}-\mathrm{BP}_{234}=\mathrm{IP}_{1}$

Similarly, $\quad B P_{1234}-B P_{134}=I P_{2}$

$$
\mathrm{BP}_{1234}-\mathrm{BP}_{124}=\mathrm{IP}_{3}
$$

$$
B P_{1234}-B P_{123}=\mathrm{IP}_{4}
$$

$$
\mathrm{IP}_{1234}=I \mathrm{IP}_{1}+I \mathrm{I}_{2}+I \mathrm{IP}_{3}+\mathrm{IP}_{4}
$$

$$
\mathrm{FP}_{1234}=\mathrm{IP}_{1234}-\mathrm{BP}_{1234}
$$

## Retardation Test

In this test, the engine is run at required speed with no load condition. Now the engine fuel is cut-off and the time taken for ' N ' speed drop from required speed is noted. Again the
engine is run at required speed, but with some load on the engine. Now fuel is cut - off and time taken for the same ' N ' speed drop is noted.

Let $\mathrm{t}_{1}=$ time taken for ' N ' speed drop with no load.
Let $t_{2}=$ time taken for ' $N$ ' speed drop with load.
$\mathrm{W}=$ load applied on the engine.
We know that, $B P=2 \pi N T_{B} / 60$

$$
\mathrm{FP}=2 \pi \mathrm{NT}_{\mathrm{F}} / 60
$$

$\mathrm{T}_{\mathrm{B}}=$ Braking torque $=\mathrm{WR}_{\mathrm{m}}$ (or) WR
$\mathrm{T}_{\mathrm{F}}=$ Friction torque.
With no load
Torque on the engine $(T)=I \alpha$
$\mathrm{I}=$ Mass moment of inertia.
$\alpha=$ angular acceleration (or) retardation.

$$
\mathrm{T}=\mathrm{T}_{\mathrm{F}}=\mathrm{I}\left(\omega_{1}-\omega_{2}\right) / \mathrm{t}_{1} \quad \rightarrow \text { (1) }
$$

With load

$$
\mathrm{T}=\mathrm{T}_{\mathrm{B}}+\mathrm{T}_{\mathrm{F}}=\mathrm{I}\left(\omega_{1}-\omega_{2}\right) / \mathrm{t}_{2} \quad \rightarrow(2)
$$

(2) / (1)

$$
\begin{aligned}
\frac{T_{B}+T_{F}}{T_{F}} & =\frac{t_{1}}{t_{2}} \\
\frac{T_{B}}{T_{F}}+1 & =\frac{t_{1}}{t_{2}} \\
\frac{T_{B}}{T_{F}} & =\frac{t_{1}}{t_{2}}-1=\frac{t_{1}-t_{2}}{t_{2}}
\end{aligned}
$$

$$
T_{F}=T_{B}\left[\frac{t_{2}}{t_{1}-t_{2}}\right]
$$

$$
F_{P}=\frac{2 \pi N T_{F}}{60}
$$

Note: Only Tachometer and stop watch are required for this test.

## Heat balance sheet

A balance sheet is an account of heat supplied and heat utilized in various ways in the system. The heat balance is generally done on second or minute or hour basis.

To draw the heat balance sheet for IC engine, it is run at constant load.
Let, $\quad$ TFC = Total fuel consumption in kg/s
CV = Calorific value of fuel used in $\mathrm{J} / \mathrm{kg}$
$m_{w}=$ Mass flow rate of cooling water in $\mathrm{kg} / \mathrm{s}$
$m_{a}=$ Mass flow rate of air in $\mathrm{kg} / \mathrm{s}$
$m_{g}=$ Mass flow rate of exhaust gas in $\mathrm{kg} / \mathrm{s}$
$\mathrm{T}_{\mathrm{wi}}$ = Inlet cooling water temperature
$\mathrm{T}_{\mathrm{wo}}=$ Outlet cooling water temperature
$\mathrm{T}_{\mathrm{g}}=$ Exhaust gas temperature
T = Torque in Nm
$\mathrm{N}=$ Speed of the engine in rpm
$\mathrm{C}_{\mathrm{pw}}=$ Specific heat of water in J/kgK
$\mathrm{C}_{\mathrm{pg}}=$ Specific heat of exhaust gas in $\mathrm{J} / \mathrm{kgK}$
Input heat $\left(Q_{i}\right)=$ TFC $\times$ CV ----- J/s

$$
\% Q_{i n}=\frac{Q_{i n}}{Q_{i}} \times 100=100
$$

Useful work $\left(Q_{\text {use }}\right)=B P=2 \pi N T / 60$

$$
\% Q_{\text {use }}=\frac{Q_{\text {use }}}{Q_{i}} \times 100
$$

Heat taken away by the cooling water $\left(Q_{w}\right)=m_{w} C_{p w}\left(T_{w o}-T_{w i}\right)$

$$
\% Q_{w}=\frac{Q_{w}}{Q_{i}} \times 100
$$

Heat carried away by the exhaust gases $\left(Q_{g}\right)=m_{g} C_{p g}\left(T_{g}-T_{a}\right)$

$$
\% Q_{g}=\frac{Q_{g}}{Q_{i}} \times 100
$$

Unaccounted losses $\left(Q_{u n}\right)=Q_{i}-\left(Q_{u s e}+Q_{w}+Q_{g}\right)$

$$
\% Q_{u n}=\frac{Q_{u n}}{Q_{i}} \times 100
$$

The value of $\mathrm{m}_{\mathrm{g}}$ can be determined by two methods:

1. Air flow measurement method
2. Exhaust gas calorimeter method

| Item | $\mathrm{J} / \mathrm{s}$ | $\%$ |
| :--- | :--- | :--- |
| Heat input | $\mathrm{Q}_{\mathrm{i}}$ | 100 |
| Useful heat | $\mathrm{Q}_{\mathrm{use}}$ | $\%_{\mathrm{use}}$ |
| Heat taken away by the cooling water | $\mathrm{Q}_{\mathrm{w}}$ | $\%_{\mathrm{w}}$ |
| Heat carried away by the exhaust gases | $\mathrm{Q}_{\mathrm{g}}$ | $\%_{\mathrm{g}}$ |
| Unaccounted losses | $\mathrm{Q}_{\mathrm{un}}$ | $\%_{\mathrm{un}}$ |
| Total | $\mathrm{Q}_{\mathrm{i}}$ | 100 |

## PROBLEMS

1. A four cylinder four stroke petrol engine has a size 65 mm diameter and 95 mm stroke. On test, it developed a torque of 64 Nm when running at 3000 rpm . If clearance volume in each cylinder is $63 \mathrm{~cm}^{3}$, the brake efficiency ratio based on air standard efficiency is 0.5 and calorific value of petrol is $42000 \mathrm{~kJ} / \mathrm{kg}$. Determine the fuel consumption in $\mathrm{kg} / \mathrm{h}$ and brake mean effective pressure.
Given:
Four stroke petrol engine
Number of cylinders ( $Z$ ) $=4$
Number of working cycles ( $n$ ) $=\mathrm{N} / 2$
Diameter (D)

$$
=65 \mathrm{~mm}
$$

Stroke (L)

$$
=95 \mathrm{~mm}
$$

Torque (T)

$$
=64 \mathrm{Nm}
$$

Speed (N)

$$
=3000 \mathrm{rpm}
$$

Clearance volume $\left(\mathrm{V}_{2}\right) \quad=63 \mathrm{~cm}^{3}=63 \times 10^{-6} \mathrm{~m}^{3}$
Efficiency ratio $=$ Relative efficiency $\left(\eta_{r}\right)=0.5$
Calorific value of fuel (CV) $=42000 \mathrm{~kJ} / \mathrm{kg}$
Required: Mass flow of fuel $\left(\mathrm{m}_{\mathrm{f}}\right)$, bmep

## Solution:

$$
\eta_{\mathrm{bt}}=B P /\left(\mathrm{m}_{\mathrm{f}} \times \mathrm{CV}\right)
$$

$B P=$ Brake power $=2 \pi N T / 60$

$$
\begin{aligned}
& =2 \times \pi \times 3000 \times 64 / 60=20106.2 \mathrm{~W} \\
\eta_{\mathrm{r}} & =\eta_{\mathrm{bt}} / \eta_{\text {air }} \\
\eta_{\text {air }} & =1-1 / \mathrm{r}^{r-1}
\end{aligned}
$$

$r=V_{1} / V_{2}$


To find $V_{1}-V_{2}$

$$
\begin{aligned}
V_{1}-V_{2} & =(\pi / 4) D^{2} L=(\pi / 4) \times 0.065^{2} \times 0.095=315.239 \times 10^{-6} \mathrm{~m}^{3} \\
V_{1}-63 \times 10^{-6} & =315.239 \times 10^{-6} \\
V_{1} & =378.239 \times 10^{-6}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad r & =378.239 \times 10^{-6} / 63 \times 10^{-6}=6 \\
\therefore \quad \eta_{\text {air }} & =1-1 / 6^{1.4-1}=0.5125 \\
\eta_{\text {bt }} & =0.5125 \times 0.5=0.2562 \\
0.2562 & =20106.2 /\left(\mathrm{m}_{\mathrm{f}} \times 42000 \times 10^{3}\right) \\
\mathrm{m}_{\mathrm{f}} & =1.8685 \times 10^{-3} \mathrm{~kg} / \mathrm{s}=6.726 \mathrm{~kg} / \mathrm{h} \text {--- Ans }
\end{aligned}
$$

Also, $B P=Z p_{m b} L A n / 60$

$$
\begin{aligned}
20106.2 & =\left[4 \times p_{\mathrm{mb}} \times 0.095 \times(\pi / 4) \times 0.065^{2} \times 3000 / 2\right] / 60 \\
\mathrm{p}_{\mathrm{mb}} & =637807.7 \mathrm{~N} / \mathrm{m}^{2}=6.37 \text { bar }-- \text { Ans }
\end{aligned}
$$

2. The following particulars refer to a $\mathbf{2}$ stroke diesel engine:

| Bore | $=10 \mathrm{~cm}$ |
| :--- | :--- |
| Stroke | $=15 \mathrm{~cm}$ |
| Piston speed | $=300 \mathrm{~m} / \mathrm{min}$ |
| Torque developed | $=58 \mathrm{Nm}$ |
| Mechanical efficiency | $=80 \%$ |
| Indicated thermal efficiency | $=40 \%$ |
| Calorific value of fuel used | $=44000 \mathrm{~kJ} / \mathrm{kg}$ |

Determine (a) Indicated power (b) Indicated mean effective pressure and (c) Fuel consumption per kWh on brake power basis.
Given:
Number of working cycles ( n ) $=\mathrm{N}$
Required: (a) IP (b) $\mathrm{p}_{\mathrm{mi}}$ (c) SFC
Solution:
(a)

$$
\begin{aligned}
& I P=Z p_{m i} L A n / 60 \\
& B P=2 \pi N T / 60
\end{aligned}
$$

Piston speed $=2 \mathrm{~L} \mathrm{~N}$

$$
\begin{aligned}
& \mathrm{N}=300 /(2 \times 0.15)=1000 \mathrm{rpm} \\
& \quad \therefore \mathrm{BP}=2 \times \pi \times 1000 \times 58 / 60=6073 \mathrm{~W}
\end{aligned}
$$

Mechanical efficiency $\left(\eta_{m}\right)=B P / I P$

$$
\begin{aligned}
0.8 & =6073 / \mathrm{IP} \\
\mathrm{IP} & =7592.1 \mathrm{~W} \text {--. Ans }
\end{aligned}
$$

(b)

$$
\mathrm{IP}=\mathrm{Z} \mathrm{p}_{\mathrm{mi}} \mathrm{~L} \mathrm{An} / 60
$$

$$
7592.1=\left[1 \times \mathrm{p}_{\mathrm{mi}} \times 0.15 \times(\pi / 4) \times 0.1^{2} \times 1000\right] / 60
$$

$$
\mathrm{p}_{\mathrm{mi}}=386666.7 \mathrm{~N} / \mathrm{m}^{2}=3.86 \text { bar }-- \text { Ans }
$$

(c)

$$
\mathrm{SFC}=\mathrm{m}_{\mathrm{f}} / \mathrm{BP}
$$

BP in kW \& $\mathrm{m}_{\mathrm{f}}$ in $\mathrm{kg} / \mathrm{h}$

$$
\begin{aligned}
\eta_{\text {it }} & =\mathrm{IP} /\left(\mathrm{m}_{\mathrm{f}} \mathrm{CV}\right) \\
0.4 & =7592.1 /\left(\mathrm{m}_{\mathrm{f}} \times 44000 \times 10^{3}\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{m}_{\mathrm{f}} & =4.3137 \times 10^{-4} \mathrm{~kg} / \mathrm{s}=1.5529 \mathrm{~kg} / \mathrm{h} \\
\mathrm{SFC} & =1.5529 / 6.073=0.2557 \mathrm{~kg} / \mathrm{kWh} \text {--- Ans }
\end{aligned}
$$

3. The following observations relate to the test on a 4 - stroke diesel engine. Determine indicated mep and indicated power. Area of indicator diagram = 420 $\mathrm{mm}^{2}$; Length of the indicator diagram $=62 \mathrm{~mm}$; spring constant $=1.2 \mathrm{bar} / \mathrm{mm}$; bore $=100 \mathrm{~mm}$; stroke $=150 \mathrm{~mm}$; speed $=\mathbf{4 5 0} \mathrm{rpm}$.
Given
4 stroke engine.
Bore (D)

$$
=0.1 \mathrm{~m}
$$

Stroke (L)

$$
=0.15 \mathrm{~m}
$$

Speed (N) $\quad=450 \mathrm{rpm}$
Area of the indicator diagram $=420 \mathrm{~mm}^{2}$
Length of the indicator diagram $=62 \mathrm{~mm}$
Spring constant $\quad=1.2 \mathrm{bar} / \mathrm{mm}$
Required : $\mathrm{p}_{\mathrm{m}}$ \& IP

## Solution

$$
\begin{aligned}
& p_{m}=\stackrel{\text { Area of the indicator diagram }}{\text { Length of the indicator diagram }} \times \text {--------------- Spring constant } \\
& =-\frac{420}{62} \times 1.2=8.129 \text { bar }-- \text { Ans } \\
& I P=n p_{m} L A N / 120 \\
& =1 \times\left(8.129 \times 0.15 \times \pi / 4(0.1)^{2} \times 450 / 120\right) \\
& =3591.28 \mathrm{~W} \text {---- Ans }
\end{aligned}
$$

4. Four strike engine has a piston displacement of $2210 \mathrm{~cm}^{3}$. The compression ratio is 6.4. The fuel consumption is $0.13 \mathrm{~kg} / \mathrm{min}$. The CV of fuel is $45000 \mathrm{~kJ} / \mathrm{kg}$. The brake power developed while running at 2500 rpm is 50.25 kW . Determine the BMEP and relative efficiency based on brake thermal efficiency.

Given
Four stroke engine
Displacement of piston (stroke volume)

$$
\begin{aligned}
& =\mathrm{V}_{1}-\mathrm{V}_{2}=2210 \mathrm{~cm}^{3} \\
& =2210 \times 10^{-6} \mathrm{~m}^{3} \\
& =6.4 \\
& =0.13 \mathrm{~kg} / \mathrm{min}=0.13 / 60 \mathrm{~kg} / \mathrm{s} \\
& =45000 \mathrm{~kJ} / \mathrm{kg} \\
& =50.25 \mathrm{~kW} \\
& =2500 \mathrm{rpm}
\end{aligned}
$$

Required: $\mathrm{p}_{\mathrm{m}(\mathrm{B})} \& \eta_{\mathrm{R}}$
Solution

$$
\mathrm{p}_{\mathrm{m}(\mathrm{~B})}=\mathrm{BMEP}
$$

We can write ,

$$
\begin{aligned}
B P & =Z p_{m(B)} L A n / 60 \\
& =Z p_{m(B)}(V 1-V 2) N / 120
\end{aligned}
$$

$$
\begin{aligned}
& 50.25 \times 10^{3}=1 \times \mathrm{p}_{\mathrm{m}(\mathrm{~B})} \times 2210 \times 10^{-6} \times 2500 / 120 \\
& \therefore \quad \mathrm{p}_{\mathrm{m}(\mathrm{~B})}=109140.7 \mathrm{~N} / \mathrm{m}^{2}=10.914 \text { bar } \quad \text {--- Ans. }
\end{aligned}
$$

Relative efficiency $\left(\eta_{R}\right)=\eta_{B T} / \eta_{\text {air }}$

$$
\begin{aligned}
\eta_{\mathrm{BT}}= & \mathrm{BP} \\
& \\
& =50.25 /(0.13 / 60 \times 4500)=0.5154 \\
& \\
\eta_{\text {air }} & =1-\left[1 /(r)^{\gamma-1}\right]=1-\left[1 /(6.4)^{1.4-1}\right]=0.524 \\
\therefore \quad \eta_{R} & =0.5154 / 0.524=0.983 \quad-\cdots \text { Ans. }
\end{aligned}
$$

5. The following observations were recorded in a test of one hour duration on a single cylinder oil engine working on four stroke cycle.

| Bore | $=300 \mathrm{~mm}$ |
| :---: | :---: |
| Stroke | $=450 \mathrm{~mm}$ |
| Fuel used | $=8.8 \mathrm{~kg}$ |
| C.V. of fuel | $=41800 \mathrm{~kJ} / \mathrm{kg}$ |
| Average speed | $=200 \mathrm{rpm}$ |
| Mep | $=5.8 \mathrm{bar}$ |
| Brake friction load | $=1860 \mathrm{~N}$ |
| Quantity of cooling water | $=650 \mathrm{~kg}$ |
| Temperature rise | $=22^{\circ} \mathrm{C}$ |
| Diameter of the brake whe | $\mathrm{el}=1.22 \mathrm{~m}$ |
| Calculate (i) $\eta_{\mathrm{m}}$, (ii) $\eta_{\mathrm{BT}}$ \& | i) draw heat balance sheet. |
| Given |  |
| Bore (D) | $=0.3 \mathrm{~m}$ |
| Stroke (L) | $=0.45 \mathrm{~m}$ |
| Fuel used (TFC) | $=8.8 \mathrm{~kg} / \mathrm{h}=8.8 / 3600 \mathrm{~kg} / \mathrm{s}$ |
| CV | $=41800 \mathrm{~kJ} / \mathrm{kg}$ |
| Average speed (N) | $=200 \mathrm{rpm}$ |
| $\operatorname{Mep}\left(\mathrm{p}_{\mathrm{m}}\right)$ | $=5.8 \mathrm{bar}$ |

Load (W) $\quad=1860 \mathrm{~N}$
Mass flow rate of cooling water $\left(\mathrm{m}_{\mathrm{w}}\right)=650 \mathrm{~kg} / \mathrm{h}$
$=650 / 3600 \mathrm{~kg} / \mathrm{s}$
Temperature rise $=\mathrm{T}_{\text {wo }}-\mathrm{T}_{\text {wi }} \quad=22^{\circ} \mathrm{C}$
Diameter of the brake wheel $\left(\mathrm{D}_{\mathrm{B}}\right) \quad=1.22 \mathrm{~m}$
Required: (i) $\eta_{m}$, (ii) $\eta_{\boldsymbol{B}}$, \& (iii) to draw heat balance sheet
Solution

$$
\begin{align*}
\eta_{m} & =\mathrm{BP} / \mathrm{IP}  \tag{i}\\
\mathrm{BP} & =2 \pi \mathrm{~N}(\mathrm{WR}) / 60=2 \pi \times 200 \times(1860 \times 1.22 / 2) / 60 \\
& =23763 \mathrm{~W} \\
\mathrm{IP} & =\mathrm{Z} \mathrm{p} \mathrm{p}_{\mathrm{m}} \mathrm{LAn} / 60 \\
& =1 \times\left(5.8 \times 10^{5} \times 0.45 \times \pi / 4 \times 0.3^{2} \times 200 / 120\right) \\
& =30748.34 \mathrm{~W} \\
\therefore \quad \eta_{\mathrm{m}} & =23763 / 30748.34=0.773 \quad--- \text { Ans }
\end{align*}
$$

(ii)

$$
\begin{aligned}
\eta_{\text {BT }}= & B P \\
& (T F----------1 \\
= & (23763 / 1000) /[(8.8 / 3600) \times 41800] \\
= & \mathbf{0 . 2 8 2 5} \quad--- \text { Ans }
\end{aligned}
$$

(iii) Heat balance sheet ( second basis )

$$
\text { Useful work }=Q_{u}=I P=30748.34 \mathrm{~W}
$$

(If IP is known, it should be taken as $Q_{u}$ )
Heat carried away by cooling water $=m_{w} C_{p w}\left(T_{w o}-T_{w i}\right)$

$$
\begin{aligned}
& =(650 / 3600) \times 4186 \times(22) \\
& =16627.7 \mathrm{~W}
\end{aligned}
$$

Heat carried away by exhaust gases $=\mathrm{m}_{\mathrm{g}} \mathrm{C}_{\mathrm{Pg}}\left(\mathrm{T}_{\mathrm{g}}-\mathrm{T}_{\mathrm{a}}\right)$
This loss can't be found out, since sufficient data is not available.

$$
\begin{aligned}
\text { Heat input }=\mathrm{Q}_{\mathrm{i}} & =\text { TFC } \times \mathrm{CV} \\
& =(8.6 / 600) \times 41800 \times 10^{3} \\
& =99855.5 \mathrm{~W}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \% Q_{u} & =\left(Q_{u} / Q_{i}\right) \times 100=(30748.34 / 99855.5) \times 100 \\
& =30.8 \% \\
\% Q_{w} & =\left(Q_{w} / Q_{i}\right) \times 100=(16627.7 / 99855.5) \times 100 \\
& =16.65 \% \\
\text { Unaccounted loss } & =Q_{u n}=Q_{i}-\left(Q_{u}+Q_{w}\right) \\
& =99855.5-(30748.54+16627.7) \\
& =52479.46 \mathrm{~W} \\
\therefore \% Q_{u n} & =Q_{u n} / Q_{l} \\
& =(52479.46 / 99855.5) \times 100 \\
& =52.55 \%
\end{aligned}
$$

| Item | $\mathrm{W}(\mathrm{kJ} / \mathrm{s})$ | $\%$ |
| :--- | :---: | :---: |
| Heat input | 99855.5 | 100 |
| 1. Heat absorbed in IP | 30748.34 | 30.8 |
| 2. Heat carried away by cooling water. | 16627.7 | 16.65 |
| 3. Unaccounted loss | 52479.46 | 52.55 |
| Total | 99855.5 | 100 |

6. In a test on a 4-stroke, 4-cylinder diesel engine, the following data are observed. Draw up the energy balance sheet. Test duration $=30 \mathrm{~min}$; Brake torque $=160 \mathrm{Nm}$; Rpm = 1500; Fuel consumption = 4.125 lit; CV $=43000 \mathrm{~kJ} / \mathrm{kg}$; Density of fuel $=0.85$ gm/cc; Cooling water consumed = 5.3 lit in 10 sec ; Rise in temperature of cooling water $=9^{\circ} \mathrm{C} ; \mathrm{A} / \mathrm{F}=25: 1 ; \mathrm{C}_{\mathrm{p} \text { (gas) }}=1.115 \mathrm{~kJ} / \mathrm{kgK}$; Temperature of exhaust gas $=$ $300^{\circ} \mathrm{C}$; $\mathrm{T}_{\text {room }}=25^{\circ} \mathrm{C}$.
Given
Four stroke engine
No of cylinders (Z) = 4
Test duration $\quad=30 \mathrm{~min}$
Brake torque $=\left(T_{B}\right)=160 \mathrm{Nm}$
Speed (N) $\quad=1500 \mathrm{rpm}$
Quantity of fuel $\left(\mathrm{Q}_{\mathrm{f}}\right) \quad=4.125 \mathrm{lit} / 30 \mathrm{~min}$

$$
\begin{aligned}
& =4.125 \times 10^{-3} \mathrm{~m}^{3} / 30 \mathrm{~min} \\
& =0.1375 \times 10^{-3} \mathrm{~m}^{3} / \mathrm{min}
\end{aligned}
$$

CV $\quad=43000 \mathrm{~kJ} / \mathrm{kg}$
Density of fuel $(\rho) \quad=0.85 \mathrm{gm} / \mathrm{cc}=850 \mathrm{~kg} / \mathrm{m}^{3}$
Quantity of water $\left(\mathrm{V}_{\mathrm{w}}\right)=5.3$ lit in 10 sec .
Mass of water $\left(m_{w}\right)=5.3 \mathrm{~kg}$ in 10 sec .

$$
\begin{array}{ll} 
& =0.53 \mathrm{~kg} / \mathrm{s} \\
\text { Rise in temperature } & =\mathrm{T}_{\mathrm{wo}}-\mathrm{T}_{\mathrm{wi}}=9^{\circ} \mathrm{C} \\
\mathrm{~A} / \mathrm{F} & =25: 1 \\
\mathrm{C}_{\mathrm{p} \text { (gas) }} & =1.115 \mathrm{~kJ} / \mathrm{kgK}
\end{array}
$$

Temperature of exhaust gas $\left(\mathrm{T}_{\mathrm{g}}\right)=300^{\circ} \mathrm{C}$
$\mathrm{T}_{\text {room }} \quad=25^{\circ} \mathrm{C}$
Required: to draw Heat balance sheet.

## Solution

Let us draw heat balance in second basis.

$$
\begin{aligned}
\text { Heat input }=Q_{i} & =T F C \times C V \\
T F C & =V_{f} \times \rho \\
& =0.1375 \times 10^{-3} \times 850 \\
& =0.116785 \mathrm{~kg} / \mathrm{min} \\
& =1.9479 \times 10^{-3} \mathrm{~kg} / \mathrm{s} \\
\therefore \quad Q_{\mathrm{i}} & =1.9479 \times 10^{-3} \times 43000 \times 1000 \\
& =83759.7 \mathrm{~W}
\end{aligned}
$$

$$
\begin{aligned}
\text { Useful work }=Q_{u}=B P & =2 \pi N T / 60=2 \pi \times 1500 \times 160 / 60 \\
& =2513279 \mathrm{~W} \\
\therefore \quad \% Q_{u} & =\left(Q_{u} / Q_{i}\right) \times 100 \\
& =(25132.7 / 83759.7) \times 100 \\
& =23.84 \%
\end{aligned}
$$

Heat carried by the engine exhaust, $Q_{g}=m_{g} C_{p g}\left(T_{g}-T_{a}\right)$

$$
\begin{aligned}
\mathrm{m}_{\mathrm{g}} & =\mathrm{TFC}+\mathrm{m}_{\mathrm{a}} \\
\mathrm{~m}_{\mathrm{g}} / \mathrm{TFC} & =1+\left(\mathrm{m}_{\mathrm{a}} / \mathrm{TFC}\right) \\
\therefore \mathrm{m}_{\mathrm{g}} & =\mathrm{TFC}[1+(\mathrm{A} / \mathrm{F})] \\
\mathrm{m}_{\mathrm{g}} & =1.9479 \times 10^{-3}(1+25) \\
& =0.0506454 \mathrm{~kg} / \mathrm{s} \\
\therefore \mathrm{Q}_{\mathrm{g}} & =0.0506454 \times 1115(300-25) \\
& =15529.14 \mathrm{~W}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad \% Q_{g}=\left(Q_{g} / Q_{i}\right) \times 100 & =(15529.14 / 83759.7) \times 100 \\
& =18.54 \%
\end{aligned}
$$

$$
\text { Unaccounted loss } \begin{aligned}
\left(Q_{u n}\right) & =Q_{i}-\left(Q_{u}+Q_{w}+Q_{g}\right) \\
& =83759.7-(25132.7+19967.22+15529.14) \\
& =23130.64 \mathrm{~W} \\
\therefore \% Q_{\mathrm{un}} & =\left(Q_{\mathrm{un}} / Q_{\mathrm{i}}\right) \times 100 \\
& =(23130.64 / 83759.7) \times 100=27.61 \%
\end{aligned}
$$

| Item | W | $\%$ |
| :--- | :---: | :---: |
| Heat input | 83759.7 | 100 |
| 1. Heat absorbed in IP | 25132.7 | 30.0 |
| 2. Heat carried away by cooling water. | 19967.22 | 23.84 |
| 3. Heat carried away by exhaust gases. | 15529.14 | 18.54 |
| 4. Unaccounted loss. | 23130.64 | 27.61 |
| Total | 83759.7 | 99.99 |

7. In a test of a 4 cylinder, 4 stroke engine 75 mm bore and 100 mm stroke, the following results were obtained at full throttle at a particular constant speed and with fixed setting of fuel supply of $6.0 \mathrm{~kg} / \mathrm{hour}$.
BP with all cylinders working $=15.6 \mathrm{~kW}$
$B P$ with cylinder (1) cut-out $=11.1 \mathrm{~kW}$
BP with cylinder (2) cut-out $=11.03 \mathrm{~kW}$
BP with cylinder (3) cut-out $=10.88 \mathrm{~kW}$
BP with cylinder (4) cut-out $=10.66 \mathrm{~kW}$
If the CV of the fuel is $83600 \mathrm{~kJ} / \mathrm{kg}$ and clearance volume is $0.0001 \mathrm{~m}^{\mathbf{3}}$, calculate, (i) $\eta_{m}$, (ii) $\eta_{I T}, \&$ (iii) $\eta_{\text {Air. }}$
Given
Bore (D) $\quad=0.775 \mathrm{~m}$
Stroke (L) $\quad=6.1 \mathrm{~m}$
TFC $\quad=6 \mathrm{~kg} / \mathrm{h}=6 / 3600 \mathrm{~kg} / \mathrm{s}$
CV $\quad=83600 \mathrm{~kJ} / \mathrm{kg}$
Clearance volume $\left(\mathrm{V}_{2}\right)=0.0001 \mathrm{~m}^{3}$
Required: (i) $\eta_{\mathrm{m}}$ (ii) $\eta_{\mathrm{IT}}$ (iii) $\eta_{\text {air }}$

## Solution

(i)

$$
\begin{aligned}
& \eta_{\mathrm{m}}=\frac{\mathrm{BP}_{1234}}{\mathrm{I}-----} \\
& \mathrm{IP}_{1234}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{IP}_{1} & =\mathrm{BP}_{1234}-\mathrm{BP}_{234} \\
& =15.6-11.1=4.5 \mathrm{~kW} \\
\mathrm{IP}_{2} & =\mathrm{BP}_{1234}-\mathrm{BP}_{134} \\
& =15.6-11.03=4.57 \mathrm{~kW} \\
\mathrm{IP}_{3} & =\mathrm{BP}_{1234}-\mathrm{BP}_{124} \\
& =15.6-10.88=4.72 \mathrm{~kW} \\
\mathrm{IP}_{4} & =\mathrm{BP}_{1234}-\mathrm{BP}_{123} \\
& =15.6-10.66=4.94 \mathrm{~kW} \\
\mathrm{IP}_{1234} & =4.5+4.57+4.72+4.94 \\
& =18.73 \mathrm{~kW}
\end{aligned}
$$

$$
\begin{equation*}
\therefore \quad \eta_{\mathrm{m}}=15.6 / 18.73=0.8328 \quad--- \text { Ans } \tag{ii}
\end{equation*}
$$

(iii)

$$
\begin{aligned}
\eta_{\text {air }} & =1-\left[1 / \mathrm{r}^{\gamma-1}\right] \\
\mathrm{r} & =\mathrm{V}_{1} / \mathrm{V}_{2} \\
\mathrm{~V}_{1}-\mathrm{V}_{2} & =(\pi / 4) \times 0.075^{2} \times 0.1 \\
\mathrm{~V}_{1} & =0.00054178 \mathrm{~m}^{3} \\
\therefore \quad \mathrm{r} & =(0.00054178 / 0.0001)=5.4 \\
\therefore \eta_{\text {Air }} & =1-\left[1 / 5.4^{1.4-1}\right]=0.4906 \quad---- \text { Ans }
\end{aligned}
$$

8. A petrol engine has a fuel consumption of $10 \mathrm{l} / \mathrm{h}$. The air fuel ratio supplied through the carburettor is 15 . The choke has a throat diameter of 20 mm . Determine the diameter of the jet of the carburettor if the top of the jet is 5 mm above the fuel level in the float chamber. The barometer reads 750 mm of Hg and the temperature is $32^{\circ} \mathrm{C}$. Neglect compressibility of air. Assume $\mathrm{C}_{\mathrm{a}}=0.85$ and $\mathrm{C}_{\mathrm{f}}=$ $0.7, \rho_{f}=700 \mathrm{~kg} / \mathrm{m}^{3}$.

## Given:

Volume flow rate of fuel $=10 \mathrm{l} / \mathrm{h}$
Mass flow rate of fuel $\left(\mathrm{m}_{\mathrm{f}}\right)=10 \times 10^{-3}$ (volume flow in $\left.\mathrm{m}^{3}\right) \times 700$ (density) / 3600

$$
=1.9444 \times 10^{-3} \mathrm{~kg} / \mathrm{s}
$$

Air fuel ratio $\left(\mathrm{m}_{\mathrm{a}} / \mathrm{m}_{\mathrm{f}}\right) \quad=15$
Throat diameter $\left(\mathrm{d}_{\mathrm{f}}\right) \quad=20 \mathrm{~mm}$

Level difference $(z) \quad=5 \mathrm{~mm}=0.005 \mathrm{~m}$
Atmospheric pressure $\left(p_{\mathrm{a}}\right)=750 \mathrm{~mm}$ of $\mathrm{Hg}=750 \times 1.01325 / 760=0.9999$ bar
Atmospheric temperature $\left(\mathrm{T}_{\mathrm{a}}\right)=32^{\circ} \mathrm{C}=305 \mathrm{~K}$
$\mathrm{C}_{\mathrm{a}}=0.85$
$\mathrm{C}_{\mathrm{f}}=0.7$
$\rho_{\mathrm{f}}=700 \mathrm{~kg} / \mathrm{m}^{3}$
Required: Diameter of the jet

## Solution

Mass flow rate of fuel is given by, $\mathrm{m}_{\mathrm{f}}=C_{d f} A_{f} \sqrt{2 \rho_{f}\left(p_{1}-p_{2}\right)-g z \rho_{f}}$
Mass flow rate of air is given by, $\mathrm{m}_{\mathrm{a}}=C_{d a} A_{a} \sqrt{2 \rho_{a}\left(p_{1}-p_{2}\right)}$
Mass flow rate of air $\left(\mathrm{m}_{\mathrm{a}}\right)=\mathrm{A} / \mathrm{F} \times \mathrm{m}_{\mathrm{f}}=15 \times 1.9444 \times 10^{-3}=29.1666 \times 10^{-3} \mathrm{~kg} / \mathrm{s}$

$$
\begin{aligned}
& \rho_{a}=\frac{p_{a}}{R T_{a}}=\frac{0.9999 \times 10^{5}}{287 \times 305}=1.142 \mathrm{~kg} / \mathrm{m}^{3} \\
& A_{a}=\frac{\pi}{4} D^{2}=\frac{\pi}{4} \times 0.02^{2}
\end{aligned}
$$

Therefore, $29.166 \times 10^{-3}=0.85 \times \frac{\pi}{4} \times 0.02^{2} \times \sqrt{2 \times 1.142 \times\left(p_{1}-p_{2}\right)}$

$$
\mathrm{p}_{1}-\mathrm{p}_{2}=5223 \mathrm{~N} / \mathrm{m}^{2}
$$

Substituting in (1), $1.9444 \times 10^{-3}=0.7 \times A_{f} \sqrt{2 \times 700 \times 5223-9.81 \times 0.005 \times 700}$

$$
\begin{aligned}
\mathrm{A}_{\mathrm{f}} & =1.03061 \times 10^{-6} \mathrm{~m}^{2} \\
& =\frac{\pi}{4} d_{a}^{2} \\
\mathrm{~d}_{\mathrm{a}} & =1.1455 \times 10^{-3} \mathrm{~m}=1.1455 \mathrm{~mm}---- \text { Ans }
\end{aligned}
$$

9. An engine having a single jet carburettor consumes 6.5 kg of fuel per hour. The fuel density is $700 \mathrm{~kg} / \mathrm{m}^{3}$. The level of fuel in the float chamber is 3 mm below the top of the jet when the engine is not running. Ambient conditions are 1.01325 bar and $17^{\circ} \mathrm{C}$. The jet diameter is 1.25 mm and its discharge coefficient is 0.6 . The coefficient of air is $\mathbf{0 . 8 5}$. Air fuel ratio is 15 . Determine the critical air velocity and the throat diameter (effective). Express the pressure depression in mm of water. Neglect the compressibility of air.
Given:

| Mass flow rate of fuel $\left(\mathrm{m}_{\mathrm{f}}\right)$ | $=6.5 \mathrm{~kg} / \mathrm{h}=1.805 \times 10^{-3} \mathrm{~kg} / \mathrm{s}$ |
| :--- | :--- |
| Ambient pressure of air $\left(\mathrm{p}_{\mathrm{a}}\right)$ | $=1.01325 \mathrm{bar}=\mathrm{p}_{1}$ |
| Ambient temperature of air $\left(\mathrm{T}_{\mathrm{a}}\right)$ | $=17^{\circ} \mathrm{C}=290 \mathrm{~K}$ |
| Jet discharge coefficient $\left(\mathrm{C}_{\mathrm{f}}\right)$ | $=0.6$ |
| Coefficient of air $\left(\mathrm{C}_{\mathrm{a}}\right)$ | $=0.85$ |
| Level difference $(\mathrm{z})$ | $=3 \mathrm{~mm}=0.003 \mathrm{~m}$ |

Jet diameter $\left(\mathrm{d}_{\mathrm{f}}\right)$
Air fuel ratio
Density of petrol $\left(\rho_{f}\right)$

$$
\begin{aligned}
& =1.25 \mathrm{~mm}=0.00125 \mathrm{~m} \\
& =15 \\
& =700 \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

Required: Critical velocity, throat diameter and pressure depression

## Solution

Mass flow rate of air $\left(\mathrm{m}_{\mathrm{a}}\right)=\mathrm{A} / \mathrm{F} \times \mathrm{m}_{\mathrm{f}}=15 \times 1.805 \times 10^{-3}=0.027075 \mathrm{~kg} / \mathrm{s}$

$$
\rho_{a}=\frac{p_{a}}{R T_{a}}=\frac{1.01325 \times 10^{5}}{287 \times 290}=1.217 \mathrm{~kg} / \mathrm{m}^{3}
$$

Critical velocity of air $\left(\mathrm{C}_{\mathrm{a}}\right)_{\text {critical }}=C_{d a} \sqrt{\frac{2 g z \rho_{f}}{\rho_{a}}}$

$$
=0.85 \sqrt{\frac{2 \times 9.81 \times 0.003 \times 700}{1.217}}=4.945 \mathrm{~m} / \mathrm{s}--- \text { Ans }
$$

Mass flow rate of fuel is given by, $\mathrm{m}_{\mathrm{f}}=C_{d f} A_{f} \sqrt{2 \rho_{f}\left(p_{1}-p_{2}\right)-g z \rho_{f}}$

$$
\begin{aligned}
1.805 \times 10^{-3} & =0.6 \times \frac{\pi}{4} \times 0.00125^{2} \sqrt{2 \times 700 \times\left(p_{1}-p_{2}\right)-9.81 \times 0.003 \times 700} \\
\mathrm{p}_{1}-\mathrm{p}_{2} & =42925 \mathrm{~N} / \mathrm{m}^{2}
\end{aligned}
$$

Mass flow rate of air is given by, $\mathrm{m}_{\mathrm{a}}=C_{d a} A_{a} \sqrt{2 \rho_{a}\left(p_{1}-p_{2}\right)}$

Therefore,

$$
\begin{aligned}
0.027075 & =0.85 \times \frac{\pi}{4} \times d_{a}^{2} \times \sqrt{2 \times 1.217 \times 4292.5} \\
d_{a} & =0.0199 \mathrm{~m}=19.9 \mathbf{~ m m} \cdots \text { Ans }
\end{aligned}
$$

Pressure depression $\left(p_{1}-p_{2}\right)=4292.5 / 9.81=437.5 \mathrm{~mm}$ of water ---- Ans

## UNIT-III

## STEAM NOZZLES AND TURBINES

## Steam Nozzles

When a fluid flows through a passage or channel of varying cross section, its velocity varies from point to point along the passage. If the velocity increases, the passage is called a Nozzle. The steam nozzle is device designed to increase the velocity of steam. The fluid enters the nozzle at high pressure and expands to lower pressure. If the cross section of the nozzle decreases continuously from the entrance to exit, it is called Convergent nozzle. The maximum Mach number at the exit of the convergent nozzle is 1 . If the cross section of the nozzle increases, it is called Divergent nozzle. If the cross section of the nozzle, first decreases and then increases, it is called Convergent-divergent nozzle. At the throat, i.e., at the narrowest cross section the Mach number is 1 .


Use
It is used to convert pressure energy into kinetic energy.

## Velocity of steam leaving the nozzle

Steady flow energy equation is given by,

$$
h_{1}+\frac{C_{1}^{2}}{2}+g Z_{1}+q=h_{2}+\frac{C_{2}^{2}}{2}+g Z_{2}+w
$$

Flow through the nozzle is work free isentropic.

$$
\therefore \mathrm{q}=0, \quad \mathrm{w}=0 \quad \text { Also, } \mathrm{Z}_{1}=\mathrm{Z}_{2}
$$

h


Therefore, $\quad h_{1}+\frac{C_{1}^{2}}{2}=h_{2}+\frac{C_{2}^{2}}{2}$

$$
C_{2}=\sqrt{2\left(h_{1}-h_{2}\right)+C_{1}^{2}}
$$

$\mathrm{C}_{1}=$ Inlet velocity of steam
$\mathrm{C}_{2}=$ Exit velocity of steam from nozzle
Let $\quad \eta_{n}=$ Nozzle efficiency

$$
\eta_{n}=\frac{\text { Actual enthalpy drop }}{\text { Isentropic enthalpic drop }}=\frac{h_{1}-h_{2}}{h_{1}-h_{2^{\prime}}}
$$

1-2 $\rightarrow$ Actual (adiabatic) expansion
$1-2 \rightarrow$ Isentropic expansion

$$
\begin{aligned}
\therefore \quad \mathrm{h}_{1}-\mathrm{h}_{2} & =\eta_{\mathrm{n}}\left(\mathrm{~h}_{1}-\mathrm{h}_{2^{\prime}}\right) \\
C_{2} & =\sqrt{2 \eta_{n}\left(h_{1}-h_{2^{\prime}}\right)+C_{1}^{2}}
\end{aligned}
$$

## Mass flow rate of steam through the nozzle

Consider a convergent-divergent nozzle.
Mass flow rate of steam (m) $=\rho_{1} A_{1} C_{1}$

$$
\begin{aligned}
& =\rho_{2} A_{2} C_{2} \\
& =\rho_{3} A_{3} C_{3}
\end{aligned}
$$

$\mathrm{C}_{1}=$ Velocity of steam at inlet
$\mathrm{C}_{2}=$ Velocity of steam at throat
$\mathrm{C}_{3}=$ Velocity of steam at exit
The flow of steam through the nozzle is isentropic or adiabatic with index of expansion ' $n$ '. There is no work transfer during the expansion of steam through nozzle. The expansion in the nozzle increases the kinetic energy of steam.

If the increase in kinetic energy is utilized for work (flow work), we can write,
Between inlet and throat, $\quad \frac{C_{2}^{2}-C_{1}^{2}}{2}=\frac{n\left(p_{1} v_{1}-p_{2} v_{2}\right)}{n-1}$
Between throat and exit, $\quad \frac{C_{3}^{2}-C_{2}^{2}}{2}=\frac{n\left(p_{2} v_{2}-p_{3} v_{3}\right)}{n-1}$
Between inlet and exit, $\quad \frac{C_{1}^{2}-C_{3}^{2}}{2}=\frac{n\left(p_{1} v_{1}-p_{3} v_{3}\right)}{n-1}$
$\mathrm{p}_{1}=$ Pressure of the steam at inlet
$\mathrm{v}_{1}=$ Specific volume of steam at the throat
$\mathrm{p}_{2}=$ Pressure of steam at throat
$\mathrm{v}_{2}=$ Specific volume of steam at throat
$p_{3}=$ Pressure of steam at exit
$v_{3}=$ Specific volume of steam at exit

Consider section 1 - 2 .
Generally, $\mathrm{C}_{1}=0$

$$
\therefore \quad \begin{aligned}
\frac{C_{2}^{2}}{2} & =\frac{n\left(p_{1} v_{1}-p_{2} v_{2}\right)}{n-1} \\
C_{2} & =\sqrt{\frac{2 n}{n-1}\left(p_{1} v_{1}-p_{2} v_{2}\right)} \\
& =\sqrt{\frac{2 n}{n-1} p_{1} v_{1}\left[1-\frac{p_{2} v_{2}}{p_{1} v_{1}}\right]}
\end{aligned}
$$

But, $\mathrm{p}_{1} \mathrm{v}_{1}{ }^{\mathrm{n}}=\mathrm{p}_{2} \mathrm{v}_{2}{ }^{\mathrm{n}}$

$$
\text { Or } \frac{v_{2}}{v_{1}}=\left(\frac{p_{2}}{p_{1}}\right)^{-1 / n}
$$

Therefore,

$$
\begin{aligned}
C_{2} & =\sqrt{\frac{2 n}{n-1} p_{1} v_{1}\left[1-\left(\frac{p_{2}}{p_{1}}\right)\left(\frac{p_{2}}{p_{1}}\right)^{-1 / n}\right]} \\
& =\sqrt{\frac{2 n}{n-1} p_{1} v_{1}\left[1-\left(\frac{p_{2}}{p_{1}}\right)^{n-1 / n}\right]}
\end{aligned}
$$

$$
\therefore \quad \quad \quad *=\rho_{2} A_{2} \sqrt{\frac{2 n}{n-1} p_{1} v_{1}\left[1-\left(\frac{p_{2}}{p_{1}}\right)^{n-1 / n}\right]}
$$

$$
\frac{\stackrel{*}{m}}{A_{2}}=\rho_{2} \sqrt{\frac{2 n}{n-1} p_{1} v_{1}\left[1-\left(\frac{p_{2}}{p_{1}}\right)^{n-1 / n}\right]}
$$

$$
=\frac{1}{v_{2}} \sqrt{\frac{2 n}{n-1} p_{1} v_{1}\left[1-\left(\frac{p_{2}}{p_{1}}\right)^{n-1 / n}\right]}
$$

But, $v_{2}=v_{1}\left(\frac{p_{2}}{p_{1}}\right)^{-1 / n}$

Therefore,

$$
\begin{aligned}
\frac{\stackrel{*}{m}}{A_{2}} & =\frac{1}{v_{1}}\left(\frac{p_{2}}{p_{1}}\right)^{1 / n} \sqrt{\frac{2 n}{n-1} p_{1} v_{1}\left[1-\left(\frac{p_{2}}{p_{1}}\right)^{n-1 / n}\right]} \\
& =\sqrt{\frac{2 n}{n-1} \frac{p_{1}}{v_{1}}\left[\left(\frac{p_{2}}{p_{1}}\right)^{2 / n}-\left(\frac{p_{2}}{p_{1}}\right)^{n+1 / n}\right]}
\end{aligned}
$$

Considering section $1-3, \quad \frac{\stackrel{m}{A_{3}}}{A_{3}}=\sqrt{\frac{2 n}{n-1} \frac{p_{1}}{v_{1}}\left[\left(\frac{p_{3}}{p_{1}}\right)^{2 / n}-\left(\frac{p_{3}}{p_{1}}\right)^{n+1 / n}\right]}$

## Critical pressure ratio $\left(p_{2} / p_{1}\right)$

The narrowest section in the nozzle is throat (2). At the throat the value of $\binom{*}{\frac{m}{A}}$ is maximum. This ratio is function of pressure ratio $\left(\frac{p_{2}}{p_{1}}\right)$. The critical pressure ratio is the pressure ratio at which the $\left(\frac{\stackrel{*}{m}}{A_{2}}\right)$ will be maximum.

$$
\text { To get }\left(\frac{\stackrel{m}{m}}{A_{2}}\right)_{\max } \text { differentiate } \frac{\stackrel{*}{m}}{A_{2}}=\sqrt{\frac{2 n}{n-1} \frac{p_{1}}{v_{1}}\left[\left(\frac{p_{2}}{p_{1}}\right)^{2 / n}-\left(\frac{p_{2}}{p_{1}}\right)^{n+1 / n}\right]} \text { with respect }
$$

to $\left(p_{2} / p_{1}\right)$ and equate to zero.

$$
\left.\frac{d\left(\frac{*}{m}\right.}{A_{2}}\right),
$$

$\mathrm{p}_{1}, \mathrm{v}_{1}$ and n are constants.

$$
\begin{aligned}
\frac{d\left[\left(\frac{p_{2}}{p_{1}}\right)^{2 / n}-\left(\frac{p_{2}}{p_{1}}\right)^{n+1 / n}\right]}{d\left(\frac{p_{2}}{p_{1}}\right)} & =0 \\
\frac{2}{n}\left(\frac{p_{2}}{p_{1}}\right)^{\frac{2}{n}-1}-\frac{n+1}{n}\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n+1}{n}-1} & =0 \\
\frac{\left(\frac{p_{2}}{p_{1}}\right)^{\frac{1}{n}}}{\left(\frac{p_{2}}{p_{1}}\right)^{\frac{2-n}{n}}} & =\frac{2}{n+1} \\
\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}} & =\frac{2}{n+1} \\
\left(\frac{p_{2}}{p_{1}}\right) & =\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}
\end{aligned}
$$


$\mathrm{n}=1.035+0.1 \mathrm{x}_{1}$ for wet steam at inlet
$=1.135$ for dry steam at inlet
$=1.3$ for superheated steam at inlet

## Effect of friction in Nozzle

1. Friction reheats the steam and reduces the exit velocity.
2. Steam velocity does not reach the local sonic velocity at the nozzle throat, but at a short distance beyond it.

## Nozzle Efficiency ( $\eta_{\mathrm{n}}$ )

$\eta_{n}=\frac{\text { Actual (adiabatic)enthalpy drop }}{\text { Isentropic enthalpy drop }}=\frac{h_{1}-h_{2}}{h_{1}-h_{2^{\prime}}}$

$$
=\frac{h_{1}-h_{3}}{h_{1}-h_{3^{\prime}}} \rightarrow \text { For Con-Div nozzle }
$$

## Relationship between Area, Velocity and Pressure in nozzle flow

$$
\frac{d A}{A}=\frac{1}{\gamma} \frac{d p}{p}\left[\frac{1-M^{2}}{M^{2}}\right]
$$

Where, $\mathrm{M}=$ Mach number $=\frac{\text { Local velocity of fluid }}{\text { Velocity of sound }}$
For Nozzles, $d p / p$ is negative.
(i) $M<1, \quad d A / A$ is negative $\rightarrow$ this correspond to convergent part
(ii) $M=1, \quad d A / A$ is zero $\rightarrow$ this corresponds to throat
(iii) $\mathrm{M}>1, \quad \mathrm{dA} / \mathrm{A}$ is positive $\rightarrow$ this corresponds to divergent part

## Equilibrium \& Supersaturated Flow in Nozzle

When a super heated vapour expands slowly and isentropically, condensation within the vapour begins to form when the saturated vapour line is reached. As the expansion continues below this line into the wet region, condensation proceeds
gradually with the progressive decrease of quality ( $x$ ) and increase in the degree of wetness.


Point ' $A$ ' represents the point at which condensation within the vapour just starts. If the condensation occurs at point ' $A$ ' on saturation curve, the flow is called equilibrium flow.

However, when the steam expands from super heated state to the two phase region in the nozzle, the expansion occurs so rapidly that the vapour does not condense immediately as it crosses the dry saturated line, but somewhat later (at $x=0.96$ to 0.97), when all the vapour suddenly condenses into liquid. Beyond the dry saturation line till the state when the vapour condenses, the flow is said to be supersaturated and the system is in metastable equilibrium, which means that it is stable to small disturbances but unstable to large disturbances.

Wilson line ( $x=0.96$ to 0.97 ) is the locus of states below the dry saturation line where condensation within the vapour occurs at different pressures.

## Effect of supersaturation

(i) there is an increase in the entropy and specific volume of steam.
(ii) the exit velocity of steam is reduced due to decreased enthalpy drop.
(iii) there is an increase in mass of steam discharged.
(iv) the dryness fraction of steam is improved.

## Useful relations for solving problems

## EQUILIBRIUM FLOW

## 1. Velocity of steam leaving the nozzle (Convergent type)

$$
C_{2}=\sqrt{2\left(h_{1}-h_{2}\right)+C_{1}^{2}}
$$

$\mathrm{h}_{1}=$ Specific enthalpy of steam at nozzle inlet from Mollier chart in ' $\mathrm{J} / \mathrm{kgK}$ '
$\mathrm{h}_{2}=$ Specific enthalpy of steam at nozzle outlet from Mollier chart in 'J/kgK'
$\mathrm{C}_{1}=$ Velocity of steam at nozzle inlet $-\mathrm{m} / \mathrm{s}$ (If $\mathrm{C}_{1}$ is not given in the problem, it can be neglected)

## 2. Velocity of steam at the throat (Convergent-Divergent type)

$$
C_{2}=\sqrt{2\left(h_{1}-h_{2}\right)+C_{1}^{2}}
$$

$\mathrm{h}_{1}=$ Specific enthalpy of steam at nozzle inlet from Mollier chart in ' $\mathrm{J} / \mathrm{kgK}$ '
$h_{2}=$ Specific enthalpy of steam at nozzle throat from Mollier chart in ' $\mathrm{J} / \mathrm{kgK}$ '
$\mathrm{C}_{1}=$ Velocity of steam at nozzle inlet $-\mathrm{m} / \mathrm{s}$ (If $\mathrm{C}_{1}$ is not given in the problem, it can be neglected)
3. Velocity of steam leaving the nozzle (Convergent-Divergent type)

$$
C_{3}=\sqrt{2\left(h_{1}-h_{3}\right)+C_{1}^{2}}
$$

$\mathrm{h}_{1}=$ Specific enthalpy of steam at nozzle inlet from Mollier chart in ' $\mathrm{J} / \mathrm{kgK}$ '
$\mathrm{h}_{3}=$ Specific enthalpy of steam at nozzle outlet from Mollier chart in 'J/kgK'
$\mathrm{C}_{1}=$ Velocity of steam at nozzle inlet $-\mathrm{m} / \mathrm{s}$ (If $\mathrm{C}_{1}$ is not given in the problem, it can be neglected)

## 4. Mass flow rate of steam

$$
\stackrel{*}{m}=\frac{A_{1} C_{1}}{v_{1}}=\frac{A_{2} C_{2}}{v_{2}}-\frac{A_{3} C_{3}}{v_{3}}
$$

$\mathrm{A}_{1}=$ Area of nozzle at inlet $-\mathrm{m}^{2}$
$\mathrm{C}_{1}=$ Velocity of steam at inlet $-\mathrm{m} / \mathrm{s}$
$\mathrm{v}_{1}=$ Specific volume of steam at inlet from steam table or chart - $\mathrm{m}^{3} / \mathrm{kg}$
$A_{2}, C_{2}, v_{2} \& A_{3}, C_{3}, v_{3}$ are similar values at the throat \& outlet
5. Nozzle efficiency ( $\eta_{\mathrm{n}}$ )
$\eta_{n}=\frac{\text { Actual (adiabatic)enthalpy drop }}{\text { Isentropic enthalpy drop }}=\frac{h_{1}-h_{2}}{h_{1}-h_{2^{\prime}}}$

$$
=\frac{h_{1}-h_{3}}{h_{1}-h_{3^{\prime}}} \rightarrow \text { For Con-Div nozzle }
$$

## SATURATED FLOW

6. Degree of under-cooling $=T_{2}-T_{2}$
$T_{2}=$ Saturation temperature at given ' $p_{2}$ ' from steam table

$$
\mathrm{T}_{2^{\prime}} \rightarrow \frac{T_{2^{\prime}}}{T_{1}}=\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}
$$

Note: The unit of temp is ' $K$ '
7. Degree of supersaturation $=p_{2} / p_{2}$,

$$
\mathrm{p}_{2^{\prime}} \Leftarrow \text { From steam table at } \mathrm{T}_{2^{\prime}}
$$

8. Loss in available heat drop $=\left(h_{1}-h_{2}\right)_{\text {chart }}-\left(h_{1}-h_{2}\right)_{\text {equ }}$

$$
\begin{aligned}
& \left(h_{1}-h_{2}\right)_{\text {equ }}=\frac{n}{n-1} p_{1} v_{1}\left[1-\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}\right] \quad-----\mathrm{J} / \mathrm{kg} \\
& \mathrm{v}_{1}=\text { Specific volume of steam from chart }-\mathrm{m}^{3} / \mathrm{kg} \\
& \mathrm{n}=1.3
\end{aligned}
$$

9. Increase in entropy $=\frac{\text { Loss in available heat drop }}{T_{2}}$
10. Mass flow rate of supersaturated flow $=\frac{A_{2} C_{2^{\prime}}}{v_{2^{\prime}}}$

$$
\begin{aligned}
& \mathbf{v}_{2^{\prime}} \rightarrow \frac{v_{2^{\prime}}}{v_{1}}=\left(\frac{p_{1}}{p_{2}}\right)^{\frac{1}{n}} \\
& \mathrm{C}_{2^{\prime}} \rightarrow \mathrm{C}_{2^{\prime}}^{2} / 2=\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)_{\mathrm{equ}}
\end{aligned}
$$

11. Number of nozzles $=$ Total area $/$ Area per nozzle

## PROBLEMS

1. Steam approaches a nozzle with velocity of $250 \mathrm{~m} / \mathrm{s}$, pressure of 3.5 bar and dryness fraction of 0.95 . If the isentropic expansion in the nozzle proceeds till the pressure of the exit is 2 bar, determine the change in enthalpy and the dryness fraction of steam. Calculate also the exit velocity from the nozzle and the area of the exit of the nozzle for the flow of $0.75 \mathrm{~kg} / \mathrm{s}$.

## Given

Type $\quad=$ Convergent
Velocity of steam at inlet $\left(\mathrm{V}_{1}\right)=250 \mathrm{~m} / \mathrm{s}$
Pressure at inlet ( $\mathrm{p}_{1}$ )
$=3.5$ bar with $\mathrm{x}_{1}=0.95$
Pressure at outlet ( $p_{2}$ )

$$
=2 \mathrm{bar}
$$

Mass flow rate (m) $\quad=0.75 \mathrm{~kg} / \mathrm{s}$
Required: $\left(h_{1}-h_{2}\right), x_{2}, V_{2} \& A_{2}$

## Solution

h

s

From Chart,

$$
\begin{aligned}
& \mathrm{h}_{1}=2625 \mathrm{~kJ} / \mathrm{kg} \\
& \mathrm{~h}_{2}=2540 \mathrm{~kJ} / \mathrm{kg} \\
& \mathrm{x}_{2}=0.92 \text {--- Ans } \\
& \therefore\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)=2625-2540=85 \mathrm{~kJ} / \mathrm{kg} \quad---- \text { Ans } \\
& \left(\mathrm{V}_{2}^{2}-\mathrm{V}_{1}^{2}\right) / 2=\mathrm{h}_{1}-\mathrm{h}_{2} \\
& \left(\mathrm{~V}_{2}^{2}-250^{2}\right) / 2=85 \times 10^{3} \\
& \therefore \quad V_{2}=482.2 \mathrm{~m} / \mathrm{s} \quad-- \text { Ans }
\end{aligned}
$$

$\mathrm{A}_{2}=$ Area of the nozzle at outlet

$$
\mathrm{m}=\mathrm{A}_{2} \mathrm{~V}_{2} / \mathrm{v}_{2}
$$

$$
\mathrm{v}_{2}=0.8 \mathrm{~m}^{3} / \mathrm{kg} \quad \text { from chart at point (2) }
$$

$$
\begin{aligned}
\therefore \quad 0.75 & =\mathrm{A}_{2} \times 482.2 / 0.8 \\
\mathrm{~A}_{2} & =0.0012443 \mathrm{~m}^{2}=12.443 \mathrm{~cm}^{2} \ldots-- \text { Ans }
\end{aligned}
$$

2. Dry saturated steam at pressure of 8 bar flows through nozzles at the rate of $4.6 \mathrm{~kg} / \mathrm{s}$ and discharges at a pressure of 1.5 bar . The loss due to friction occurs only in the diverging portion of the nozzle and its magnitude is $12 \%$ of the total isentropic enthalpy drop. Assume the isentropic index of expansion $\mathrm{n}=$ 1.135, determine the cross sectional area at the throat and exit of the nozzles.

Given

| Type | $=$ Con-div type |
| :--- | :--- |
| Inlet pressure $\left(p_{1}\right)$ | $=8 \mathrm{bar}$, dry |
| Mass flow rate of steam $(\mathrm{m})$ | $=4.6 \mathrm{~kg} / \mathrm{s}$ |
| Discharge pressure $\left(\mathrm{p}_{3}\right)$ | $=1.5 \mathrm{bar}$ |
| $\mathrm{h}_{3}-\mathrm{h}_{3^{\prime}}$ | $=0.12\left(\mathrm{~h}_{1}-\mathrm{h}_{3^{\prime}}\right)$ |
| n | $=1.135$ |

Required: $\mathrm{A}_{2}, \mathrm{~A}_{3}$
Solution

$$
\mathrm{M}=\mathrm{A}_{2} \mathrm{~V}_{2} / \mathrm{v}_{2}=\mathrm{A}_{3} \mathrm{~V}_{3} / \mathrm{v}_{3}
$$

Inlet velocity is not given, $\quad \therefore \quad \mathrm{V}_{1}=0$

$$
\therefore \quad \mathrm{V}_{2}^{2} / 2=\mathrm{h}_{1}-\mathrm{h}_{2}
$$

From chart,

$$
\mathrm{h}_{1}=2770 \mathrm{~kJ} / \mathrm{kg}
$$

h


We can write,

$$
\begin{aligned}
\mathrm{p}_{2} / \mathrm{p}_{1} & =\left[--\frac{2}{n+--]^{n /(n+1)}}\right. \\
\therefore \quad \mathrm{p}_{2} / 8 & =[2 /(1.135+1)]^{1.135 . /(1.135-1)} \\
\mathrm{p}_{2} & =4.62 \mathrm{bar}
\end{aligned}
$$

From Chart, $\quad \mathrm{h}_{2}=2545 \mathrm{~kJ} / \mathrm{kg}$

From chart, $\quad \mathrm{h}_{3^{\prime}}=2455 \mathrm{~kJ} / \mathrm{kg}$

$$
\begin{aligned}
\mathrm{v}_{3^{\prime}}=1.1 \mathrm{~m}^{3} / \mathrm{kg} & \approx \mathrm{v}_{3} \\
\mathrm{~h}_{3}-\mathrm{h}_{3^{\prime}} & =0.12\left(\mathrm{~h}_{1}-\mathrm{h}_{3^{\prime}}\right) \\
\mathrm{h}_{3}-2455 & =0.12(2770-2455) \\
\mathrm{h}_{3} & =2492.8 \mathrm{~kJ} / \mathrm{kg} \\
\mathrm{~V}_{3}^{2} / 2 & =(2770-2492.8) \times 10^{3} \\
\mathrm{~V}_{3} & =744.6 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

$$
\therefore 4.6=\mathrm{A}_{3} \times 744.6 / 1.1
$$

$$
A_{3}=0.0067956 \mathrm{~m}^{2}=67.956 \mathrm{~cm}^{2}--- \text { Ans }
$$

3. Steam at a pressure of 10 bar and dryness fraction of 0.98 is discharged through a convergent divergent nozzle to a back pressure of 0.1 bar. The mass flow rate is $10 \mathrm{~kg} / \mathrm{kWh}$. If the power developed is 200 kW , determine, (a) Pressure at the throat (b) Number of nozzles required, if each nozzles has a throat of rectangular cross section of $5 \mathrm{~mm} \times 10 \mathrm{~mm}$ and (c) exit area of nozzle if $\mathbf{1 0} \%$ the overall isentropic enthalpy drop reheats the steam by friction in the divergent portion.

Given

| Type | $=$ Con-div |
| :--- | :--- |
| Inlet pressure $\left(p_{1}\right)$ | $=10 \mathrm{bar}$ with $x_{1}=0.98$ |
| Back pressure $\left(p_{3}\right)$ | $=0.1 \mathrm{bar}$ |
| Mass flow rate $(m)$ | $=10 \mathrm{~kg} / \mathrm{kWh}$ |
| Power $(P)$ | $=200 \mathrm{~kW}$ |
| Size of nozzle | $=5 \mathrm{~mm} \times 10 \mathrm{~mm}$ |
| $\mathrm{~h}_{3}-\mathrm{h}_{3^{\prime}}$ | $=0.1\left(\mathrm{~h}_{1}-\mathrm{h}_{3^{\prime}}\right)$ |

Required: (a) $p_{2}$ (b) Number of nozzles (c) $A_{3}$
Solution

$$
\begin{aligned}
& \mathrm{v}_{2}=0.42 \mathrm{~m}^{3} / \mathrm{kg} \\
& \therefore \quad \mathrm{~V}_{2}{ }^{2} / 2=(2770-2545) \times 10^{3} \\
& \mathrm{~V}_{2}=670.8 \mathrm{~m} / \mathrm{s} \\
& \therefore \quad 4.6=\mathrm{A}_{2} \times 670.8 / 0.42 \\
& \mathrm{~A}_{2}=0.00288 \mathrm{~m}^{2}=\mathbf{2 8 . 8} \mathbf{c m}^{2} \text {---- Ans }
\end{aligned}
$$


s
(a) Pressure at throat $\left(\mathrm{p}_{2}\right)$

$$
\begin{aligned}
& \mathrm{p}_{2} / \mathrm{p}_{1}\left.=\frac{2}{2}-----\right]^{\mathrm{n} /(\mathrm{n+1)}} \\
& \mathrm{n}+1 \\
&=1.035+0.1\left(\mathrm{x}_{1}\right) \\
&=1.035+(0.1 \times 0.98)=1.133 \\
& \therefore \mathrm{p}_{2} / 10=[2 /(1.133+1)]^{1.133 . /(1.133-1)} \\
& \mathrm{p}_{2}=5.78 \text { bar }--- \text { Ans }
\end{aligned}
$$

(b) Number of nozzles $=$ Total area $/$ Area per nozzle $=\mathrm{A}_{2} / \mathrm{A}_{2}$ per nozzle

$$
\begin{gathered}
\mathrm{A}_{2} / \text { nozzle }=0.005 \times 0.01=0.00005 \mathrm{~m}^{2} \\
\mathrm{~m}=\mathrm{A}_{2} \mathrm{~V}_{2} / \mathrm{v}_{2} \\
\mathrm{~m}(\mathrm{~kg} / \mathrm{s})=-\cdots(\mathrm{kg} / \mathrm{kWh}) \times \mathrm{P} \\
3600 \\
=10 \times 200 / 3600=0.5556 \mathrm{~kg} / \mathrm{s} \\
\mathrm{~V}_{2}^{2} / 2=\mathrm{h}_{1}-\mathrm{h}_{2}
\end{gathered}
$$

From chart, $\quad \mathrm{h}_{1}=2735 \mathrm{~kJ} / \mathrm{kg}$

$$
\begin{aligned}
& \mathrm{h}_{2}=2625 \mathrm{~kJ} / \mathrm{kg} \\
& \mathrm{v}_{2}=0.32 \mathrm{~m}^{3} / \mathrm{kg}
\end{aligned}
$$

$$
\therefore \mathrm{V}_{2}^{2}=(27735-2625) \times 10^{3}
$$

$$
V_{2}=469.04 \mathrm{~m} / \mathrm{s}
$$

$$
\therefore \quad 0.5556=\mathrm{A}_{2} \times 469.04 / 0.32
$$

$$
\mathrm{A}_{2}=0.00037905 \mathrm{~m}^{2}
$$

$\therefore \quad$ Number of nozzles $=0.00037905 / 0.00005=7.58=8$---- Ans

$$
\begin{aligned}
\mathrm{m} & =\mathrm{A}_{3} \mathrm{~V}_{3} / \mathrm{V}_{3} \\
\mathrm{~V}_{3}^{2} / 2 & =\mathrm{h}_{1}-\mathrm{h}_{3}
\end{aligned}
$$

From chart, $\quad \mathrm{h}_{3^{\prime}}=2055 \mathrm{~kJ} / \mathrm{kg}$

$$
\begin{aligned}
\mathrm{v}_{3^{\prime}}=13 \mathrm{~m}^{3} / \mathrm{kg} & =\mathrm{v}_{3} \\
\mathrm{~h}_{3}-\mathrm{h}_{3^{\prime}} & =0.1\left(\mathrm{~h}_{1}-\mathrm{h}_{3^{\prime}}\right) \\
\mathrm{h}_{3}-2055 & =0.1 \times(2735-2055) \\
\mathrm{h}_{3} & =2123 \mathrm{~kJ} / \mathrm{kg} \\
\therefore \quad \mathrm{~V}_{3}^{2} / 2 & =(2735-2123) \times 10^{3} \\
\mathrm{~V}_{3} & =1106.3 \mathrm{~m} / \mathrm{s} \\
\therefore \quad 0.5556 & =\mathrm{A}_{3} \times 1106.3 / 13 \\
& \\
\mathrm{~A}_{3} & =0.0065276 \mathrm{~m}^{2} \cdots \text { Ans }
\end{aligned}
$$

4. Steam enters a nozzle passing a mass flow of $14 \mathrm{~kg} / \mathrm{s}$ at a pressure of 30 bar and a temperature of $300^{\circ} \mathrm{C}$. After expansion to a exit pressure of 5 bar, the exit velocity was $800 \mathrm{~m} / \mathrm{s}$. (i) Determine the nozzle efficiency and the exit area (ii) If the losses occur only in the divergent portion, determine the velocity of steam at the throat.

## Given

Type
Mass flow rate (m)
Inlet pressure $\left(\mathrm{p}_{1}\right)$
Inlet temperature ( $\mathrm{T}_{1}$ )
Exit pressure ( $\mathrm{p}_{3}$ )
Velocity of steam at outlet $\left(\mathrm{V}_{3}\right)$

$$
\begin{aligned}
& =\text { Con-div } \\
& =14 \mathrm{~kg} / \mathrm{s} \\
& =30 \mathrm{bar} \\
& =300^{\circ} \mathrm{C}=573 \mathrm{~K} \\
& =5 \mathrm{bar} \\
& =800 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Required: (i) $\eta_{\mathrm{n}}, \mathrm{A}_{3}$ (ii) $\mathrm{V}_{2}$
Solution

(i) Nozzle efficiency $\left(\eta_{n}\right) \quad=\begin{aligned} & h_{1}--h_{3} \\ & h_{1}----h_{3}\end{aligned}$

From chart, $\quad h_{1}=2990 \mathrm{~kJ} / \mathrm{kg}$

$$
\begin{aligned}
& \mathrm{h}_{3^{\prime}}=2625 \mathrm{~kJ} / \mathrm{kg} \\
& \mathrm{v}_{3^{\prime}}=0.38 \mathrm{~m}^{3} / \mathrm{kg}=\mathrm{v}_{3}
\end{aligned}
$$

To find $h_{3}$

$$
\begin{aligned}
& \left(\mathrm{h}_{1}-\mathrm{h}_{3}\right) \times 10^{3}=\mathrm{V}_{3}{ }^{2} / 2 \\
& \left(2990-h_{3}\right) \times 10^{3}=800^{2} / 2 \\
& \mathrm{~h}_{3}=2670 \mathrm{~kJ} / \mathrm{kg} \\
& \therefore \quad \eta_{\mathrm{n}}=\frac{2990-2670}{2990-------2625}=0.8767 \text {---- Ans } \\
& m=A_{3} V_{3} / v_{3} \\
& 14=\mathrm{A}_{3} \times 800 / 0.38 \\
& A_{3}=0.00665 \mathrm{~m}^{2}=66.5 \mathrm{~cm}^{2}--- \text { Ans }
\end{aligned}
$$

$$
\begin{align*}
\mathrm{V}_{2}^{2} / 2 & =\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)  \tag{ii}\\
\mathrm{p}_{2} / \mathrm{p}_{1} & =\left[\frac{2}{\mathrm{n}+--]^{\mathrm{n} /(\mathrm{n}+1)}}\right. \\
\mathrm{p}_{2} / 30 & =[2 /(1.3+1)]^{1.3 /(1.3-1)} \\
\mathrm{p}_{2} & =16.372 \text { bar }
\end{align*}
$$

From chart, $\quad h_{2}=2840 \mathrm{~kJ} / \mathrm{kg}$

$$
\mathrm{v}_{2}=0.13 \mathrm{~m}^{3} / \mathrm{kg}
$$

$$
\begin{aligned}
\therefore \quad \mathrm{V}_{2}^{2} / 2 & =(2990-2840) \times 10^{3} \\
\mathrm{~V}_{2} & =547.7 \mathrm{~m} / \mathrm{s}--- \text { Ans }
\end{aligned}
$$

5. A set of 16 nozzles for an impulse turbine receives steam at $16 \mathrm{bar}, 300^{\circ} \mathrm{C}$. The pressure of steam at exit is 10 bar. If the total discharge is $245 \mathrm{~kg} / \mathrm{min}$ and nozzles efficiency is $90 \%$, find the cross sectional area of the exit of each nozzle. If the steam has a velocity of $100 \mathrm{~m} / \mathrm{s}$ at entry to the nozzles, find the $\%$ increase in discharge.

## Given

| Type | $=$ Con |
| :--- | :--- |
| Number of nozzles | $=16$ |
| Inlet pressure $\left(p_{1}\right)$ | $=16 \mathrm{bar}$ |
| Inlet temperature $\left(T_{1}\right)$ | $=300^{\circ} \mathrm{C}$ |
| Exit pressure $\left(p_{2}\right)$ | $=10 \mathrm{bar}$ |
| Mass flow rate $(\mathrm{m})$ | $=245 \mathrm{~kg} / \mathrm{min}=4.083 \mathrm{~kg} / \mathrm{s}$ |

Required: $A_{2} /$ nozzle if $\mathrm{V}_{1}=0, \%$ increases in discharge if $\mathrm{V}_{1}=100 \mathrm{~m} / \mathrm{s}$
Solution
h


$$
\begin{aligned}
\mathrm{m} & =\mathrm{A}_{2} \mathrm{~V}_{2} / \mathrm{V}_{2} \\
\left(\mathrm{~V}_{2}^{2}-\mathrm{V}_{1}^{2}\right) & =\mathrm{h}_{1}-\mathrm{h}_{2}
\end{aligned}
$$

From chart, $\quad \mathrm{h}_{1}=3035 \mathrm{~kJ} / \mathrm{kg}$
$\mathrm{h}_{2^{\prime}}=2925 \mathrm{~kJ} / \mathrm{kg}$
$\mathrm{v}_{2}=0.22 \mathrm{~m}^{3} / \mathrm{kg}$

$$
\begin{aligned}
& =---------\quad h_{1} \\
& \text { Nozzle efficiency }\left(\eta_{n}\right) \\
& \therefore 0.9=\frac{\mathrm{h}_{1}--\mathrm{h}_{2}}{3035------2925} \\
& \therefore \quad \mathrm{~h}_{1}-\mathrm{h}_{2}=99 \mathrm{~kJ} / \mathrm{kg} \\
& \mathrm{~V}_{2}{ }^{2} / 2=99 \times 10^{3} \\
& \mathrm{~V}_{2}=444.97 \mathrm{~m} / \mathrm{s} \\
& \therefore \quad 4.083=\mathrm{A}_{2} \times 444.97 / 0.22 \\
& \mathrm{~A}_{2}=0.0020186 \mathrm{~m}^{2} \\
& \therefore \quad \mathrm{~A}_{2} / \mathrm{noz}=0.0020186 / 16=\mathbf{1 . 2 6 1 6} \times \mathbf{1 0}^{-4} \mathrm{~m}^{\mathbf{2}} \ldots-\mathrm{Ans}
\end{aligned}
$$

$$
\begin{aligned}
& \left(\mathrm{V}_{2}{ }^{2}-\mathrm{V}_{1}{ }^{2}\right) / 2=\mathrm{h}_{1}-\mathrm{h}_{2} \\
& \left(V_{2}{ }^{2}-100^{2}\right) / 2=99 \times 10^{3} \\
& \mathrm{~V}_{2}=456.07 \mathrm{~m} / \mathrm{s} \\
& \mathrm{~m}=\mathrm{A}_{2} \mathrm{~V}_{2} / \mathrm{V}_{2}=0.0020186 \times 456.07 / 0.22 \\
& =4.1846 \mathrm{~kg} / \mathrm{s}
\end{aligned}
$$

4.1846 - 4.083
$\therefore \quad \%$ increase in mass flow rate $=----------------\times 100=2.49 \%$--- Ans 4.083
6. Steam enters a nozzle in a dry saturated condition and expands from a pressure of 2 bar to a pressure of 1 bar. It is observed that supersaturated flow is taking place and the steam flow reverts to a normal flow at 1 bar. What is the degree of under-cooling, degree of supersaturation, increase in entropy and loss in the available heat drop due to irreversibility.

Given
Inlet pressure $\left(p_{1}\right)=2$ bar, Dry
Outlet pressure $\left(p_{2}\right)=1$ bar
Flow = Supersaturated
Required: $\left(T_{2}-T_{2}\right),\left(p_{2} / p_{2}\right),(\Delta s)$, Loss in availability

## Solution

Degree of undercooling $\quad=T_{2}-T_{2}$
$\mathrm{T}_{2}=99.63^{\circ} \mathrm{C}$ from steam table at $\mathrm{p}_{2}=1$ bar
To find $\underline{T}_{\underline{2}}$

$$
\mathrm{T}_{2} / \mathrm{T}_{1}=\left[\mathrm{p}_{2} / \mathrm{p}_{1}\right]^{(\mathrm{n}-1) / \mathrm{n}}
$$

$\mathrm{T}_{1}=120.2^{\circ} \mathrm{C}$ from steam table at $\mathrm{p}_{1}$

$$
\therefore \mathrm{T}_{2^{\prime} /(120.2+273)}=[1 / 2]^{(1.3-1) / 1.3}
$$

$$
\mathrm{T}_{2^{\prime}}=335.07 \mathrm{~K}=62.07^{\circ} \mathrm{C}
$$

$\therefore$ Degree of undercooling $=99.63-62.07=37.56^{\circ} \mathrm{C}$---- Ans
Degree of supersaturation $=\quad p_{2} / p_{2}$

$$
\mathrm{p}_{2^{\prime}}=0.21838 \text { bar from steam table at } \mathrm{T}_{2^{\prime}}=60.07^{\circ} \mathrm{C}
$$

$\therefore$ Degree of supersaturation $=1 / 0.21838=4.579$--- Ans
Loss in availability $\quad=\left(h_{1}-h_{2}\right)_{\text {chart }}-\left(h_{1}-h_{2}\right)_{\text {equ }}$

From chart, $\quad \mathrm{h}_{1}=2710 \mathrm{~kJ} / \mathrm{kg}$

$$
\begin{array}{cl} 
& h_{2}=2590 \mathrm{~kJ} / \mathrm{kg} \\
\therefore & \left(\mathrm{~h}_{1}-\mathrm{h}_{2}\right)_{\text {chart }}=2710-2590=120 \mathrm{~kJ} / \mathrm{kg} \\
& \left(h_{1}-h_{2}\right)_{\text {equ }}=n /(n-1) p_{1} \mathrm{v}_{1}\left[1-\left(p_{2} / p_{1}\right)^{(n-1) / n}\right]
\end{array}
$$

$$
v_{1}=0.87 \mathrm{~m}^{3} / \mathrm{kg} \text { from chart }
$$

$$
\begin{aligned}
\therefore \quad\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)_{\text {equ }} & =1.3 /(1.3-1) \times 2 \times 10^{5} \times 0.87 \times\left[1-(1 / 2)^{(1.3-1) / 1.3}\right] \\
& =111456 \mathrm{~J} / \mathrm{kg}=111.5 \mathrm{~kJ} / \mathrm{kg} \\
\therefore \quad \text { Loss } & =120-111.5=8.5 \mathrm{~kJ} / \mathrm{kg}--- \text { Ans } \\
& =\text { Loss } / \mathrm{T}_{2} \\
\text { Increase in entropy } \quad & 8.5 /(99.63+273)=\mathbf{0 . 0 2 2 8 1} \mathbf{~ k J} / \mathrm{kgK} \text {--- Ans }
\end{aligned}
$$

7. Steam is supplied to a group of 4 nozzles at 18 bar and $250^{\circ} \mathrm{C}$. It is expanded down to 4 bar and friction loss may be neglected. If the expansion is metastable, calculate for a flow of $2.5 \mathrm{~kg} / \mathrm{s}$, the exit dimensions of nozzles if they are rectangular in shape and have length to breath ratio of $3: 1$. What is the degree of undercooling and degree of supersaturation.

## Given

No of nozzles $=4$
Inlet pressure $\left(p_{1}\right)=18$ bar
Inlet temperature $\left(\mathrm{T}_{1}\right) \quad=250^{\circ} \mathrm{C}$
Exit pressure $\left(\mathrm{p}_{2}\right)=4$ bar
Mass flow rate $(\mathrm{m}) \quad=2.5 \mathrm{~kg} / \mathrm{s}$
I: b
= 3 : 1
Flow = Supersaturated
Required: $\mathrm{I}_{2}, \mathrm{~b}_{2},\left(\mathrm{~T}_{2}-\mathrm{T}_{2^{\prime}}\right), \mathrm{p}_{2} / \mathrm{p}_{2}$
Solution
Mass flow rate (m) $\quad=\mathrm{A}_{2} \mathrm{~V}_{2^{\prime}} / \mathrm{V}_{2^{\prime}}$
To find $\mathrm{v}_{\underline{2}}$

$$
v_{2^{\prime}} / v_{1}=\left(p_{1} / p_{2}\right)^{1 / n}
$$

$\mathrm{v}_{1}=0.12 \mathrm{~m}^{3} / \mathrm{kg}$ from chart
$\mathrm{n}=1.3$ for supersaturated flow

$$
\begin{aligned}
\therefore \quad \mathrm{v}_{2^{\prime}} / 0.12 & =(18 / 4)^{1.3} \\
\mathrm{v}_{2^{\prime}} & =0.38164 \mathrm{~m}^{3} / \mathrm{kg}
\end{aligned}
$$

$$
\begin{aligned}
V_{2} / 2 & =\left(h_{1}-h_{2}\right)_{\text {equ }} \\
\left(h_{1}-h_{2}\right)_{\text {equ }} & =n /(n-1) p_{1} v_{1}\left[1-\left(p_{2} / p_{1}\right)^{(n-1) / n}\right] \\
& =1.3 /(1.3-1) \times 18 \times 10^{5} \times 0.12 \times\left[1-(4 / 18)^{(1.3-1) / 1.3}\right] \\
& =274493 \mathrm{~J} / \mathrm{kg}
\end{aligned}
$$

$$
\therefore \mathrm{V}_{2} / 2=274493
$$

$$
\therefore \quad \mathrm{V}_{2^{\prime}}=740.93 \mathrm{~m} / \mathrm{s}
$$

$$
\therefore \quad 2.5=\mathrm{A}_{2} \times 740.93 / 0.38164
$$

$$
\mathrm{A}_{2}=0.001288 \mathrm{~m}^{2}
$$

$$
\mathrm{A}_{2} / \mathrm{noz}=0.001288 / 4=0.000322 \mathrm{~m}^{2}
$$

$$
\text { Let } a=\text { Breath }
$$

$$
\therefore \text { Length }=3 \mathrm{a}
$$

and

$$
\begin{array}{rlrl} 
& 3 a \times a & =3 a^{2}=0.000322 \\
& \therefore & a & =0.01036 \mathrm{~m}
\end{array}---- \text { Ans }
$$

and $\quad$ Length $=3 a 3 \times 0.01036=0.3108 \mathbf{m}$---- Ans
Degree of undercooling $\quad=T_{2}-T_{2}$
$\mathrm{T}_{2}=143.6^{\circ} \mathrm{C}$ from steam table at $\mathrm{p}_{2}=4$ bar

## To find $T_{2}$

$$
\begin{aligned}
\mathrm{T}_{2^{\prime}} / \mathrm{T}_{1} & =\left[\mathrm{p}_{2} / \mathrm{p}_{1}\right]^{(\mathrm{n}-1) / \mathrm{n}} \\
\therefore \quad \mathrm{~T}_{2^{\prime}} /(250+273) & =[4 / 18]^{(1.3-1) / 1.3} \\
\mathrm{~T}_{2^{\prime}} & =369.6 \mathrm{~K}=96.6^{\circ} \mathrm{C}
\end{aligned}
$$

$\therefore$ Degree of undercooling $=143.6-96.6=47^{\circ} \mathrm{C}$---- Ans
Degree of supersaturation $=\quad \mathrm{p}_{2} / \mathrm{p}_{2}$
$\mathrm{p}_{2^{\prime}}=0.8964$ bar from steam table at $\mathrm{T}_{2^{\prime}}=96.6^{\circ} \mathrm{C}$
$\therefore$ Degree of supersaturation $=4 / 0.8964=4.4623$--- Ans
8. In an installation $5 \mathrm{~kg} / \mathrm{s}$ of steam at 35 bar and $350^{\circ} \mathrm{C}$ is supplied to group of 6 nozzles in a wheel chamber maintained at 5 bar. (a) Determine the dimensions of the nozzles of rectangular cross sectional area with aspect ratio $3: 1$. The expansion may be considered metastable and friction is neglected. (b) Also calculate, (i) degree of undercooling and supersaturation (ii) loss in available
heat drop due to irreversibility (iii) increase in entropy and (iv) ratio of mass flow rate with metastable expansion to that if expansion in thermal equilibrium. Given

| No of nozzles | $=6$ |
| :--- | :--- |
| Inlet pressure $\left(p_{1}\right)$ | $=35 \mathrm{bar}$ |
| Inlet temperature $\left(\mathrm{T}_{1}\right)$ | $=350^{\circ} \mathrm{C}$ |
| Exit pressure $\left(p_{2}\right)$ | $=5 \mathrm{bar}$ |
| Mass flow rate $(\mathrm{m})$ | $=5 \mathrm{~kg} / \mathrm{s}$ |
| $1: \mathrm{b}$ | $=3: 1$ |
| Flow | $=$ Supersaturated |

Required: (a) $I_{2}, b_{2}$, (b)(i) $\left(T_{2}-T_{2^{\prime}}\right), p_{2} / p_{2^{\prime}}$ (ii) Loss (iii) Inc. in entropy (iv) $m_{t} / m_{m}$
Solution
(a) Mass flow rate $\left(\mathrm{m}_{\mathrm{m}}\right) \quad=\mathrm{A}_{2} \mathrm{~V}_{2^{\prime}} / \mathrm{V}_{2^{\prime}}$

To find $v_{2}$

$$
\begin{aligned}
& \mathrm{v}_{2^{\prime}} / \mathrm{v}_{1}=\left(\mathrm{p}_{1} / \mathrm{p}_{2}\right)^{1 / \mathrm{n}} \\
& \mathrm{v}_{1}=0.075 \mathrm{~m}^{3} / \mathrm{kg} \text { from chart } \\
& \mathrm{n}=1.3 \text { for supersaturated flow } \\
& \therefore \quad \mathrm{v}_{2^{\prime}} / 0.075=(35 / 5)^{1.3} \\
& \mathrm{v}_{2^{\prime}}=0.335 \mathrm{~m}^{3} / \mathrm{kg}
\end{aligned}
$$

To find $V_{2}$

$$
\begin{aligned}
\mathrm{V}_{2^{\prime} / 2} & =\left(h_{1}-h_{2}\right)_{\text {equ }} \\
\left(h_{1}-h_{2}\right)_{\text {equ }} & =n /(n-1) p_{1} v_{1}\left[1-\left(p_{2} / p_{1}\right)^{(n-1) / n}\right] \\
& =1.3 /(1.3-1) \times 35 \times 10^{5} \times 0.075 \times\left[1-(5 / 35)^{(1.3-1) / 1.3}\right] \\
& =411513.1 \mathrm{~J} / \mathrm{kg} \\
\therefore \quad \mathrm{~V}_{2^{2}} / 2 & =411513.1 \\
\therefore \quad V_{2^{\prime}} & =907.2 \mathrm{~m} / \mathrm{s} \\
\therefore \quad 5 & =A_{2} \times 907.2 / 0.335 \\
A_{2} & =0.00184634 \mathrm{~m}^{2} \\
\mathrm{~A}_{2} / \text { noz } & =0.00184634 / 6=0.000307723 \mathrm{~m}^{2} \\
\text { Let } \quad \mathrm{a}= & \text { Breath } \\
\therefore \text { Length } & =3 \mathrm{a}
\end{aligned}
$$

and

$$
3 a \times a=3 a^{2}=0.000307723
$$

$$
\therefore \quad a=0.0101279 \mathrm{~m}----- \text { Ans }
$$

and

$$
\text { Length }=3 a 3 \times 0.010129=0.30384 \mathbf{m}--- \text { Ans }
$$

(b) (i) Degree of undercooling $=\mathrm{T}_{2}-\mathrm{T}_{2}$,
$\mathrm{T}_{2}=151.8^{\circ} \mathrm{C}$ from steam table at $\mathrm{p}_{2}=5$ bar
To find $T_{\underline{2}}$

$$
\begin{aligned}
\mathrm{T}_{2^{\prime}} / \mathrm{T}_{1} & =\left[\mathrm{p}_{2} / \mathrm{p}_{1}\right]^{(\mathrm{n}-1) / \mathrm{n}} \\
\therefore \quad \mathrm{~T}_{2^{\prime} /(350+2}(373) & =[5 / 35]^{(1.3-1) / 1.3} \\
\mathrm{~T}_{2^{\prime}} & =379.6 \mathrm{~K}=124.6^{\circ} \mathrm{C}
\end{aligned}
$$

$\therefore$ Degree of undercooling $=151.8-124.6=\mathbf{2 7 . 2} \mathbf{2}^{\circ} \mathrm{C}$---- Ans
Degree of supersaturation $=\quad p_{2} / p_{2}$
$\mathrm{p}_{2^{\prime}}=2.29327$ bar from steam table at $\mathrm{T}_{2^{\prime}}=124.6^{\circ} \mathrm{C}$
$\therefore$ Degree of supersaturation $=5 / 2.29327=2.1803$--- Ans
(ii) Loss in availability

$$
=\left(h_{1}-h_{2}\right)_{\text {chart }}-\left(h_{1}-h_{2}\right)_{\text {equ }}
$$

From chart,

$$
\begin{aligned}
& \mathrm{h}_{1}=3105 \mathrm{~kJ} / \mathrm{kg} \\
& \mathrm{~h}_{2}=2690 \mathrm{~kJ} / \mathrm{kg}
\end{aligned}
$$

$$
\therefore \quad\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)_{\text {chart }}=3105-2690=415 \mathrm{~kJ} / \mathrm{kg}
$$

$$
\left(h_{1}-h_{2}\right)_{\text {equ }}=411.5131 \mathrm{~kJ} / \mathrm{kg}
$$

$$
\therefore \quad \text { Loss }=415-411.5131=3.4869 \mathrm{~kJ} / \mathrm{kg} \quad---- \text { Ans }
$$

(iii) Increase in entropy $\quad=$ Loss $/ T_{2}$

$$
=3.4869 /(151.8+273)=0.008208 \mathrm{~kJ} / \mathrm{kgK} \text {--- Ans }
$$

(iv) Let $\quad m_{t}=$ Mass flow rate in thermal equilibrium flow

$$
m_{m}=\text { Mass flow rate in metastable flow }
$$

$$
\mathrm{m}_{\mathrm{m}}=5 \mathrm{~kg} / \mathrm{s} \text { (given in problem) }
$$

To find $\mathrm{m}_{\mathrm{t}}$

$$
\begin{aligned}
\mathrm{m}_{\mathrm{t}} & =\mathrm{A}_{2} \mathrm{~V}_{2} / \mathrm{v}_{2} \\
\mathrm{~V}_{2}^{2} / 2 & =\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)_{\text {chart }} \\
\therefore \quad \mathrm{V}_{2}^{2} / 2 & =(3105-2690) \times 10^{3}
\end{aligned}
$$

$$
V_{2}=911.04 \mathrm{~m} / \mathrm{s}
$$

$\mathrm{v}_{2}=0.4 \mathrm{~m}^{3} / \mathrm{kg}$ from chart at $\mathrm{p}_{2}=5$ bar

$$
\begin{aligned}
\mathrm{m}_{\mathrm{t}} & =0.00184634 \times 911.04 / 0.4=4.205 \mathrm{~kg} / \mathrm{s} \\
\therefore \quad \mathrm{~m}_{\mathrm{m}} / \mathrm{m}_{\mathrm{t}} & =5 / 4.205=1.19 \text {--- Ans }
\end{aligned}
$$

## STEAM TURBNES

The steam turbine is a prim-mover in which the potential energy of steam is transferred into kinetic energy and later in its turn transferred into the mechanical energy of rotation of the turbine shaft.

Based on action of steam the steam turbines may be classified as
(i) Impulse turbine
(ii) Reaction turbine
(iii) Impulse and reaction turbine

According to the direction of steam flow
(i) Axial flow turbine
(ii) Radial flow turbine

According to the number of stages
(i) Single stage turbine
(ii) Multi stage turbine

## Simple Impulse turbine



An impulse turbine runs by the impulse of steam jet. In this turbine the steam is first made to flow through a nozzle. Then the steam jet impinges on the turbine blades. The steam jet after impinging on the rotor blades glides over the concave surface of the blades and finally leaves he turbine.

A De-Laval turbine is the simple impulse turbine and is commonly used with fixed nozzles and a rotor with a ring of blades inside a casing. The surface of the blades are
generally very smooth to minimize the frictional losses. The blades are generally made of special steel alloys. Steam supplied to an impulse turbine expands completely in the nozzle.

As the steam flows through the nozzle its pressure falls from steam chest pressure to condenser pressure. Due to this relatively higher ratio of expansion of steam in the nozzles the steam leaves the nozzle with a very high velocity. It can be observed that the velocity of the steam leaving the moving blades is comparatively higher. The loss of energy due to this higher exit velocity is called "Carry over loss" or "leaving loss". This loss may amount to 3 to $4 \%$ of the nozzle velocity.

The moving blades of impulse turbine are 'constant flow area profile type blades'. Therefore the pressure remains constant during the flow of steam through the moving blades of impulse turbine.

## Reaction turbine



In this type of turbine, there is a gradual pressure drop and takes place continuously over the fixed and moving blades. The function of the fixed blades is that they alter the direction of the steam as well as allow it expand to a larger velocity. As the steam passes over the moving blades its kinetic energy is absorbed by them. Instead of a set of nozzles, steam is admitted for whole of the circumference and therefore there is all-round admission. In passing through the first row of fixed blades, the steam undergoes a small drop in pressure and its velocity is increased. It then enters the first row of moving blades and it suffers a change in direction and therefore momentum. This gives impulse to the blades. But the moving blades are of aerofoil type and hence there is also a pressure drop in the moving blades.

The reaction turbines which are used these days are really impulse-reaction turbines. Pure reaction turbines are not in general use. The expansion of steam and heat drop occur both in fixed and moving blades. The velocity of steam in this type of turbines
is comparatively low, the maximum being about equal to blade velocity. This type of turbine is very successful in practice. It is also called "Parson's Reaction Turbine".

## Difference between Impulse and Reaction turbines

| S.No. | Particulars | Impulse turbine | Reaction turbine |
| :---: | :---: | :---: | :---: |
| 1. | Pressure drop | Only in nozzles and not in moving blades. | In fixed blades(nozzles) aswell as in moving blades. |
| 2. | Area of blade channels | Constant. | Varying (converging type). |
| 3. | Blades | Profile type. | Aerofoil type. |
| 4. | Admission of steam | Not all round or complete. | All round or complete. |
| 5. | Nozzles/fixed blades | Diaphram contains the nozzle, | Fixed blades similar to moving blades attached to the casing serve as nozzles and guide the steam. |
| 6. | Power | Not much power can be developed. | Much power can be developed. |
| 7. | Space | Requires less space for same power. | Requires more space for same power. |
| 8. | Efficiency | Low. | High. |
| 9. | Suitability | Suitable for small power requirements. | Suitable for medium and higher power requirements. |
| 10. | Blade manufacture | Not difficult. | Difficult. |

## Methods of reducing rotor speed

In case of simple impulse turbine, the steam is expanded from the boiler pressure to the condenser pressure in one stage only. Hence the speed of the rotor becomes very high for practical purposes. In order to make the rotor speed practicable compounding of steam turbine is done. Compounding is the method of reducing rotor speed by adding stages to a simple impulse turbine without affecting the turbine work output. The rotor speed can be reduced by the following methods.
(i) Velocity compounding
(ii) Pressure compounding
(iii) Pressure-Velocity compounding
(iv) Reaction turbine

## Velocity Compounding

Steam is expanded through a stationary nozzle from the boiler or inlet pressure to condenser pressure. So, the pressure in the nozzle drops, the kinetic energy of the steam increases due to increase in velocity. A portion of this available energy is absorbed by a row of moving blades. The steam (whose velocity has decreased while moving over the moving blades) then flows through the second row of blades which are fixed. The function of these fixed blades is to redirect the steam flow without altering its velocity to the following next row moving blades where again work is done on them and steam leaves the turbine with a low velocity. Fig shows a cut away section of such a stage and changes in pressure and velocity as the steam passes through the nozzle, fixed and moving blades. Though this method has the advantage that the initial cost is low due to lesser number of stages yet its efficiency is low.


## Pressure Compounding



Fig shows rings of fixed blades incorporated between the rings of moving blades. The steam at boiler pressure enters the first set of nozzles and expands partially. The kinetic energy of the steam thus obtained is absorbed by the moving blades. The steam then expands partially in the second set of nozzles where its pressure again falls and the velocity increases; the kinetic energy so obtained is absorbed by the second ring of
moving blades (stage-2). This is repeated in stage-3 and steam finally leaves the turbine at low velocity and pressure. The number of stages depends on the number of rows of nozzles through which the steam must pass.

This method of compounding is used in Rateau and Zoelly turbine. This is most efficient turbine since the speed ratio remains constant but it is expensive owing to a large number of stages.

## Pressure-Velocity Compounding

This method of compounding is the combination of pressure and velocity compounding. The total drop in steam pressure is divided into stages and the velocity obtained in each stage is also compounded. The rings of nozzles are fixed at the beginning of each stage and pressure remains constant during each stage. The changes in pressure and velocity are shown. This method of compounding is used in Curtis and Moore turbine.


## Steam turbine governing and control

The method of maintaining the speed of the turbine constant irrespective of variation of the load on the turbine is known as governing of turbines. For this purpose governor is used, which regulates the supply of steam to the turbine in such a way that the speed of the turbine is maintained as far as possible a constant under varying load conditions. The various methods of governing of steam turbines are,
(i) Throttle governing
(ii) Nozzle control governing
(iii) By-pass governing
(iv) Combination of (i) and (ii) and (i) and (iii)

## Throttle Governing

The throttle governing of a steam turbine is a method of controlling its speed by varying the quantity of steam entering the turbine.


The centrifugal governor is driven from the main shaft of the turbine. The control valve controls the direction of flow of oil either in the pipe-AA or BB. The relay cylinder has a piston whose motion is connected to a spear which moves inside the nozzle.

When the load on the turbine increases, the speed of the turbine decreases. The fly balls of the governor will come down. The fly balls bring down the sleeve. The downward movement of the sleeve will raise the control valve rod. The mouth of the pipe-AA will open. Now the oil under pressure will rush from the control valve to the right side of the piston in the relay cylinder through the pipe-AA. This will move the piston and spear towards the left which will open more area of nozzle. As a result the steam flow rate into the turbine increases, which inturn brings the speed of the turbine to the normal range.

## Nozzle control governing

The efficiency of a steam turbine is considerably reduced if throttle governing is carried out at low loads. An alternative, and more efficient form of governing is by means of nozzle control. In this method of governing, the nozzles are grouped together 3 to 5 or more groups and supply of steam to each group is controlled by regulating valves. Under full load conditions the valves remain fully open.

When the load on the turbine becomes more or less than the design value, the supply of steam to a group of nozzles may be varied accordingly so as to restore the original speed.

Nozzle control can only be applied to the first stage of a turbine. It is suitable for simple impulse turbine and larger units which have an impulse stage followed by an impulse-reaction turbine. In pressure compounded impulse turbines, there will be some drop in pressure at entry to second stage when some of the first stage nozzles are cut out.


## By-pass governing

The steam turbines are designed to work at economic load is desirable to have full admission of steam in the high pressure stages. At the maximum load, which is greater than the economic load, the additional steam required could not pass through the first stage since additional nozzles are not available. By-pass regulation allows for this in a turbine which is throttle governed, by means of a second by-pass valve in the first stage nozzle. This valve opens when throttle valve has opened a definite amount. Steam is by-passed through the second valve to a lower stage to the turbine. When bypass valve operates it is under the control of the turbine governor. The secondary and tertiary supplies of steam in the lower stages increase the work output in these stage, but there is a loss in efficiency.


## Velocity diagram for moving blades of impulse turbine



Let
$\alpha_{1}$ - Angle with which the steam enters the moving blade (Nozzle angle)
$\alpha_{2}$ - Angle with which the steam leaves the moving blade
$\beta_{1}$ - Blade angle at inlet
$\beta_{2}$ - Blade angle at outlet
$\mathrm{C}_{1}$ - Absolute velocity of steam at inlet of the blade
$\mathrm{C}_{2}$ - Absolute velocity of steam leaving the blade
$\mathrm{C}_{\mathrm{w} 1}$ - Tangential component of absolute velocity at inlet (Whirl velocity)
$\mathrm{C}_{\mathrm{w} 2}$ - Tangential component of absolute velocity at outlet
$\mathrm{C}_{\mathrm{f} 1}$ - Axial component of absolute velocity at inlet (Flow velocity)
$\mathrm{C}_{\mathrm{f} 2}$ - Axial component of absolute velocity at exit
$\mathrm{C}_{\mathrm{r} 1}$ - Relative velocity of steam at inlet
$\mathrm{C}_{\mathrm{r} 2}$ - Relative velocity of steam at exit
$C_{b}$ - Blade velocity
The steam jet issuing from the nozzle at a velocity of $\mathrm{C}_{1}$ impinges on the blade at an angle $\alpha_{1}$. The tangential component of this jet ( $\mathrm{C}_{\mathrm{w} 1}$ ) performs work on the blades. The axial component does no work, but causes the steam to flow through the turbine.

The relative velocity at outlet $\left(\mathrm{C}_{\mathrm{r} 2}\right)$ is same as the relative velocity at inlet $\left(\mathrm{C}_{\mathrm{r} 1}\right)$ if there is no frictional loss at the blade.

Work done on the blade
Work done
= Force $\times$ Distance traveled

Work done per second

$$
\begin{aligned}
& =\text { Force } \mathrm{x} \text { distance traveled per second } \\
& =\text { mass } \mathrm{x} \text { acceleration } \\
& =\text { mass } \mathrm{x} \text { rate of change of velocity } \\
& =\text { mass flow rate } \mathrm{x} \text { change of velocity } \\
& =\stackrel{*}{m}_{s}\left(C_{w 1}-\left(-C_{w 2}\right)\right)=\stackrel{*}{m_{s}}\left(C_{w 1}+C_{w 2}\right)
\end{aligned}
$$

Distance traveled per second $=C_{b}$
Therefore $\quad W=\stackrel{*}{m}\left(C_{w 1}+C_{w 2}\right) C_{b}$
Blade or Diagram efficiency $\left(\eta_{b}\right)$

$$
\eta_{b}=\frac{\text { Work done on the blade }}{\text { Energysup plied to the blade }}
$$

Energy supplied to the blade $=$ Kinetic energy of the steam at inlet $=1 / 2 \mathrm{~m}_{\mathrm{s}} \mathrm{C}_{1}{ }^{2}$

$$
\begin{aligned}
\eta_{b} & =\frac{\stackrel{*}{m_{s}}\left(C_{w 1}+C_{w 2}\right) C_{b}}{\frac{1}{2} \stackrel{*}{s}_{s}^{2}} \\
& =\frac{2\left(C_{w 1}+C_{w 2}\right) C_{b}}{C_{1}^{2}}
\end{aligned}
$$

Stage efficiency $\left(\eta_{s t}\right)$

## UNIT-IV

## AIR COMPRESSORS

## Air Compressors

An air compressor takes in atmospheric air, compresses it and delivers the high pressure air to a storage vessel from which it may be conveyed by the pipe line to wherever the supply of compressed air is required.

## Classification

1. According to working,
a) Reciprocating compressors
b) Rotary compressors
2. According to action,
a) Single acting compressors
b) Double acting compressors
3. According to number of stages,
a) Single stage compressors
b) Multi stage compressors

Single stage Reciprocating air compressor


A reciprocating compressor consists of a cylinder, piston, inlet and outlet valves. The arrangement of the compressor is shown in fig.

During the downward motion of the piston, the pressure inside the cylinder falls below the atmospheric pressure and the inlet valve is opened due to the pressure difference. The air is taken into the cylinder until the piston reaches bottom dead centre (BDC) position.

As the piston starts moving upwards, the inlet valve closed and the pressure starts increasing continuously until the pressure inside the cylinder is above the pressure of the delivery side which is connected to the receiver. At the end of the delivery stroke, small volume of high pressure air is left in the clearance space. The high pressure air left in the clearance space expands as the piston starts moving downwards and the pressure of the air falls until it is just below the atmospheric pressure. The inlet valve opens as the pressure inside the cylinder falls below the atmospheric pressure and the air from outside is taken in and the cycle is repeated.

The suction, compression and delivery of the air takes place within two strokes of the piston or one revolution of the crank.

## Definitions



Inlet pressure $\left(\mathbf{p}_{1}\right)$ : It is the absolute pressure of air at the inlet of the compressor.
Discharge pressure $\left(p_{2}\right)$ : It is the absolute pressure of air at the outlet of the compressor.

## Compression ratio (or) Pressure ratio ( $p_{2} / p_{1}$ )

$$
=\frac{\text { Disch arg e pressure }}{\text { Inlet pressure }}=\frac{p_{2}}{p_{1}}
$$

## Compressor capacity

It is the volume of free air delivered by the compressor, and is expressed in ' $\mathrm{m}^{3} / \mathrm{s}^{\prime}$ or ' $\mathrm{m}^{3} / \mathrm{min}$ '.

## Free air delivery (FAD)

It is the actual volume of air delivered by a compressor when reduced to the 'Intake' or 'Normal' or 'Required' temperature and pressure conditions.

Let, $p_{f}=$ Free air pressure

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{f}}=\text { Free air temperature } \\
& \mathrm{V}_{\mathrm{f}}=\text { Free air delivery }
\end{aligned}
$$

And $p_{1}, V_{1} \& T_{1}$ are corresponding values of air at inlet.

$$
\begin{aligned}
& \frac{p_{f} V_{f}}{T_{f}}=\frac{p_{1} V_{1}}{T_{1}} \rightarrow \text { Compressor with no clearance volume } \\
& \frac{p_{f} V_{f}}{T_{f}}=\frac{p_{1}\left(V_{1}-V_{4}\right)}{T_{1}} \rightarrow \text { Compressor with clearance volume }
\end{aligned}
$$

## Swept volume (or) Stroke volume $\left(\mathrm{V}_{1}-\mathrm{V}_{3}\right)$

It is the theoretical volume of air that can be sucked by the compressor during suction stroke.

$$
\begin{aligned}
V_{1}-V_{3} & =(\pi / 4) D^{2} L N / 60-\text { for single acting } \\
& =2 \times(\pi / 4) D^{2} L N / 60-\text { for double acting } \\
& =Z(\pi / 4) D^{2} L N / 60-\text { for multi cylinder compressor }
\end{aligned}
$$

Where, $\quad \mathrm{D}=$ Diameter of the piston (or) Bore
L = Stroke
$\mathrm{N}=$ Speed of the compressor in 'rpm'
Z = number of cylinders

## Clearance ratio (k)

$$
k=\frac{\text { Clearance volume }}{\text { Swept volume }}=\frac{V_{3}}{V_{1}-V_{3}}
$$

## Mean effective pressure ( $p_{m}$ )

= -------------------

## Workdone in a single stage reciprocating air compressor

(1) Without clearance volume


Let, $p_{1}=$ Intake pressure of the air $\mathrm{p}_{2}=$ Delivery pressure of the air
$\mathrm{T}_{1}=$ Intake air temperature
$\mathrm{T}_{2}=$ Delivery air temperature

## Workdone when the compression is Isothermal

Workdone $=$ Area enclosed by the curves on $\mathrm{p}-\mathrm{V}$ diagram.

$$
\begin{aligned}
W & =\text { Area 1-2-3-4-1 } \\
& =p_{2} V_{2}+p_{1} V_{1} \ln \left(\frac{V_{1}}{V_{2}}\right)-p_{1} V_{1}
\end{aligned}
$$

For isothermal process, $\mathrm{p}_{1} \mathrm{~V}_{1}=\mathrm{p}_{2} \mathrm{~V}_{2}$

$$
\therefore \quad W=p_{1} V_{1} \ln \left(\frac{V_{1}}{V_{2}}\right)
$$

## Workdone when the compression is polytropic

Workdone $=$ Area enclosed by the curves on $\mathrm{p}-\mathrm{V}$ diagram.

$$
\begin{aligned}
W & =\text { Area 1-2-3-4-1 } \\
& =p_{2} V_{2}+\frac{p_{2} V_{2}-p_{1} V_{1}}{n-1}-p_{1} V_{1} \\
& =\left(p_{2} V_{2}-p_{1} V_{1}\right)\left[1+\frac{1}{n-1}\right] \\
& =\frac{n}{n-1}\left(p_{2} V_{2}-p_{1} V_{1}\right) \\
& =\frac{n}{n-1} p_{1} V_{1}\left[\frac{p_{2} V_{2}}{p_{1} V_{1}}-1\right]
\end{aligned}
$$

We know for polytropic process, $\mathrm{p}_{1} \mathrm{~V}_{1}{ }^{\mathrm{n}}=\mathrm{p}_{2} \mathrm{~V}_{2}{ }^{\mathrm{n}}$

$$
\begin{aligned}
& \operatorname{Or} \quad \frac{V_{2}}{V_{1}}=\left(\frac{p_{2}}{p_{1}}\right)^{-1 / n} \\
W & =\frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)\left(\frac{p_{2}}{p_{1}}\right)^{-1 / n}-1\right] \\
& =\frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{1-1 / n}-1\right] \\
& =\frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
\end{aligned}
$$

Workdone when the compression is isentropic

$$
W=\frac{\gamma}{\gamma-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(\gamma-1) / \gamma}-1\right]
$$

## (2) With clearance volume



1-2 $\rightarrow$ Compression
2-3 $\rightarrow$ Discharge
3-4 $\rightarrow$ Expansion
4-1 $\rightarrow$ Actual suction
In practice all reciprocating compressors will have a clearance volume. The clearance volume is that volume which remains in the cylinder after the piston has reached the end of its inward stroke or upward stroke.

$$
\begin{aligned}
\mathrm{V}_{1}-\mathrm{V}_{3} & =\text { Stroke volume (Theoretical volume) } \\
\mathrm{V}_{1}-\mathrm{V}_{4} & =\text { Actual volume of air sucked by the compressor } \\
\mathrm{V}_{3} & =\text { Clearance volume } \\
\mathrm{W} & =\text { Area 1-2-3-5-6-4-1 - Area 4-3-5-6-4 } \\
& =\frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]-\frac{n}{n-1} p_{1} V_{4}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right] \\
& =\frac{n}{n-1}\left(p_{1} V_{1}-p_{1} V_{4}\right)\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right] \\
& =\frac{n}{n-1} p_{1}\left(V_{1}-V_{4}\right)\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
\end{aligned}
$$

## Volumetric efficiency ( $\eta_{v}$ )

It is the ratio of effective swept volume to the swept volume.

$$
\eta_{v}=\frac{V_{1}-V_{4}}{V_{1}-V_{3}}
$$

It is the ratio of free air delivered to the displacement of the compressor.

$$
\begin{aligned}
\eta_{v} & =\frac{V_{f}}{V_{1}-V_{3}} \\
\eta_{v} & =\frac{V_{1}-V_{4}}{V_{1}-V_{3}} \\
& =\frac{\left(V_{1}-V_{3}\right)+\left(V_{3}-V_{4}\right)}{\left(V_{1}-V_{3}\right)} \\
& =1+\frac{V_{3}-V_{4}}{V_{1}-V_{3}}=1+\frac{V_{3}}{V_{1}-V_{3}}-\frac{V_{4}}{V_{1}-V_{3}} \\
& =1+\frac{V_{3}}{V_{1}-V_{3}}-\frac{V_{3}}{V_{1}-V_{3}}\left(\frac{V_{4}}{V_{3}}\right) \\
& =1+k-k\left(\frac{V_{4}}{V_{3}}\right)^{2} \\
& =1+k-k\left(\frac{V_{1}}{V_{2}}\right)^{1 / n} \\
& =1+k-k\left(\frac{p_{2}}{p_{1}}\right)^{1 / n}
\end{aligned}
$$

Note : Since expansion \& Compression follows the same law,

$$
\frac{V_{4}}{V_{3}}=\frac{V_{1}}{V_{2}}
$$

## Factors Reducing Volumetric efficiency

- Very high speed.
- Leakage past the piston.
- Too large a clearance volume.
- Obstruction at inlet valves.
- Overheating of air by contact with the hot cylinder walls.
- Inertia effect of air in suction pipe.


## Effect of clearance volume

Because of presence of clearance volume, volumetric efficiency is always less than unity.

The greater the pressure ratio through the reciprocating compressor, then the greater will be the effect of the clearance volume since the clearance air will now expand through the greater volume before intake conditions are reached. The cylinder size and stroke being fixed, the effective swept volume will reduce as the pressure ratio increases and thus the volumetric efficiency reduces.

## Actual pV-diagram of reciprocating compressor

In practical compressors, the pressure during suction is less than the atmospheric and will not be constant. Similarly, during delivery process the pressure is greater than the reservoir pressure and also fluctuating.


## Multistage compression

## Disadvantages of single stage compression

If the delivery pressure is increased too far, certain disadvantages appear in the single stage reciprocating air compressors.

- The size of the cylinder will be too large, if high pressure air is to be delivered.
- Due to compression, there is a rise in temperature of the air. It is difficult to reject heat from the air in the small time available during compression.
- Sometimes, the temperature of the air at the end of compression is too large. It may heat up the cylinder head or burn the lubricating oils.
- Clearance air expansion will be more as the pressure ratio increases, i.e., effective swept volume decreases, and thereby reducing mass flow rate of air through the compressor.
- As the delivery pressure increases, delivery temperature increases. Any increase in temperature increases the power required to drive the compressor.
- If high pressure is to be delivered by a single stage compressor, then it will require heavy working parts in order to accommodate the high pressure ratio through the machine. This will also increase the balancing problem.


## Advantages of Multistage compression

- The air can be cooled at pressures intermediate between intake \& delivery pressures.
- The power required to drive a multistage machine is less than single stage machine for same delivery pressure and quantity of air discharged.
- Multistage compressors have better mechanical balance.
- The pressure range may be kept within desirable limits. This results in (i) Reduced losses due to air leakage (ii) Improved lubrication due to lower temperatures (iii) Improved volumetric efficiency.
- The cylinder in a single stage machine must be robust enough to withstand the delivery pressure. The down pressure cylinders of a multistage machine may be lighter in construction since the maximum pressure is low in that cylinders.


## Disadvantages of Multistage compression

- Initial cost is very high since it employs inter-coolers.


## Multistage Reciprocating air compressor

Process 5-1 represents the suction stroke of the low pressure stage. The air is compressed to an intermediate pressure of ' $p_{2}$ ' according to the law $p V^{n}=C$. The LP cylinder then discharges the air at ' $p_{2}$ ' to the inter cooler where the air is cooled at constant pressure to the initial temperature ' $T_{1}$ ' by circulating cold water. This is shown by ' $2-2$ '. The air from the inter cooler enters the HP cylinder where the pressure of the air is increased to final delivery temperature of ' $p_{3}$ '.


## Complete (or) Perfect Inter-cooling

If the temperature of the air leaving the inter cooler is equal to the original atmospheric air temperature, the inter-cooling is Complete or Perfect inter-cooling.

$$
\mathrm{T}_{2}^{\prime}=\mathrm{T}_{1}
$$

## Incomplete Inter-cooling

If the temperature of air leaving the inter-cooler is more than the original atmospheric temperature, the inter-cooling is known as Incomplete inter-cooling.

$$
\mathrm{T}_{2}^{\prime}>\mathrm{T}_{1}
$$

## Workdone in a Two stage Compressor

## Assumption

- The effect of clearance is neglected.
- There is no pressure drop in the inter-cooler.
- The compression in both the cylinders is polytropic.
- The suction \& delivery of air takes place at constant pressure.
(i) Incomplete Inter-cooling

Workdone in LP cylinder is given by,

$$
W_{L P}=\frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
$$

Workdone in HP cylinder is given by,

$$
W_{H P}=\frac{n}{n-1} p_{2} V_{2^{\prime}}\left[\left(\frac{p_{3}}{p_{2}}\right)^{(n-1) / n}-1\right]
$$

$\therefore$ Total workdone $=\mathrm{W}=\mathrm{W}_{\mathrm{LP}}+\mathrm{W}_{\mathrm{HP}}$

$$
\begin{aligned}
= & \frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]+ \\
& \frac{n}{n-1} p_{2} V_{2^{\prime}}\left[\left(\frac{p_{3}}{p_{2}}\right)^{(n-1) / n}-1\right]
\end{aligned}
$$

## (ii) Complete inter-cooling

In perfect inter-cooling, $\mathrm{T}_{2}{ }^{\prime}=\mathrm{T}_{1}$

$$
\begin{gathered}
\mathrm{p}_{1} \mathrm{~V}_{1}=\mathrm{p}_{2} \mathrm{~V}_{2}^{\prime} \\
W=\frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}+\left(\frac{p_{3}}{p_{2}}\right)^{(n-1) / n}-2\right]
\end{gathered}
$$

## Condition for minimum work

Compression with perfect inter-cooling gives minimum work.

$$
W=\frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}+\left(\frac{p_{3}}{p_{2}}\right)^{(n-1) / n}-2\right]
$$

- Here, $p_{1} \& p_{3}$ are constants.
- The only variable is ' $p_{2}$ '.

Therefore differentiate the above equation with respect to ' $p_{2}$ ' and equate to zero, for getting condition for minimum work.

$$
\begin{aligned}
& \text { i.e., } \frac{d W}{d p_{2}}=0 \\
& \frac{d}{d p_{2}}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}+\left(\frac{p_{3}}{p_{2}}\right)^{(n-1) / n}-2\right]=0
\end{aligned}
$$

Let, $a=\frac{n-1}{n}$

$$
\begin{aligned}
& \frac{d}{d p_{2}}\left[\left(\frac{p_{2}}{p_{1}}\right)^{a}+\left(\frac{p_{3}}{p_{2}}\right)^{a}-2\right]=0 \\
& \frac{1}{p_{1}^{a}} a\left(p_{2}\right)^{a-1}+p_{3}^{a}(-a)\left(p_{2}\right)^{-a-1}=0 \\
& a\left(p_{2}\right)^{a-1} p_{1}^{-a}-a p_{3}^{a}\left(p_{2}\right)^{-a-1}=0 \\
& a\left(p_{2}\right)^{a-1} p_{1}^{-a}=a p_{3}^{a}\left(p_{2}\right)^{-a-1} \\
&\left(p_{2}\right)^{a-1+a+1}=p_{3}^{a} p_{1}^{a} \\
& p_{2}^{2 a}=\left(p_{1} p_{3}\right)^{a} \\
& p_{2}^{2}=p_{1} p_{3} \\
& p_{2}=\sqrt{p_{1} p_{3}}
\end{aligned}
$$

$$
\text { (or) } \frac{p_{2}}{p_{1}}=\frac{p_{3}}{p_{2}}=---=\frac{p_{x+1}}{p_{x}}
$$

i.e., The pressure ratio for each stage is equal.
$x=$ number of stages
Minimum Workdone

$$
\begin{aligned}
W & =\frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}+\left(\frac{p_{2}}{p_{2}}\right)^{(n-1) / n}-2\right] \\
& =\frac{2 n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
\end{aligned}
$$

From, $\quad p_{2}=\sqrt{p_{1} p_{3}}$

$$
\begin{aligned}
& \qquad \begin{aligned}
\frac{p_{2}}{p_{1}} & =\frac{\sqrt{p_{1} p_{3}}}{p_{1}} \\
& =\sqrt{\frac{p_{3}}{p_{1}}} \\
W & =\frac{2 n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{3}}{p_{1}}\right)^{(n-1) / 2 n}-1\right] \\
\text { For 'x' stages, } & \frac{p_{2}}{p_{1}}=\left(\frac{p_{x+1}}{p_{1}}\right)^{1 / x} \\
W & =\frac{x n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{x+1}}{p_{1}}\right)^{(n-1) / x n}-1\right]
\end{aligned}
\end{aligned}
$$

## Rotary compressors

Whenever large quantity of air or gas is required at relatively low pressure, rotary compressors are employed.

## Classification

1. Displacement compressors

* Roots Blower
* Sliding vane compressors

2. Steady flow compressors

* Centrifugal compressors
* Axial flow compressors


## Displacement compressors

Displacement compressors are those compressors in which air is compressed by being tapped in the reduced space formed by two sets of engaging surfaces.

## Roots Blower

A roots blower compressor consists of two rotors with lobes rotating in an air tight casing which has inlet and outlet ports. They generally have two or three lobes.

The mechanical energy is provided to one of the rotor from some external source, while the other is gear driven from the first. As the rotors rotate, the air at atmospheric pressure is tapped in the pockets formed between the lobes and casing. The rotating motion of the lobes delivers the entrapped air into the receiver. Thus more and more flow of air at a higher pressure is delivered from the receiver.


Air in

## Back flow

When the rotating lobes uncover the exit port, some air (under high pressure) flows back into the pocket from the receiver. It is known as 'back flow process'. The air, which flows from the receiver to the pocket, gets mixed up with the entrapped air. The back flow of air continues, till the pressure in the pocket and receiver is equalised. Thus the pressure of air entrapped in the pocket is increased qt constant volume entirely by the back flow.

## Vane blower compressor

A vane blower compressor consists of disc rotating eccentrically in an air tight casing with inlet and outlet ports. The disc has number of slots (generally 4 to 8 ) containing vanes. When the rotor rotates the disc, the vanes are pressed against the casing due to centrifugal force, and form air tight pockets.


The mechanical energy is provided to the disc from some external source. As the disc rotates, the air is trapped in the pockets formed between the vanes and casing.

First the rotating motion of the vanes compresses the air. When the rotating vane uncovers exit port, some air (under high pressure) flows back in the pocket. Thus the pressure of the entrapped air in the pocket is increased by decreasing the volume and by back flow of air.

## Steady flow compressors

Steady flow compressors are those compressors in which compression occurs by transfer of kinetic energy from the rotor.

## Centrifugal Compressors

A centrifugal compressor consists of a rotor (impeller) to which a number of vanes are fitted systematically. The rotor rotates in an air tight volute casing with inlet and outlet points.

In the casing kinetic energy is converted into pressure energy. The mechanical energy is provided to the rotor from some external source. As the rotor rotates, it sucks air through its eye, increases its pressure due to centrifugal force and forces the air to flow over the diffuser. The pressure of air is further increased during its flow over the diffuser.


Axial flow compressors


An axial flow compressor consists of a number of rotating blades and fixed blades. The drum rotates inside an air tight casing to which the fixed blades are fixed. The mechanical energy is provided to the rotating shaft, which rotates the
drum. The air enters the left side of the compressor. As the drum rotates, the air flows through the alternately arranged stator and rotor and it gets compressed.

Comparison of Reciprocating \& Rotary compressors

| Reciprocating | Rotary |
| :---: | :---: |
| 1. Maximum delivery pressure is 1000 bar. | 10 bar |
| 2. Maximum FAD $=300 \mathrm{~m}^{3} / \mathrm{min}$ | $3000 \mathrm{~m}^{3} / \mathrm{min}$ |
| 3. Suitable for low discharge at very high pressure. | Suitable for high discharge at low Pressure. |
| 4. The speed of the compressor is low. | The speed is very high. |
| 5. The air supply is intermittent. | Air supply is continuous. |
| 6. For given discharge, size of the compressor is large. | Size is small. |
| 7. The balancing is major problem. | No balancing problem. |
| 8. Lubrication system is complicated. | Simple. |
| 9. The air delivered is less clean as it comes in contact with lubricating oil. | More clean as it does not come in contact with the lubricating oil. |

## PROBLEMS

1. Determine the power required for the compression of air at the rate of 0.6 $\mathrm{m}^{3} / \mathrm{s}$ from 1 bar and $20^{\circ} \mathrm{C}$ to 6 bar if the compression (i) is isentropic (ii) follows the law $\mathrm{pV}^{1.3}=\mathrm{C}$ and (iii) is isothermal.

## Given

Initial volume of air $\left(\mathrm{V}_{1}\right) \quad=0.6 \mathrm{~m}^{3} / \mathrm{s}$
Initial pressure of air $\left(\mathrm{p}_{1}\right)=1 \mathrm{bar}$
Initial temperature of air $\left(T_{1}\right)=20^{\circ} \mathrm{C}=293 \mathrm{~K}$
Final pressure of the air $\left(p_{2}\right)=6$ bar
Index of compression ( n ) $=1.3$
Required : Power (P)

## Solution

This is the compression with no clearance volume.
(i) Isentropic compression.

$$
\begin{aligned}
\mathrm{P} & =\mathrm{W} \text { if } \mathrm{V} \text { is in ' } \mathrm{m}^{3} / \mathrm{s}^{\prime} \text { or } \mathrm{m} \text { is in ' } \mathrm{kg} / \mathrm{s} \text { ' } \\
W & =\frac{\gamma}{\gamma-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(\gamma-1) / \gamma}-1\right] \\
& =\frac{1.4}{1.4-1} \times 1 \times 10^{5} \times 0.6\left[\left(\frac{6}{1}\right)^{(1.4-1) / 1.4}-1\right] \\
& =140387 \mathrm{~W} \text {---- Ans }
\end{aligned}
$$

(ii)

$$
\begin{aligned}
P & =\frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right] \\
& =\frac{1.3}{1.3-1} \times 1 \times 10^{5} \times 0.6\left[\left(\frac{6}{1}\right)^{(1.3-1) / 1.3}-1\right] \\
& =133140 \mathrm{~W}--- \text { Ans }
\end{aligned}
$$

(iii)

$$
\begin{aligned}
P & =p_{1} V_{1} \ln \left(\frac{V_{1}}{V_{2}}\right) \\
& =p_{1} V_{1} \ln \left(\frac{p_{2}}{p_{1}}\right) \\
& =1 \times 10^{5} \times 0.6 \ln \left(\frac{6}{1}\right) \\
& =107506 \mathbf{W} \text {--- Ans }
\end{aligned}
$$

2. A double acting compressor running at 210 rpm has a bore of $\mathbf{2 5} \mathbf{c m}$ and a stroke of 36 cm . The inlet air is at 0.95 bar and $40^{\circ} \mathrm{C}$. The volumetric efficiency of the compressor is $72 \%$. Determine the power required if the delivery is at 5 bar and the index of compression is 1.3. Also determine the delivery temperature and FAD in $\mathrm{m}^{3} / \mathrm{h}$ referred to 1 bar $20^{\circ} \mathrm{C}$.

## Given

Double acting compressor
Speed of the compressor ( N ) $=210 \mathrm{rpm}$
Bore (D)
Stroke (L) $=0.25 \mathrm{~m}$
0.36 m

Inlet pressure $\left(p_{1}\right) \quad=0.95$ bar
Inlet temperature $\left(\mathrm{T}_{1}\right) \quad=40^{\circ} \mathrm{C}=313 \mathrm{~K}$
Volumetric efficiency $\left(\eta_{v}\right) \quad=0.72$
Delivery pressure $\left(p_{2}\right)=5$ bar
Index of compression ( n ) $=1.3$
Free air pressure $\left(p_{f}\right) \quad=1$ bar
Free air temperature $\left(T_{f}\right) \quad=20^{\circ} \mathrm{C}=293 \mathrm{~K}$
Required: $\mathrm{P}, \mathrm{T}_{2}$ \& $\mathrm{V}_{\mathrm{f}}$ in $\mathrm{m}^{3} / \mathrm{h}$

## Solution

This is the compressor with clearance volume ( $\eta_{v}=0.72$ )


$$
P=\frac{n}{n-1} p_{1}\left(V_{1}-V_{4}\right)\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
$$

To find $\left(V_{1}-V_{4}\right)$

$$
\begin{aligned}
\eta_{v} & =\frac{V_{1}-V_{4}}{V_{1}-V_{3}} \\
\mathrm{~V}_{1}-\mathrm{V}_{3} & =2 \times \pi / 4 \times \mathrm{D}^{2} \times \mathrm{L} \times \mathrm{N} / 60 \rightarrow \text { for double acting compressor } \\
\mathrm{V}_{1}-\mathrm{V}_{4} & =\eta_{v}\left(2 \times \pi / 4 \times \mathrm{D}^{2} \times \mathrm{L} \times \mathrm{N} / 60\right) \\
& =0.72 \times 2 \times \pi / 4 \times 0.25^{2} \times 0.36 \times 210 / 60 \\
& =0.089 \mathrm{~m}^{3} / \mathrm{s} \\
P & =\frac{1.3}{1.3-1} \times 0.95 \times 10^{5} \times 0.089\left[\left(\frac{5}{0.95}\right)^{(1.3-1) / 1.3}-1\right] \\
& =17111.5 \mathbf{~ W} \text {--- Ans }
\end{aligned}
$$

Let $\mathrm{V}_{\mathrm{f}}=$ Free air delivered

$$
\begin{aligned}
& \therefore \quad \begin{array}{c}
0.95 \times 0.089 \\
313
\end{array} \\
& V_{f}=284.93 \mathrm{~m}^{3} / \mathrm{h} \cdots \text {---- Ans } \\
& T_{2} / T_{1}=\left[p_{2} / p_{1}\right]^{(n-1) / n} \\
& \mathrm{~T}_{2} / 313=[5 / 0.95]^{(1.3-1) / 1.3}
\end{aligned}
$$

$$
\mathrm{T}_{2}=459.2 \mathrm{~K}----\mathrm{Ans}
$$

3. A compressor running at 410 rpm has a bore of 9 cm and a stroke of 10 cm . Two cylinders are used. The clearance is $4 \%$ of stroke. If compression and expansion follows the law $\mathrm{pV}^{\gamma}=\mathrm{C}$, determine the power developed and the FAD in $\mathrm{m}^{3} / \mathrm{h}$. Inlet conditions are 0.96 bar and $40^{\circ} \mathrm{C}$ and delivery is at 6 bar. Assume effects other than clearance can be neglected in arriving at volumetric efficiency. Standard conditions for free air are 1 bar \& $\mathbf{2 0}^{\circ} \mathrm{C}$.

## Given

Two cylinder compressor.
Speed of the compressor $(N)=410 \mathrm{rpm}$

Bore (D)
Stroke (L)
Inlet pressure ( $\mathrm{p}_{1}$ )
Inlet temperature ( $\mathrm{T}_{1}$ )
Clearance

$$
=0.09 \mathrm{~m}
$$

$$
=0.1 \mathrm{~m}
$$

$$
=0.96 \mathrm{bar}
$$

$$
=40^{\circ} \mathrm{C}=313 \mathrm{~K}
$$

$$
=4 \% \text { of stroke } ; \mathrm{V}_{2}=0.04\left(\mathrm{~V}_{1}-\mathrm{V}_{3}\right)
$$

$$
\text { i.e., } k=0.04
$$

Delivery pressure $\left(p_{2}\right)=6$ bar
Index of compression ( n ) $=1.3$
Free air pressure $\left(\mathrm{p}_{\mathrm{f}}\right) \quad=1$ bar
Free air temperature $\left(T_{f}\right) \quad=20^{\circ} \mathrm{C}=293 \mathrm{~K}$
Required: $\mathrm{P}, \& \mathrm{~V}_{\mathrm{f}}$ in $\mathrm{m}^{3} / \mathrm{h}$

## Solution

This is the compressor with clearance volume ( $k=0.04$ )

$$
P=\frac{n}{n-1} p_{1}\left(V_{1}-V_{4}\right)\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
$$

To find $\left(V_{1}-V_{4}\right)$

$$
\eta_{v}=\frac{V_{1}-V_{4}}{V_{1}-V_{3}}=1+k-k\left(\frac{p_{2}}{p_{1}}\right)^{1 / n}
$$

$V_{1}-V_{3}=2 \times \pi / 4 \times D^{2} \times L \times N / 60 \rightarrow$ for two cylinder compressor


$$
\begin{aligned}
\mathrm{V}_{1}-\mathrm{V}_{4} & =\left(2 \times \pi / 4 \times \mathrm{D}^{2} \times \mathrm{L} \times \mathrm{N} / 60\right) \times\left[1+\mathrm{k}-\mathrm{k}\left(\mathrm{p}_{2} / \mathrm{p}_{1}\right)^{1 / n}\right] \\
& =\left(2 \times \pi / 4 \times 0.09^{2} \times 0.1 \times 410 / 60\right) \times\left(1+0.04-0.04(6 / 0.96)^{1 / 1.3}\right. \\
& =0.007618 \mathrm{~m}^{3} / \mathrm{s} \\
P & =\frac{1.3}{1.3-1} \times 0.96 \times 10^{5} \times 0.007618\left[\left(\frac{6}{0.96}\right)^{(1.3-1) / 1.3}-1\right] \\
& =1668.16 \mathrm{~W}--- \text { Ans }
\end{aligned}
$$

Let $\mathrm{V}_{\mathrm{f}}=$ Free air delivered

$$
\begin{aligned}
& \therefore \begin{array}{c}
\mathrm{p}_{1}\left(\mathrm{~V}_{1}-\mathrm{V}_{4}\right) \\
--------\quad \mathrm{T}_{1} \mathrm{~V}_{\mathrm{f}} \\
\mathrm{~T}_{\mathrm{f}}
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
& V_{f}=0.0068459 \mathrm{~m}^{3} / \mathrm{s} \\
& V_{f}=24.645 \mathrm{~m}^{3} / \mathrm{h} \cdots \text {---- Ans }
\end{aligned}
$$

4. A single stage single acting reciprocating air compressor has a bore of 200 mm and a stroke of 300 mm . It receives air at 1 bar and $20^{\circ} \mathrm{C}$ and delivers at 5.5 bar. If the compression follows the law $\mathrm{pV}^{1.3}=\mathrm{C}$ and clearance volume is $5 \%$ of the stroke volume, determine the power required to drive the compressor at 500 rpm .
Given
Speed of the compressor ( N ) $=500 \mathrm{rpm}$
Bore (D)

$$
\begin{aligned}
& =0.2 \mathrm{~m} \\
& =0.3 \mathrm{~m} \\
& =1 \mathrm{bar} \\
& =20^{\circ} \mathrm{C}=293 \mathrm{~K}
\end{aligned}
$$

Stroke (L)
Inlet pressure $\left(\mathrm{p}_{1}\right)$
Inlet temperature ( $\mathrm{T}_{1}$ )
Clearance volume $\left(V_{2}\right)=5 \%$ of stroke; $V_{2}=0.05\left(V_{1}-V_{3}\right)$
i.e., $k=0.05$

Delivery pressure $\left(p_{2}\right) \quad=5.5$ bar
Index of compression ( n ) $=1.3$
Required: P

## Solution

This is the compressor with clearance volume ( $k=0.05$ )

$$
P=\frac{n}{n-1} p_{1}\left(V_{1}-V_{4}\right)\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
$$

To find $\left(V_{1}-V_{4}\right)$

$$
\begin{aligned}
& \eta_{v}=\frac{V_{1}-V_{4}}{V_{1}-V_{3}}=1+k-k\left(\frac{p_{2}}{p_{1}}\right)^{1 / n} \\
& \mathrm{~V}_{1}-\mathrm{V}_{3}=\pi / 4 \times \mathrm{D}^{2} \times \mathrm{L} \times \mathrm{N} / 60 \rightarrow \text { for single acting compressor } \\
& 18
\end{aligned}
$$

$$
\begin{aligned}
V_{1}-V_{4} & =\left(\pi / 4 \times D^{2} \times L \times N / 60\right) \times\left(1+k-k\left(p_{2} / p_{1}\right)^{1 / n}\right. \\
& =\left(\pi / 4 \times 0.2^{2} \times 0.3 \times 500 / 60\right) \times\left(1+0.05-0.05(5.5 / 1)^{1 / 1.3}\right. \\
& =0.06788 \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$



$$
\begin{aligned}
P & =\frac{1.3}{1.3-1} \times 1 \times 10^{5} \times 0.06788\left[\left(\frac{5.5}{1}\right)^{(1.3-1) / 1.3}-1\right] \\
& =14178.34 \mathbf{W}--- \text { Ans }
\end{aligned}
$$

5. A single stage double acting compressor has a FAD of $14 \mathrm{~m}^{3} / \mathrm{min}$ measured at 1 bar \& $15^{\circ} \mathrm{C}$. The pressure \& temperature during compression are 0.95 bar $\& 32^{\circ} \mathrm{C}$. The delivery pressure is 7 bar and index of compression is 1.3. The clearance volume is $5 \%$ of the swept volume. Calculate indicated power and volumetric efficiency of the compressor.

## Given

Double acting compressor.
Free air delivered $\left(\mathrm{V}_{\mathrm{f}}\right) \quad=14 \mathrm{~m}^{3} / \mathrm{min}=0.2333 \mathrm{~m}^{3} / \mathrm{s}$
Inlet pressure $\left(p_{1}\right) \quad=0.95$ bar
Inlet temperature $\left(\mathrm{T}_{1}\right) \quad=32^{\circ} \mathrm{C}=305 \mathrm{~K}$
Clearance volume $\left(\mathrm{V}_{3}\right) \quad=5 \%$ of stroke ; $\mathrm{V}_{3}=0.05\left(\mathrm{~V}_{1}-\mathrm{V}_{3}\right)$
i.e., $k=0.05$

Delivery pressure $\left(\mathrm{p}_{2}\right) \quad=7 \mathrm{bar}$
Index of compression ( n ) $=1.3$
Free air pressure $\left(p_{f}\right) \quad=1$ bar
Free air temperature $\left(\mathrm{T}_{\mathrm{f}}\right) \quad=15^{\circ} \mathrm{C}=288 \mathrm{~K}$
Required : $\mathrm{P}, \& \eta_{\mathrm{v}}$

## Solution

This is the compressor with clearance volume ( $k=0.05$ )

$$
P=\frac{n}{n-1} p_{1}\left(V_{1}-V_{4}\right)\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
$$

To find $\left(V_{1}-V_{4}\right)$
Let $\mathrm{V}_{\mathrm{f}}=$ Free air delivered

$$
\begin{aligned}
& \frac{p_{1}\left(V_{1}-V_{4}\right)}{T_{1}}=---------\quad T_{f}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{V}_{1}-\mathrm{V}_{4}=0.266007 \mathrm{~m}^{3} / \mathrm{s} \\
& P=\frac{1.3}{1.3-1} \times 0.95 \times 10^{5} \times 0.26007\left[\left(\frac{7}{0.95}\right)^{(1.3-1) / 1.3}-1\right] \\
& \text { = 62683.7 W ---- Ans } \\
& \eta_{v}=1+0.05-0.05(7 / 0.95)^{1 / 1.3} \\
& =0.817 \text {---- Ans }
\end{aligned}
$$

Since we know FAD, we can calculate $\eta_{v}$ based on FAD.

$$
\therefore \quad \eta_{v}=\frac{V_{f}}{V_{1}----V_{3}}
$$

To find $V_{1}-V_{3}$

$$
\begin{aligned}
& \eta_{v} & =\frac{V_{1}-V_{4}}{V_{1}-V_{3}}=1+k-k\left(\frac{p_{2}}{p_{1}}\right)^{1 / n}=0.817 \\
& \therefore \quad V_{1}-V_{3} & =0.26007 / 0.817=0.3183 \mathrm{~m}^{3} / \mathrm{s} \\
& \therefore \quad \eta_{v} & =0.2333 / 0.3183=0.733-- \text { Ans }
\end{aligned}
$$

6. A single stage, single acting reciprocating air compressor has a bore and stroke of 150 mm . The clearance volume is $6 \%$ of swept volume. The speed is $150 \mathrm{rev} / \mathrm{min}$. Intake pressure is $100 \mathrm{kN} / \mathrm{m}^{2}$ and the delivery pressure is $600 \mathrm{kN} / \mathrm{m}^{2}$. The index of compression is 1.3 . Determine ( i ) the volume of air delivered/s at $600 \mathrm{kN} / \mathrm{m}^{2}$ and (ii) the power required by the compressor.
Given
Speed of the compressor $(\mathrm{N})=150 \mathrm{rpm}$

| Bore (D) | $=0.15 \mathrm{~m}$ |
| :---: | :---: |
| Stroke (L) | $=0.15 \mathrm{~m}$ |
| Inlet pressure ( $\mathrm{p}_{1}$ ) | $=100 \mathrm{kN} / \mathrm{m}^{2}=1 \mathrm{bar}$ |
| Clearance volume ( $\mathrm{V}_{3}$ ) i.e., | $\begin{aligned} & =6 \% \text { of stroke } ; V_{3}=0.06\left(V_{1}-V_{3}\right) \\ k & =0.06 \end{aligned}$ |
| Delivery pressure ( $\mathrm{p}_{2}$ ) | $=600 \mathrm{kN} / \mathrm{m}^{2}=6 \mathrm{bar}$ |
| Index of compression (n) | $=1.3$ |
| Required: Volume at $\mathrm{p}_{2}$ \& |  |

Solution

$$
\text { Volume at } p_{2}=V_{2}-V_{3}
$$



To find $V_{3}$

$$
\begin{aligned}
& k=\frac{V_{3}}{V_{1}------V_{3}} \\
& 0.06=----------------------------150 / 60 \\
& V_{3}=0.0003976 \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

To find $V_{2}$

$$
\begin{aligned}
\mathrm{p}_{1} \mathrm{~V}_{1}^{\mathrm{n}} & =\mathrm{p}_{2} \mathrm{~V}_{2}^{\mathrm{n}} \\
\mathrm{~V}_{1} / \mathrm{V}_{2} & =\left(\mathrm{p}_{2} / \mathrm{p}_{1}\right)^{1 / \mathrm{n}} \\
\mathrm{~V}_{1}-\mathrm{V}_{3} & =(\pi / 4) \times 0.15^{2} \times 0.15 \times 150 / 60 \\
& =0.006626 \mathrm{~m}^{3} / \mathrm{s} \\
\therefore \quad \mathrm{~V}_{1} & =0.006626+0.0003976 \\
& =0.0070236 \mathrm{~m}^{3} / \mathrm{s} \\
0.0070236 / \mathrm{V}_{2} & =[1 / 16]^{1 / 1.3} \\
\mathrm{~V}_{2} & =0.00177 \mathrm{~m}^{3} / \mathrm{s} \\
\therefore \quad \mathrm{~V}_{2}-\mathrm{V}_{3} & =0.00177-0.0003976
\end{aligned}
$$

$$
\begin{aligned}
& =0.0013727 \mathbf{m}^{3} / \mathrm{s} \\
P & =--- \text { Ans } \\
n-1 & p_{1}\left(V_{1}-V_{4}\right)\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
\end{aligned}
$$

To find $V_{1}-V_{4}$

$$
\begin{aligned}
\eta_{v} & =\frac{V_{1}-V_{4}}{V_{1}-V_{3}}=1+k-k\left(\frac{p_{2}}{p_{1}}\right)^{1 / n} \\
& =\frac{V_{1}-V_{4}}{0.006626}=1+0.06-0.06\left(\frac{6}{1}\right)^{1 / 1.3} \\
\mathrm{~V}_{1}-\mathrm{V}_{4} & =0.005446 \mathrm{~m}^{3} / \mathrm{s} \\
P & =\frac{1.3}{1.3-1} \times 1 \times 10^{5} \times 0.005446\left[\left(\frac{6}{1}\right)^{(1.3-1) / 1.3}-1\right] \\
& =1208.5 \mathrm{~W}----\mathrm{Ans}
\end{aligned}
$$

7. Air is to be compressed in a single stage reciprocating compressor from 1.013 bar and $15^{\circ} \mathrm{C}$ to 7 bar. Calculate the indicated power required for a free air delivery of $0.3 \mathrm{~m}^{3} / \mathrm{min}$, when the compression process is (a) Isentropic (b) Polytropic with $\mathbf{n}=1.25$.
Given
Intake pressure $\left(p_{1}\right)=1.013$ bar
Intake temperature $\left(\mathrm{T}_{1}\right)=15^{\circ} \mathrm{C}=288 \mathrm{~K}$
Delivery pressure $\left(p_{2}\right)=7$ bar
Free air delivered $\left(\mathrm{V}_{\mathrm{f}}\right)=0.3 \mathrm{~m}^{3} / \mathrm{min}=0.005 \mathrm{~m}^{3} / \mathrm{s}$
Required: $P$ if (a) $\mathrm{pV}^{\gamma}=C$ (b) $\mathrm{pV}^{1.25}=\mathrm{C}$

## Solution

(a)

$$
P=\frac{\gamma}{\gamma-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(\gamma-1) / \gamma}-1\right]
$$

$\mathrm{V}_{1}=\mathrm{V}_{\mathrm{f}}$ since reference conditions for free air is not given \& it is taken as intake condition.
(b)

$$
P=\frac{1.4}{1.4-1} \times 1.013 \times 10^{5} \times 0.005\left[\left(\frac{7}{1.013}\right)^{(1.25-1) / 1.25}-1\right]
$$

$$
=1306.9 \mathrm{~W} \text {---- Ans }
$$

$$
P=\frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
$$

$$
=\frac{1.25}{1.25-1} \times 1.013 \times 10^{5} \times 0.005\left[\left(\frac{7}{1.013}\right)^{(1.25-1) / 1.25}-1\right]
$$

$$
=1195.25 \mathrm{~W} \text {--- Ans }
$$

8. $2 \mathrm{~kg} / \mathrm{s}$ of air enter the LP cylinder of 2 stage compressor. The overall pressure ratio is 9:1. The air at inlet to the compressor is at $100 \mathbf{k P a}$ and $35^{\circ} \mathrm{C}$. The index of compression in each cylinder is 1.3 . Find the intercooler pressure for perfect inter-cooling. Also find the maximum power required for compression and percentage power saved over single stage compression.

Given
Two stage compressor, i.e., $x=2$

Mass flow rate of air (m)
Overall pressure ratio ( $\mathrm{p}_{\mathrm{N}+1} / \mathrm{p}_{1}$ )
Inlet pressure $\left(\mathrm{p}_{1}\right)$
Inlet temperature ( $\mathrm{T}_{1}$ )
Compression index ( n )

$$
\begin{aligned}
& =2 \\
& =2 \mathrm{~kg} / \mathrm{s} \\
& =9 \\
& =100 \mathrm{kPa}=1 \mathrm{bar} \\
& =35^{\circ} \mathrm{C}=308 \mathrm{~K} \\
& =1.3
\end{aligned}
$$

Required : $\mathrm{p}_{2}, \mathrm{P}_{\text {min }} \& \%$ save in power

## Solution

Inter-cooler pressure for perfect inter-cooling is given by,

$$
\begin{aligned}
& p_{2}
\end{aligned}=\left(p_{1} p_{3}\right)^{1 / 2} .
$$



Minimum power required $\left(P_{\text {min }}\right)$

$$
\begin{aligned}
P_{\min } & =\frac{x n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{x+1}}{p_{1}}\right)^{(n-1) / x n}-1\right] \\
& =\frac{x n}{n-1} \stackrel{*}{m} R T_{1}\left[\left(\frac{p_{x+1}}{p_{1}}\right)^{(n-1) / x n}-1\right] \\
& =\frac{2(1.3)}{1.3-1} \times 2 \times 287 \times 308\left[\left(\frac{9}{1}\right)^{(1.3-1) /(2 \times 1.3)}-1\right] \\
& =442132.04 \mathbf{~ W}--- \text { Ans }
\end{aligned}
$$

For single stage

$$
\begin{aligned}
P & =\frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right] \\
& =\frac{n}{n-1} \stackrel{*}{m} R T_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
\end{aligned}
$$

Here $p_{1}=1$ bar $\& p_{2}=9$ bar

$$
\begin{aligned}
& P=\frac{1.3}{1.3-1} \times 2 \times 287 \times 308\left[\left(\frac{9}{1}\right)^{(1.3-1) / 1.3}-1\right] \\
& =505923.02 \mathrm{~W} \\
& \therefore \quad \% \text { save }=\frac{505923.02--------------------------102}{505923.02} \times 100 \\
& =12.608 \text { \% ----- Ans }
\end{aligned}
$$

9. What is the percentage saving in work by compressing air in two stages to 6 bar instead of in one stage? Assume compression index of 1.3 and that the inter-cooling restores the air to its original temperature.

Given
Two stage air compressor, $x=2$
Delivery pressure $\left(p_{3}\right)=6$ bar
Compression index ( n ) $=1.3$
Perfect inter-cooling
Required : \% save in power
Solution
For two stage compressor
Take $\mathrm{p}_{1}=1$ bar


$$
\begin{aligned}
& =\frac{2(1.3)}{1.3-1} p_{1} V_{1}\left[\left(\frac{6}{1}\right)^{(1.3-1) /(2 \times 1.3)}-1\right] \\
& =1.9904 \mathrm{p}_{1} \mathrm{~V}_{1} \mathrm{~W}
\end{aligned}
$$

For single stage

$$
P=\frac{n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
$$

Here $p_{1}=1$ bar $\& p_{2}=6$ bar

$$
\begin{aligned}
& P=\frac{1.3}{1.3-1} p_{1} V_{1}\left[\left(\frac{6}{1}\right)^{(1.3-1) / 1.3}-1\right] \\
& =2.21899 \mathrm{p}_{1} \mathrm{~V}_{1} \mathrm{~W}
\end{aligned}
$$

$$
\begin{aligned}
& =10.3 \% \text {----- Ans }
\end{aligned}
$$

10. Air enters a single stage double acting air compressor at 100 kPa and $29^{\circ} \mathrm{C}$. The index of compression is 1.3. The compression ratio is $6: 1$. The speed of the compressor is 550 rpm . The volume rate measured at suction condition is $5 \mathrm{~m}^{3} / \mathrm{min}$. Find the motor power required if the mechanical efficiency is $90 \%$.
Given
Single stage double acting compressor.
```
Intake pressure (p
```



```
Compression ratio (p}\mp@subsup{p}{2}{}/\mp@subsup{p}{1}{})=
Index of compression (n) = 1.3
Volume flow rate of air ( }\mp@subsup{V}{1}{})=5\mp@subsup{\textrm{m}}{}{3}/\textrm{min}=5/60\mp@subsup{\textrm{m}}{}{3}/\textrm{s
Mechanical efficiency (( }\mp@subsup{\eta}{m}{})=0.
```

Required: P

## Solution

For single stage double acting compressor,

$$
P=\frac{2 n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
$$

Considering mechanical efficiency,

$$
\begin{aligned}
& P=\frac{1}{\eta_{m}} \frac{2 n}{n-1} p_{1} V_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right] \\
& =\left(\frac{1}{0.96}\right) \frac{2(1.3)}{1.3-1} \times 1 \times 10^{5} \times(5 / 60)\left[(6)^{(1.3-1) / 1.3}-1\right]
\end{aligned}
$$

$$
=41092.447 \mathrm{~W} \quad---\quad \text { Ans }
$$

11. Determine the size of cylinder dimensions of 4 cylinder double acting compressor required to compress $25 \mathrm{~m}^{3} / \mathrm{min}$ of air at 1 bar and $25^{\circ} \mathrm{C}$ upto a pressure of 15 bar with the following additional data:
Clearance volume = 5 \%
n ------------------- = 1.35
Length ------------ = $1.2 \times$ diameter
Rpm --------------- = 300
Mechanical efficiency $=0.8$
Given
4 cylinder double acting compressor with clearance volume.

| Inlet volume $\left(\mathrm{V}_{1}-\mathrm{V}_{4}\right)$ <br> Inlet pressure $\left(\mathrm{p}_{1}\right)$ | $=25 \mathrm{~m}^{3} / \mathrm{min}=25 / 60 \mathrm{~m}^{3} / \mathrm{s}$ |
| :---: | :---: |
|  | $=1 \mathrm{bar}$ |
| Delivery pressure ( $\mathrm{p}_{2}$ ) | = 15 bar |
| Clearance ovum $=5 \%$ i.e. $\mathrm{k}=0.05$ |  |
|  | $\mathrm{n}=1.35$ |
| Length of cylinder (L) | $=1.2 \mathrm{D}$ |
| Rpm ( N ) | $=300$ |
| Mechanical efficiency ( $\eta_{\mathrm{m}}$ ) | $=0.8$ |

Required: D \& L

## Solution

Stroke volume $=V_{1}-V_{3}=\pi / 4 D^{2} L$ N/60 $\quad \rightarrow$ for single cylinder single acting

$$
\begin{aligned}
& =2(\pi / 4) D^{2} L N / 60 \rightarrow \text { for single cylinder double acting } \\
& =2(\pi / 4) D^{2} L(N / 60) \times 4 \rightarrow \begin{array}{c}
\text { for four cylinder double } \\
\text { acting }
\end{array}
\end{aligned}
$$



Total volume $\left(\mathrm{V}_{1}\right)=2 \times(\pi / 4) \mathrm{D}^{2} \mathrm{~L}(\mathrm{~N} / 60) \times 4$

To find $V_{1}$
Clearance volume $\left(\mathrm{V}_{3}\right)=0.05\left(\mathrm{~V}_{1}-\mathrm{V}_{3}\right)$

$$
=0.05 \mathrm{~V}_{1}-0.05 \mathrm{~V}_{3}
$$

or $\quad \mathrm{V}_{3}+0.05 \mathrm{~V}_{3}=0.05 \mathrm{~V}_{1}$

$$
V_{1}=(1.05 / 0.05) V_{3}=21 V_{3}
$$

Substituting in (1),

$$
21 V_{3}-V_{3}=0.6145
$$

And

$$
V_{3}=0.030725 \mathrm{~m}^{3} / \mathrm{s}
$$

$$
V_{1}=0.645225 \mathrm{~m}^{3} / \mathrm{s}
$$

Substituting in (2),

$$
\begin{aligned}
0.645225 & =2 \times(\pi / 4) \times D^{2}(1.2 \mathrm{D}) \times(300 / 60) \times 4 \\
\mathrm{D} & =0.2577 \mathrm{~m}--- \text { Ans } \\
\therefore \quad \mathrm{L} & =1.2 \times 0.2577=0.30924 \mathrm{~m} \text {---- Ans }
\end{aligned}
$$

12. A single stage reciprocating air compressor receives air at $25 \mathrm{~m}^{3} / \mathrm{min}$ at 1 bar and discharges it at $\mathbf{1 5}$ bar. Assume the value of ' $n$ ' for compression as 1.35 , and volumetric efficiency as 0.75 . Determine (i) the theoretical power required (ii) the piston displacement per min and (iii) maximum air temperature.
Given
Single stage compressor with clearance volume.
Intake pressure $\left(p_{1}\right) \quad=1$ bar
Discharge pressure $\left(\mathrm{p}_{2}\right) \quad=15 \mathrm{bar}$
Index of compression (n) $\quad=1.35$
Volume flow rate of air $\left(V_{1}-V_{4}\right)=25 \mathrm{~m}^{3} / \mathrm{min}=25 / 60 \mathrm{~m}^{3} / \mathrm{s}$
Volumetric efficiency $\left(\eta_{\mathrm{m}}\right) \quad=0.75$
Required : (i) P (ii) Piston displacement/min (iii) Maximum air temperature

## Solution

(i) Theoretical power (P)

$$
\begin{aligned}
P & =\frac{n}{n-1} p_{1}\left(V_{1}-V_{4}\right)\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right] \\
& =\frac{1.35}{1.35-1} \times 1 \times 10^{5} \times(25 / 60)\left[\left(\frac{15}{1}\right)^{(1.35-1) / 1.35}-1\right] \\
& =163600.7 \mathbf{W}---- \text { Ans }
\end{aligned}
$$

(ii) Piston displacement = Stroke volume

$$
=(\pi 4) D^{2} L N / 60
$$

$$
\begin{aligned}
& =\mathrm{V}_{1}-\mathrm{V}_{3} \\
& \eta_{v}=\frac{V_{1}-V_{4}}{V_{1}-----V_{3}} \\
& 0.75=\begin{array}{c}
25 / 60 \\
V_{1}------- \\
V_{3}
\end{array} \\
& \mathrm{~V}_{1}-\mathrm{V}_{3}=0.5556 \mathrm{~m}^{3} / \mathrm{s} \\
& =33.334 \mathrm{~m}^{3} / \mathrm{min} \text {----- Ans }
\end{aligned}
$$

(iii) Maximum air temperature $\left(\mathrm{T}_{2}\right)$

We can write,

$$
\frac{T_{2}}{T_{1}}=\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}
$$

For finding out $T_{1}$, we have to assume mass value. But, it is better to assume inlet temperature. Therefore take $\mathrm{T}_{1}=30^{\circ} \mathrm{C}$.

$$
\begin{aligned}
\therefore \quad \frac{T_{2}}{(30+273)} & =\left(\frac{15}{1}\right)^{(1.35-1) / 1.35} \\
\mathbf{T}_{2} & =611.5 \mathrm{~K} \ldots-\mathrm{Ans}
\end{aligned}
$$

13. A single cylinder, single acting reciprocating air compressor with a bore of 12 cm and stroke of 16 cm runs at 410 rpm . At the beginning of compression, the pressure and temperature in the cylinder are 0.98 and $40^{\circ} \mathrm{C}$. The delivery pressure is 6 bar. The index of compression is 1.32 . the clearance is $6 \%$. Determine the volume of air delivered referred to 1 bar and $20^{\circ} \mathrm{C}$. What is compressor power required?
Given
Single cylinder compressor with clearance volume.
Speed of the compressor ( N ) = 410 rpm
Bore (D)

$$
\begin{aligned}
& =0.12 \mathrm{~m} \\
& =0.16 \mathrm{~m} \\
& =0.98 \mathrm{bar} \\
& =40^{\circ} \mathrm{C}=313 \mathrm{~K}
\end{aligned}
$$

Stroke (L)
Inlet pressure $\left(p_{1}\right)$
Inlet temperature ( $\mathrm{T}_{1}$ )
Clearance $\quad=6 \% ; \mathrm{V}_{2}=0.06\left(\mathrm{~V}_{1}-\mathrm{V}_{3}\right)$
i.e., $k=0.06$

Delivery pressure $\left(p_{2}\right)=6$ bar
Index of compression (n) $\quad=1.32$
Free air pressure $\left(\mathrm{p}_{\mathrm{f}}\right) \quad=1$ bar
Free air temperature $\left(\mathrm{T}_{\mathrm{f}}\right) \quad=20^{\circ} \mathrm{C}=293 \mathrm{~K}$
Required: $V_{f} \& P$

## Solution

This is the compressor with clearance volume ( $k=0.06$ )

$$
P=\frac{n}{n-1} p_{1}\left(V_{1}-V_{4}\right)\left[\left(\frac{p_{2}}{p_{1}}\right)^{(n-1) / n}-1\right]
$$



To find $\left(V_{1}-V_{4}\right)$

$$
\begin{aligned}
\eta_{v} & =\frac{V_{1}-V_{4}}{V_{1}-V_{3}}=1+k-k\left(\frac{p_{2}}{p_{1}}\right)^{1 / n} \\
\mathrm{~V}_{1}-\mathrm{V}_{3} & =\pi / 4 \times \mathrm{D}^{2} \times \mathrm{L} \times \mathrm{N} / 60 \\
\mathrm{~V}_{1}-\mathrm{V}_{4} & =\left(\pi / 4 \times \mathrm{D}^{2} \times \mathrm{L} \times \mathrm{N} / 60\right) \times\left[1+\mathrm{k}-\mathrm{k}\left(\mathrm{p}_{2} / \mathrm{p}_{1}\right)^{1 / n}\right] \\
& =\left(\pi / 4 \times 0.15^{2} \times 0.16 \times 410 / 60\right) \times\left(1+0.06-0.06(6 / 0.98)^{1 / 1.3}\right. \\
& =0.01017 \mathrm{~m}^{3} / \mathrm{s} \\
P & =\frac{1.32}{1.32-1} \times 0.98 \times 10^{5} \times 0.01017\left[\left(\frac{6}{0.98}\right)^{(1.32-1) / 1.32}-1\right] \\
& =\mathbf{2 2 6 7 . 6} \mathbf{~ W}--- \text { Ans }
\end{aligned}
$$

Let $V_{f}=$ Free air delivered

$$
\begin{aligned}
& \left.\therefore \quad \begin{array}{c}
0.98 \times 0.01017 \\
313
\end{array}\right)=-------------- \\
& V_{f}=0.0093297 \mathrm{~m}^{3} / \mathrm{s} \text {----- Ans }
\end{aligned}
$$

14. In a single stage single acting two stage reciprocating air compressor 4.5 kg of air per min is compressed from 1.013 bar and $15^{\circ} \mathrm{C}$ through a pressure ratio of $9: 1$. Both stages have the same pressure ratio and the law of compression and expansion in both stages is $\mathrm{pV}^{1.3}=\mathrm{C}$. If the intercooling is complete, calculate (i) indicated power and (ii) the cylinder swept volumes required. Assume that the clearance volumes of both stages are $5 \%$ of their respective swept volumes and that the compressor runs at 300 rpm.

Given

```
Type \(\quad=\) Two stage, single acting, complete inter-cooling
Mass flow rate of air \(=4.5 \mathrm{~kg} / \mathrm{min}\)
Inlet pressure \(\left(\mathrm{p}_{1}\right) \quad=1.013 \mathrm{bar}\)
Inlet temperature \(\left(T_{1}\right)=15^{\circ} \mathrm{C}\)
Pressure ratio \(\left(p_{3} / p_{1}\right)=9\)
Law of comp \& exp \(\quad=\mathrm{pV}^{1.3}=\mathrm{C}\)
Clearance ratio (k) \(=0.05\) (for both cylinders)
Speed (N) \(\quad=300 \mathrm{rpm}\)
```

Required: (i) P (ii) $\left(\mathrm{V}_{1}-\mathrm{V}_{3}\right),\left(\mathrm{V}_{2}-\mathrm{V}_{6}\right)$
Solution


V
(i) Indicated power (IP)

$$
\begin{aligned}
I P & =\frac{2 n}{n-1} p_{1} V_{a c t}\left[\left(\frac{p_{3}}{p_{1}}\right)^{(n-1) / 2 n}-1\right] \\
& =\frac{2 n}{n-1} \stackrel{*}{m} R T_{1}\left[\left(\frac{p_{3}}{p_{1}}\right)^{(n-1) / 2 n}-1\right] \\
& =\frac{2(1.3)}{1.3-1} \times(4.5 / 60) \times 287 \times 288\left[(9)^{(1.3-1) / 2(1.3)}-1\right] \\
& =15503.3 \mathbf{W}--- \text { Ans }
\end{aligned}
$$

(ii) Volumetric efficiency of I-Cylinder $\left(\eta_{v 1}\right)=1+k-k\left(p_{2} / p_{1}\right)=\frac{V_{1}--V_{4}}{V_{1}----V_{3}}$ Volumetric efficiency of II-Cylinder $\left(\eta_{\mathrm{v} 2}\right)=1+\mathrm{k}-\mathrm{k}\left(\mathrm{p}_{3} / \mathrm{p}_{2}\right)=\frac{\mathrm{V}_{2^{\prime}}-\mathrm{V}_{7}}{\mathrm{~V}_{2^{\prime}}--\mathrm{V}_{6}}$ For complete inter-cooling , the work will be minimum \& $p_{2} / p_{1}=\left[p_{3} / p_{1}\right]^{1 / 2}$

$$
\therefore \mathrm{p}_{2} / \mathrm{p}_{1}=\mathrm{p}_{3} / \mathrm{p}_{2}=[9]^{1 / 2}=3
$$

$$
\begin{aligned}
\eta_{v 1} & =1+k-k\left(p_{2} / p_{1}\right) \\
& =1+0.05-0.05[3]=0.9779 \\
\eta_{v 1} & =1+k-k\left(p_{3} / p_{2}\right) \\
& =1+0.05-0.05[3]=0.9779
\end{aligned}
$$

To find $\left(V_{1}-V_{4}\right)$

$$
\begin{aligned}
\mathrm{p}_{1}\left(\mathrm{~V}_{1}-\mathrm{V}_{4}\right) & =\mathrm{mR} \mathrm{~T}_{1} \\
1.013 \times 10^{5} \times\left(\mathrm{V}_{1}-\mathrm{V}_{4}\right) & =4.5 / 60 \times 287 \times 288 \\
\therefore \quad\left(\mathrm{~V}_{1}-\mathrm{V}_{4}\right) & =0.061196 \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

To find $\left(V_{2^{\prime}}-V_{7}\right)$

$$
\mathrm{p}_{2}\left(\mathrm{~V}_{2^{\prime}}-\mathrm{V}_{7}\right)=\mathrm{mR}_{2^{\prime}}
$$

$\mathrm{T}_{2^{\prime}}=\mathrm{T}_{1}=288 \mathrm{~K}$ for perfect intercooling

$$
\mathrm{p}_{2}=3 \times 1.013=3.039 \mathrm{bar}
$$

$3.039 \times 10^{5} \times\left(V_{2^{\prime}}-V_{7}\right)=4.5 / 60 \times 287 \times 288$

$$
\therefore\left(\mathrm{V}_{2^{\prime}}-\mathrm{V}_{7}\right)=0.020399 \mathrm{~m}^{3} / \mathrm{s}
$$

$$
\therefore \quad 0.9779=0.061196 / \mathrm{V}_{1}-\mathrm{V}_{3}
$$

$$
V_{1}-V_{3}=0.062578 \mathrm{~m}^{3} / \mathrm{s}
$$

$$
=0.062578 \times 60 / \mathrm{N}
$$

$$
=0.062578 \times 60 / 300=0.0125 \mathbf{m}^{3}---- \text { Ans }
$$

and

$$
\begin{aligned}
0.9779 & =0.020399 / \mathrm{V}_{2^{\prime}}-\mathrm{V}_{6} \\
\mathrm{~V}_{1}-\mathrm{V}_{3} & =0.02086 \mathrm{~m}^{3} / \mathrm{s} \\
& =0.02086 \times 60 / \mathrm{N} \\
& =0.02086 \times 60 / 300=\mathbf{0 . 0 0 4 1 7 2} \mathbf{m}^{\mathbf{3}} \text {---. Ans }
\end{aligned}
$$

## UNIT-V

## REFRIGERATION AND AIR CONDITIONING

## Refrigeration

It is the process of maintaining the temperature of the system below the temperature of the surroundings.

## Vapour Compression Refrigerator



The major difference in theory and treatment of vapour refrigeration system as compared to the air refrigeration system is that, the vapour alternately undergoes a change of phase from vapour to liquid and liquid to vapour during the completion of cycle. The latent heat of vapourisation is utilized for carrying heat from the refrigerator which is quite high compared with the air cycle, which depends only upon the sensible heat of air.

The substances used do not leave the plant are circulated through the system alternately after condensing and re-evaporating. During evaporation, it absorbs its latent heat and in condensing it gives out its latent heat, therefore this machine is also called "Latent heat pump".

Main components:

- Evaporator
- Compressor
- Condenser
- Expansion device (Throttle valve (or) Control valve)

The arrangement of the components of vapour compression refrigeration system is shown in fig.

The refrigerant vapour is compressed by means of compressor to a pressure at which temperature obtained at the end of compression will be more than atmospheric temperature, so that at this temperature it will reject heat to the atmosphere and will then get condensed. This condensate is then allowed to pass through the heat exchanger and filter to expansion device. The refrigerant from condenser is cooled in the heat exchanger by means of vapour refrigerant from evaporator. The function of filter is to remove the moisture from the refrigerant in order to prevent the freezing of air in the expansion device. In the expansion device the pressure and temperature are lowered. The pressure of the refrigerant when it leaves the expansion device is maintained above the atmospheric pressure, whereas the temperature of refrigerant will be corresponding to the saturation temperature. The low temperature, low pressure refrigerant is allowed to pass through Flash chamber to the evaporator. Flash chamber separates vapour from the liquid + vapour mixture and allows pure liquid refrigerant to evaporator which increases the COP. In the evaporator, the refrigerant absorbs latent heat from the substance to be cooled. From the evaporator the refrigerant is taken to the compressor through the Accumulator and heat exchanger. Accumulator separates liquid refrigerant from vapour + liquid mixture and liquid refrigerant is taken back to evaporator. Thus the cycle is completed.

## Control systems in Vapour compression refrigeration system

1. Capillary tube
2. Thermo static expansion valve

Capillary tube is used universally as a refrigerant control device. Its resistance to flow permits the capillary to be used as a pressure reducing device to meter the flow of refrigerant given to evaporator. This device is only used for small capacity units like domestic refrigerators, water coolers and small commercial freezers. It is a small diameter tube connected between condenser and evaporator. The required pressure drop is caused due to heavy frictional resistance offered by a small diameter tube.

As the load increases in summer, the tube supplies more quantity of flow as an effect of increased condenser pressure. Similarly when the load decreases in winter, the flow through the tube decreases as an effect of decreased condenser pressure. The capillary is a self compensating device.

Thermo static expansion valve controls the flow of refrigerant through the evaporator.

## Effect of Sub-cooling \& Superheating



3' $-3 \rightarrow$ Sub-cooling
$\rightarrow$ Increases refrigeration effect
$\rightarrow$ No change in compressor work
$\rightarrow$ Therefore increases the COP
1' $-1 \rightarrow$ Superheating
$\rightarrow$ Increases the refrigeration effect
$\rightarrow$ Increases the compressor work
$\rightarrow$ But increase in RE < increase in W
$\rightarrow$ Therefore it decreases COP

T-s \& p-h diagrams for various conditions
Refrigerant is dry at the end of compression with no sub cooling



1-2 $\rightarrow$ Compressor (Isentropic compression)
2-3 $\rightarrow$ Condenser (Constant pressure condensation)
3-4 $\rightarrow$ Expansion device (Isenthalpic expansion)
4-1 $\rightarrow$ Evaporator (Constant pressure evaporation)
Refrigerant is dry at the beginning of compression with no sub cooling


Refrigerant is superheated (superheating) at the beginning of compression with no subcooling

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## Wet compression \& Dry compression

If the condition of vapour after compression is wet, the compression is called 'Wet compression'.

- Wet compression damages the valves and cylinder head.
- Wet compression wash out the lubricating oil from the walls of the cylinder, thus accelerating the wear.
If the condition of vapour before compression is dry, the compression is called 'Dry compression'.


## Definitions

## Refrigeration effect

The cooling effect produced by removal of heat from cold body is called 'refrigeration effect'.

Refrigeration effect $=m_{f}\left(h_{1}-h_{4}\right)$

## Coefficient of performance (COP)

$$
\begin{aligned}
& \text { Refrigeration effect } \\
& \text { COP }=\text {------------------------- } \\
& =\frac{h_{1}-h_{4}}{h_{1}---h_{2}} \rightarrow \text { Vapour compression refrigerator } \\
& =\frac{T_{2}}{T_{1}---T_{2}} \rightarrow \text { Carnot refrigerator } \\
& \text { Actual heat extraction } \\
& (\mathrm{COP})_{\text {act }}=\text {-------------------------- }
\end{aligned}
$$

$$
(\mathrm{COP})_{\text {rel }}=\frac{(\mathrm{COP})_{\text {act }}}{(\mathrm{COP})_{\text {the }}}
$$

Note: The COP found out from refrigeration chart or table is called (COP) $)_{\text {the }}$.
Capacity of the refrigerator
It is expressed in 'Ton of refrigeration'. It is defined as the rate of heat added to 1 ton of ice at $0^{\circ} \mathrm{C}$ to convert it into water at $0^{\circ} \mathrm{C}$ in 24 hour.

1 Ton of Refrigeration (TR) $=210 \mathrm{~kJ} / \mathrm{min}=3.5 \mathrm{~kJ} / \mathrm{s}$
1 Tonne of Refrigeration (TR) $=3.87 \mathrm{~kJ} / \mathrm{s}$
Power required to drive the compressor

$$
\mathrm{P}=\mathrm{m}_{\mathrm{f}}\left(\mathrm{~h}_{2}-\mathrm{h}_{1}\right)
$$

## Applications

| Refrigeration <br> application | Short descriptions | Typical <br> refrigerants used |
| :--- | :--- | :--- |
| Domestic <br> refrigeration | Appliances used for keeping food in dwelling <br> units | R-600a, R-134a |
| Commercial <br> refrigeration | Holding and displaying frozen and fresh food in <br> retail outlets | R-134a, R-404A, <br> R-507 |
| Food processing <br> and cold storage | Equipment to preserve, process and store food <br> from its source to the wholesale distribution point | R-134a, R-407C, <br> R-410A, R-507 |
| Industrial <br> refrigeration | Large equipment, typically 25 kW to 30 MW, <br> used for chemical processing, cold storage, food <br> processing and district heating and cooling | R-134a, R-404A, <br> R-507, R-717 |
| Transport <br> refrigeration | Equipment to preserve and store goods, primarily <br> foodstuffs, during transport by road, rail, air and <br> sea | R-134a, R-407C, <br> R-410A |
| Electronic cooling | Low-temperature cooling of CMOS circuitry and <br> other components in large computers and <br> servers | R-134a, R-404A, <br> $R-507$ |
| Medical <br> refrigeration | R-134a, R-404A, |  |
| Cryogenic <br> refrigeration | R-507 |  |

## Unit of refrigeration

Domestic and commercial refrigerators may be rated in $\mathrm{kJ} / \mathrm{s}$, or Btu/h of cooling. Commercial refrigerators in the US are mostly rated in tons of refrigeration, but elsewhere in kW . One ton of refrigeration capacity can freeze one short ton of water at 0 ${ }^{\circ} \mathrm{C}\left(32^{\circ} \mathrm{F}\right)$ in 24 hours. Based on that:

Latent heat of ice (i.e., heat of fusion) $=333.55 \mathrm{~kJ} / \mathrm{kg} \approx 144$ Btu/lb
One short ton $=2000 \mathrm{lb}$
Heat extracted $=(2000)(144) / 24 \mathrm{hr}=288000 \mathrm{Btu} / 24 \mathrm{hr}=12000 \mathrm{Btu} / \mathrm{hr}=200 \mathrm{Btu} / \mathrm{min}$
1 ton refrigeration $=200 \mathrm{Btu} / \mathrm{min}=3.517 \mathrm{~kJ} / \mathrm{s}=3.517 \mathrm{~kW}$
A much less common definition is: 1 tonne of refrigeration is the rate of heat removal required to freeze a metric ton (i.e., 1000 kg ) of water at $0^{\circ} \mathrm{C}$ in 24 hours. Based on the heat of fusion being $333.55 \mathrm{~kJ} / \mathrm{kg}$, 1 tonne of refrigeration $=13,898 \mathrm{~kJ} / \mathrm{h}=3.861 \mathrm{~kW}$. As can be seen, 1 tonne of refrigeration is $10 \%$ larger than 1 ton of refrigeration.

Most residential air conditioning units range in capacity from about 1 to 5 tons of refrigeration.

## Properties of good refrigerant

- Low boiling point.
- High critical temperature.
- High latent heat of evaporation.
- Low specific heat of liquid.
- Low specific volume of vapour.
- Non-corrosive to metal.
- Non-flammable and non-explosive.
- Non-toxic.
- Easy to liquefy at moderate pressure \& temperature.
- Low cost.


## Vapour Absorption Refrigeration System

The major drawback of the vapour compression refrigeration system is that it requires a compressor to compress large volume of refrigerant vapour which requires large mechanical power for its operation. If some methods are used to reduce this volume before compression, there would be considerable reduction in weight of the system and power requirement for its operation.

Some liquids have great affinity of absorbing large quantities of certain vapours and reducing them to liquid state, this reduces the total volume significantly.

The absorption system differs fundamentally from vapour compression system only in the method employed for compressing the refrigerant. In the absorption system, the compressor is replaced by "an absorber, a generator and a pump".

## Ammonia Absorption system

The basic components are (i) Generator (ii) Analyser (iii) Rectifier (iv) Condenser (v) Heat exchanger (vi) Receiver (vii) Expansion valve (viii) Evaporator (ix) Absorber (x) Pump etc.

The low pressure refrigerant vapour from the evaporator is absorbed by the weak solution of the refrigerant in water which is sprayed in the absorber. Absorption of ammonia lowers the pressure in the absorber, which in turn draws more ammonia vapour from the evaporator. Usually some form of cooling is employed in the absorber to remove the heat of solution evolved there. (during absorption of vapour by water spray evolves heat). The cooling of hot ammonia solution is necessary to increase its absorption capacity, because at high temperature, water absorbs less ammonia vapour. The strong ammonia solution thus formed is then pumped into the generator. The pump increases the pressure of the solution about 10 bar. The strong solution of ammonia is heated by some external means (steam or gas), and in the heating process, the refrigerant vapour is driven out of solution and the vapour is allowed to enter the condenser through Rectifier. The rectifier removes the water particles from the vapour. If there is water with ammonia vapour, this water will freeze in the expansion device and will affect the performance of the system. The dry ammonia vapour in the condenser is converted into liquid ammonia. The weak solution of ammonia left in the generator after ammonia has driven off, is first throttled to a low pressure by an expansion valve and then returned to the absorber through heat exchanger. In the heat exchanger, the strong solution from the absorber is heated by weak solution from generator. The high pressure liquid ammonia is passed through a throttle valve where the pressure and temperature of the liquid ammonia reduced and is passed to the evaporator. In the evaporator, liquid ammonia absorbs latent heat from the space (or) substance to be cooled and dry ammonia vapour coming out of the evaporator is allowed to mix with the weak solution of ammonia sprayed in the absorber. This completes the cycle.


## Lithium-Bromide Absorption Refrigeration system

In this system the water itself works as a refrigerant and $\mathrm{Li}-\mathrm{Br}$ salt solution as an absorber.

For air conditioning in which refrigeration temperatures below $0^{\circ} \mathrm{C}$ are not required, an absorption system of this type has been successfully developed and has achieved great commercial success.

The $\mathrm{Li}-\mathrm{Br}$ solution has a strong affinity for water vapour because of its very low vapour pressure, so that if water and $\mathrm{Li}-\mathrm{Br}$ solution are placed adjacent to each other in a closed evacuated system, the water will evaporate.

The arrangement of this system is shown in fig.


In the evaporator, the water will evaporate absorbing its latent heat from the remaining water and lowering its temperature to $2^{\circ} \mathrm{C}$ as the pressure in the evaporator is 5.5 cm of Hg . The cooled water in evaporator is used to cool the water used for air conditioning purposes. This formed water vapour will be absorbed by the strong $\mathrm{Li}-\mathrm{Br}$ salt solution which is sprayed in the absorber, maintaining very low pressure in the evaporator.

The water + Li-Br solution is pumped to generator where it is heated using steam and part of water is removed in the form of vapour and makes $\mathrm{Li}-\mathrm{Br}$ solution strong. This strong solution is again passed into the absorber through heat exchanger. The generated water vapour from the generator is further passed into the condenser where it is condensed by cold water supplied externally. This condensed water is again passed into the evaporator to compensate the water which was evaporated in the evaporator and it competes the cycle.

Refrigerants for Vapour absorption system

1. Ammonia
2. Water

Properties of ideal refrigerant for Vapour absorption system

1. High critical temperature.
2. Large latent heat of evaporation.
3. Low specific heat.
4. Stability in complete cycle.

Different types of absorbents

1. Water
2. Lithium bromide

Properties of ideal absorbent

1. It should remain in liquid condition under operating conditions.
2. It should have greater affinity for the refrigerant.
3. Heat liberated should be minimum when the refrigerant is absorbed by absorbent.
4. It should Kave high boiling point.
5. It should have chemical stability.

## PROBLEMS

1. An ammonia refrigerator works on Vapour compression cycle. The temperature range in the compressor is $25^{\circ} \mathrm{C}$ to $-15^{\circ} \mathrm{C}$. The vapour is dry saturated at the end of compression and an expansion valve is used. Find COP. Take the following properties for ammonia.

| Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Specific Enthalpy <br> $(\mathrm{kJ} / \mathrm{kg})$ |  | Specific entropy <br> $(\mathrm{kJ} / \mathrm{kgK})$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Liquid | Vapour | Liquid | Vapour |
|  | 100.04 | 1319.22 | 0.3473 | 4.4852 |
| -15 | -54.56 | 1304.99 | -2.1338 | 5.0585 |

Given
Temperature at the inlet of the compressor $\left(\mathrm{T}_{1}\right)=-15^{\circ} \mathrm{C}$
Temperature at the outlet of the compressor $\left(\mathrm{T}_{2}\right)=25^{\circ} \mathrm{C}$
Vapour is dry at the end of the compression
Required: COP

## Solution

$$
1-2 \rightarrow \text { Compressor }
$$

2-3 $\rightarrow$ Condenser
3-4 $\rightarrow$ Expansion valve
4-1 $\rightarrow$ Evaporator



$$
\mathrm{COP}=\frac{\mathrm{h}_{1}-\mathrm{h}_{4}}{--------}
$$

$h_{1}=$ specific enthalpy of refrigerant at compressor inlet

$$
=\mathrm{h}_{\mathrm{f} 1}+\mathrm{x}_{1} \mathrm{~h}_{\mathrm{fg} 1}=\mathrm{h}_{\mathrm{f} 1}+\mathrm{x}_{1}\left(\mathrm{~h}_{\mathrm{g}}-\mathrm{h}_{\mathrm{f}}\right)_{1}
$$

$$
\begin{aligned}
\mathrm{h}_{\mathrm{f} 1} & =\text { specific enthalpy of liquid at }-15^{\circ} \mathrm{C} \\
& =-54.56 \mathrm{~kJ} / \mathrm{kg} \\
\mathrm{~h}_{\mathrm{g} 1} & =\text { specific enthalpy of vapour at } 25^{\circ} \mathrm{C} \\
& =1304.99 \mathrm{~kJ} / \mathrm{kg}
\end{aligned}
$$

To find ' $x$ '
$1-2 \rightarrow$ Isentropic process

$$
\begin{aligned}
\mathrm{s}_{1} & =\mathrm{s}_{2} \\
\mathrm{~s}_{2} & =\text { specific entropy at compressor outlet } \\
& =\text { specific entropy of vapour at } 25^{\circ} \mathrm{C} \\
& =4.4852 \mathrm{~kJ} / \mathrm{kgK} \\
\mathrm{~s}_{1} & =\text { specific entropy at compressor inlet } \\
& =\text { specific entropy of wet vapour at }-15^{\circ} \mathrm{C} \\
& =\mathrm{s}_{\mathrm{f} 1}+\mathrm{x}_{1} \mathrm{~s}_{\mathrm{fg} 1} \\
& =\mathrm{s}_{\mathrm{f} 1}+\mathrm{x}_{1}\left(\mathrm{~s}_{\mathrm{g}}-\mathrm{s}_{\mathrm{f}}\right)_{1} \\
\mathrm{~s}_{\mathrm{f} 1} & =\mathrm{specific} \mathrm{entropy} \mathrm{of} \mathrm{liquid} \mathrm{at}-15^{\circ} \mathrm{C} \\
& =-2.1338 \mathrm{~kJ} / \mathrm{kgK} \\
\mathrm{~s}_{\mathrm{g}} & =\mathrm{specific} \mathrm{entropy} \text { of vapour at }-15^{\circ} \mathrm{C} \\
& =5.0585 \mathrm{~kJ} / \mathrm{kgK} \\
\therefore \quad 4.4858 & =-2.1338+\mathrm{x}_{1}(5.0585-(-2.1338)) \\
\mathrm{x} & =0.92 \\
\mathrm{~h}_{1} & =-54.56+0.92(1304.99-(-54.56)) \\
& =1196.226 \mathrm{~kJ} / \mathrm{kg}
\end{aligned}
$$

$\mathrm{h}_{2}=$ specific enthalpy at compressor outlet
$=$ specific enthalpy of vapour at $25^{\circ} \mathrm{C}$
$=1319.22 \mathrm{~kJ} / \mathrm{kg}$
$h_{3}=$ specific enthalpy at condenser outlet
$=$ specific enthalpy of liquid at $25^{\circ} \mathrm{C}$
$=100.04 \mathrm{~kJ} / \mathrm{kg}$

$$
\therefore \quad \text { COP }=\frac{1196.226----------------------100.04}{1319.22-1196.226}=8.9 \quad \text { Ans }
$$

2. A vapour compression refrigerator uses methyl chloride and operates between temperature limits of $-10^{\circ} \mathrm{C}$ and $45^{\circ} \mathrm{C}$. At entry to the compressor, the refrigerant is dry saturated and after compression it acquired a temperature of $60^{\circ} \mathrm{C}$. Find the COP of the refrigerator. Take the following properties :

| Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Specific Enthalpy <br> $(\mathrm{kJ} / \mathrm{kg})$ |  | Specific entropy <br> $(\mathrm{kJ} / \mathrm{kgK})$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Liquid | Vapour | Liquid | Vapour |
|  | 133.0 | 483.6 | 0.485 | 1.587 |
| -10 | 45.4 | 460.7 | 0.183 | 1.637 |

Given
Temperature at the inlet of the compressor $\left(\mathrm{T}_{1}\right)=-10^{\circ} \mathrm{C}$
Temperature at the outlet of the compressor $\left(\mathrm{T}_{2}\right)=60^{\circ} \mathrm{C}$
Vapour is dry at the beginning of the compression.
Required: COP

## Solution

1-2 $\rightarrow$ Compressor
2-3 $\rightarrow$ Condenser
3-4 $\rightarrow$ Expansion valve
4-1 $\rightarrow$ Evaporator



$$
\mathrm{COP}=\frac{\mathrm{h}_{1}-\mathrm{h}_{4}}{\mathrm{~h}_{2}---\mathrm{h}_{1}}
$$

$h_{1}=$ specific enthalpy of refrigerant at compressor inlet

$$
\begin{aligned}
& =\text { specific enthalpy of vapour at }-10^{\circ} \mathrm{C} \\
& =460.7 \mathrm{~kJ} / \mathrm{kg} \\
\mathrm{~h}_{2} & =\text { specific enthalpy at compressor outlet } \\
& =\text { specific enthalpy of super heated vapour at } 60^{\circ} \mathrm{C} \\
& =\mathrm{h}_{\mathrm{g} 2^{2}}+\mathrm{C}_{\mathrm{p}}\left(\mathrm{~T}_{\text {sup }}-\mathrm{t}_{\mathrm{s}}\right) \\
\mathrm{h}_{\mathrm{g} 2^{\prime}} & =\text { specific enthalpy of saturated vapour at } 45^{\circ} \mathrm{C} \\
& =483.6 \mathrm{~kJ} / \mathrm{kg} \\
\mathrm{C}_{\mathrm{p}} & =\text { specific heat of refrigerant }
\end{aligned}
$$

To find $\mathrm{C}_{\mathrm{g}}$
$1-2 \rightarrow$ Isentropic process

$$
\begin{aligned}
& \mathrm{S}_{1}=\mathrm{s}_{2} \\
& \mathrm{~s}_{2}=\text { specific entropy at compressor outlet } \\
& =\text { specific entropy of superheated vapour at } 60^{\circ} \mathrm{C} \\
& =\mathrm{s}_{\mathrm{g} 2^{\prime}}+\mathrm{C}_{\mathrm{p}} \ln \left[\mathrm{~T}_{\text {sup }} / \mathrm{t}_{\mathrm{s}}\right] \\
& \mathrm{s}_{1}=\text { specific entropy at compressor inlet } \\
& =\text { specific entropy of saturated vapour at }-10^{\circ} \mathrm{C} \\
& =1.637 \mathrm{~kJ} / \mathrm{kgK} \\
& =\mathrm{s}_{\mathrm{f} 1}+\mathrm{X}_{1}\left(\mathrm{~s}_{\mathrm{g}}-\mathrm{s}_{\mathrm{f}}\right)_{1} \\
& \mathrm{~s}_{\mathrm{f} 1}=\text { specific entropy of liquid at }-10^{\circ} \mathrm{C} \\
& =-2.1338 \mathrm{~kJ} / \mathrm{kgK} \\
& \mathrm{~s}_{\mathrm{g}}=\text { specific entropy of vapour at }-10^{\circ} \mathrm{C} \\
& =5.0585 \mathrm{~kJ} / \mathrm{kgK} \\
& \therefore \quad 4.4858=-2.1338+\mathrm{x}_{1}(5.0585-(-2.1338)) \\
& X_{1}=0.92 \\
& \therefore \quad \text { COP }=\frac{1196.226-100.04}{1319.22---------------1196.226}=8.9 \quad--- \text { Ans }
\end{aligned}
$$

## PSYCHROMETRY

Atmospheric air always contains water vapour. The water vapour content in air plays an important role in comfort air conditioning. The partial pressure of water vapour in atmospheric air is very low and the vapour exists either in superheated or saturated.

The science which deals with the study of the behaviour of air and water vapour mixture is known as Psychrometry. The properties of air and water vapour mixture are known as psychrometric properties.
Dry air : Air with no water vapour is called 'dry air'.

$$
\mathrm{O}_{2}=21 \% ; \quad \mathrm{N}_{2}=78.1 \% ; \quad \mathrm{CO}_{2}=0.03 \%
$$

Since dry air is never found, it always may contain some water vapour.
Moisture : The water vapour present in the air is known as moisture.
Moist air : It is a mixture of dry air and water vapour.
Unsaturated and Saturated air : The moist air in which the water vapour exists in superheated state, is known as Unsaturated air and such air will be invisible

If the water vapour is added to dry air or unsaturated air, a limit will be reached when the air will be saturated and can hold no more water vapour. Such air will also be invisible and the air is called 'saturated air'. But if more water is added, the drops of water may remain in suspension and make air misty or foggy. Thus such drops are the condensed particles of water vapour. This will happen only beyond saturation limit.

According to the Dalton's law of partial pressures,

$$
p_{b}=p_{a}+p_{v}
$$

$\mathrm{p}_{\mathrm{b}}=$ Barometric pressure
$\mathrm{p}_{\mathrm{a}}=$ Partial pressure of dry air
$p_{v}=$ Partial pressure of water vapour
Dry Bulb Temperature (DBT) : The temperature of the air measured by ordinary thermometer whose bulb is dry, is known as DBT of the air.
Wet Bulb Temperature (WBT) : The temperature of the air measured by a thermometer when its bulb is covered with wet cloth and is exposed to a current of air is known as WBT of air.

## Wet bulb depression = DBT - WBT

Dew Point Temperature (DPT) : It is the temperature of air at which water vapour in the air starts condensing when the is cooled. Thus this temperature will corresponds to the saturation temperature at partial pressure of water vapour. DPT can be obtained from steam table at $p_{v}$.

Dew point depression = DBT - DPT
Note:
For saturated air, $\quad \mathrm{DBT}=\mathrm{WBT}=\mathrm{DPT}$

## Specific humidity (W)

It is the mass of water vapour present with one kg of dry air.

$$
W=0.622 \frac{p_{v}}{p_{b}-p_{v}} \mathrm{~kg} / \mathrm{kg} \text { dry air }
$$

Also,

$$
\begin{aligned}
W & =\frac{C_{p a}\left(T_{w}-T_{d}\right)+W_{w} h_{f g W}}{h_{g d}-h_{f w}}=\frac{\rho_{v}}{\rho_{a}}=\frac{m_{v}}{m_{a}} \\
W_{w} & =0.622 \frac{p_{s w}}{p_{b}-p_{s w}}
\end{aligned}
$$

$\mathrm{p}_{\mathrm{sw}}=$ Saturation pressure at WBT
$\mathrm{h}_{\text {fgw }}=\mathrm{h}_{\text {fg }}$ at WBT in $\mathrm{kJ} / \mathrm{kg}$
$h_{g d}=h_{g}$ at DBT in $\mathrm{kJ} / \mathrm{kg}$
$h_{f w}=h_{f}$ at WBT in $\mathrm{kJ} / \mathrm{kg}$
$\mathrm{C}_{\mathrm{pa}}=$ Specific heat of air $=1.005 \mathrm{~kJ} / \mathrm{kgK}$
$\rho_{v}=$ Density of water vapour in the mixture
$\rho_{\mathrm{a}}=$ Density of dry air in the mixture
$m_{v}=$ Mass of water vapour
$m_{a}=$ Mass of dry air
Density of dry air and water vapour
Density of dry air can be calculated as,
$\rho_{\mathrm{a}}=\mathrm{p}_{\mathrm{a}} /\left(\mathrm{R}_{\mathrm{a}} \mathrm{T}_{\mathrm{d}}\right)$
$\mathrm{T}_{\mathrm{d}}=$ Dry bulb temperature in K
$R_{\mathrm{a}}=287 \mathrm{~J} / \mathrm{kgK}$
$\mathrm{p}_{\mathrm{a}}=$ Partial pressure of dry air in Pa
Density of water vapour can be calculated from,
$\rho_{\mathrm{v}}=\mathrm{W} \rho_{\mathrm{a}}$

## Mass of dry air and water vapour

Mass of dry air $\left(m_{a}\right)$ is calculated from,

$$
\mathrm{p}_{\mathrm{a}} \mathrm{~V}=\mathrm{m}_{\mathrm{a}} \mathrm{R}_{\mathrm{a}} \mathrm{~T}_{\mathrm{d}}
$$

Mass of water vapour $\left(m_{v}\right)$ is calculated from,

$$
p_{v} V=m_{v} R_{v} T_{d}
$$

If gas constant $\left(R_{v}\right)$ for water vapour is not available, $m_{v}$ can be calculated from,

$$
m_{v}=W m_{a}
$$

## Specific humidity of saturated air ( $W_{s}$ )

It is the mass of water vapour present with 1 kg of dry air when the air is saturated.

$$
W_{s}=0.622 \frac{p_{s}}{p_{b}-p_{s}} \mathrm{~kg} / \mathrm{kg} \text { dry air }
$$

$\mathrm{p}_{\mathrm{s}} \rightarrow$ Saturation pressure at DBT
Degree of saturation (or) Saturation ratio ( $\mu$ )

$$
\mu=\frac{W}{W_{s}}=\frac{p_{v}\left(p_{b}-p_{s}\right)}{p_{s}\left(p_{b}-p_{v}\right)}
$$

If $\Phi=0, p_{v}=0, W=0, \mu=0$
If $\Phi=100 \%, p_{v}=p_{s}, W=W_{s}, \mu=1$
Therefore $\mu$ varies between 0 and 1 .

## Relative humidity ( $\phi$ )

It is the ratio of actual mass of water vapour in a given volume of moist air to the mass of water vapour in the same volume of saturated air at the same temperature \& pressure.

$$
\phi=\frac{p_{v}}{p_{s}}=\frac{\mu}{1-(1-\mu) \frac{p_{s}}{p_{b}}}
$$

$\mathrm{p}_{\mathrm{v}}$ can be calculated from, Carrier's equation,

$$
p_{v}=p_{s w}-\frac{\left(p_{b}-p_{s w}\right)\left(T_{d}-T_{w}\right)}{1527.4-1.3 T_{w}}
$$

Note: $\mathrm{T}_{\mathrm{w}} \rightarrow$ WBT in ${ }^{\circ} \mathrm{C}$

$$
\mathrm{p}_{\mathrm{sw}}=\text { Saturation pressure at } \mathrm{WBT} \quad \mathrm{p}_{\mathrm{b}}=\text { Barometric pressure }
$$

$\mathrm{p}_{\mathrm{v}}$ also can be determined from, $W=0.622 \frac{p_{v}}{p_{b}-p_{v}}$ knowing W .

## Enthalpy of moist air

$$
h=C_{p a} T_{d}+W\left[h_{g d}+1.88\left(T_{d}-T_{d p}\right)\right]
$$

$\mathrm{T}_{\mathrm{d}}=$ DBT in ${ }^{\circ} \mathrm{C} \quad \mathrm{C}_{\mathrm{pa}}=1.005 \mathrm{~kJ} / \mathrm{kgK} \quad \mathrm{W}=$ Specific humidity in $\mathrm{kg} / \mathrm{kg}$ dry air
$\mathrm{T}_{\mathrm{dp}}=$ DPT in ${ }^{\circ} \mathrm{C}$

## Psychrometric Chart



DBT - lines $\rightarrow$ Uniformly spaced vertical lines.
W - lines $\quad \rightarrow$ Uniformly spaced horizontal lines.
DPT - lines $\rightarrow$ Non-uniformly spaced horizontal lines.
WBT - lines $\rightarrow$ Non-uniformly spaced inclined lines.
$\phi$ - lines $\quad \rightarrow$ Curved lines.


## Psychrometric processes

## Sensible cooling

Refrigerant out


Refrigerant in

The cooling of air without change in its specific humidity is known as 'Sensible cooling'.

$$
\text { By pass factor }(B P F)=\frac{T_{d 2}-T_{d 3}}{T_{d 1}-T_{d 3}}
$$

$\mathrm{t}_{\mathrm{d} 3}=$ Coil temperature
Coil efficiency $\left(\eta_{c}\right)=1$ - BPF

## Sensible heating

The heating of air without change in its specific humidity is known as 'Sensible heating'.


By pass factor $(B P F)=\frac{T_{d 3}-T_{d 2}}{T_{d 3}-T_{d 1}}$
$\mathrm{T}_{\mathrm{d} 3}=$ Coil temperature

$$
\text { Coil efficiency }\left(\eta_{\mathrm{h}}\right)=1-\mathrm{BPF}
$$

## Dehumidification

The removal of moisture from the air, without change in its dry bulb temperature is known as 'dehumidification'. Perforated plate is used as 'dehumidifier' which removes water particles from the air.


## Chemical dehumidification

Some substances like silica gel and activated alumina have great affinity with water vapour. They are called absorbents. When air passes through a bed of silica gel, water vapour molecules get absorbed on its surface. Latent heat of condensation is released.

For vapour to condense, it has to lose its heat to surrounding air. So, the DBT of air increases. This process is called 'Chemical dehumidification'.


## Humidification

Addition of moisture to the air, without change in its dry bulb temperature is known as 'Humidification'.


## Adiabatic dehumidification and humidification



Dehumidification or humidification of air at constant enthalpy with no heat transfer is called 'adiabatic dehumidification' or 'adiabatic humidification'.

## Cooling \& Dehumidification

This process is generally used in Summer air conditioning to cool and dehumidify the air.
Sensible heat factor $($ SHF $)=\frac{\text { Sensible heat }}{\text { Total heat }}$

$$
=\frac{h_{1}-h_{2}}{-------}
$$



## Heating \& Humidification

This process is generally used in Winter air conditioning to warm \& humidify the air.


$$
\begin{aligned}
\text { Sensible heat factor (SHF) }= & \begin{array}{l}
\text { Sensible } \\
\text { Total }
\end{array} \\
& =---------- \\
& h_{3}-h_{1}
\end{aligned}
$$

## Adiabatic evaporative cooling



A large quantity of water is constantly circulated through a spray chamber. The moist air is passed through this spray water. Some water evaporates into moist air. During this evaporation the water absorbs latent heat from the air reducing air temperature and increasing air specific humidity. This process is called 'Evaporative cooling' of air. If the air is cooled in a insulated chamber, then the cooling is known as 'adiabatic evaporative cooling'.

Cooling tower utilizes the phenomenon of evaporative cooling to cool warm water below the DBT of the air.

## Adiabatic mixing of two streams


$\mathrm{m}_{1}=$ Mass flow rate of dry air of First stream
$h_{1}=$ Specific enthalpy of moist air/kg of dry air of first stream
$\mathrm{W}_{1}=$ Specific humidity of moist air of first stream
$m_{2}, h_{2}, W_{2} \rightarrow$ Corresponding parameters of second stream
$m_{3}, h_{3}, W_{3} \rightarrow$ Corresponding parameters of mixed stream
We can write,

$$
\begin{align*}
\mathrm{m}_{1}+\mathrm{m}_{2} & =\mathrm{m}_{3}  \tag{1}\\
\mathrm{~m}_{1} \mathrm{~h}_{1}+\mathrm{m}_{2} \mathrm{~h}_{2} & =\mathrm{m}_{3} \mathrm{~h}_{3}  \tag{2}\\
\mathrm{~m}_{1} \mathrm{~W}_{1}+\mathrm{m}_{2} \mathrm{~W}_{2} & =\mathrm{m}_{3} \mathrm{~W}_{3} \tag{3}
\end{align*}
$$

From (1) \& (2), $\quad m_{1} W_{1}+m_{2} W_{2}=\left(m_{1}+m_{2}\right) W_{3}$

$$
m_{1}\left(W_{1}-W_{3}\right)=m_{2}\left(W_{3}-W_{2}\right)
$$

$$
\frac{m_{1}}{----}=\frac{W_{3}-W_{2}}{m_{2}}=\frac{------}{W_{1}-W_{3}}
$$

And similarly we can write,

$$
\frac{m_{1}}{----}=\frac{h_{3}-h_{2}}{m_{2}}=\frac{------}{h_{1}-h_{3}}
$$



## Adiabatic saturation


(a) Adiabatic saturation arrangement

(b) Representation of adiabatic saturation on psychrometric chart

When unsaturated air flows over a long sheet of water in an insulated chamber, the water evaporates, and the specific humidity of the air increases. Both the air and water are cooled as evaporation takes places. The process continues until the energy transferred from the air to the water is equal to the energy required to evaporate the water. When this point is reached, thermal equilibrium exists with respect to the water, air and water vapour, and correspondingly the air is saturated. This process is called adiabatic saturation. And the enthalpy is constant as there is no heat transfer during the process.

## PROBLEMS

1. 5 gm of water vapour per kg of atmospheric air at $35^{\circ} \mathrm{C}, 60 \% \mathrm{RH}$ and 1.013 bar is removed and the temperature of air after removing the water vapour is $25^{\circ} \mathrm{C}$ DBT. Determine RH and DPT.

Given
Mass of water vapour $\left(\mathrm{m}_{\mathrm{v}}\right)=5 \mathrm{gm}$
Initial DBT ( $\mathrm{T}_{\mathrm{d} 1}$ )
$=35^{\circ} \mathrm{C}$
Initial RH ( $\Phi_{1}$ )
= 60\%
Total pressure of air $\left(\mathrm{p}_{\mathrm{b}}\right)$
$=1.013 \mathrm{bar}$
Final DBT ( $\mathrm{T}_{\mathrm{d} 2}$ )
$=25^{\circ} \mathrm{C}$
Required: $\Phi_{2}$ and $\mathrm{T}_{\mathrm{dp} 2}$

## Solution



$$
\phi_{1}=\frac{p_{v 1}}{p_{s 1}}
$$

$p_{s 1}=0.05628$ bar at $\mathrm{T}_{\mathrm{d} 1}=35^{\circ} \mathrm{C}$ from steam table
Therefore,

$$
\begin{aligned}
& 0.6=\frac{p_{v 1}}{0.05628} \\
& \begin{aligned}
& \mathrm{p}_{\mathrm{v} 1}=0.03377 \mathrm{bar} \\
& \begin{aligned}
W_{1} & =0.622 \frac{p_{v 1}}{p_{b}-p_{v 1}} \\
W_{1} & =0.622 \frac{0.03377}{1.013-0.03377}
\end{aligned} \\
&=0.02145 \mathrm{~kg} / \mathrm{kg} \mathrm{dry} \text { air } \\
&=21.45 \mathrm{gm} / \mathrm{kg} \text { dry air }
\end{aligned}
\end{aligned}
$$

Therefore,

$$
\mathrm{W}_{2}=21.45-5=16.45 \mathrm{gm} / \mathrm{kg} \text { dry air }
$$

$\mathrm{p}_{\mathrm{s} 2}=0.03169$ bar at $\mathrm{T}_{\mathrm{d} 2}=25^{\circ} \mathrm{C}$ from steam table

$$
W_{2}=0.622 \frac{p_{v 2}}{p_{b}-p_{v 2}}
$$

$$
\begin{aligned}
& 0.01645=0.622 \times \frac{p_{\mathrm{v} 2}}{1.013-p_{v 2}} \\
& 0.02679-0.026447 \mathrm{p}_{\mathrm{v} 2}=\mathrm{p}_{\mathrm{v} 2} \\
& \mathrm{p}_{\mathrm{v} 2}=0.0261 \mathrm{bar} \\
& \phi_{1}=\frac{p_{v 2}}{p_{\mathrm{s} 2}}=\frac{0.0261}{0.03169}=0.823=82.3 \% \quad--- \text { Ans }
\end{aligned}
$$

From steam table, $\quad \mathrm{T}_{\mathrm{dp} 2}=22^{\circ} \mathrm{C}$ at $\mathrm{p}_{\mathrm{v} 2}=0.0261$ bar ---- Ans
2. The sling psychrometer reads $40^{\circ} \mathrm{C}$ DBT and $28^{\circ} \mathrm{C}$ WBT. Calculate, (i) specific humidity, (ii) relative humidity (iii) vapour density of air, (iv) DPT and (v) enthalpy of mixture. Assume atmospheric pressure to be 1.013 bar.

## Given

Atmospheric pressure $\left(p_{b}\right) \quad=1.013$ bar
Dry bulb temperature $\left(\mathrm{T}_{\mathrm{d}}\right) \quad=40^{\circ} \mathrm{C}$
Wet bulb temperature $\left(\mathrm{T}_{\mathrm{w}}\right) \quad=28^{\circ} \mathrm{C}$
Required: (i) W (ii) $\Phi$ (iii) $\rho_{v} \quad$ (iv) $\mathrm{T}_{\mathrm{dp}} \quad$ (v) h

## Solution

$$
\begin{equation*}
W=0.622 \frac{p_{v}}{p_{b}-p_{v}} \tag{i}
\end{equation*}
$$

Also,

$$
\begin{aligned}
W & =\frac{C_{p a}\left(T_{w}-T_{d}\right)+W_{w} h_{f g W}}{h_{g d}-h_{f w}} \\
W_{w} & =0.622 \frac{p_{s w}}{p_{b}-p_{s w}}
\end{aligned}
$$

$\mathrm{p}_{\mathrm{sw}}=$ Saturation pressure at $\mathrm{WBT}=0.03778$ bar from steam table

$$
W_{w}=0.622 \frac{0.03778}{1.013-0.03778}=0.024096 \mathrm{~kg} / \mathrm{kg} \mathrm{dryair}
$$

$\mathrm{h}_{\mathrm{fgw}}=\mathrm{h}_{\mathrm{fg}}$ at $\mathrm{WBT}=2435.4 \mathrm{~kJ} / \mathrm{kg}$ from steam table at $28^{\circ} \mathrm{C}$
$\mathrm{h}_{\mathrm{gd}}=\mathrm{h}_{\mathrm{g}}$ at DBT $=2574.4 \mathrm{~kJ} / \mathrm{kg}$ from steam table at $40^{\circ} \mathrm{C}$
$\mathrm{h}_{\mathrm{fw}}=\mathrm{h}_{\mathrm{f}}$ at WBT $=117.3 \mathrm{~kJ} / \mathrm{kg}$ from steam table at $28^{\circ} \mathrm{C}$
$C_{p a}=$ Specific heat of air $=1.005 \mathrm{~kJ} / \mathrm{kgK}$

$$
W=\frac{1.005(28-40)+0.024096(2435.4)}{2574.4-117.3}=0.018975 \mathrm{~kg} / \mathrm{kg} \mathrm{dry} \text { air }
$$

(ii)

$$
\begin{aligned}
\phi & =\frac{p_{v}}{p_{s}} \\
W & =0.622 \frac{p_{v}}{p_{b}-p_{v}}
\end{aligned}
$$

$$
\begin{aligned}
0.018975 & =0.622 \times \frac{p_{v}}{1.013-p_{v}} \\
p_{v} & =0.02998 \mathrm{bar}
\end{aligned}
$$

$\mathrm{p}_{\mathrm{s}}=0.07375$ bar at $40^{\circ} \mathrm{C}$ from steam table
Therefore, $\Phi=0.02998 / 0.07375=0.4065=40 \%----$ Ans
(iii) Density of water vapour can be found out from, $W=\rho_{v} / \rho_{a}$

We know that, $\quad \rho_{a}=\frac{p_{a}}{R_{a} T_{a}}$
$\mathrm{R}_{\mathrm{a}}=287 \mathrm{~J} / \mathrm{kgK}$
$\mathrm{T}_{\mathrm{a}}=$ DBT
$\mathrm{p}_{\mathrm{a}}=$ Partial pressure of dry air $=\mathrm{p}_{\mathrm{b}}-\mathrm{p}_{\mathrm{v}}$
$p_{a}=1.013-0.02998=0.98302$ bar

$$
\rho_{a}=\frac{0.98302 \times 10^{5}}{287 \times(40+273)}=1.0943 \mathrm{~kg} / \mathrm{m}^{3}
$$

Therefore, $\rho_{\mathrm{v}}=0.018975 \times 1.0943=\mathbf{0 . 0 2 0 7 6 4} \mathbf{~ k g} / \mathbf{m}^{\mathbf{3}}$----- Ans
(iv) At $p_{v}=0.020764$ bar from steam table, $\mathrm{T}_{\mathrm{dp}}=24^{\circ} \mathrm{C}----$ Ans

$$
\begin{align*}
h & =C_{p a} T_{d}+W\left[h_{g d}+1.88\left(T_{d}-T_{d p}\right)\right]  \tag{v}\\
& =1.005(40)+0.018975[2574.4+1.88(40-24)] \\
& =\mathbf{8 9 . 6 2} \mathbf{~ k J} / \mathbf{k g} \text { dry air }--- \text { Ans }
\end{align*}
$$

Note: Substitute $\mathrm{T}_{\mathrm{d}}$ in ${ }^{\circ} \mathrm{C}$
3. Atmospheric air at 1.0132 bar has a DBT of $32^{\circ} \mathrm{C}$ and WBT of $26^{\circ} \mathrm{C}$. Compute (a) the partial pressure of water vapour, (b) the specific humidity, (c) the DPT, (d) the RH, (e) the degree of saturation, (f) the density of air in the mixture, $(\mathrm{g})$ the density of vapour in the mixture and (h) the enthalpy of the mixture.

## Given

Atmospheric pressure $\left(p_{b}\right) \quad=1.0132$ bar
Dry bulb temperature $\left(\mathrm{T}_{\mathrm{d}}\right) \quad=32^{\circ} \mathrm{C}$
Wet bulb temperature $\left(T_{w}\right) \quad=26^{\circ} \mathrm{C}$
Required: (a) $p_{v}$ (b) $W$ (c) $T_{d p}$ (d) $\Phi$ (e) $\mu$ (f) $\rho_{a}$ (g) $\rho_{v} \quad$ (h) $h$

## Solution

(a)

I-Method (Using Carrier's equation)

$$
p_{v}=p_{s w}-\frac{\left(p_{b}-p_{s w}\right)\left(T_{d}-T_{w}\right)}{1527.4-1.3 T_{w}}
$$

Note: $\mathrm{T}_{\mathrm{w}} \rightarrow$ WBT in ${ }^{\circ} \mathrm{C}$
$\mathrm{p}_{\mathrm{sw}}=$ Saturation pressure at $\mathrm{WBT}=0.03360$ bar from steam table

$$
\begin{aligned}
p_{v} & =0.03360-\frac{(1.0132-0.03360)(32-26)}{1527.4-1.3(26)} \\
& =\mathbf{0 . 0 2 9 6} \text { bar }---- \text { Ans }
\end{aligned}
$$

II-Method

$$
\begin{aligned}
W & =\frac{C_{p a}\left(T_{w}-T_{d}\right)+W_{w} h_{f g W}}{h_{g d}-h_{f w}} \\
W_{w} & =0.622 \frac{p_{s w}}{p_{b}-p_{s w}}
\end{aligned}
$$

$\mathrm{p}_{\mathrm{sw}}=$ Saturation pressure at $\mathrm{WBT}=0.03360$ bar from steam table

$$
W_{w}=0.622 \frac{0.03360}{1.0132-0.03360}=0.021334 \mathrm{~kg} / \mathrm{kg} \text { dry air }
$$

$\mathrm{h}_{\mathrm{fgw}}=\mathrm{h}_{\mathrm{fg}}$ at $\mathrm{WBT}=2440.2 \mathrm{~kJ} / \mathrm{kg}$ from steam table at $26^{\circ} \mathrm{C}$
$\mathrm{h}_{\mathrm{gd}}=\mathrm{h}_{\mathrm{g}}$ at DBT $=2560 \mathrm{~kJ} / \mathrm{kg}$ from steam table at $32^{\circ} \mathrm{C}$
$\mathrm{h}_{\mathrm{fw}}=\mathrm{h}_{\mathrm{f}}$ at WBT $=108.9 \mathrm{~kJ} / \mathrm{kg}$ from steam table at $26^{\circ} \mathrm{C}$
$\mathrm{C}_{\mathrm{pa}}=$ Specific heat of air $=1.005 \mathrm{~kJ} / \mathrm{kgK}$

$$
W=\frac{1.005(26-32)+0.021334(2440.2)}{2560-108.9}=0.01878 \mathrm{~kg} / \mathrm{kg} \text { dry air }
$$

Also, $\quad W=0.622 \frac{p_{v}}{p_{b}-p_{v}}$ $0.01878=0.622 \times \frac{p_{v}}{1.0132-p_{v}}$

$$
p_{v}=0.0297 \text { bar ---- Ans }
$$

(b)

$$
\begin{aligned}
W & =0.622 \frac{p_{v}}{p_{b}-p_{v}} \\
& =0.622 \frac{0.0296}{1.0132-0.0296}=0.01872 \mathrm{~kg} / \mathrm{kg} \text { dry air ---- Ans }
\end{aligned}
$$

(c) $T_{d p}=24^{\circ} \mathrm{C}$ at $p_{v}$ from steam table ----- Ans
(d)

$$
\phi=\frac{p_{v}}{p_{s}}
$$

$p_{s}=0.04753$ bar at $32^{\circ} \mathrm{C}$ from steam table
Therefore, $\Phi=0.0296 / 0.04753=0.623=62.3 \%-----$ Ans
(e) $\quad \mu=\frac{W}{W_{s}}=\frac{p_{v}\left(p_{b}-p_{s}\right)}{p_{s}\left(p_{b}-p_{v}\right)}$

$$
=\frac{0.0296(1.0132-0.04753)}{0.04753(1.0132-0.0296)}=0.6114 \quad---- \text { Ans }
$$

(f) We know that, $\quad \rho_{a}=\frac{p_{a}}{R_{a} T_{a}}$
$\mathrm{R}_{\mathrm{a}}=287 \mathrm{~J} / \mathrm{kgK}$
$\mathrm{T}_{\mathrm{a}}=\mathrm{DBT}$
$p_{a}=$ Partial pressure of dry air $=p_{b}-p_{v}$
$p_{a}=1.0132-0.0296=0.9836$ bar

$$
\rho_{a}=\frac{0.9836 \times 10^{5}}{287 \times(32+273)}=1.1236 \mathrm{~kg} / \mathrm{m}^{3} \ldots-- \text { Ans }
$$

(g) Density of water vapour can be found out from, $\mathrm{W}=\rho_{\mathrm{v}} / \rho_{\mathrm{a}}$

Therefore, $\rho_{v}=0.01872 \times 1.1236=\mathbf{0 . 0 2 1} \mathbf{~ k g} / \mathbf{m}^{\mathbf{3}} \boldsymbol{- - - - -}$ Ans
(h)

$$
h=C_{p a} T_{d}+W\left[h_{g d}+1.88\left(T_{d}-T_{d p}\right)\right]
$$

At $p_{v}=0.0296$ bar from steam table, $T_{d p}=24^{\circ} \mathrm{C}----$ Ans

$$
\begin{aligned}
h & =1.005(32)+0.01872[2560+1.88(32-24)] \\
& =\mathbf{8 0 . 3 6 5} \mathbf{~ k J} / \mathbf{k g} \text { dry air }--- \text { Ans }
\end{aligned}
$$

Note: Substitute $\mathrm{T}_{\mathrm{d}}$ in ${ }^{\circ} \mathrm{C}$
4. Air at $20^{\circ} \mathrm{C}, 40 \% \mathrm{RH}$ is mixed adiabatically with air at $40^{\circ} \mathrm{C}, 40 \% \mathrm{RH}$ in the ratio of 1 kg of the former with 2 kg of latter (on dry basis). Find the final condition of air. Draw the process in chart also as diagram.

## Given

| DBT of I-stream $\left(T_{\mathrm{d} 1}\right)$ | $=20^{\circ} \mathrm{C}$ |
| :--- | :--- |
| RH of I-stream $\left(\Phi_{1}\right)$ | $=40 \%$ |
| DBT of II-stream $\left(T_{\mathrm{d} 2}\right)$ | $=40^{\circ} \mathrm{C}$ |
| RH of II-stream $\left(\Phi_{2}\right)$ | $=40 \%$ |
| $\mathrm{~m}_{\mathrm{a} 1} / \mathrm{m}_{\mathrm{a} 2}$ | $=1 / 2$ |
| Mass of dry air in the I-stream $\left(\mathrm{m}_{\mathrm{a} 1}\right)$ | $=1 \mathrm{~kg}$ |
| Mass of dry air in the II-stream $\left(\mathrm{m}_{\mathrm{a} 2}\right)$ | $=2 \mathrm{~kg}$ |

Required: Final condition of air $\left(h_{3}, W_{3}\right)$

## Solution

We can write,

$$
\begin{aligned}
m_{a 1} h_{1}+m_{a 2} h_{2} & =\left(m_{a 1}+m_{a 2}\right) h_{3} \\
m_{a 1} W_{1}+m_{a 2} W_{2} & =\left(m_{a 1}+m_{a 2}\right) W_{3}
\end{aligned}
$$

Using Steam table and equations

$$
\phi_{1}=\frac{p_{v 1}}{p_{s 1}}
$$

$p_{\mathrm{s} 1}=0.02337$ bar at $\mathrm{T}_{\mathrm{d} 1}=20^{\circ} \mathrm{C}$ from steam table

$$
0.4=\frac{p_{v 1}}{0.02337}
$$



$$
\begin{aligned}
W_{1} & =0.622 \frac{p_{v 1}}{p_{b}-p_{v 1}} \\
& =0.622 \times \frac{0.009348}{1.0132-0.009348}=0.0058 \mathrm{~kg} / \mathrm{kg} \text { dry air }
\end{aligned}
$$

$$
\phi_{2}=\frac{p_{v 2}}{p_{s 2}}
$$

$p_{\mathrm{s} 2}=0.07375$ bar at $\mathrm{T}_{\mathrm{d} 2}=40^{\circ} \mathrm{C}$ from steam table

$$
\begin{aligned}
0.4 & =\frac{p_{v 2}}{0.07375} \\
\mathrm{p}_{\mathrm{v} 2} & =0.0295 \text { bar } \\
W_{2} & =0.622 \frac{p_{v 2}}{p_{b}-p_{v 2}} \\
& =0.622 x \frac{0.0295}{1.0132-0.0295}=0.01865 \mathrm{~kg} / \mathrm{kg} \text { dry air } \\
h_{1} & =C_{p a} T_{d 1}+W_{1}\left[h_{g d 1}+1.88\left(T_{d 1}-T_{d p 1}\right)\right]
\end{aligned}
$$

At $p_{\mathrm{v} 1}=0.09348$ bar from steam table, $\mathrm{T}_{\mathrm{dp} 1}=6^{\circ} \mathrm{C}$
$\mathrm{h}_{\mathrm{gd} 1}=2538.2 \mathrm{~kJ} / \mathrm{kg}$

$$
h_{1}=1.005(20)+0.0058[2538.2+1.88(20-6)]
$$

$$
\begin{aligned}
& =34.97 \mathrm{~kJ} / \mathrm{kg} \text { dry air } \\
h_{2} & =C_{p a} T_{d 2}+W_{2}\left[h_{g d 2}+1.88\left(T_{d 2}-T_{d p 2}\right)\right]
\end{aligned}
$$

At $\mathrm{p}_{\mathrm{v} 2}=0.0295$ bar from steam table, $\mathrm{T}_{\mathrm{dp} 2}=24^{\circ} \mathrm{C}$
$\mathrm{h}_{\mathrm{gd} 2}=2574.4 \mathrm{~kJ} / \mathrm{kg}$

$$
\begin{aligned}
h_{2} & =1.005(40)+0.01865[2574.4+1.88(40-24)] \\
& =88.8 \mathrm{~kJ} / \mathrm{kg} \text { dry air } \\
1(34.97)+2(88.8) & =(1+2) \mathrm{h}_{3} \\
\mathrm{~h}_{3} & =70.86 \mathrm{~kJ} / \mathrm{kg} \text { dry air ----- Ans } \\
1(0.0058)+2(0.01865) & =(1+2) \mathrm{W}_{3} \\
\mathrm{~W}_{3} & =0.01436 \mathrm{~kg} / \mathrm{kg} \text { dry air }-.-- \text { Ans }
\end{aligned}
$$

Using chart

- Locate the point (1) on the chart at $\Phi_{1}=40 \%$ and $T_{d 1}=20^{\circ} \mathrm{C}$
- Locate the point (2) at $\Phi_{2}=40 \%$ and $T_{d 2}=40^{\circ} \mathrm{C}$
- Get $h_{1}, h_{2}, W_{1}$ and $W_{2}$ from chart

$$
\begin{aligned}
& \mathrm{h}_{1}=35 \mathrm{~kJ} / \mathrm{kg} \text { dry air } \mathrm{W}_{1}=0.0058 \mathrm{~kg} / \mathrm{kg} \text { dry air } \\
& \mathrm{h}_{2}=90 \mathrm{~kJ} / \mathrm{kg} \text { dry air } \mathrm{W}_{2}=0.0187 \mathrm{~kg} / \mathrm{kg} \text { dry air } \\
& 1(35)+2(90)=(1+2) \mathrm{h}_{3} \\
& \mathrm{~h}_{3}=71.67 \mathrm{~kJ} / \mathrm{kg} \text { dry air ----- Ans } \\
& 1(0.0058)+2(0.0187)=(1+2) \mathrm{W}_{3} \\
& \mathrm{~W}_{3}=0.0144 \mathrm{~kg} / \mathrm{kg} \text { dry air }-\ldots-- \text { Ans }
\end{aligned}
$$

We can also get other properties from chart.

- Locate point (3) on the chart at $W_{3}$ or $h_{3}$
- Point (3) lies between (1) and (2) and on the line joining (1) and (2)
- Get WBT and DPT on the saturation curve
- Get DBT on the x-axis
- Get RH and specific volume at point (3)
$\mathrm{T}_{\mathrm{w} 3}=23.8^{\circ} \mathrm{C}, \quad \mathrm{T}_{\mathrm{d} 3}=33.1^{\circ} \mathrm{C}, \quad \mathrm{T}_{\mathrm{dp} 3}=19.6^{\circ} \mathrm{C}, \quad \mathrm{v}_{3}=0.887 \mathrm{~m}^{3} / \mathrm{kg}$ dry air, $\quad \Phi_{3}=$ 46\%


5. Air-water vapour mixture at $0.1 \mathrm{MPa}, 30^{\circ} \mathrm{C}, 80 \% \mathrm{RH}$ has a volume of $50 \mathrm{~m}^{3}$. Calculate the specific humidity, DPT, WBT, mass of dry air and mass of water vapour.

## Given

Total pressure of mixture $\left(\mathrm{p}_{\mathrm{b}}\right)=0.1 \mathrm{MPa}=1 \mathrm{bar}$
DBT of mixture $\left(T_{d}\right) \quad=30^{\circ} \mathrm{C}$
RH of mixture ( $\Phi$ ) $=80 \%$
Volume of mixture (V) $\quad=50 \mathrm{~m}^{3}$
Required: W, DPT, WBT, $m_{a}$ and $m_{w}$

## Solution

$$
\begin{aligned}
W & =0.622 \frac{p_{v}}{p_{b}-p_{v}} \\
\phi & =\frac{p_{v}}{p_{s}}
\end{aligned}
$$

$p_{s}=0.04242$ bar at $\mathrm{T}_{\mathrm{d}}=30^{\circ} \mathrm{C}$ from steam table
$p_{v}=0.8(0.04242)=0.03394$ bar

$$
\begin{aligned}
& W=0.622 \times \frac{0.03394}{1-0.03394}=0.02185 \mathrm{~kg} / \mathrm{kg} \text { dry air }---\mathrm{Ans} \\
& \mathrm{~T}_{\mathrm{dp}}=26^{\circ} \mathrm{C} \text { at } \mathrm{p}_{\mathrm{v}}=0.03394 \text { bar from steam table }- \text {--- }
\end{aligned}
$$

## Ans

- Using Carrier's equation, only by trial and error we can find WBT. Therefore it is advisable to refer chart for obtaining the WBT.
- Locate the point on the chart at $30^{\circ} \mathrm{C}$ and $80 \% \mathrm{RH}$
- Get WBT on saturation curve

$$
\mathrm{T}_{\mathrm{w}}=27^{\circ} \mathrm{C}---- \text { Ans }
$$

To find mass of dry air

$$
\begin{aligned}
\mathrm{p}_{\mathrm{a}} & =\mathrm{p}_{\mathrm{b}}-\mathrm{p}_{\mathrm{v}}=1-0.03394=0.96606 \mathrm{bar} \\
\mathrm{p}_{\mathrm{a}} \vee & =\mathrm{m}_{\mathrm{a}} \mathrm{R}_{\mathrm{a}} \mathrm{~T}_{\mathrm{a}} \\
0.96606 \times 10^{5} \times 50 & =\mathrm{m}_{\mathrm{a}} \times 287 \times(30+273) \\
\mathrm{m}_{\mathrm{a}} & =55.54 \mathrm{~kg} \quad--- \text { Ans }
\end{aligned}
$$

To find mass of water vapour

$$
\begin{aligned}
W & =\frac{\rho_{v}}{\rho_{a}}=\frac{m_{v}}{m_{a}} \\
\mathrm{~m}_{\mathrm{v}} & =0.02185(55.54)=1.2136 \mathrm{~kg} \quad--- \text { Ans }
\end{aligned}
$$

Note: If we know the value of $R_{w}, m_{w}$ can be found out from, $p_{w} V=m_{w} R_{w} T_{v}$

$$
\mathrm{T}_{\mathrm{a}}=\mathrm{T}_{\mathrm{v}}=\mathrm{T}_{\mathrm{d}}
$$

6. Saturated air at $20^{\circ} \mathrm{C}$ at a rate of $1.16 \mathrm{~m}^{3} / \mathrm{s}$ is mixed adiabatically with the outside air at $35^{\circ} \mathrm{C}$ and $50 \% \mathrm{RH}$ at a rate of $0.5 \mathrm{~m}^{3} / \mathrm{s}$. Assuming adiabatic mixing condition at 1 atm, determine specific humidity, relative humidity, dry bulb temperature and volume flow rate of the mixture.

## Given

| Saturated air, i.e., $\Phi_{1}$ | $=100 \%$ |
| :--- | :--- |
| DBT of I-stream $\left(T_{d 1}\right)$ | $=20^{\circ} \mathrm{C}$ |
| Volume flow rate of I-stream $\left(\mathrm{V}_{1}\right)$ | $=1.16 \mathrm{~m}^{3} / \mathrm{s}$ |
| DBT of II-stream $\left(T_{d 2}\right)$ | $=35^{\circ} \mathrm{C}$ |
| RH of II-stream $\left(\Phi_{2}\right)$ | $=50 \%$ |
| Volume flow rate of II-stream $\left(\mathrm{V}_{2}\right)$ | $=0.5 \mathrm{~m}^{3} / \mathrm{s}$ |
| Total pressure $\left(\mathrm{p}_{\mathrm{b}}\right)$ | $=1 \mathrm{~atm}=1.01325 \mathrm{bar}$ |

Required: $\mathrm{W}_{3}, \Phi_{3} \mathrm{~T}_{\mathrm{d} 3}$ and $\mathrm{V}_{3}$

## Solution



We can write,

$$
\begin{aligned}
m_{a 1} h_{1}+m_{a 2} h_{2} & =\left(m_{a 1}+m_{a 2}\right) h_{3} \\
m_{a 1} W_{1}+m_{a 2} W_{2} & =\left(m_{a 1}+m_{a 2}\right) W_{3}
\end{aligned}
$$

Using chart

- Locate the point (1) on the chart at $\Phi_{1}=100 \%$ and $T_{d 1}=20^{\circ} \mathrm{C}$
- Locate the point (2) at $\Phi_{2}=50 \%$ and $T_{\mathrm{d} 2}=35^{\circ} \mathrm{C}$
- Get $h_{1}, h_{2}, W_{1}$ and $W_{2}$ from chart

$$
\begin{array}{lll}
\mathrm{h}_{1}=58 \mathrm{~kJ} / \mathrm{kg} \text { dry air } & \mathrm{W}_{1}=0.015 \mathrm{~kg} / \mathrm{kg} \text { dry air } & \mathrm{V}_{1}=0.851 \mathrm{~m}^{3} / \mathrm{kg} \text { dry } \\
\text { air } \\
\mathrm{h}_{2}=81.5 \mathrm{~kJ} / \mathrm{kg} \text { dry air } \quad \mathrm{W}_{2}=0.018 \mathrm{~kg} / \mathrm{kg} \text { dry air } & \mathrm{v}_{2}=0.898 \mathrm{~m}^{3} / \mathrm{kg} \text { dry } \\
\text { air } \\
m_{1}=\frac{V_{1}}{v_{1}}=\frac{1.16}{0.851}=1.3631 \mathrm{~kg} / \mathrm{s} &
\end{array}
$$

$$
\begin{aligned}
& m_{2}=\frac{V_{2}}{V_{2}}=\frac{0.5}{0.898}=0.5568 \mathrm{~kg} / \mathrm{s} \\
& 1.3631(0.015)+0.5568(0.018)=(1.3631+0.5568) \mathrm{W}_{3} \\
& \mathrm{~W}_{3}=0.0159 \mathrm{~kg} / \mathrm{kg} \text { dry air }---- \text { Ans }
\end{aligned}
$$

- Locate point (3) on the chart at $W_{3}$
- Point (3) lies between (1) and (2) and on the line joining (1) and (2)
- Get DBT on the x-axis
- Get RH and specific volume at point (3)

From chart,

$$
\begin{aligned}
& \Phi_{3}=80 \% \quad---- \text { Ans } \\
& \mathrm{T}_{\mathrm{d} 3}=25^{\circ} \mathrm{C} \quad---- \text { Ans } \\
& \mathrm{V}_{3}=0.866 \mathrm{~m}^{3} / \mathrm{kg} \text { dry air } \\
& \begin{aligned}
\mathrm{V}_{3} & =\mathrm{m}_{3} \mathrm{~V}_{3}
\end{aligned}=(1.3631+0.5568)(0.866) \\
& \\
& \\
& =1.6626 \mathrm{~m}^{3} / \mathrm{s} \quad---- \text { Ans }
\end{aligned}
$$

7. Air at $16^{\circ} \mathrm{C}$ and $25 \% \mathrm{RH}$ passes through a heater and then through a humidifier to reach final DBT of $30^{\circ} \mathrm{C}$ and $50 \% \mathrm{RH}$. Calculate the heat and moisture added to the air. What is the sensible heat factor?

## Given

Initial DBT of air $\left(T_{d 1}\right)=16^{\circ} \mathrm{C}$
Initial RH of air $\left(\Phi_{1}\right)=25 \%$
Final DBT of air $\left(\mathrm{T}_{\mathrm{d} 3}\right)=30^{\circ} \mathrm{C}$
Final RH of air $\left(\Phi_{3}\right)=50 \%$
Process $\quad \rightarrow$ Heating and humidification
Required: Heat added, Moisture added and SHF

## Solution



Using chart

- Locate the point (1) at $T_{d 1}$ and $\Phi_{1}$
- Locate the point (3) at $\mathrm{T}_{\mathrm{d} 3}$ and $\Phi_{3}$
- Draw the horizontal line (heating) from (1) and Vertical line (humidification) from (3) and get intersection point (2)
- Get $h_{1}, h_{2}, h_{3}, W_{1}, W_{2}$ and $W_{3}$
$h_{1}=23 \mathrm{~kJ} / \mathrm{kg}$ dry air $\mathrm{h}_{2}=38 \mathrm{~kJ} / \mathrm{kg}$ dry air $\mathrm{h}_{3}=64 \mathrm{~kJ} / \mathrm{kg}$ dry air
$W_{1}=W_{2}=0.0033 \mathrm{~kg} / \mathrm{kg}$ dry air $\mathrm{W}_{3}=0.0134 \mathrm{~kg} / \mathrm{kg}$ dry air
Heat added $=\mathrm{SH}+\mathrm{LH}=\left(\mathrm{h}_{2}-\mathrm{h}_{1}\right)+\left(\mathrm{h}_{3}-\mathrm{h}_{2}\right)$

$$
\begin{aligned}
& =h_{3}-h_{1} \\
& =64-23=41 \mathrm{~kJ} / \mathrm{kg} \text { dry air }--- \text { Ans }
\end{aligned}
$$

Moisture added $=W_{3}-W_{2}=0.0134-0.0033$

$$
=0.0101 \mathrm{~kg} / \mathrm{kg} \text { dry air }----- \text { Ans }
$$

Sensible heat factor $(\mathrm{SHF})=\frac{S H}{S H+L H}=\frac{h_{2}-h_{1}}{\left(h_{2}-h_{1}\right)+\left(h_{3}=h_{2}\right)}=\frac{h_{2}-h_{1}}{h_{3}-h_{1}}$

$$
=\frac{38-23}{64-23}=0.366 \quad----\quad \text { Ans }
$$

8. For a hall to be air conditioned, the following conditions are given:

Outdoor condition $\rightarrow 40^{\circ} \mathrm{C}$ DBT, $20^{\circ} \mathrm{C}$ WBT
Required comfort condition $\boldsymbol{\rightarrow} \mathbf{2 0}^{\circ} \mathrm{C}$ DBT, $\mathbf{6 0 \%}$ RH
Seating capacity of hall $=1500$
Amount of air supplied $=0.3 \mathrm{~m}^{3} / \mathrm{min}$ per person
If the required condition is achieved first by adiabatic humidification and then by cooling, estimate (a) capacity of the cooling coil in tones, and (b) the capacity of the humidifier.

## Given

Process $\rightarrow$ Adiabatic humidification and Cooling
Before adiabatic humidification

| DBT of air $\left(T_{d 1}\right)$ | $=40^{\circ} \mathrm{C}$ |
| :--- | :--- |
| WBT of air $\left(T_{w 1}\right)$ | $=20^{\circ} \mathrm{C}$ |

After adiabatic humidification
RH of air $\left(\Phi_{2}\right) \quad=60 \%$
After cooling (sensible)
DBT of air $\left(T_{d 3}\right) \quad=20^{\circ} \mathrm{C}$
Amount of air supplied $\left(\mathrm{V}_{\mathrm{a} 1}\right)=0.3 \mathrm{~m}^{3} / \mathrm{min}$ per person
Seating capacity $=1500$
Required: (a) Capacity of cooling coil (b) Capacity of humidifier

## Solution



- Locate the point (1) at $\mathrm{T}_{\mathrm{d} 1}=40^{\circ} \mathrm{C}$ and $\mathrm{T}_{\mathrm{w} 1}=20^{\circ} \mathrm{C}$
- During adiabatic humidification, $\mathrm{h}=$ constant. Follow the constant WBT line and get point (2) at 60\%.
- During cooling process, $\mathrm{W}=$ constant. Follow constant W line and get point (3) at $\mathrm{T}_{\mathrm{d} 3}=20^{\circ} \mathrm{C}$

From chart, $\mathrm{h}_{1}=\mathrm{h}_{2}=57 \mathrm{~kJ} / \mathrm{kg}$ dry air

$$
\begin{aligned}
\mathrm{h}_{3} & =42 \mathrm{~kJ} / \mathrm{kg} \text { dry air } \\
\mathrm{W}_{1} & =0.0065 \mathrm{~kg} / \mathrm{kg} \text { dry air } \\
\mathrm{W}_{2} & =\mathrm{W}_{3}=0.0088 \mathrm{~kg} / \mathrm{kg} \text { dry air } \\
\mathrm{V}_{1} & =0.896 \mathrm{~m}^{3} / \mathrm{kg} \text { dry air }
\end{aligned}
$$

(a) Capacity of cooling coil $=m_{a}\left(h_{2}-h_{3}\right)$

Volume flow rate of air $\left(\mathrm{V}_{\mathrm{a} 1}\right)=0.3 \times 1500 / 60=7.5 \mathrm{~m}^{3} / \mathrm{s}$
Mass flow rate of air $\left(\mathrm{m}_{\mathrm{a}}\right) \quad=\mathrm{V}_{\mathrm{a} 1} / \mathrm{v}_{1}=7.5 / 0.896=8.37 \mathrm{~kg} / \mathrm{s}$
Capacity of cooling coil $=8.37(57-42)=125.55 \mathrm{~kJ} / \mathrm{s}$
= 125.55 / 3.89 = 32.27 TR
Ans
Capacity of humidifier $\quad=m_{a}\left(W_{2}-W_{1}\right)=8.37(0.0088-0.0065)$
$=0.019251 \mathrm{~kg} / \mathrm{s}$
$=69.3 \mathrm{~kg} / \mathrm{h}$----- Ans

## Air conditioning

It is the process of controlling temperature, humidity, motion \& purity of air.

## Classification

1. According to the purpose,
(a) Comfort air conditioning system
(b) Industrial air conditioning system
2. According to season of the year,
(a) Winter air conditioning system
(b) Summer air conditioning system
(c) Year round air conditioning system
3. According to the arrangement of equipment,
(a) Unitary air conditioning system
(b) Central air conditioning system

## Winter Air Conditioning System

In winter A/C system, air is heated, which is generally accompanied by Humidification.


The outside air flows through a damper and mixes up with the re-circulated air. The mixed air passes through a filter to remove dirt, dust \& other impurities. The air now passes through a preheat coil in order to prevent the possible freezing of water and to control the evaporation of water in the humidifier. After that, the air is made to pass through a reheat coil to bring the air to the designed DBT.

Now the conditioned air is supplied to the conditioned space by a circulating fan. From the conditioned space, a part of the used air is exhausted to the atmosphere by the exhaust fans or ventilators. The remaining part of the air is again conditioned.

The outside air is sucked and made to mix with the re-circulated air, in order to make up for the loss of conditioned air through exhaust fans or ventilators from the conditioned space.

## Summer Air Conditioning System

It is the most important type of $A / C$ system, in which the air is cooled and generally dehumidified.

The outside air flows through the damper and mixes up with re-circulated air. The mixed air passes through a filter to remove dirt, dust and other impurities. The air now passes through the cooling coil. The coil has a temperature much below the required DBT of the air in the conditioned space. The cooled air passes through a 'perforated membrane' and losses its moisture in the condensed form from which is collected in a sump. After that, the air is made to pass through a heating coil which heats up the air slightly. This is done to bring the air to the designed DBT and relative humidity.


1 - Filter
2 - Cooling coil
3 - dehumidifier
4 - Water eliminators
5 - Heating coil
6 - Fan
7 - Dampers

Now the conditioned air is supplied to the conditioned space by a circulating fan. From the conditioned space, a part of the used air is exhausted to the atmosphere by the exhaust fans or ventilators. The remaining part of the air is again conditioned.

The outside air is sucked and made to mix with the re-circulated air, in order to make up for the loss of conditioned air through exhaust fans or ventilators from the conditioned space.

## Types of Air conditioning units

## 1. Central station Air conditioning system

In a central air conditioning system, all the components of the system are grouped together in one central room and conditioned air is distributed from the central room to the required places through extensive duct work. The central A/C system is generally used for the load above 25 Tons of refrigeration and $2500 \mathrm{~m}^{3} / \mathrm{min}$ of conditioned air. The unitary system can be more economically used for low capacity (below 25 Tons) units.

The central plants require the following components and all the components are assembled on the site.
(i) Cooling \& dehumidifying coils
(ii) Heat coils
(iii) Blower with motor
(iv) Sprays for cooling, dehumidifying or washing
(v) Air cleaning equipments
(vi) A control device

The central A/C plant handling $1200 \mathrm{~m}^{3} / \mathrm{min}$ as a limit is generally preferred for auditorium, departmental store and large multi stored buildings where the duct work can be easily laid down as there is more freeness to carry out the duct work and less obstructions. Office buildings, schools and similar buildings land themselves to more zoning, with air conditioning units of 300 to $1000 \mathrm{~m}^{3} / \mathrm{min}$ being in common use.

## Advantages

1. The capital cost and running cost are less per unit of refrigeration.
2. It can be located away from the air conditioned places which is useful and less costly.
3. Noise and vibration troubles are less to the people living in air conditioned places as the air conditioned plant is far away from air conditioned places.
4. Better accessibility for maintenance.

## Unitary Air conditioning system (Packaged units)

All the components of the unitary A/C system are assembles in the factory itself. These assembled units are usually installed in or immediately adjacent to a zone or space to be conditioned. It is commonly preferred for 15 Tons capacity or above or around $200 \mathrm{~m}^{3} / \mathrm{min}$ of air flow.

## Equipments used in Air Conditioning system

Circulating fan : The main function of this fan is to move air to and from the room.
(i) Axial flow fans
(ii) Centrifugal fans

Air conditioning unit : It is a unit, which consists of cooling and dehumidifying processes for Summer A/C and heating \& humidifying processes for Winter A/C.

Supply duct : It directs the conditioned air from the circulating fan to the space to be air conditioned at proper point.

Supply outlets : These are grills, which distributes the conditioned air evenly in the room.
Return outlets : These are the openings in a room surface which allow the room air to enter the return duct.

Filters : The main function of the filters is to remove dust, dirt and other harmful bacteria from the air.
(i) Dry filters
(ii) Viscous filters
(iii) Wet filters
(iv) Electric filters
(v) Centrifugal dust collectors

The purpose of all dust removing equipments is to remove or reduce the concentration of dust to a very small fraction of its original in the conditioned air.
Factors affecting comfort air conditioning

- Temperature of the air.
- Humidity of the air.
- Purity of air.
- Motion of air.


## 1. Temperature of air

In air conditioning, the control of temperature means the maintenance of any desired temperature within an enclosed space even though the temperature of outside air is above or below the desired room temperature. This is accomplished either by the addition or removal of heat from the enclosed space as and when demanded. A human being feels comfortable when the air is at $21^{\circ} \mathrm{C}$ DBT with $56 \% \mathrm{RH}$.

## 2. Humidity of air

The control of humidity of air means the increasing or decreasing of moisture amounts of air during summer or winter respectively in order to produce comfortable and healthy conditions. The control of humidity is not only necessary for human comfort but it also increases the efficiency of the workers. In summer RH should not be less than 60\% and in winter RH should not be more than $40 \%$.
3. Purity of air

A human being feels comfortable when breathing clean air. It is thus obvious that proper filtration, cleaning and purification of air is essential to keep it free from dust and other impurities.

## 4. Motion of air

This ensures constant temperature through the conditioned space.

## Cooling load

The air conditioning systems have to carry out two types of loads known as sensible heat load \& latent heat load.

1. Heat flow through the walls, ceilings, floors, windows and doors due to temperature difference between their two sides.
2. Heat due to solar radiation
(a) Heat transferred directly by radiation through glass of windows and ventilators.
(b) Heat from Sun will be absorbed by the walls and roof and later on transferred to the room by conduction \& convection.
3. Heat received from the occupants.
4. Heat received from different equipments which are commonly used in the air conditioned building.
5. Heat received from the infiltrated air from outside through cracks in doors, windows and ventilators and through their frequent openings.

## Sources of latent heat

1. The latent heat load from the air entering into the air conditioned space by infiltration.
2. The latent heat load from the occupants.
3. The latent heat from the cooking foods and from stored materials.
4. Moisture passing directly into the air conditioned space through permeable walls where the water vapour pressure is higher.

## Room Sensible Heat Factor (RSHF)

$$
\text { RSHF }=\frac{\text { RSH }}{\text { RSH }---------~ R L H ~}
$$

RSH = Room sensible heat
RLH = Room latent heat
ADP = Apparatus dew point temperature
RSHF indicates the condition of air as it moves through air conditioned hall.
Grand Sensible Heat Factor

$$
\text { GSHF }=\frac{\text { GSH }}{\text { GSH }+--------\quad \text { GLH }}
$$

GSH = Grand sensible heat
GLH = Grand latent heat
ADP = Apparatus dew point temperature
In practice, some outside air for ventilation, and some infiltrated air from outside mixes with the moist air. Thus this outside air also adds the sensible heat and latent heat to the room which is having internal sensible heat and latent heat load.

GSHF indicates the conditioning of air as it moves through cooling coil.

$$
\text { ESHF = } \frac{\text { ERSH }}{\text { ERSH }------------~ E R L H ~}
$$

ERSH = Effective room sensible heat
ERLH = Effective room latent heat
ADP = Apparatus dew point temperature

Effective room sensible heat is composed of RSH + that portion of the outdoor air sensible heat which is considered as being bypassed through the conditioning coil.

Effective room latent heat is composed of RLH + that portion of the outside air latent heat which is considered as being bypassed through the conditioning coil.
Note :

- Infiltration air $\rightarrow$ air through windows \& doors.
- Bypassed air $\rightarrow$ air through the air distribution system without touching the conditioning coil.

