SVCE
Sri Venkateswara College of Engineering
Autonomous - Affiliated to Anna University

UNIT 1

ME18405 - FLUID MECHANICS AND MACHINERY

Dr. M.GAJENDIRAN. AP/Mech.

SYLLABUS

- **UNIT I FLUID PROPERTIES AND FLOW CHARACTERISTICS**

  Units and dimensions- Properties of fluids- mass density, specific weight, specific volume, specific gravity, viscosity, compressibility, vapor pressure, surface tension and capillarity. Pressure measurement devices - U-tube manometers, pressure gauges. Flow characteristics – concept of control volume - applications of continuity equation, energy equation and momentum equation

- **UNIT II FLOW THROUGH CIRCULAR CONDUITS**

  Hydraulic and energy gradient - Laminar flow through circular conduits and circular annuli- Boundary layer concepts – types of boundary layer thickness – Darcy Weisbach equation – friction factor- Moody diagram- commercial pipes- minor losses – Flow through pipes in series and parallel.

- **UNIT III DIMENSIONAL ANALYSIS**

  Need for dimensional analysis – methods of dimensional analysis – Similitude – types of similitude - Dimensionless parameters- application of dimensionless parameters – Model analysis.

SYLLABUS

- **UNIT IV PUMPS**

  Impact of jets - Euler's equation - Theory of roto-dynamic machines – various efficiencies– velocity components at entry and exit of the rotor- velocity triangles - Centrifugal pumps– working principle - work done by the impeller - performance curves - Reciprocating pump working principle – Rotary pumps –classifications.

- **UNIT V TURBINES**

  Classification of turbines – heads and efficiencies – velocity triangles. Axial, radial and mixed flow turbines. Pelton wheel, Francis turbine and Kaplan turbines- working principles – work done by water on the runner – draft tube. Specific speed - unit quantities – performance curves for turbines – governing of turbines..

  TOTAL: 60 PERIODS

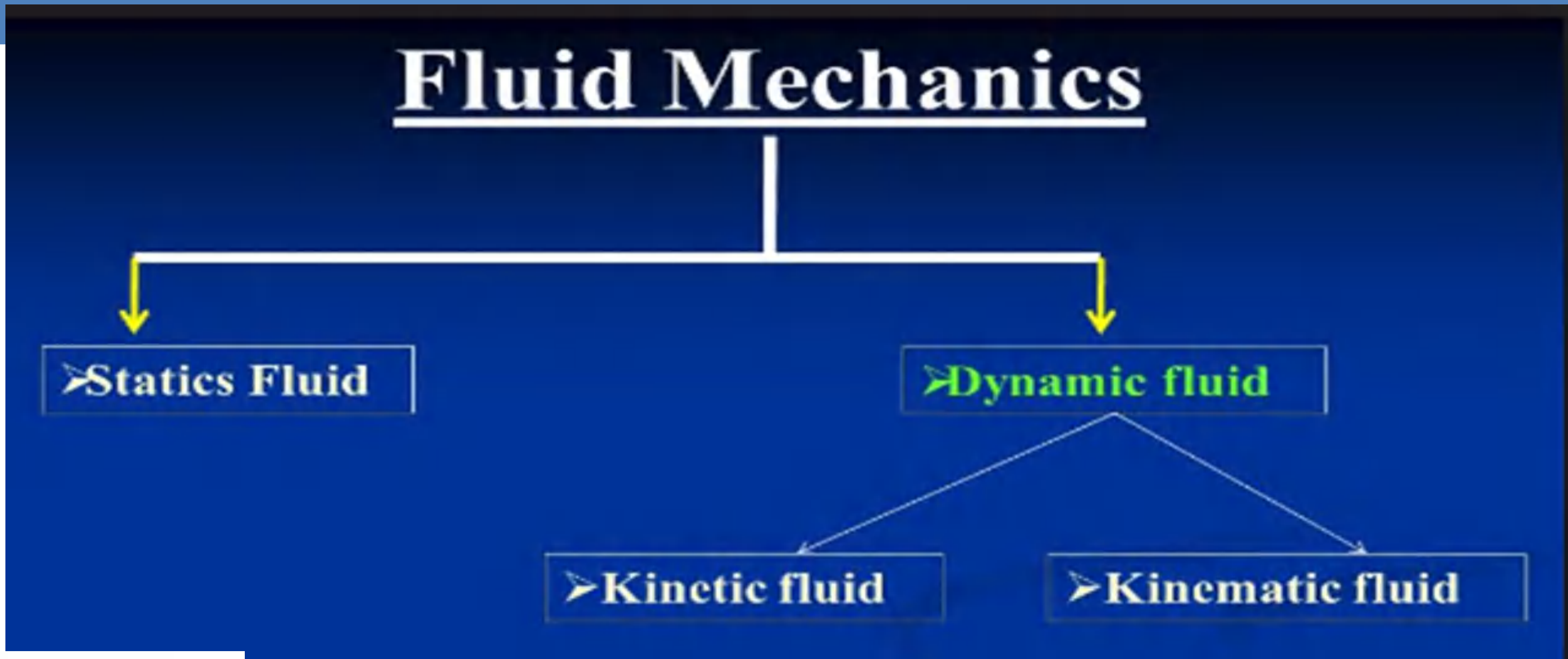
  TEXT BOOKS:

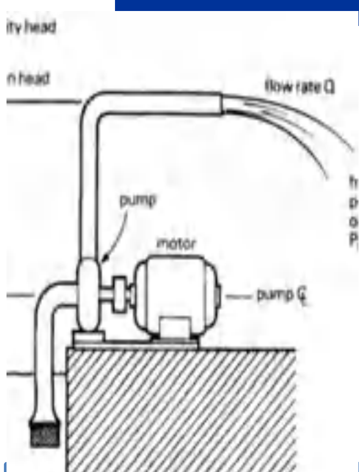
1. Bansal, R.K., "Fluid Mechanics and Hydraulic Machines", 5th edition, Laxmi Publications Pvt. Ltd, New Delhi, 2008
2. Modi P.N. and Seth, S.M. "Hydraulics and Fluid Mechanics", Standard Book House, New Delhi, 2004.

REFERENCES:

1. Fox W.R. and McDonald A.T., Introduction to Fluid Mechanics John-Wiley and Sons, Singapore, 1995.
2. Jain A. K. "Fluid Mechanics", Khanna Publishers, 2010
3. Roberson J.A and Crowe C.T., "Engineering Fluid Mechanics", Jaico Books Mumbai, 2000.
4. Streeter, V.L., and Wylie, E.B., "Fluid Mechanics", McGraw Hill, 2000.
5. White, F.M., "Fluid Mechanics", Tata McGraw Hill, 5th Edition, New Delhi, 2003.

FLUID PROPERTIES AND FLOW CHARACTERISTICS



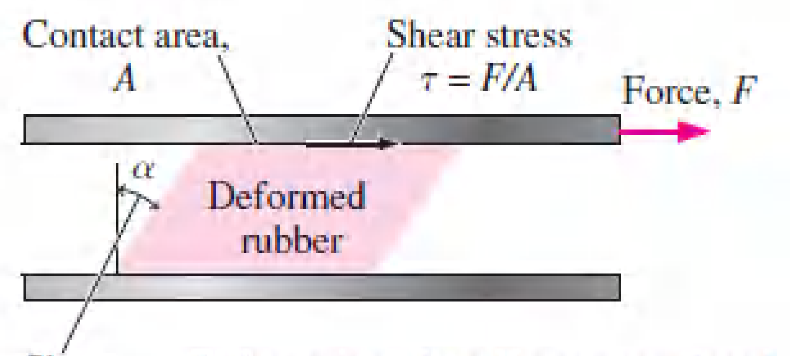


INTRODUCTION

- The branch of mechanics that deals with bodies at rest is called **statics, while the branch that deals with** bodies in motion is called **dynamics.**
- **The subcategory fluid mechanics is** defined as the science that deals with the ***interaction of fluids with solids* or other fluids behavior of fluids at rest (*fluid statics*) or in motion (*fluid dynamics*),** at the boundaries.
- Fluid mechanics itself is also divided into several categories. The study of the motion of fluids that are practically incompressible (such as liquids, especially water, and gases at low speeds) is usually referred to as **hydrodynamics.**
- A subcategory of hydrodynamics is **hydraulics, which deals with liquid** flows in pipes and open channels.
- **Gas dynamics deals with the flow of** fluids that undergo significant density changes, such as the flow of gases through nozzles at high speeds.
- **Aerodynamics deals with the** flow of gases (especially air) over bodies such as aircraft, rockets, and automobiles at high or low speeds.

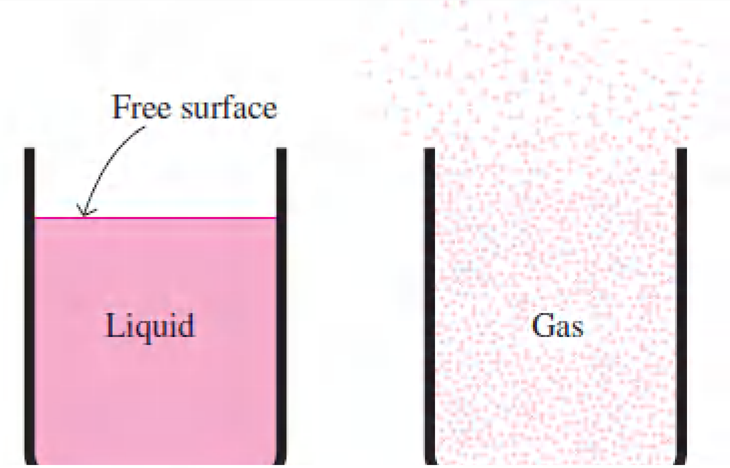
What is fluid?

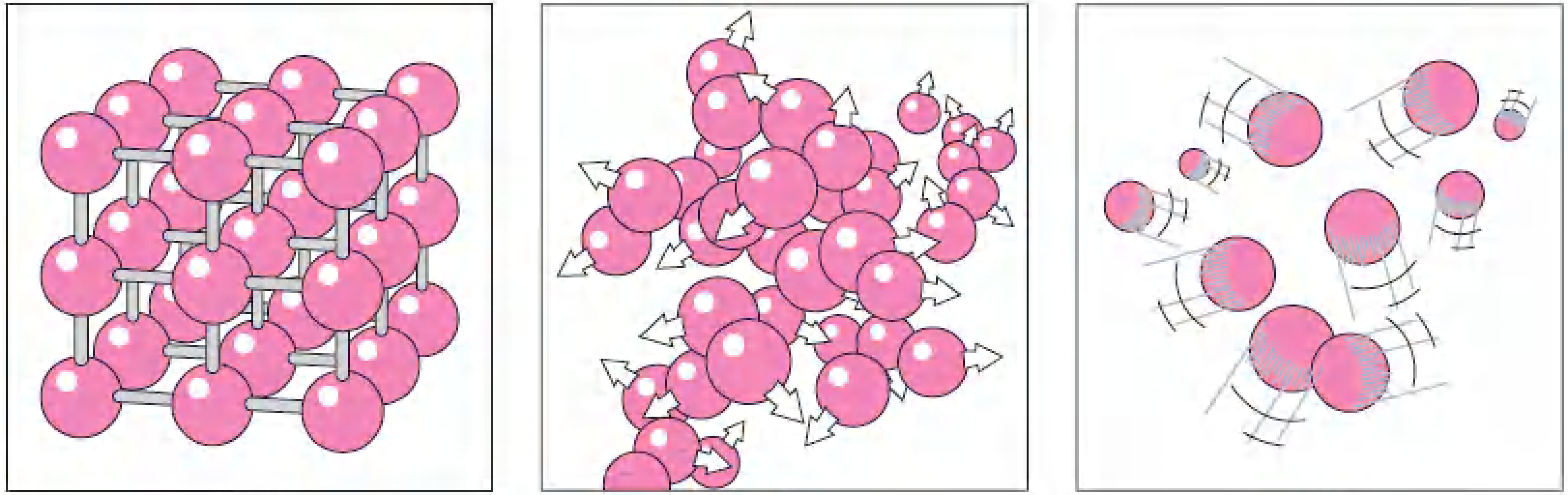
- A fluid is a substance which is capable of **flowing or which deforms continuously** when subjected to **external force (shear stress)**.
- A substance exists in three primary phases: **solid, liquid, and gas.**
- A substance in the liquid or gas phase is referred to as a **fluid.**
- **Distinction** between a solid and a fluid is made on the basis of the substance's ability to resist an applied shear (or tangential) stress that tends to change its shape.
- A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of shear stress, no matter how small.
- In solids **stress is proportional to *strain***, *but in **fluids stress is proportional** **to strain rate***.
- *When a **constant shear force is applied, a solid eventually*** **stops deforming**, at some fixed strain angle, whereas a **fluid** never stops deforming and approaches a certain rate of strain.



Deformation of a rubber eraser placed between two parallel plates under the influence of a shear force.

Unlike a liquid, a gas does not form a free surface, and it expands to fill the entire available space.





The arrangement of atoms in different phases: (*a*) molecules are at relatively fixed positions in a solid, (*b*) groups of molecules move about each other in the liquid phase, and (*c*) molecules move about at random in the gas phase.

UNITS AND DIMENSIONS

- Any physical quantity can be characterized by **dimensions.**
- **The magnitudes** assigned to the dimensions are called **units.**
- **Some basic dimensions such as** mass *m, length l, time t, and temperature T are selected as* ***primary or fundamental*** **dimensions.**
- **Velocity** *V,* ***energy E, and volume*** *V are expressed in terms of the primary dimensions and are called* ***secondary*** **dimensions, or derived dimensions.**

Standard prefixes in SI units

Multiple	Prefix
10^{12}	tera, T
10^{9}	giga, G
10^{6}	mega, M
10^{3}	kilo, k
10^{2}	hecto, h
10^{1}	deka, da
10^{-1}	deci, d
10^{-2}	centi, c
10^{-3}	milli, m
10^{-6}	micro, μ
10^{-9}	nano, n
10^{-12}	pico, p

The seven fundamental (or primary) dimensions and their units in SI

Dimension	Unit
Length	meter (m)
Mass	kilogram (kg)
Time	second (s)
Temperature	kelvin (K)
Electric current	ampere (A)
Amount of light	candela (cd)
Amount of matter	mole (mol)

Quantity	Unit symbol	Derived units
mass	kg	ton (tonne) = 1000 kg
time	s	min (60s), hr (3600s)
length	m	mm, cm, km
temperature	K, (273 + °C)	°C
force	N (newton)	kN, MN (10^6 N)
energy, work, heat	Nm, J	kJ, MJ, kNm
power	W = (Nm/s, J/s)	kW, MW
pressure	N/m², (pascal, pa)	kPa, MPa, bar (10^5Pa)

★

Secondary dimension	SI unit
Area $\{L^2\}$	m^2
Volume $\{L^3\}$	m^3
Velocity $\{LT^{-1}\}$	m/s
Acceleration $\{LT^{-2}\}$	m/s^2
Pressure or stress $\{ML^{-1}T^{-2}\}$	Pa = N/m^2
Angular velocity $\{T^{-1}\}$	s^{-1}
Energy, heat, work $\{ML^2T^{-2}\}$	J = N · m
Power $\{ML^2T^{-3}\}$	W = J/s
Density $\{ML^{-3}\}$	kg/m^3
Viscosity $\{ML^{-1}T^{-1}\}$	kg/(m · s)
Specific heat $\{L^2T^{-2}\Theta^{-1}\}$	$m^2/(s^2 \cdot K)$

Properties of Fluid

1. **Density**
2. **Specific weight**
3. **Specific volume**
4. **Specific gravity**
5. **Viscosity**
6. **Compressibility**
7. **Vapour pressure**
8. **Surface tension**
9. **capillarity**

1. DENSITY

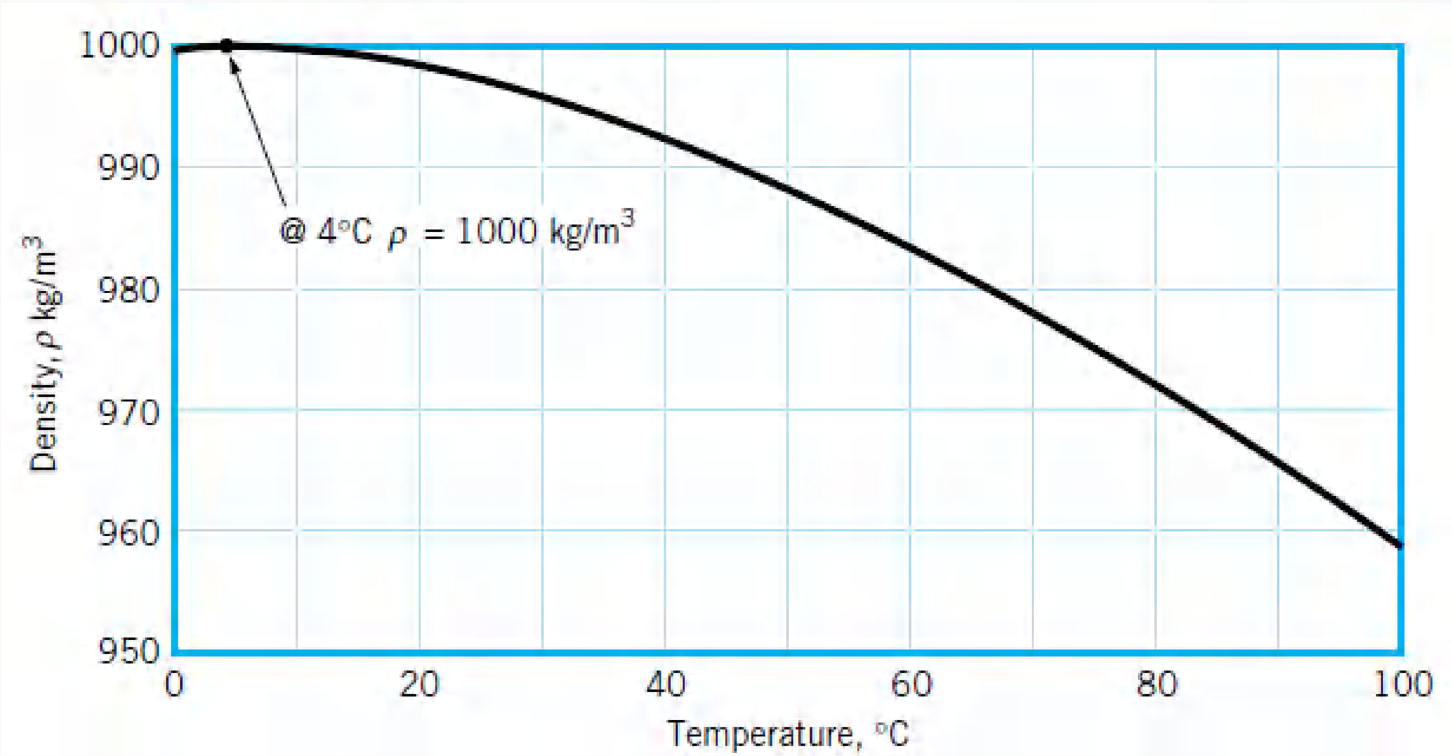
$$\rho = \frac{m}{V} \qquad (\text{kg/m}^3)$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

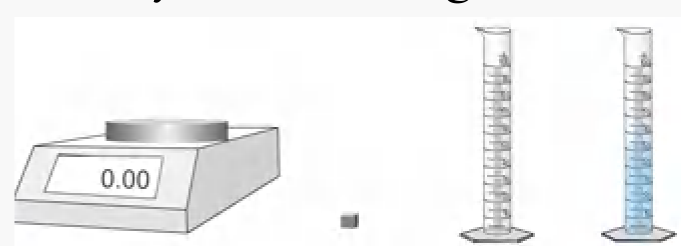
$$\text{Mass} = \text{density} \times \text{volume}$$

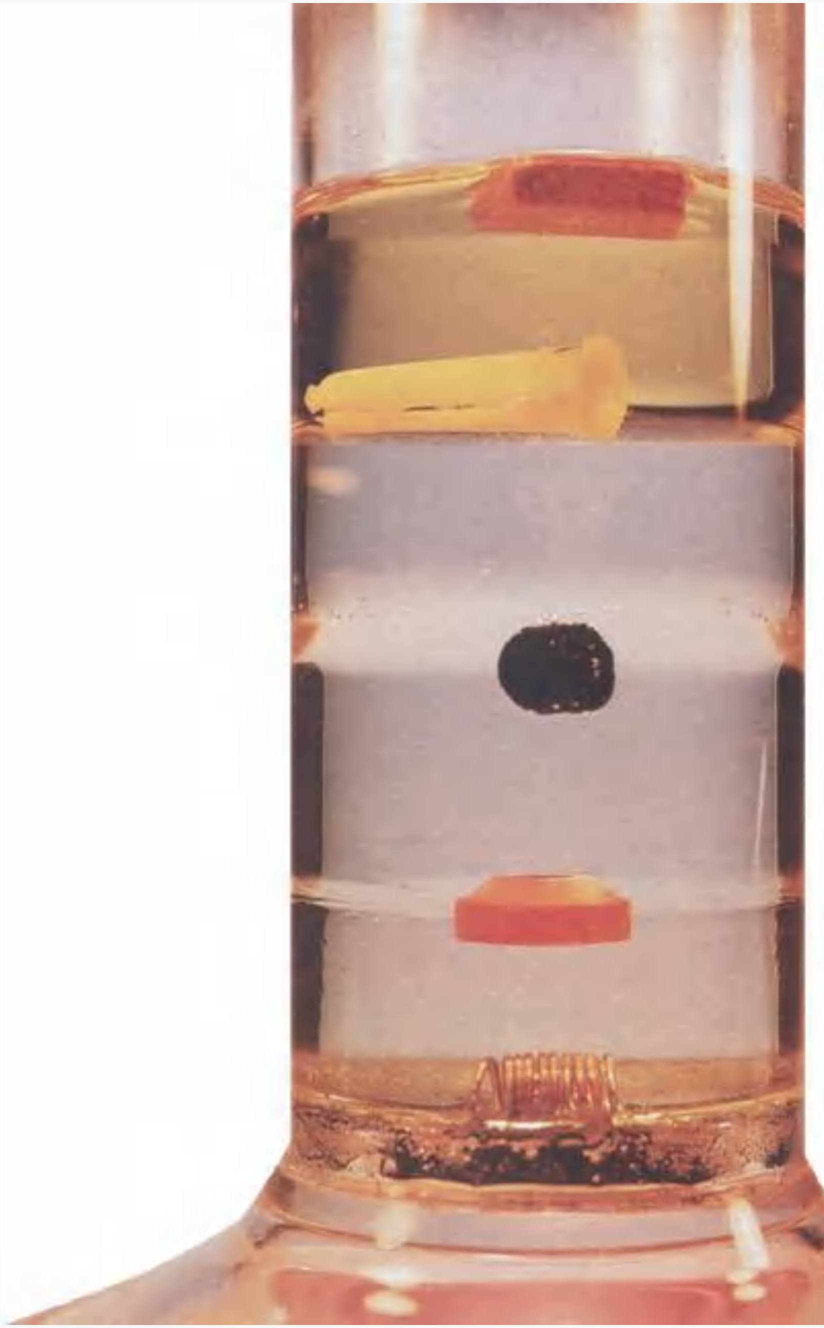
$$\text{Volume} = \frac{\text{mass}}{\text{density}}$$

- The *density of a fluid, designated by the Greek symbol ρ, is defined as its mass of fluid per* unit volume.
- Density is typically used to characterize the mass of a fluid system.
- The density of a substance depends on temperature and pressure.
- The density of most gases is proportional to pressure and inversely proportional to temperature.
- Liquids and solids are essentially incompressible substances, and the variation of their density with pressure is usually negligible.
- At 20°C, for example, the density of water changes from 998 kg/m^3 at 1 atm to 1003 kg/m^3 at 100 atm, a change of just 0.5 percent.
- The density of liquids and solids depends more strongly on temperature than it does on pressure. At 1 atm, for example, the density of water changes from 998 kg/m^3 at 20°C to 975 kg/m^3 at 75°C, a change of 2.3 percent, which can still be neglected in many engineering analyses.



- In general, liquids are about three orders of magnitude more dense than gases at atmospheric pressure.
- The heaviest common liquid is mercury, and the lightest gas is hydrogen.
- Compare their densities at 20°C and 1 atm: Mercury: 13,580 kg/m^3 Hydrogen: 0.0838 kg/m^3





Substance	Density (g/cm^3)
Wood	0.7
Corn oil	0.925
Plastic	0.93
Water	1.00
Tar ball	1.02
Glycerin	1.26
Rubber washer	1.34
Corn syrup	1.38
Copper wire	8.8
Mercury	13.6

2. SPECIFIC GRAVITY OR RELATIVE DENSITY

- It is defined as *the ratio of the density of a substance to the density of some standard substance at a specified temperature (usually water at 4°C).*

Mathematically,
$$S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

- It is a dimensionless quantity and it is denoted by SG.
- Substances with specific gravities less than 1 are lighter than water, and they would float on water.
- Engineers find these dimensionless ratio to remember the density value of various fluid.
- The numerical value of the specific gravity of a substance is exactly equal to its density in g/cm^3 or kg/L (or 0.001 times the density in kg/m^3)
- Density of water at 4°C is 1 g/cm3 = 1 kg/L = 1000 kg/m3.

Specific Gravity Measuring instrument

- Lactometer
- Hydrometer

Specific gravities of some substances at 0°C

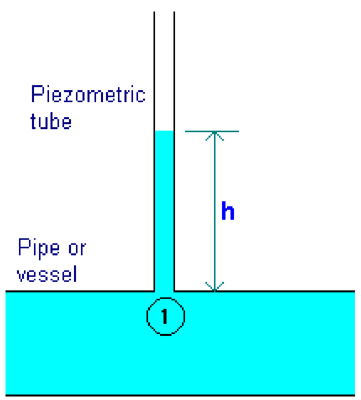
Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7–2.0
Ice	0.92
Air (at 1 atm)	0.0013

Check purity of cow's milk.

Battery condition indicator to measure the charge of the battery (~1985).

3. Specific weight of a fluid

- The *specific weight of a fluid, denoted by (lowercase Greek gamma), is its weight* per unit volume.
- It is used to characterize the weight of the system.
- It is clear that density, specific weight and specific gravity are all interrelated and from a knowledge of any one of the three the others can be calculated.
- It is very useful in the hydrostatic-pressure applications.



$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}}$$

$$= \rho \times g \qquad \left\{ \because \frac{\text{Mass of fluid}}{\text{Volume of fluid}} = \rho \right\}$$

$$w = \rho g \qquad \text{...(1.1)}$$

4. SPECIFIC VOLUME OF A FLUID

- Specific volume is defined as the volume of fluid (V) occupied per unit mass (m). It is the reciprocal of density.

$$\text{Specific Volume}, v = \frac{V}{m} \quad \frac{m^3}{kg}$$

1. Calculate the specific weight, density, specific volume and specific gravity of one litre of a liquid which weighs 7 N.

solution:

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because \; 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$$

$$(i) \quad \text{Specific weight } (w) = \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{m}^3} = 7000 \text{ N/m}^3.$$

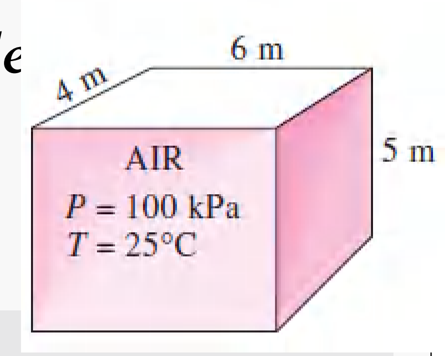
$$(ii) \quad \text{Density } (\rho) = \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = 713.5 \text{ kg/m}^3.$$

$$(iii) \quad \text{Specific gravity} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000}$$

$$= 0.7135. \text{ Ans.}$$

2. Determine the density, specific gravity, specific weight and mass of the air in a room whose dimensions are 4 m, 5 m, 6 m at 100 kPa and 25°C.

- **To find: Density, specific weight, SG and mass of the air in a room.**
- ***Assumptions:*** *At specified conditions, air can be treated as an ideal gas.*
- *Properties of air is R = 0.287 kJ/kg K.*
- ***Analysis*** *The density of air is determined from the ide*
  relation $P = \rho RT$



$$\rho = \frac{P}{RT} \qquad SG = \frac{\rho}{\rho_{H_2O}} \qquad w = \rho g \qquad m = \rho V =$$

EXAMPLE 1–3 Obtaining Formulas from Unit Considerations

A tank is filled with oil whose density is $\rho = 850\ kg/m^3$. If the volume of the tank is $V = 2\ m^3$, determine the amount of mass m in the tank.

SOLUTION The volume of an oil tank is given. The mass of oil is to be determined.

Assumptions Oil is a nearly incompressible substance and thus its density is constant.

Analysis A sketch of the system just described is given in Fig. 1–37. Suppose we forgot the formula that relates mass to density and volume. However, we know that mass has the unit of kilograms. That is, whatever calculations we do, we should end up with the unit of kilograms. Putting the given information into perspective, we have

$$\rho = 850\ \text{kg/m}^3 \quad \text{and} \quad V = 2\ \text{m}^3$$

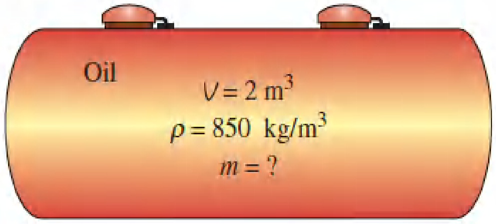
It is obvious that we can eliminate m^3 and end up with kg by multiplying these two quantities. Therefore, the formula we are looking for should be

$$m = \rho V$$

Thus,

$$m = (850\ \text{kg/m}^3)(2\ \text{m}^3) = \mathbf{1700\ kg}$$

Discussion Note that this approach may not work for more complicated formulas. Nondimensional constants also may be present in the formulas, and these cannot be derived from unit considerations alone.



VISCOSITY

- When two layers of fluid, a distance 'dy' apart, move one over the other at different velocities, say u and u+du as shown in fig.
- Viscosity together with relative velocity causes a shear stress acting between the fluid layers.

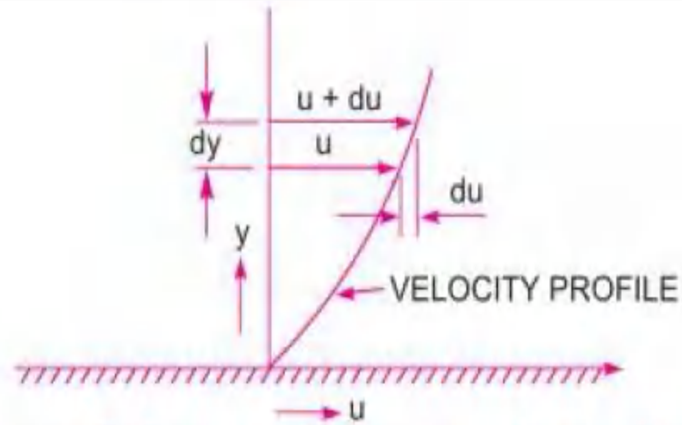


Fig. 1.1 *Velocity variation near a solid boundary.*

The top layer causes a shear on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

The shear stress is proportional to the **rate of change of velocity with respect to y.**

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

$\frac{du}{dy}$, velocity gradient

Dynamic viscosity, $\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$

One poise $= \frac{1}{10} \frac{Ns}{m^2}$.

Unit of viscosity: Ns/m^2 (Pa.s)

Newton's law of viscosity:

Newton's law of viscosity is states that **shear stress on a fluid element layer is directly proportional to the velocity gradient.**

$$\tau \propto \frac{du}{dy}$$

shear stress, $\tau = \mu \dfrac{du}{dy}$ (N/m^2)

μ: coefficient of viscosity (Dynamic viscosity)

$\dfrac{du}{dy}$ is Velocity gradient

Kinematic viscosity

Kinematic viscosity, ν is defined as the ratio of the viscosity to the density;

$$\nu = \frac{\mu}{\rho}$$

Units m^2/s

Water ν = 1.7 × 10^{-6} m^2/s.

Air ν **= 1.5** × 10^{-5} m^2/s.

- **Unit is kg/m·s** = N·s/m^2 =Pa.s .
- **Shear force acting on a Newtonian**

fluid layer (Newton's third law, the force acting on the plate) is

- The force *F required to move the upper plate at*

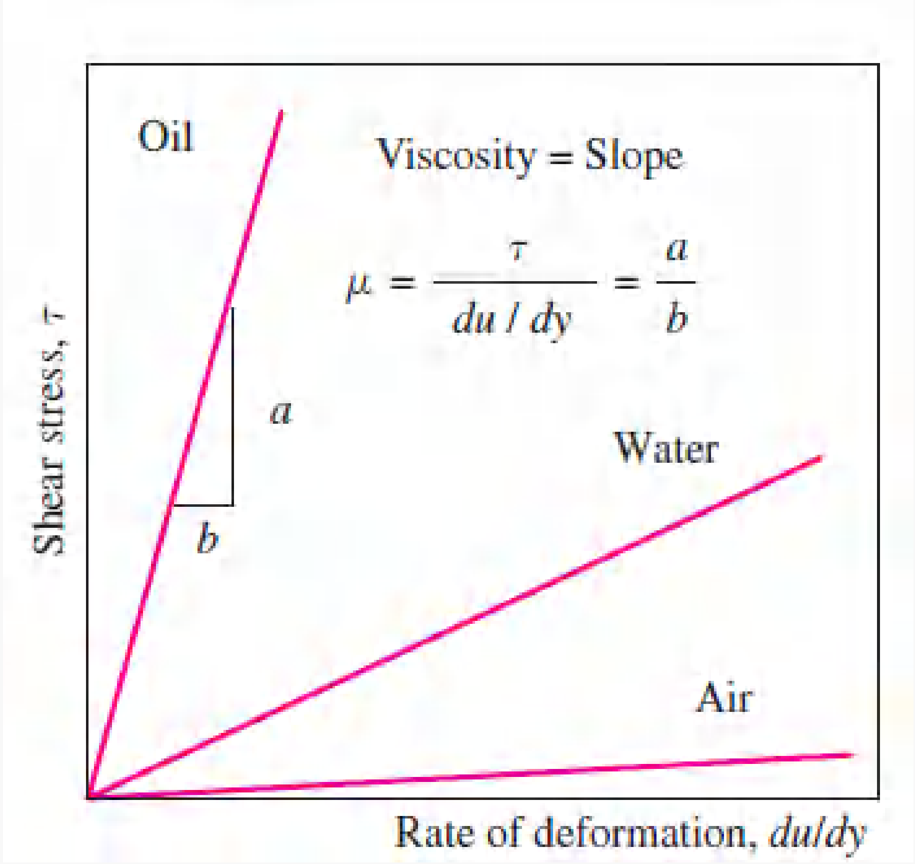
a constant velocity of *V while the lower plate*

remains stationary is

$$F = \tau A = \mu A \frac{du}{dy} \qquad \text{(N)}$$

$$F = \tau A = \mu A \frac{du}{dy} \qquad \text{(N)}$$

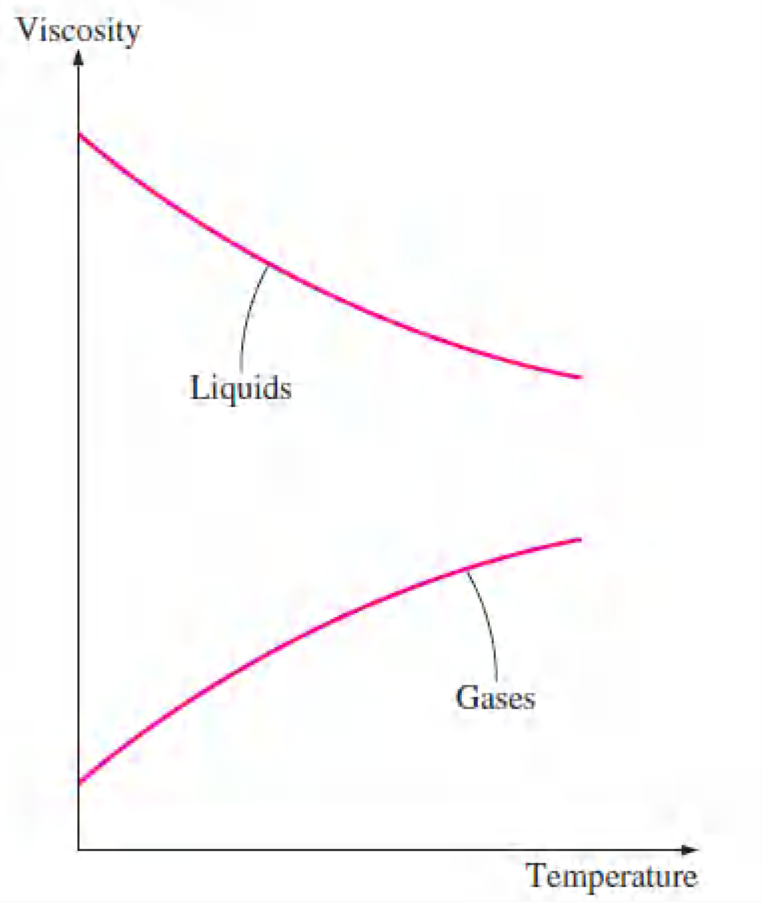
$$F = \mu A \frac{V}{\ell} \qquad \text{(N)}$$



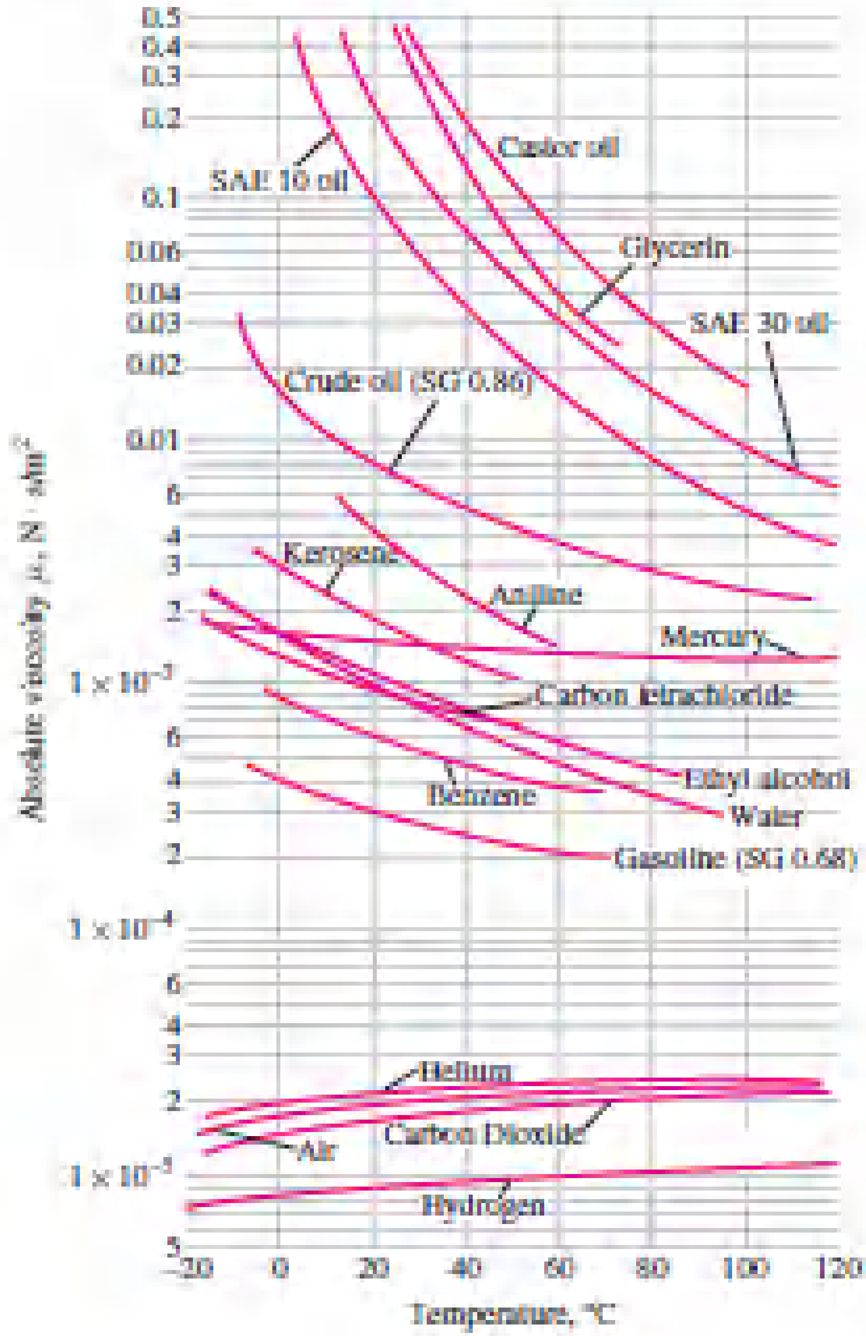
VARIATION OF VISOCITY WITH TEMPERATURE

Temperature of liquid is increases	Cohesive forces b/w the molecules decreases	Viscosity of liquid decreases
Temperature of gas is increases	Molecular momentum b/w the molecules increases	Viscosity of gas increases

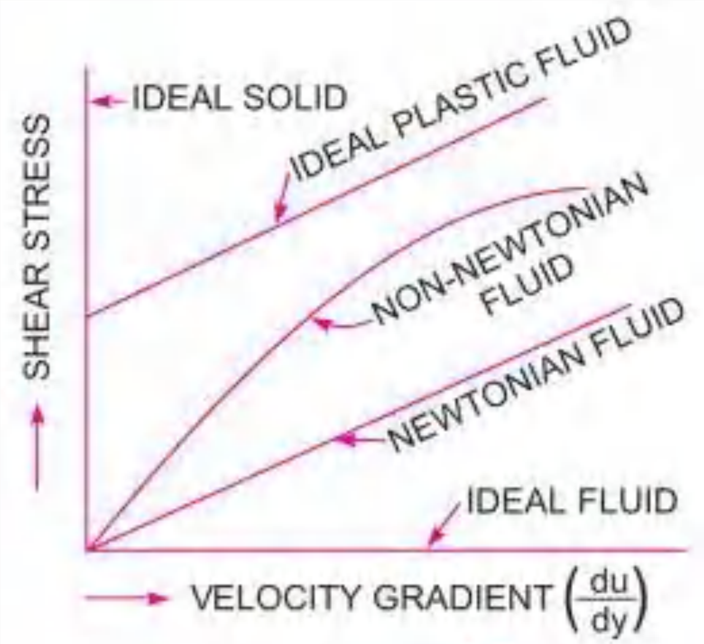
- The viscosity of a fluid is a measure of its "resistance to deformation."
- Viscosity is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other.
- Viscosity is caused by the cohesive forces between the molecules in liquids and by the molecular collisions in gases, and it varies greatly with temperature.

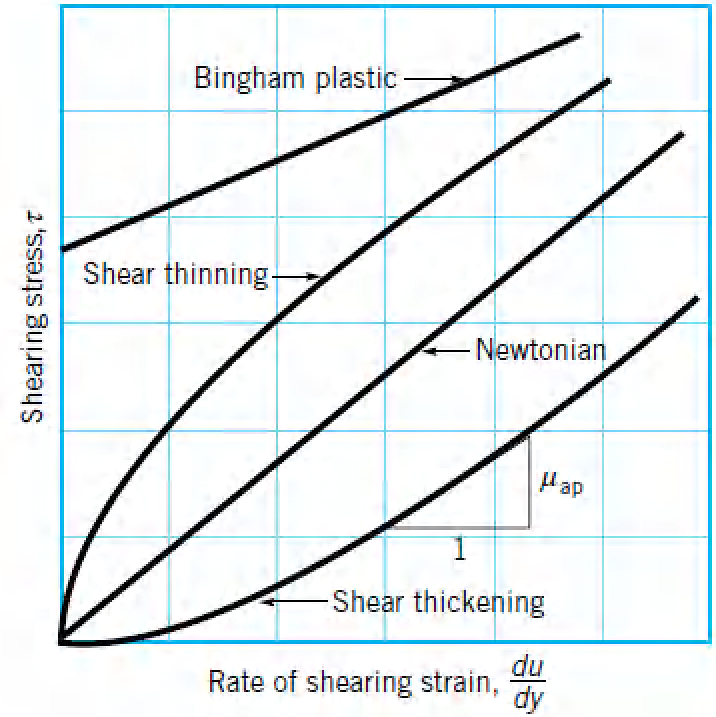


- In fluid mechanics the ratio of dynamic viscosity to density appears frequently.
- For convenience, this ratio is given the name **kinematic viscosity(nu) υ=μ/ρ**
- **Common units** are m^2/s and **stoke**
- **1 stoke = 1 cm^2/s = 0.0001 m^2/s**
- **In general, the viscosity of a fluid depends on both temperature and pressure**.
- Although the dependence on pressure is rather weak.
- For *liquids, both* the dynamic and kinematic viscosities are practically independent of pressure.
- For *gases, this is also the case for dynamic viscosity* (at low to moderate pressures), but not for kinematic viscosity since the density of a gas is proportional to its pressure.



TYPES OF FLUIDS





$\tau \propto \frac{du}{dy}$	TYPE OF FLUIDS
LINEAR	**NEWTONIAN FLUID**
NON-LINEAR	**NON-NEWTONIAN FLUID**

Non-Newtonian fluids	μ_{ap}	du/dy	Viscous	Eg
Shear Thinning Fluid	D	I	Less	Blood, milk, syrup, paint, liquid cement
Shear Thickening Fluid	I	I	More	solutions with suspended starch or sand, printing ink, Sugar in water,

- *Bingham plastic, which is r*
- Such material can wi ss without motion (therefore, it is not a flu is exceeded it flows like a fluid (hence, it is r
- Toothpaste and chocol of Bingham plastic materials.

Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)

Fluid	Dynamic Viscosity μ, kg/m · s
Glycerin:	
−20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088

1. The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 Poise. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.

Given Data:

$\mu = 6$ poise

Viscosity= $= \frac{6}{10}\frac{\text{N s}}{\text{m}^2} = 0.6\ \frac{\text{N s}}{\text{m}^2}$

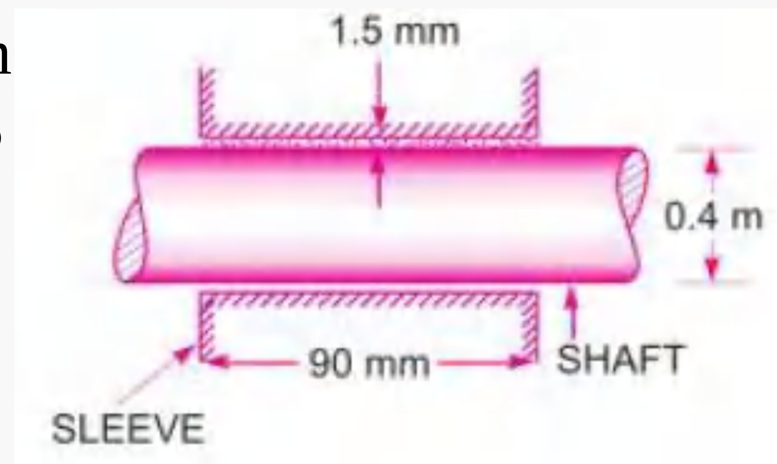
Dia. of shaft D = 0.4 m,

r = 0.2 m

Speed of shaft N = 190 rpm

Sleeve length L = 90 mm = 90 x 10^{-3} m

Thickness of oil Film t = 1.5 mm = 1.5



Solution:

Tangential velocity of shaft, $u = \frac{\pi D N}{60} = \frac{\pi \times 0.4 \times 190}{60} = 3.98$ m/s

$$\tau = \mu \frac{du}{dy}$$

where du = Change of velocity = u- 0 = u = 3.98 m/s

dy = Change of distance = t = 1.5 x 10^{-3} m = 0.0015m

Shear stress = 0.6 x (3.98/ 0.0015)= 1592 N/m^2

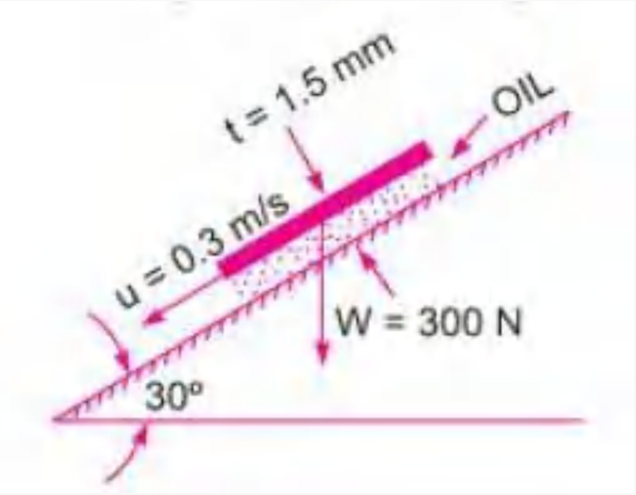
Shear force on the shaft, F = Shear stress x Area

= 1592 x π x D x L = 1592 x π x 0.4 x 90 x 10^{-3} = 180.05 N

Torque on the shaft, T = Force x R = 180.05 x 0.2 = 36.01 Nm

Power lost $= \frac{2\pi N T}{60} = \frac{2\pi \times 190 \times 36.01}{60} = 716.48$ W.

2. Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m x 0.8 m and an inclined plane with angle of inclination 30° as shown in Fig. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.



Given Data:

Area of plate A = 0.8 x 0.8 = 0.64 m^2

Angle of plane, θ= 30°

Weight of plate, W = 300 N

Velocity of plate, u = 0.3 m/s

Solution:

Thickness of oil film, = dy = 1.5 mm = 1.5 x 10^{-3} m

Let the viscosity of fluid between plate and inclined plane is μ.

Component of weight W, along the plane = W cos 60° = 300 cos 60° = 150 N

Thus the shear force, F, on the bottom surface of the plate = 150 N

Shear stress = $$\tau = \frac{F}{\text{Area}} = \frac{150}{0.64} \text{ N/m}^2$$

We know that $$\tau = \mu \frac{du}{dy}$$

where du = change of velocity = u - 0 = u = 0.3 m/s

dy = t= 1.5 x 10^{-3} m

$$\frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = \mathbf{11.7 \text{ poise.}}$$

3.A flat plate of area 1.5×10^6 mm² is pulled with a speed of 0.4 m/s relative to another plate located at a distance of 0.15 mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity as 1 poise.

Given :

Area of the plate, A= $1.5 \times 10^6 \text{ mm}^2 = 1.5 \text{ m}^2$

Speed of plate relative to another plate, $du = 0.4$ m/s

Distance between the plates, $dy = 0.15 \text{ mm} = 0.15 \times 10^{-3}$ m

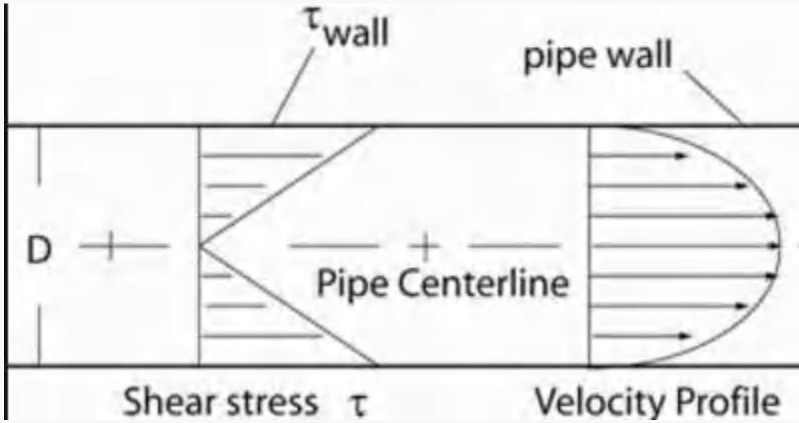
$$\mu = 1 \text{ poise} = \frac{1}{10}\frac{\text{Ns}}{\text{m}^2}.$$

$$\tau = \mu \frac{du}{dy} = \frac{1}{10} \times \frac{0.4}{.15 \times 10^{-3}} = 266.66 \frac{\text{N}}{\text{m}^2}$$

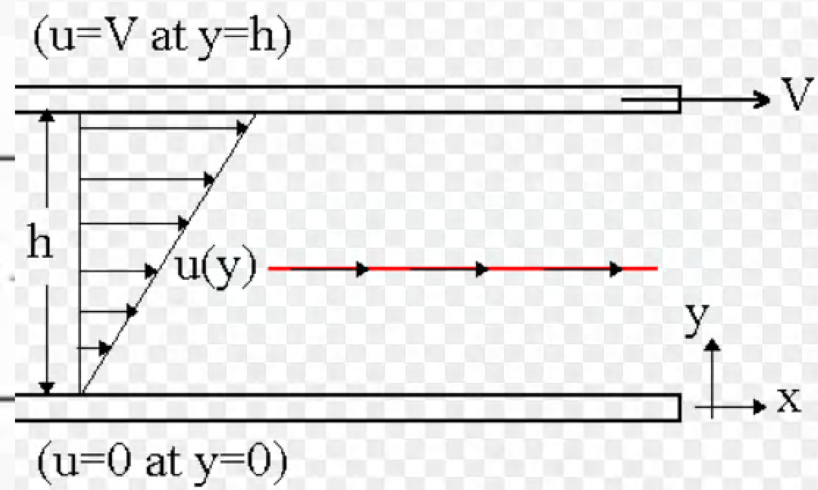
Shear force, $F = \tau \times \text{area} = 266.66 \times 1.5 = 400$ N.

Power required to move the plate at the speed 0.4 m/sec

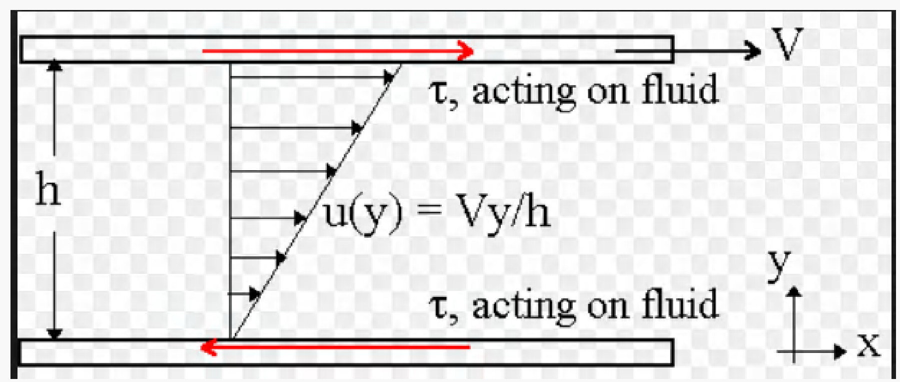
$= F \times u = 400 \times 0.4 = 160$ W



τ_{wall}
pipe wall
D
Pipe Centerline
Shear stress τ
Velocity Profile

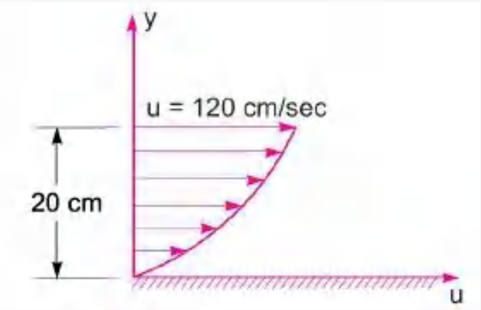


(u=V at y=h)
V
h
u(y)
y
x
(u=0 at y=0)



V
τ, acting on fluid
h
u(y) = Vy/h
y
τ, acting on fluid
x

4. If the velocity profile of a fluid over a plate is parabolic with the vertex 20 cm from the plate, where the velocity is 120 cm/sec. Calculate the velocity gradients and shear stresses at a distance of 0, 10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.



Given Data:

Distance of vertex from plate = 20 cm

Velocity at vertex $u = 120$ cm/s

Viscosity $\mu = 8.5$ poise $= 0.85\ Ns/m^2$

The velocity profile is given parabolic and equation of velocity profile is

$$u = ay^2 + by + c \text{ ----- (i)}$$

where a, b and c are constants. Their values are determined from boundary conditions as:

(a) at $y = 0$, $u = 0$

(b) at $y = 20$ cm, $u = 120$ cm/s

(c) at $y = 20$ cm, $du/dy = 0$.

Substituting boundary condition (a) in equation (i), we get

$$c = 0.$$

Boundary condition (b) on substitution in (i) gives

$120 = a(20)^2 + b(20) = 400a + 20b$ …….(ii)

Boundary condition (c) on substitution in equation (i) gives

$du/dy = 2ay + b$

or $0 = 2 \times a \times 20 + b = 40a + b$ …….(iii)

Solving equations (ii) and (iii) for a and b

From equation (iii), $b = -40a$

Substituting this value in equation ii), we get

$120 = 400a + 20 \times (-40a) = 400a - 800a =$ **$-400a$**

$a = -0.3$

$b = -40 \times (-0.3)$ $=$ 12 from equ (iii)

Substituting the values of a, b and c in equation (i),

$$\mathbf{u = -0.3y^2 + 12y}$$

- Velocity Gradient

$$\frac{du}{dy} = -0.3 \times 2y + 12 = -0.6y + 12$$

at $y = 10$ cm, $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = -6 + 12 = \mathbf{6/s}$. **Ans.**

at $y = 20$ cm, $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = -12 + 12 = \mathbf{0}$. **Ans.**

Shear stress is given by, $\tau = \mu \frac{du}{dy}$

(*i*) Shear stress at $y = 0$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.85 \times 12.0 = 10.2 \text{ N/m}^2$.

(*ii*) Shear stress at $y = 10$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=10} = 0.85 \times 6.0 = 5.1 \text{ N/m}^2$.

(*iii*) Shear stress at $y = 20$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=20} = 0.85 \times 0 = \mathbf{0}$. **Ans.**

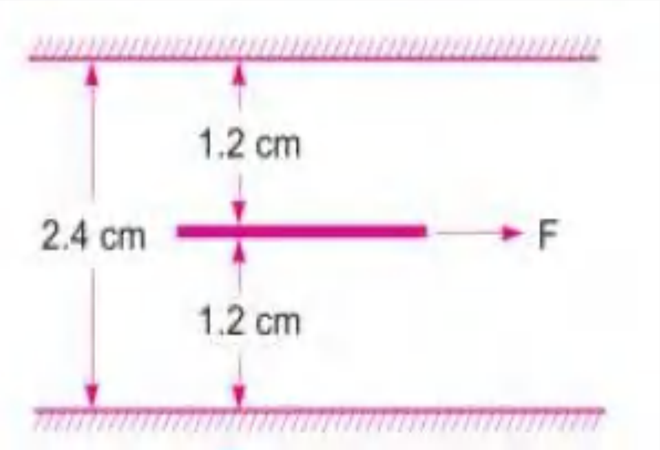
5. Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerin. What force is required to drag a very thin plate of surface area 0.5 square meter between the two large plane surfaces at a speed of 0.6 m/s, if:

(i) the thin plate is in the middle of the two plane surfaces, and
(ii) the thin plate is at a distance of 0.8 cm from one of the plane surfaces ?

Take the dynamic viscosity of glycerine = 8.10×10^{-1} N s/m^2.

Given Data:

Distance between two large surfaces = 2.4 cm
Area of thin plate, $A = 0.5$ m^2
Velocity of thin plate, $u = 0.6$ m/s
Viscosity of glycerin, $\mu = 8.10 \times 10^{-1}$ N s/m^2



Case I: When the thin plate is in the middle of the two plane surfaces
F_1 = Shear force on the upper side of the thin plate
F_2 = Shear force on the lower side of the thin plate
F = Total force required to drag the plate = $F_1 + F_2$

- The shear stress (τ) on the upper side of the thin plate is given by equation, $$\tau_1 = \mu\left(\frac{du}{dy}\right)_1$$

du = Relative velocity between thin plate and upper large plane surface

= 0.6 m/sec

dy = Distance between thin plate and upper large plane surface

= 1.2 cm = 0.012 m (plate is a thin one and hence thickness of plate is neglected)

$$\tau_1 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{.012}\right) = 40.5\ \text{N/m}^2$$

Now shear force, F_1 = Shear stress x Area

= τ_1 x A = 40.5 x 0.5 = 20.25 N

Similarly shear stress (τ_2) on the lower side of the thin plate is given by

$$\tau_2 = \mu\left(\frac{du}{dy}\right)_2 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.012}\right) = 40.5\ \text{N/m}^2$$

Shear force: $F_2 = \tau_2$ x A = 40.5 x 0.5 = 20.25 N

Total force: $F = F_1 + F_2$ = 20.25 + 20.25 = 40.5 N

CaseII:

When the thin plate is at a distance of 0.8 cm from one of the plane surfaces

Let the thin plate is at a distance 0.8 cm from the lower plane surface.

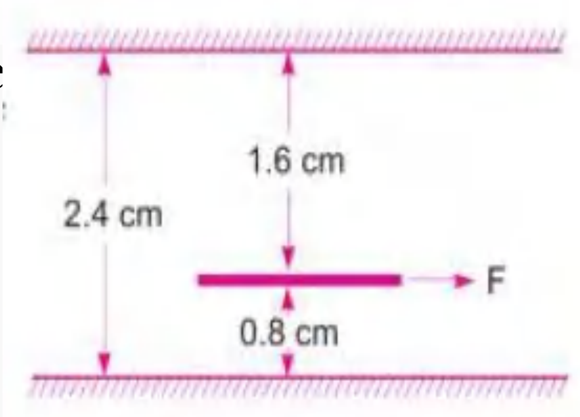
Distance of the plate from the upper plane surface = 2.4- 0.8 = 1.6 cm = 0.016 m (Neglecting thickness of the plate)

The shear force on the upper side of the thin plate,

$$F_1 = \text{Shear stress x Area} = \tau_1 \text{ x } A$$

$$= \mu \left(\frac{du}{dy}\right)_1 \times A = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.016}\right) \times 0.5 = 15.18 \text{ N}$$

The shear force on the lower side of the thin plate



$$F_2 = \tau_2 \times A = \mu \left(\frac{du}{dy}\right)_2 \times A$$

$$= 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.8/100}\right) \times 0.5 = 30.36 \text{ N}$$

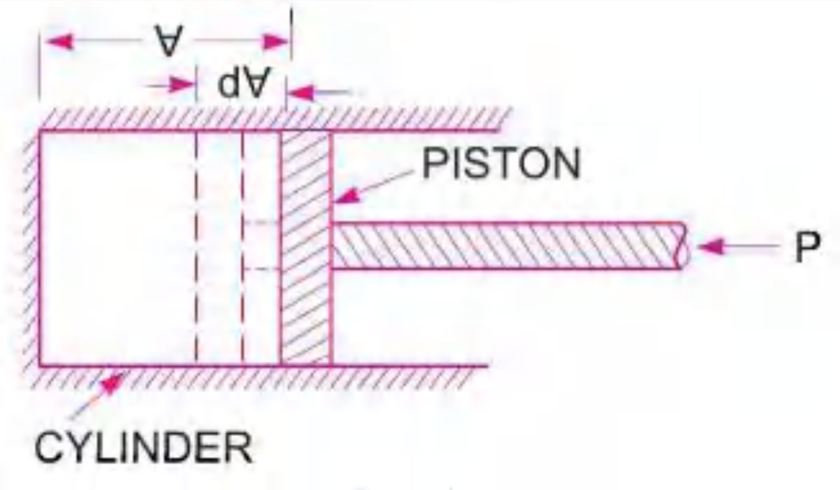
Total force required = $F_1 + F_2$ = 15.18 + 30.36 = **45.54 N. Ans.**

COMPRESSIBILITY

- Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.

Let V = Volume of a gas enclosed in the cylinder

- p = Pressure of gas when volume is V
- Let the pressure is increased to $p + dp$, the volume of gas decreases from V to V - dV.



Then increase in pressure $= dp$ kgf/m^2

Decrease in volume $= d\forall$

$\therefore$ Volumetric strain $= -\dfrac{d\forall}{\forall}$

– vc sign mcans thc volumc dccrcascs with incrcasc of prcssurc.

$\therefore$ Bulk moduius $K = \dfrac{\text{Increase of pressure}}{\text{Volumetric strain}}$

$$= \frac{dp}{\frac{-d\forall}{\forall}} = \frac{-dp}{d\forall}\forall$$

Compressibility $= \dfrac{1}{K}$

Compressibility

A fluid flow during which the **density of the fluid remains nearly constant** is called *incompressible flow*.
A flow in which **density varies significantly** is called *compressible flow*.

- All fluids compress if pressure increases resulting in an increase in density
- Compressibility is the change in volume due to a change in pressure
- A good measure of compressibility is the bulk modulus (It is inversely proportional to compressibility)

$$E_\upsilon = -\upsilon \frac{dp}{d\upsilon} \qquad \upsilon = \frac{1}{\rho} \quad (specific\ volume)$$

p *is pressure*

Compressibility

- From previous expression we may write

$$\frac{(\upsilon_{final} - \upsilon_{initial})}{\upsilon_{initial}} \approx -\frac{(p_{final} - p_{initial})}{E_\upsilon}$$

- For water at 15 *psia* and 68 degrees Farenheit, $E_\upsilon = 320{,}000\ psi$

- From above expression, increasing pressure by 1000 *psi* will compress the water by only 1/320 (0.3%) of its original volume

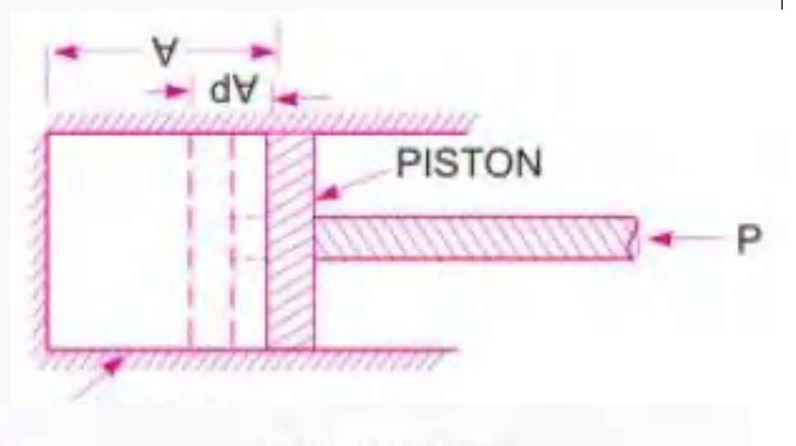
- Thus, water may be treated as incompressible (density ρ is constant)

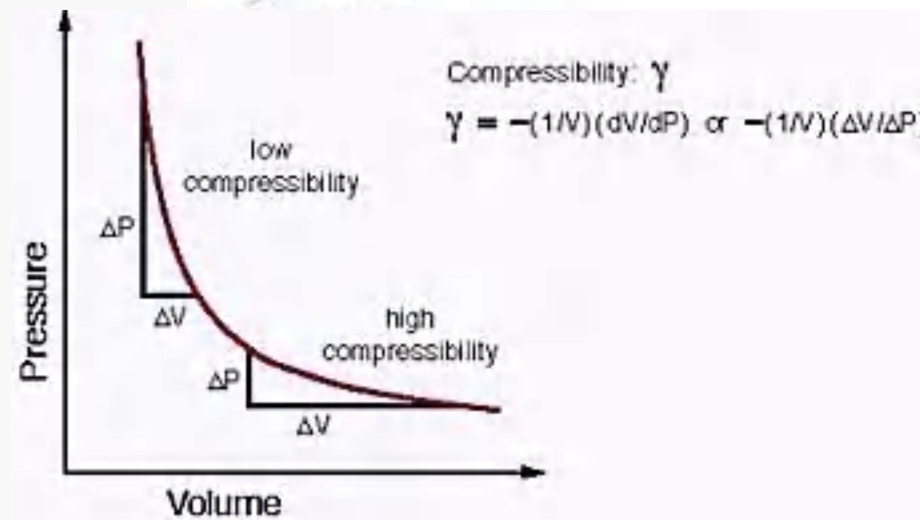
- In reality, no fluid is incompressible, but this is a good approximation for certain fluids

COMPRESSIBILITY AND BULK MODULUS

- Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.
- Consider a cylinder fitted with a piston as shown
  Let V = Volume of a gas enclosed in the cylinder
  p = Pressure of gas when volume is V
  Let the pressure is increased to p + dp, the
- volume of gas decreases from V to V - dV.
  Then increase in pressure = dp
  Decrease in volume = dV
  Volumetric strain = - dV/V
- Negative sign means the volume

decreases with increase of pressure.





Compressibility = 1/K

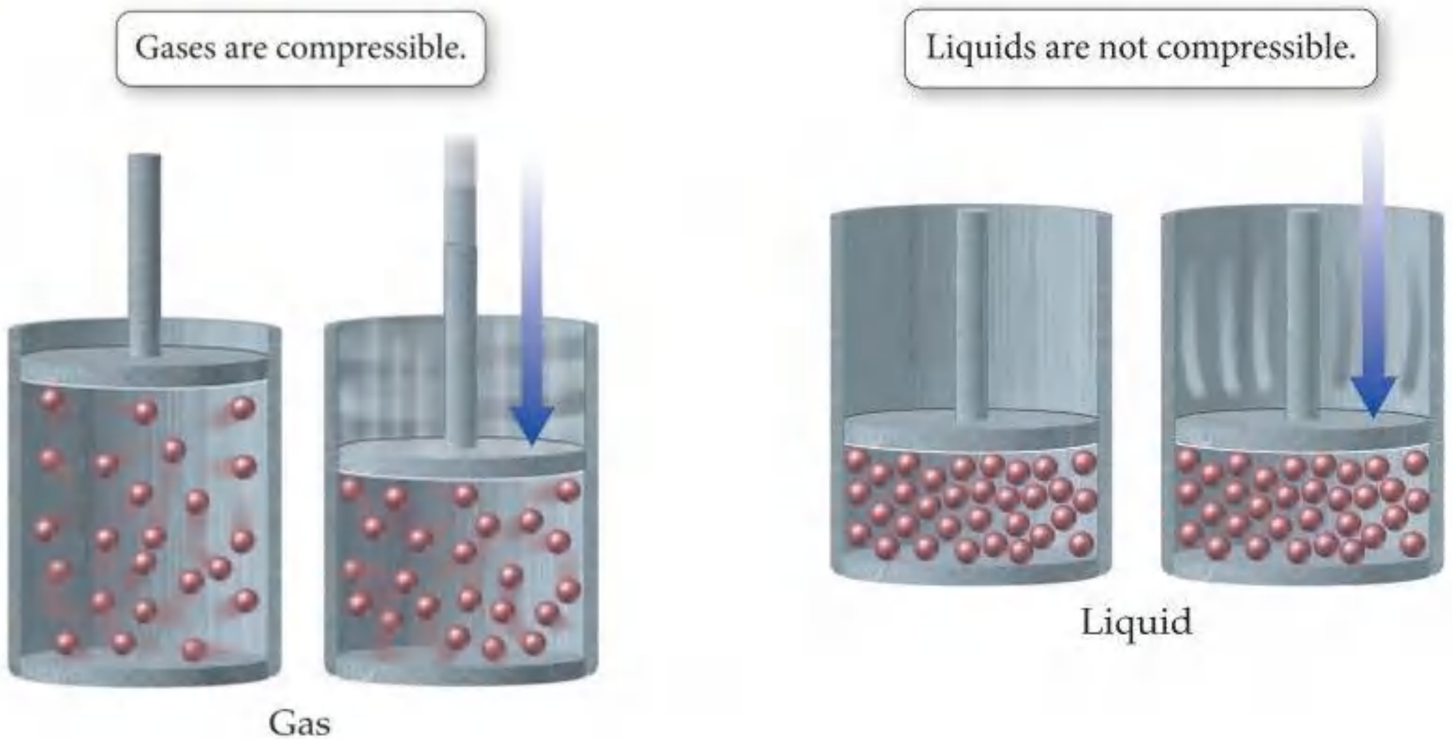
- Coefficient of compressibility k (also called the bulk modulus of compressibility or bulk modulus of elasticity) for fluids defined as increase of pressure to volumetric strain.

$$\kappa = -v\left(\frac{\partial P}{\partial v}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T \qquad \kappa \cong -\frac{\Delta P}{\Delta v/v} \cong \frac{\Delta P}{\Delta \rho/\rho} \qquad (T = \text{constant})$$

- **Coefficient of compressibility** represents the change in pressure corresponding to a fractional change in volume or density of the fluid while the temperature remains constant.
- The **inverse** of the coefficient of compressibility is called the **isothermal compressibility of a fluid** which represents the fractional change in volume or density corresponding to a unit change in pressure

$$\alpha = \frac{1}{\kappa} = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T$$

Compressibility of gases (left): Gases are compressible because there is so much empty space between gas particles.
Incompressibility of liquids (right): Liquids are not compressible because there is so little space between the liquid particles.



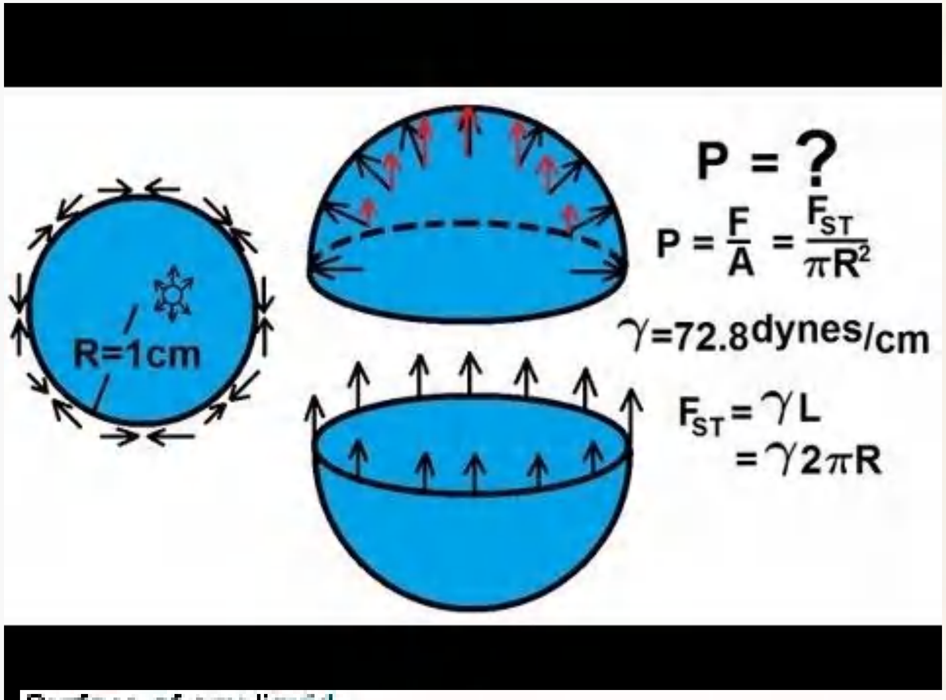
SURFACE TENSION

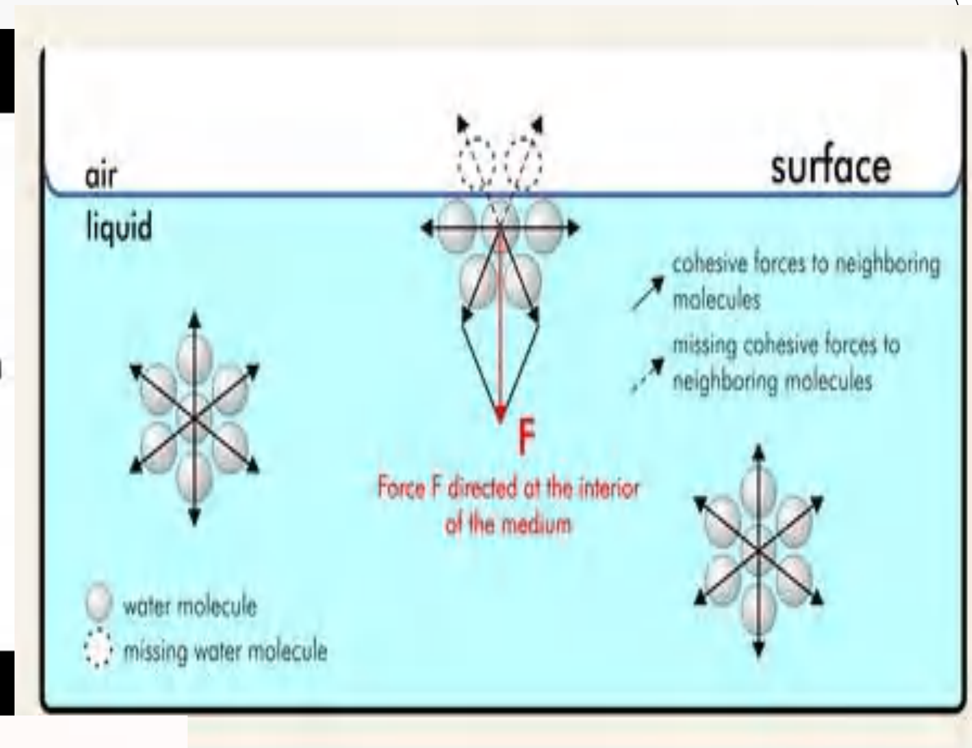
- It arises due to the two kinds of intermolecular forces;

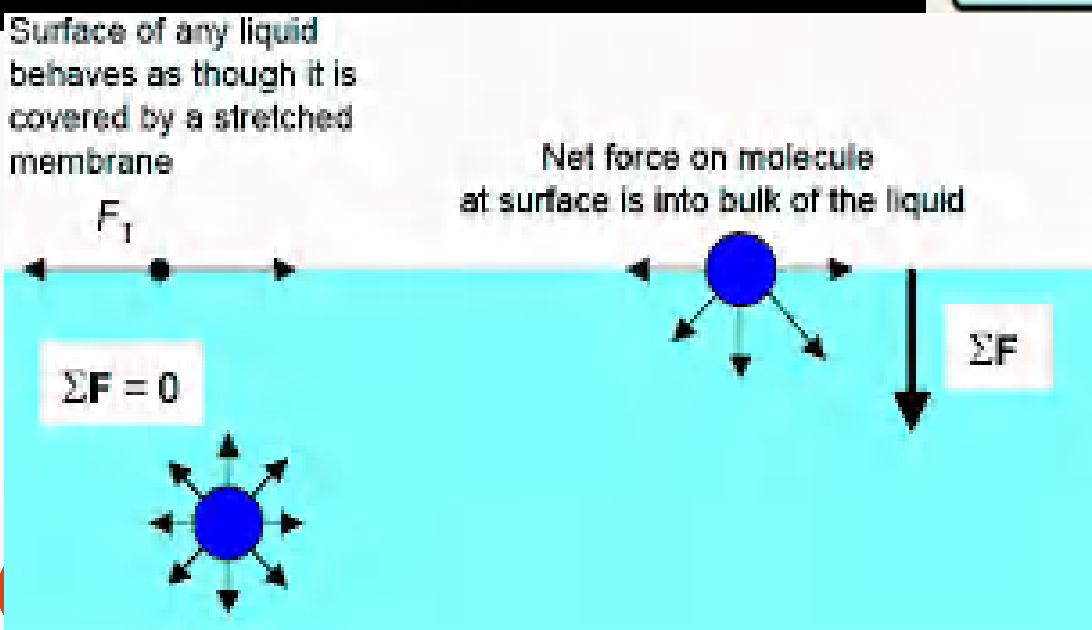
1) Cohesion (Intermolecular attraction b/w like molecules) ←ST

2) Adhesion (Intermolecular attraction b/w unlike molecules)

- At the interface between a liquid and a gas, or between two immiscible liquids, forces develop in the liquid surface which cause the surface to behave as "skin" or "membrane" stretched over the fluid mass.
- A liquid, being unable to expand freely, will form an *interface with a second liquid or* gas.
- Molecules deep within the liquid repel each other because of their close packing.
- Molecules at the surface are less dense and attract each other.
- The pulling force that causes this tension acts parallel to the surface and is due to the attractive forces between the molecules of the liquid.
- The magnitude of this force per unit length is called **surface tension** σ_s ***and is usually expressed in the unit N/m.***
- ***It is caused by the force of cohesion at the free surface.***


R=1cm
P = ?
$P = \frac{F}{A} = \frac{F_{ST}}{\pi R^2}$
γ=72.8dynes/cm
$F_{ST} = \gamma L$
$= \gamma 2\pi R$

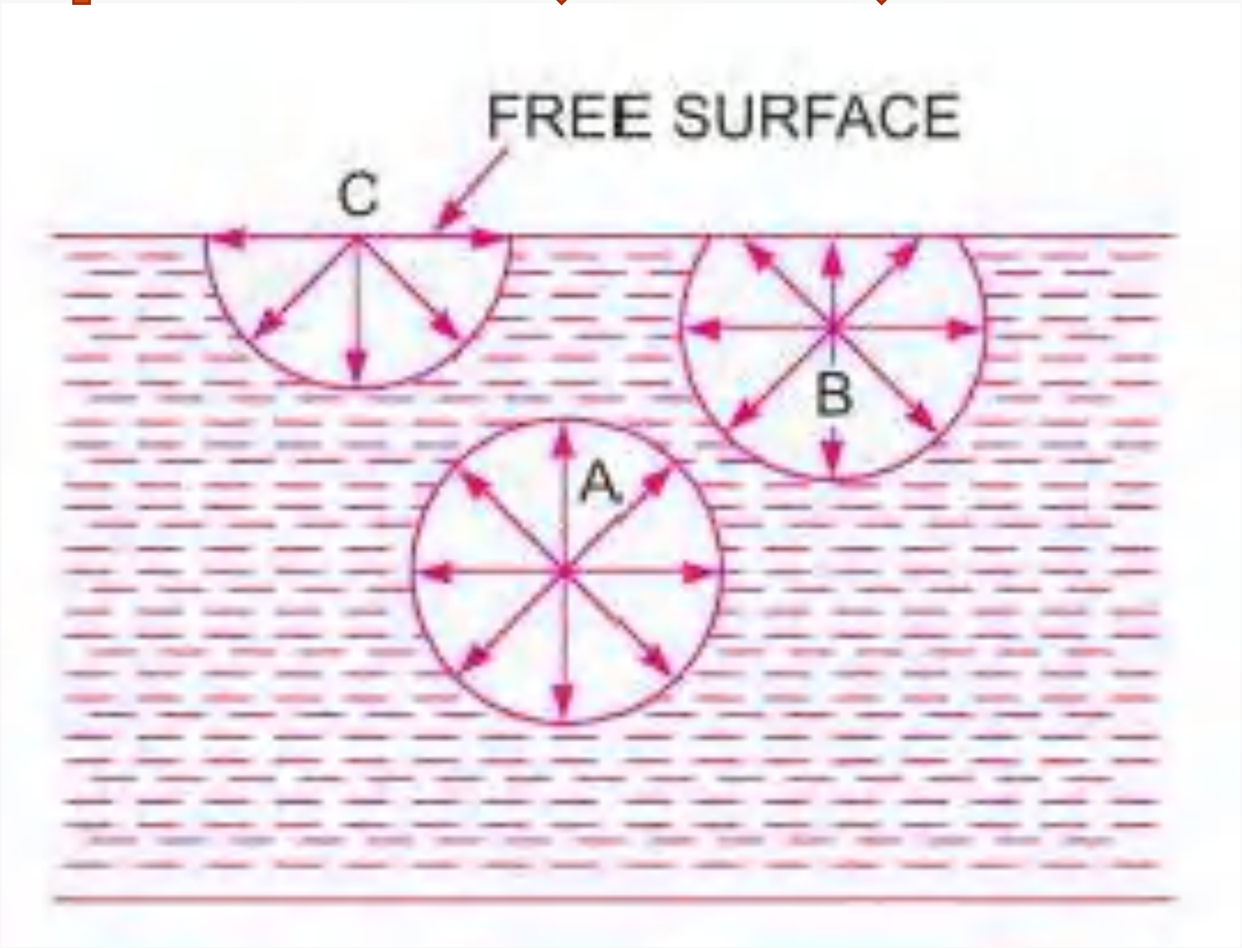

air
surface
liquid
cohesive forces to neighboring molecules
missing cohesive forces to neighboring molecules
F
Force F directed at the interior of the medium
water molecule
missing water molecule

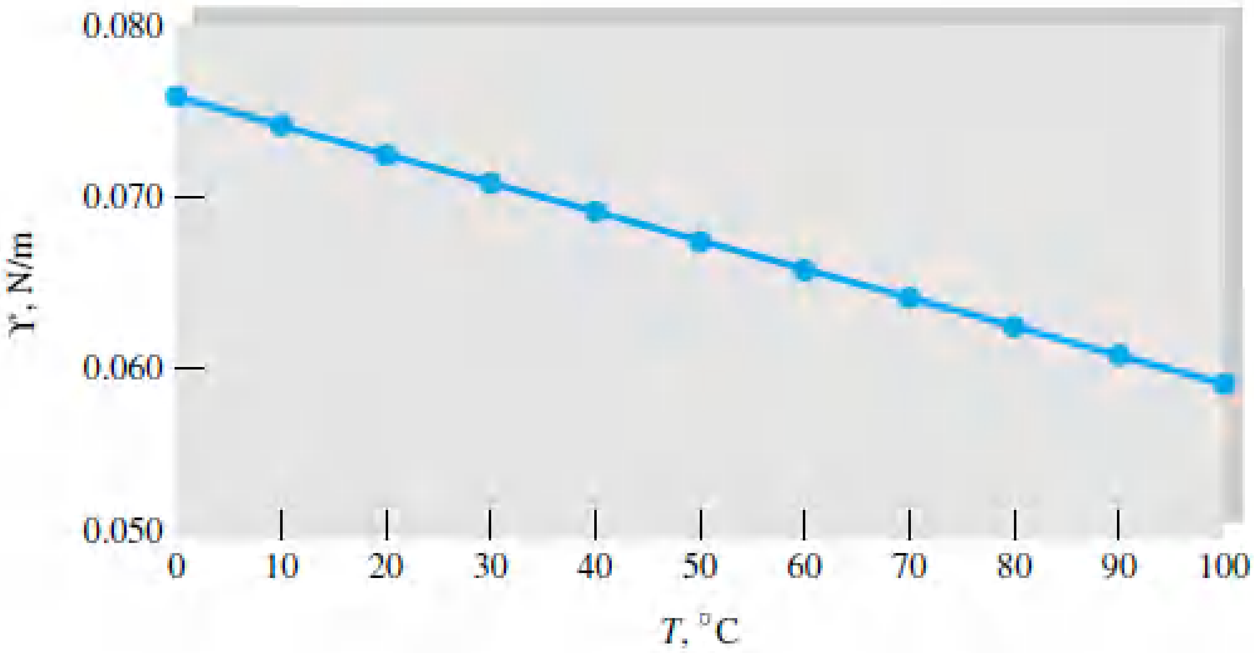

Surface of any liquid behaves as though it is covered by a stretched membrane
Net force on molecule at surface is into bulk of the liquid
F_T
ΣF = 0
ΣF

EXAMPLES

- Capillary rise and capillary fall
- Break up of liquid jets
- Collection of dust particles on water surface
- Formation of rain droplets and water bubbles
- Small needle or blade placed on the liquid surface without sinking
- Insect walk on water
- A drop of blood forms a hump on a horizontal glass
- A drop of mercury forms a near-perfect sphere and can be rolled just like a steel ball over a smooth surface
- Water droplets from rain or dew hang from branches or leaves of trees
- A liquid fuel injected into an engine forms a mist of spherical droplets
- Water dripping from a leaky valve falls as spherical droplets
- A soap bubble released into the air forms a spherical shape
- Water beads up into small drops on flower petals

- Surface tension is a binary property of the liquid and gas or two liquids which are in contact with each other and form the interface.
- Temperature ↑ Surface tension ↓ Cohesion ↓





Surface tension of some fluids in air at 1 atm and 20°C (unless otherwise stated)

Fluid	Surface Tension σ_s, N/m*
Water:	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
SAE 30 oil	0.035
Mercury	0.440
Ethyl alcohol	0.023
Blood, 37°C	0.058
Gasoline	0.022
Ammonia	0.021
Soap solution	0.025
Kerosene	0.028

- The value of σ depends on

1. Nature of the liquid
2. Nature of the surrounding matter
3. Kinetic energy of the liquid molecule
4. Temperature of the liquid molecule

1.Surface tension on liquid droplet

(*i*) tensile force due to surface tension acting around the circumference of the cut portion as shown in Fig. 1.11 (*b*) and this is equal to

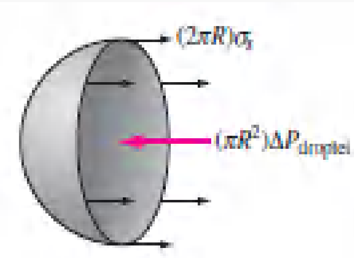
$$= \sigma \times \text{Circumference}$$

$$= \sigma \times \pi d$$

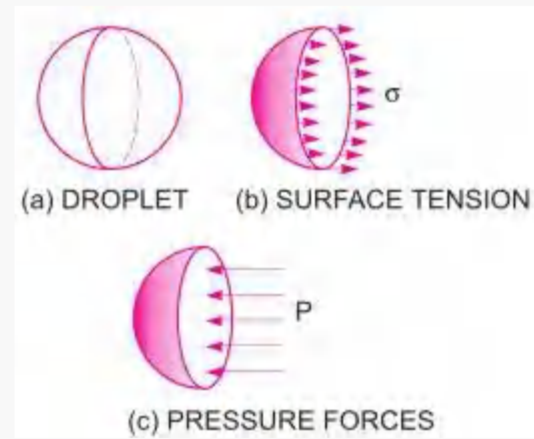
(*ii*) pressure force on the area $\frac{\pi}{4} d^2 = p \times \frac{\pi}{4} d^2$ as shown in

These two forces will be equal and opposite under equilibrium conditions, i.e.,

$$p = \frac{\sigma \times \pi d}{\frac{\pi}{4} \times d^2} = \frac{4\sigma}{d}$$



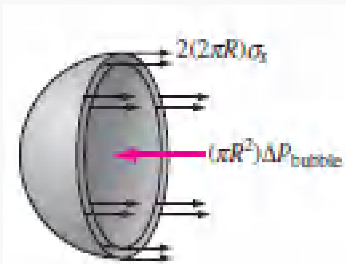
(*a*) Half a droplet



2. Surface tension on a hollow bubble

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$$p = \frac{2\sigma\pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d}$$



(*b*) Half a bubble

- 3. Surface tension on a liquid jet

Consider the equilibrium of the semi jet, we have

Force due to pressure $= p \times$ area of semi jet

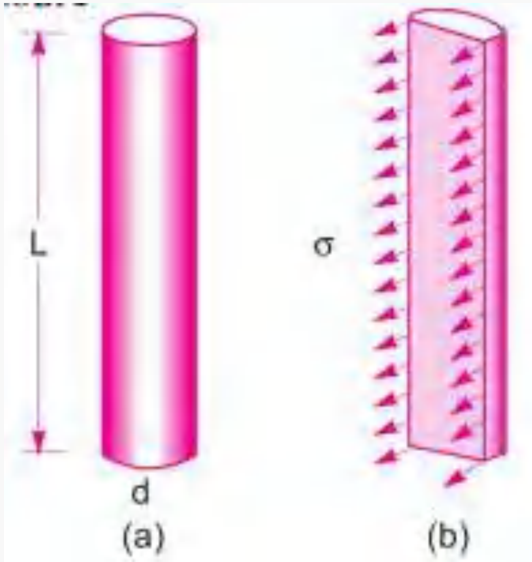
$= p \times L \times d$

Force due to surface tension $= \sigma \times 2L.$

Equating the forces, we have

$$p \times L \times d = \sigma \times 2L$$

$$\therefore \quad p = \frac{\sigma \times 2L}{L \times d}$$



1. Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m^2 above atmospheric pressure.

Given Data:

Diameter of bubble, d=40 mm = 40 x 10^{-3} m

Pressure in excess of outside, p = 2.5 N/m^2

For a soap bubble, we know that

$$p = \frac{8\sigma}{d} \quad \text{or} \quad 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m}$$

σ = 0.0125 N/m. Ans

2. The pressure outside the droplet of water of diameter 0.04 mm is 10.52 N/cm^2 (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.

Given Data;

Dia. of droplet d= 0.04 mm = .04 x 10^{-3} m

Pressure outside the droplet = 10.32 N/cm^2 = 10.32 x 10^4 N/m^2

Surface tension, 0.0725 N/m

The pressure inside the droplet, in excess of outside pressure is given by

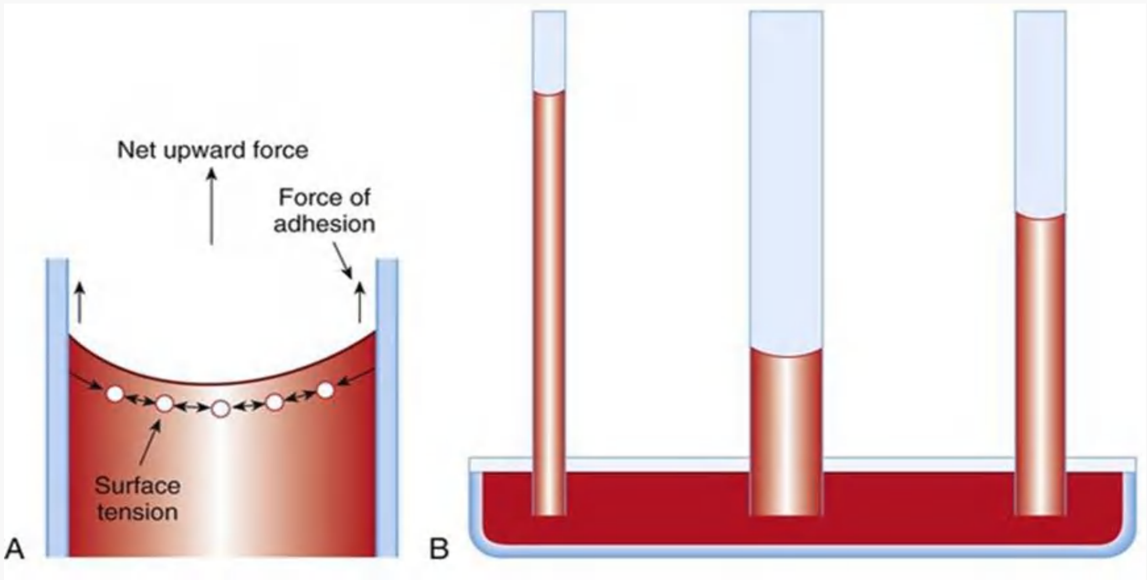
$$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250\ N/m^2 = \frac{7250\ N}{10^4\ cm^2} = 0.725\ N/cm^2$$

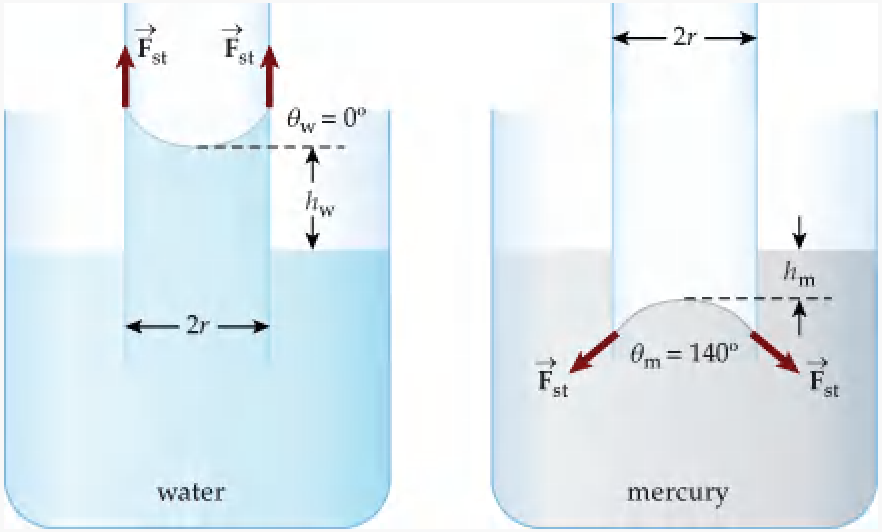
Pressure inside the droplet = p + Pressure outside the droplet

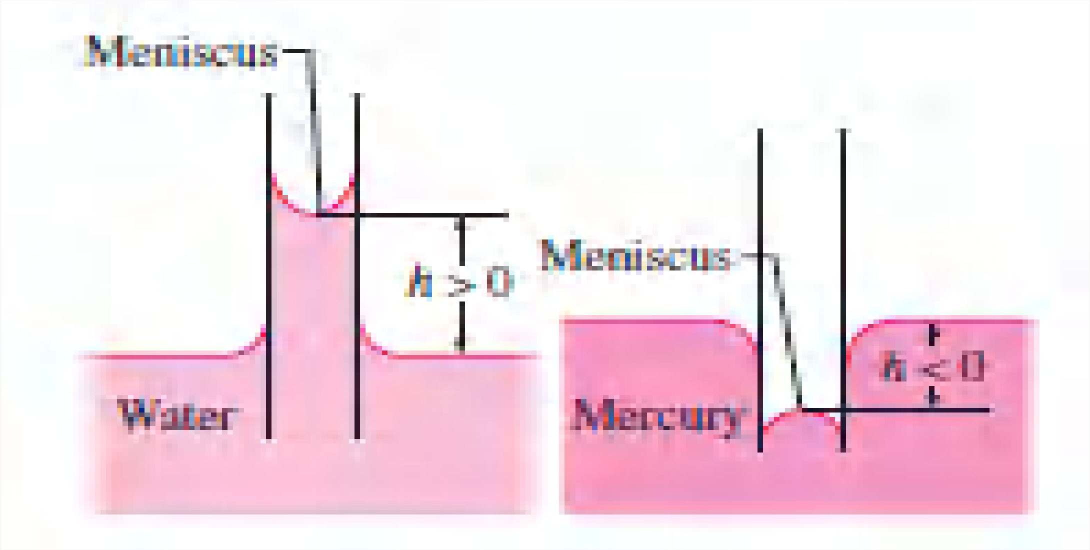
$$= 0.725 + 10.32 = \mathbf{11.045\ N/cm^2.}$$

CAPILLARITY OR CAPILLARY EFFECT

- A common phenomena associated with surface tension is the rise or fall of a liquid in a capillary tube.
- If a small open tube is inserted into water, the water level in the tube will rise above the water level outside the tube.
- The strength of the capillary effect is quantified by the **contact** (or wetting) **angle**, defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact.
- The curved free surface of a liquid in a capillary tube is called the **meniscus.**




$\vec{F}_{st}$
$\vec{F}_{st}$
$\theta_w = 0°$
h_w
$2r$
water
$2r$
h_m
$\theta_m = 140°$
$\vec{F}_{st}$
$\vec{F}_{st}$
mercury


Meniscus
$h > 0$
Water
Meniscus
$h < 0$
Mercury

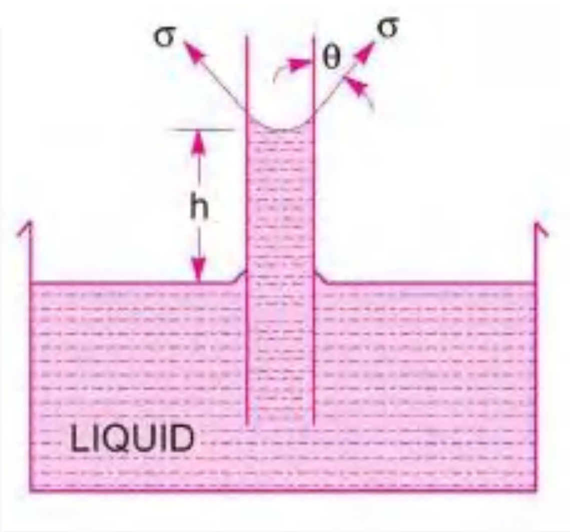
EXPRESSION FOR CAPILLARY RISE

surface tension force acts upward on water in a glass tube along the circumference, tending to pull the water up. As a result, water rises in the tube until the weight of the liquid in the tube above the liquid level of the reservoir balances the surface tension force

Let σ = Surface tension of liquid

θ = Angle of contact between liquid and glass tube.

The weight of liquid of height h in the tube = (Area of tube × h) × ρ × g



$$= \frac{\pi}{4} d^2 \times h \times \rho \times g$$

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos\theta$$
$$= \sigma \times \pi d \times \cos\theta$$

For eqnilibrium, eqnating (1.17) and (1.18), we get

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

or
$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma\cos\theta}{\rho \times g \times d}$$

- The value of θ between water and clean glass tube is approximately equal to zero and hence cos θ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d}$$

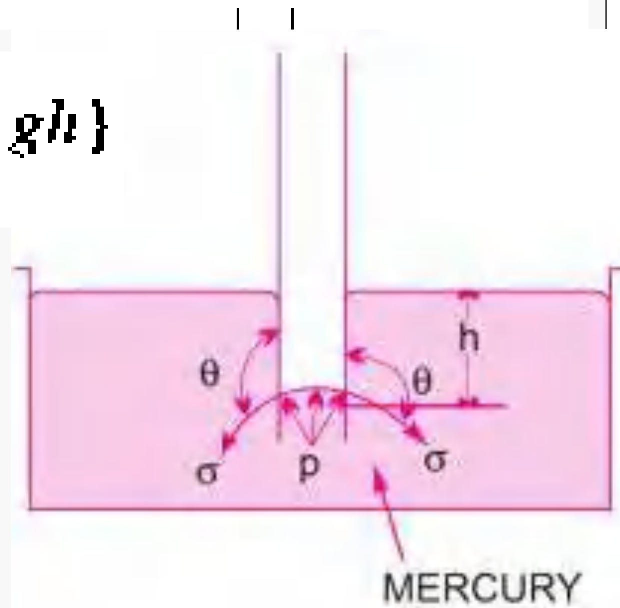
EXPRESSION FOR CAPILLARY FALL

Let h = Height of depression in tube.

Then in equilibrium, two forces are acting on the mercury inside the tube. First one is due to surface tension acting in the downward direction and is equal to $\sigma \times \pi d \times \cos\theta$.

Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth 'h' × Area

$$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \; \{\because p = \rho g h\}$$



Equating the two, we get

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

$$\therefore \qquad h = \frac{4\sigma\cos\theta}{\rho g d}$$

Value of θ for mercury and glass tube is 128°.

1.Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tensions σ= 0.0725 N/m for water and σ =0.52 N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact ϴ = 130°

Given Data:

Dia. of tube, d = 2.5 mm = 2.5 x 10 3 m

Surface tension for water = 0.0725 N/m

surface tension for mercury = 0.52 N/m

Sp. gr. of mercury = 13.6

Density = 13.6 x 1000 kg/m^3

(a) Capillary rise for water (ϴ = 0°)

$$h = \frac{4\sigma}{\rho \times g \times d} = \frac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$$

= .0118 m = 1.18 cm

(b) For mercury

Angle of contact between mercury and glass tube, ϴ = 130°

$$h = \frac{4\sigma\cos\theta}{\rho \times g \times d} = \frac{4 \times 0.52 \times \cos 130^\circ}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$$

= - 0.004 m = - 0.4 cm

The negative sign indicates the capillary depression

2. Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.075575 N/m.

Given :

Capillary rise, h = 2.0 mm = 2.0 x 10^{-3} m

Surface tension, σ = 0.073575 N/m

Let dia. of tube = d

The angle θ for water = 0°

The density for water, ρ = 1000 kg/m^3

$$h = \frac{4\sigma}{\rho \times g \times d} \text{ or } 2.0 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

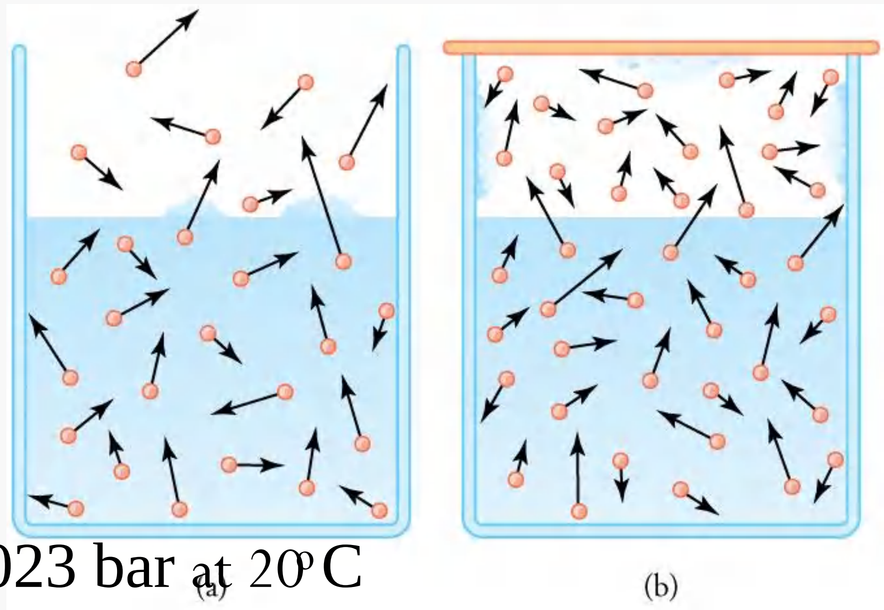
$$d = \frac{4 \times 0.073575}{1000 \times 9.81 \times 2 \times 10^{-3}} = 0.015 \text{ m} = \mathbf{1.5 \text{ cm.}}$$

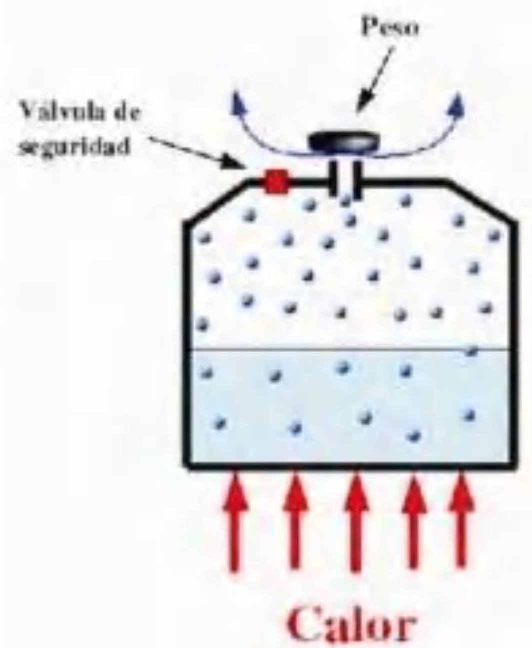
Thus minimum diameter of the tube should be **1.5 cm.**

Vapour Pressure

- The vapor pressure Pv of a pure substance is defined as the pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature.
- All liquids vaporize or evaporate due to the molecules escaping from the free surface.
- In a closed container, an equilibrium condition is reached, when the number of molecules escaping from the liquid surface is equal to the number of molecules entering the liquid through the surface.

- Units of Vapor Pressure: N/m^2
- If the pressure on the liquid is equal to or less than vapor pressure, it starts boiling.
- Why petrol vaporizes faster than water?
- Vp of petrol is 0.3 bar and water is 0.023 bar at $20^{o}C$




Peso
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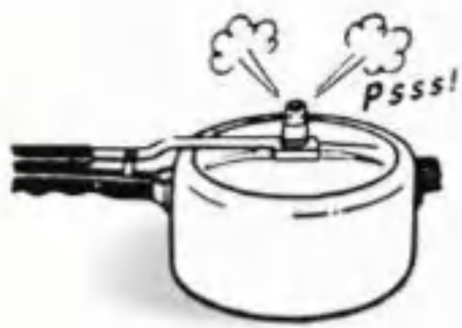
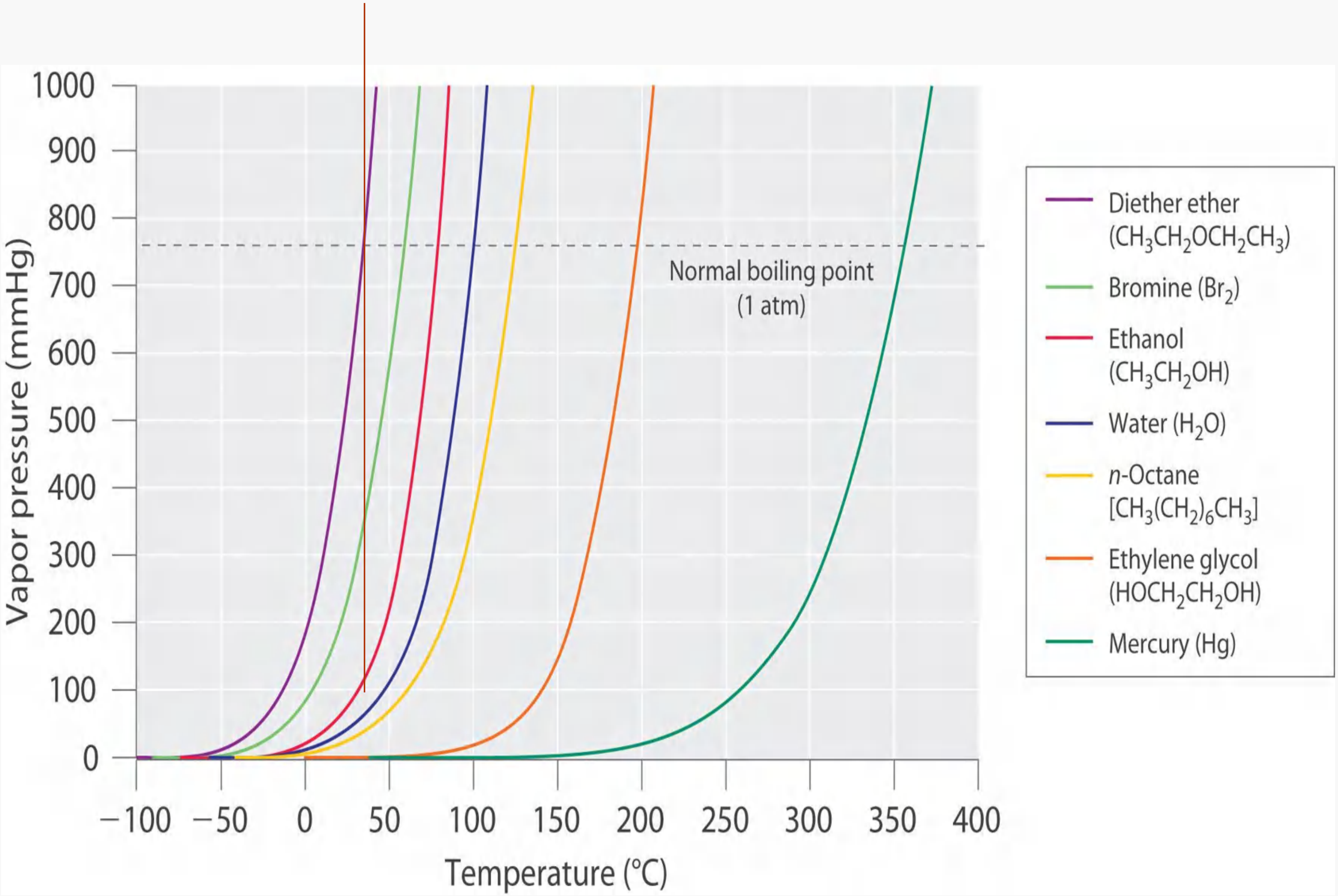
Calor


psss!

- Vapor pressure plays very important role in phenomenon called Cavitation.
- The cavitation is the phenomenon of formation of vapor bubbles of a flowing liquid in a region where the pressure of the liquid falls below the vapor pressure and sudden collapsing of these vapor bubbles in a region of higher pressure.
- When the vapour bubbles collapse, a very high pressure is created. The metallic surfaces, above which the liquid is flowing, is subjected to these high pressures, which cause pitting action on the surface.
- Liquid may vaporize and form vapour bubbles at location of tip region of impeller or suction sides of pumps, bubbles collapse at high pressure region which causes erosion of impeller blades.
- This phenomenon is important consideration in design of hydraulic turbine and pumps.
- Cavitation leads to generates vibrations, noise, damage to equipment and reduce performance.



Vapor pressure (mmHg)
1000
900
800
700
600
500
400
300
200
100
0
−100
−50
0
50
100
150
200
250
300
350
400
Temperature (°C)
Normal boiling point
(1 atm)
Diether ether
($CH_3CH_2OCH_2CH_3$)
Bromine (Br_2)
Ethanol
(CH_3CH_2OH)
Water (H_2O)
n-Octane
[$CH_3(CH_2)_6CH_3$]
Ethylene glycol
($HOCH_2CH_2OH$)
Mercury (Hg)

Vapor pressure of liquids

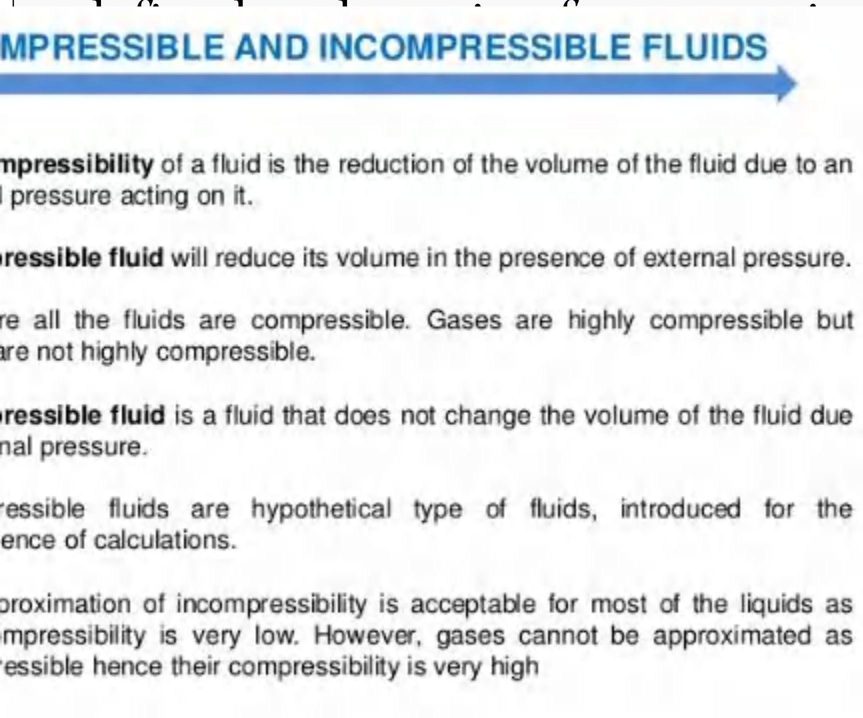
- All liquids tend to evaporate when placed in a closed container
- Vaporization will terminate when equilibrium is reached between the liquid and gaseous states of the substance in the container

i.e. Number of molecules escaping liquid surface = Number of incoming molecules

- Under this equilibrium we call the call vapor pressure as the saturation pressure
- At any given temperature, if pressure on liquid surface falls below the saturation pressure, rapid evaporation occurs (i.e. boiling)
- For a given temperature, the saturation pressure is the boiling pressure

COMPRESSIBILITY AND BULK MODULUS

- Compressibility is the reciprocal of the
stre
iston
n the
s V
- dp, the



MPRESSIBLE AND INCOMPRESSIBLE FLUIDS

mpressibility of a fluid is the reduction of the volume of the fluid due to an
l pressure acting on it.

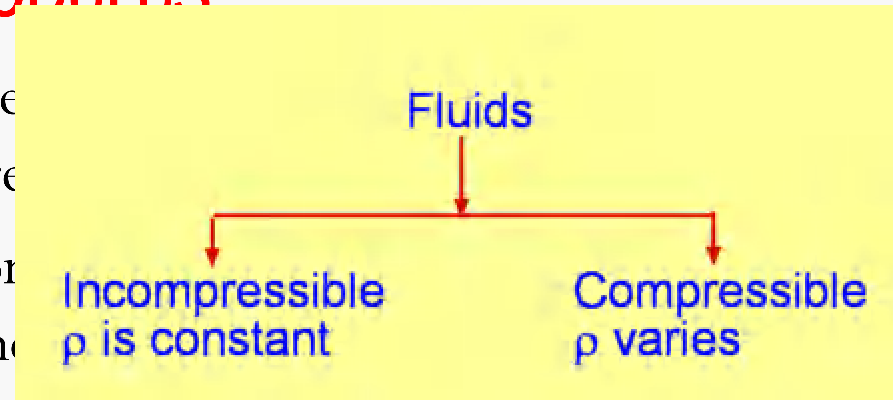
ressible fluid will reduce its volume in the presence of external pressure.

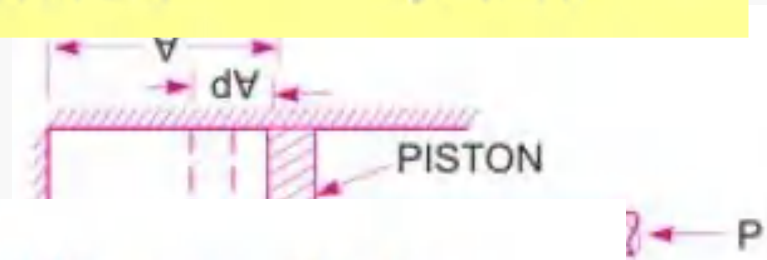
re all the fluids are compressible. Gases are highly compressible but
are not highly compressible.

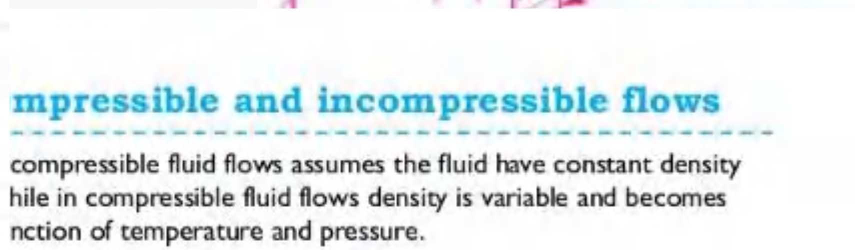
ressible fluid is a fluid that does not change the volume of the fluid due
nal pressure.

ressible fluids are hypothetical type of fluids, introduced for the
ence of calculations.

proximation of incompressibility is acceptable for most of the liquids as
mpressibility is very low. However, gases cannot be approximated as
ressible hence their compressibility is very high





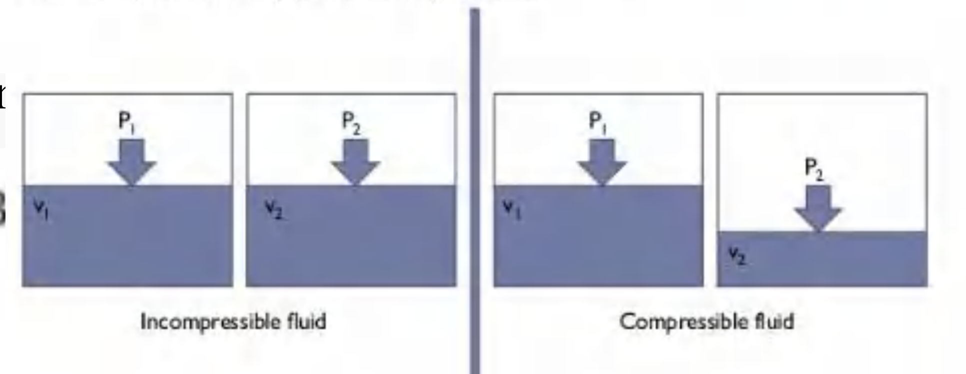


mpressible and incompressible flows

compressible fluid flows assumes the fluid have constant density
hile in compressible fluid flows density is variable and becomes
nction of temperature and pressure.

Negative sign means the volu

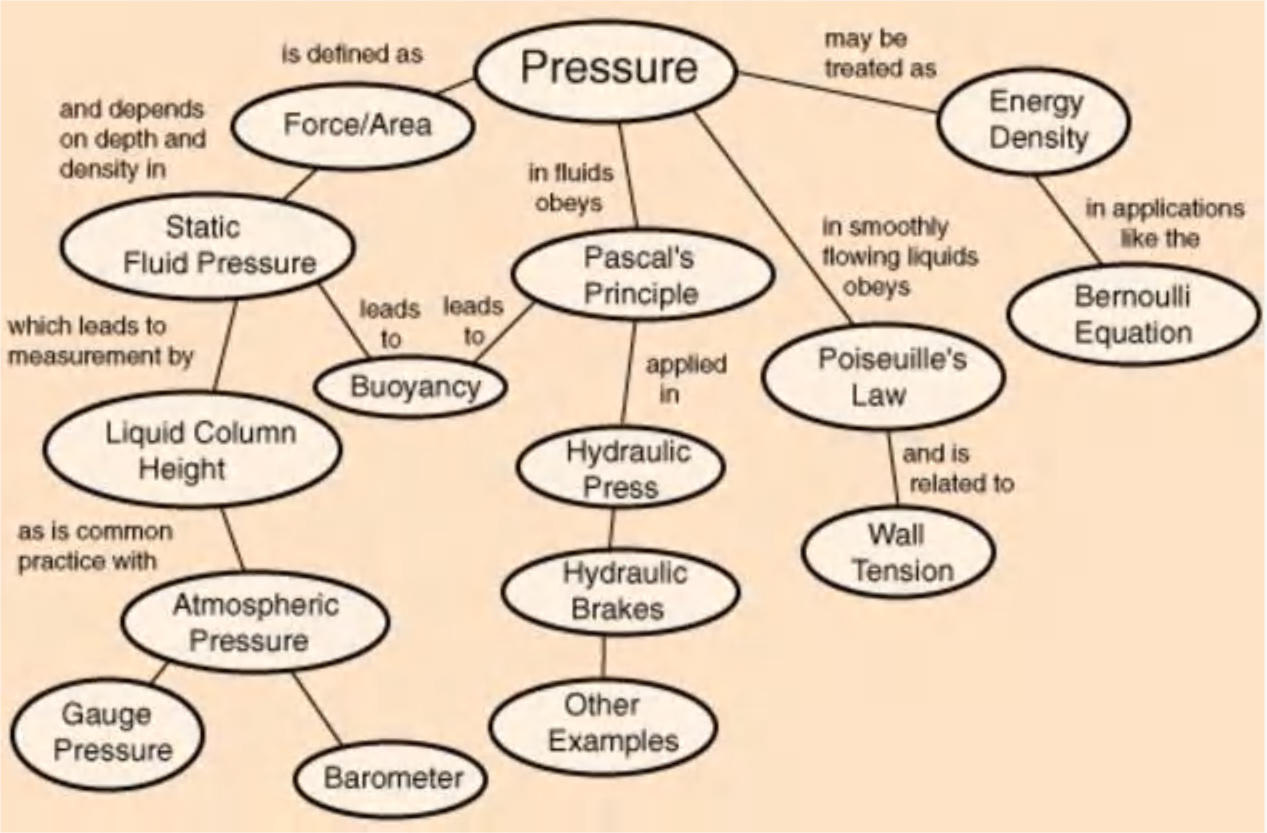
B

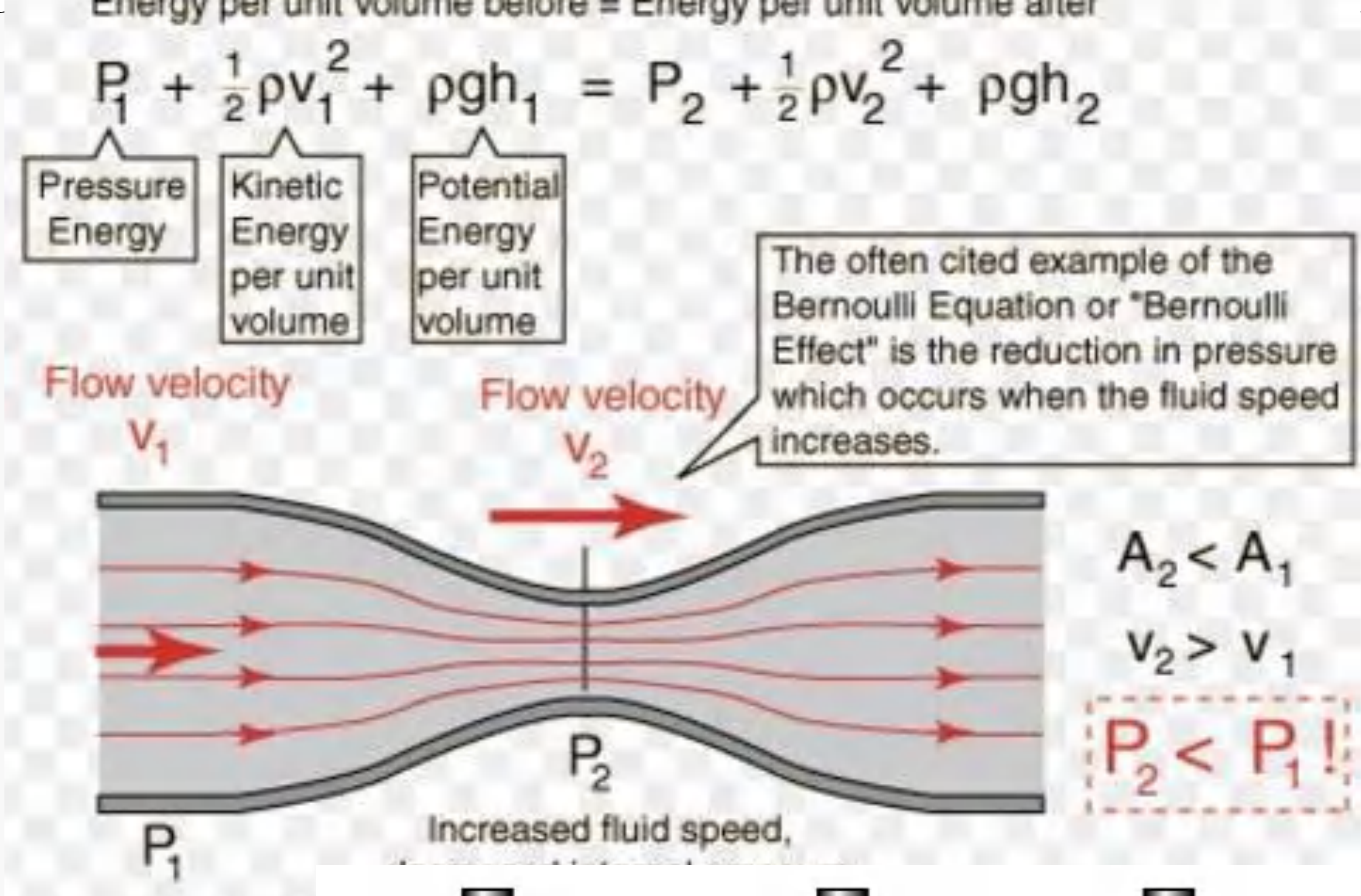


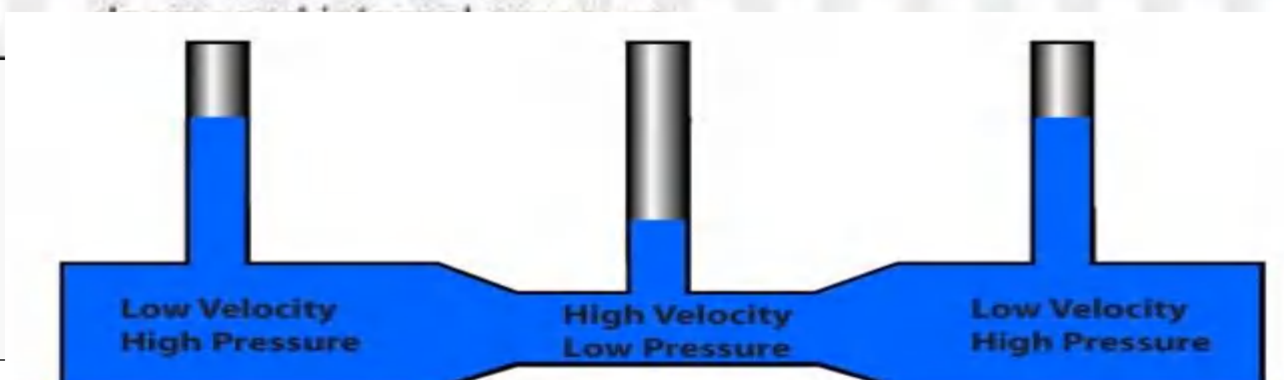
sure
ain

Compressibility = 1/K

$\frac{-dV}{V}$ dV


Pressure
is defined as
Force/Area
and depends on depth and density in
Static Fluid Pressure
which leads to measurement by
Liquid Column Height
as is common practice with
Atmospheric Pressure
Gauge Pressure
Barometer
leads to
Buoyancy
leads to
in fluids obeys
Pascal's Principle
applied in
Hydraulic Press
Hydraulic Brakes
Other Examples
may be treated as
Energy Density
in applications like the
Bernoulli Equation
in smoothly flowing liquids obeys
Poiseuille's Law
and is related to
Wall Tension


Energy per unit volume before = Energy per unit volume after
$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$
Pressure Energy
Kinetic Energy per unit volume
Potential Energy per unit volume
The often cited example of the Bernoulli Equation or "Bernoulli Effect" is the reduction in pressure which occurs when the fluid speed increases.
Flow velocity v_1
Flow velocity v_2
$A_2 < A_1$
$v_2 > v_1$
$P_2 < P_1$!
P_2
P_1
Increased fluid speed,


Low Velocity
High Pressure
High Velocity
Low Pressure
Low Velocity
High Pressure

Pressure

Pressure is defined as the force per unit area, where the force is perpendicular to the area.

$$\text{Pressure} = \frac{Force}{Area}; \qquad P = \frac{F}{A}$$

N/m^2

<u>Units of Pressure</u>

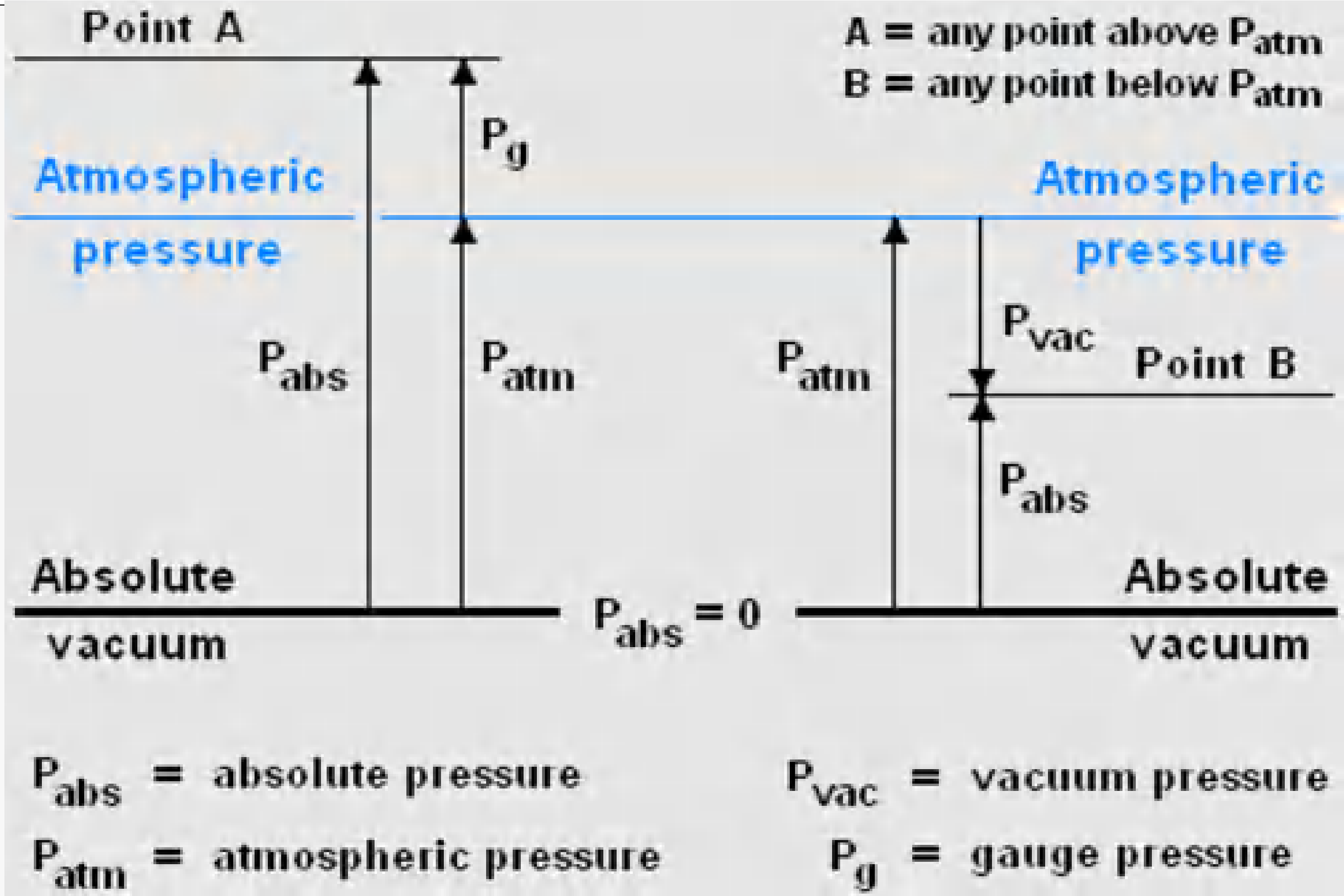
1 pascal (Pa) = 1 N/m^2

1 bar = $1*10^5$ N/m^2 = $1*10^5$ Pa

1 bar = 0.1Mpa

1 atm = 101,325 Pa = 101.325 kPa = 1.01325 bar

1bar = 1 atm =14.5psi = 760 mm of Hg



$$P_{gage} = P_{abs} - P_{atm}$$

$$P_{vac} = P_{atm} - P_{abs}$$

Find the pressure at bottom of the tank

Pressure is the *compressive force* per unit area

A = 2.25 m^2
V = 1000litre

1 Litre of water is equal to **1 kg**.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}; \qquad P = \frac{F}{A}$$

$$= \frac{ma}{A}$$

$$= 5573 \text{N/m}^2$$

75

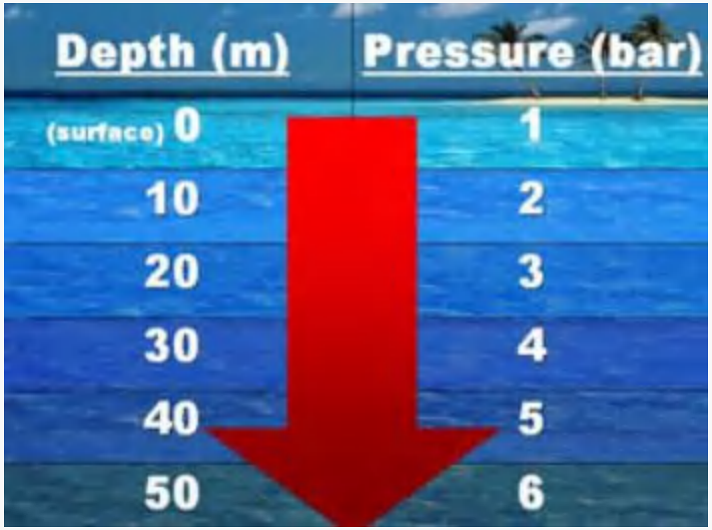
The pressure measured in your automobile tyres is the gauge pressure,35psi. 1 Pa = 1 N/m^2, in the SI system.

Pressure measuring device

1. Barometer
2. Manometer
3. Pressure Gauge
4. Pressure sensor

Pressure at a Point

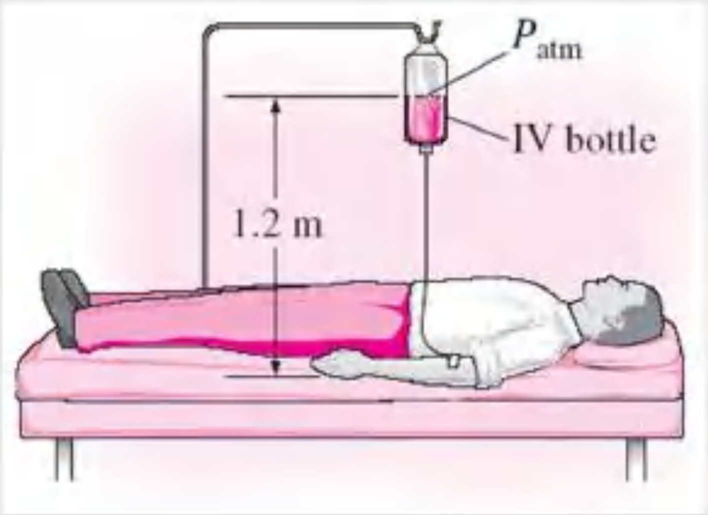
- Pressure at any point in a fluid is directly proportional to the density of the fluid and to the depth in the fluid.



EXAMPLE 3–3 **Gravity Driven Flow from an IV Bottle**

Intravenous infusions usually are driven by gravity by hanging the fluid bottle at sufficient height to counteract the blood pressure in the vein and to force the fluid into the body (Fig. 3–15). The higher the bottle is raised, the higher the flow rate of the fluid will be. (*a*) If it is observed that the fluid and the blood pressures balance each other when the bottle is 1.2 m above the arm level, determine the gage pressure of the blood. (*b*) If the gage pressure of the fluid at the arm level needs to be 20 kPa for sufficient flow rate, determine how high the bottle must be placed. Take the density of the fluid to be 1020 kg/m^3.

SOLUTION It is given that an IV fluid and the blood pressures balance each other when the bottle is at a certain height. The gage pressure of the blood and elevation of the bottle required to maintain flow at the desired rate are to be determined.



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UNIT – 2

FLOW THROUGH CIRCULAR CONDUITS

SYLLABUS

- **UNIT I FLUID PROPERTIES AND FLOW CHARACTERISTICS**

  Units and dimensions- Properties of fluids- mass density, specific weight, specific volume, specific gravity, viscosity, compressibility, vapor pressure, surface tension and capillarity. Flow characteristics – concept of control volume - application of continuity equation, energy equation and momentum equation.

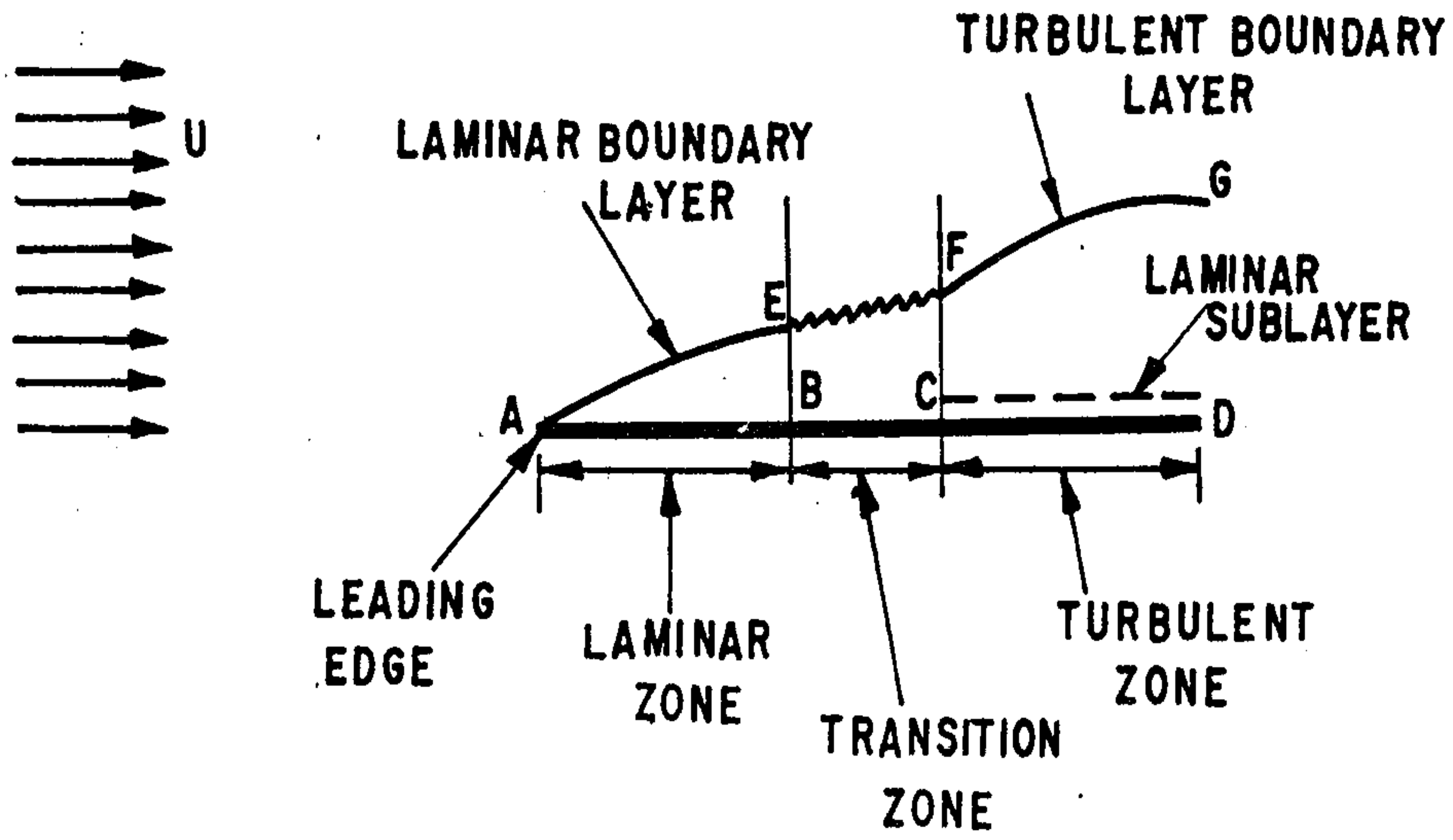
- **UNIT II FLOW THROUGH CIRCULAR CONDUITS**

  Hydraulic and energy gradient - Laminar flow through circular conduits and circular annuli-Boundary layer concepts – types of boundary layer thickness – Darcy Weisbach equation –friction factor- Moody diagram- commercial pipes- minor losses – Flow through pipes in series and parallel.

- **UNIT III DIMENSIONAL ANALYSIS**

  Need for dimensional analysis – methods of dimensional analysis – Similitude –types of similitude - Dimensionless parameters- application of dimensionless parameters – Model analysis.

LAMINAR / TURBULENT / BOUNDARY LAYER



The fluid layer near the surface in which there is a general slowing down is defined as boundary layer.

The velocity of flow in this layer increases from zero at the surface to free stream velocity at the edge of the boundary layer.

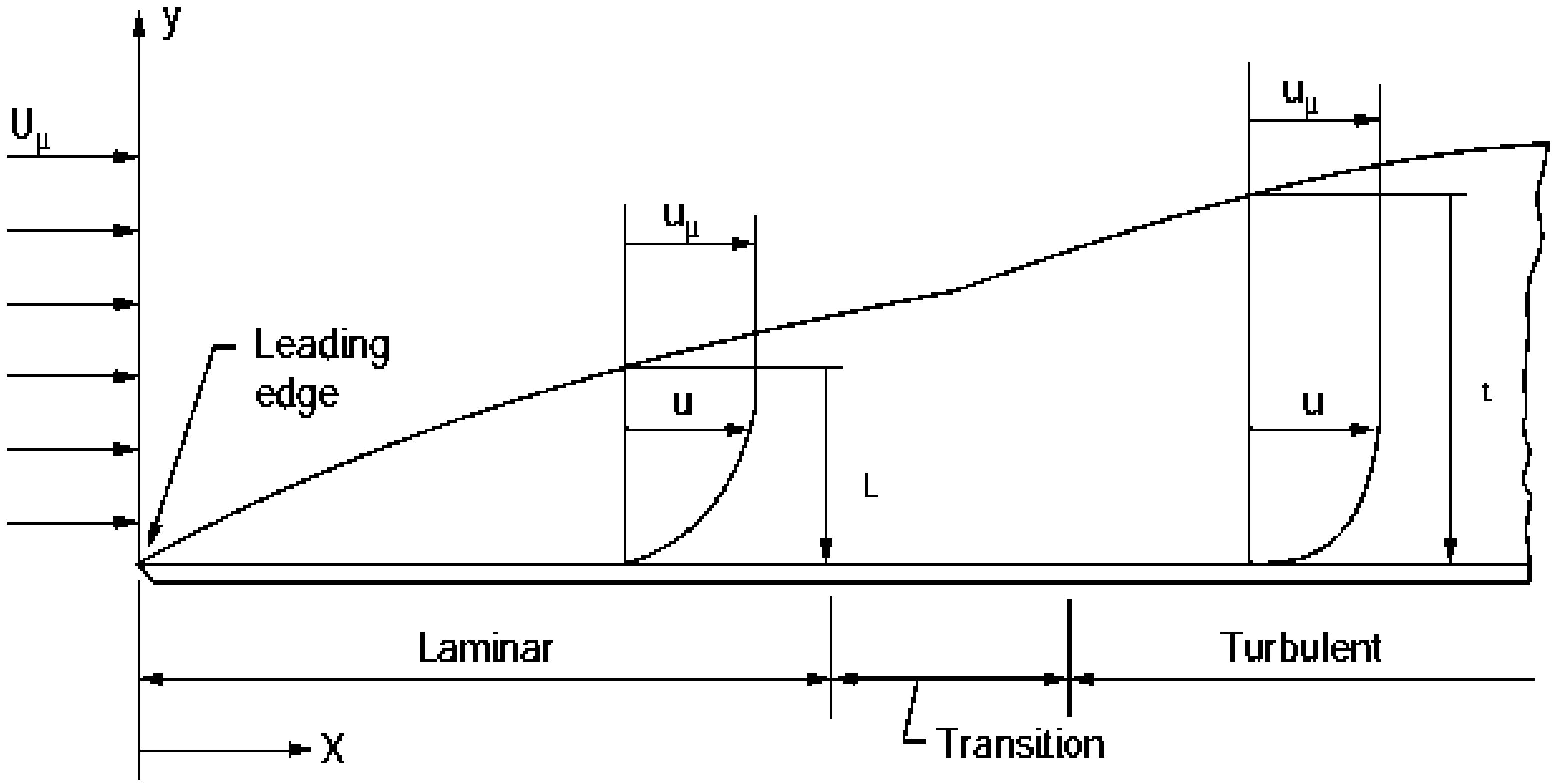
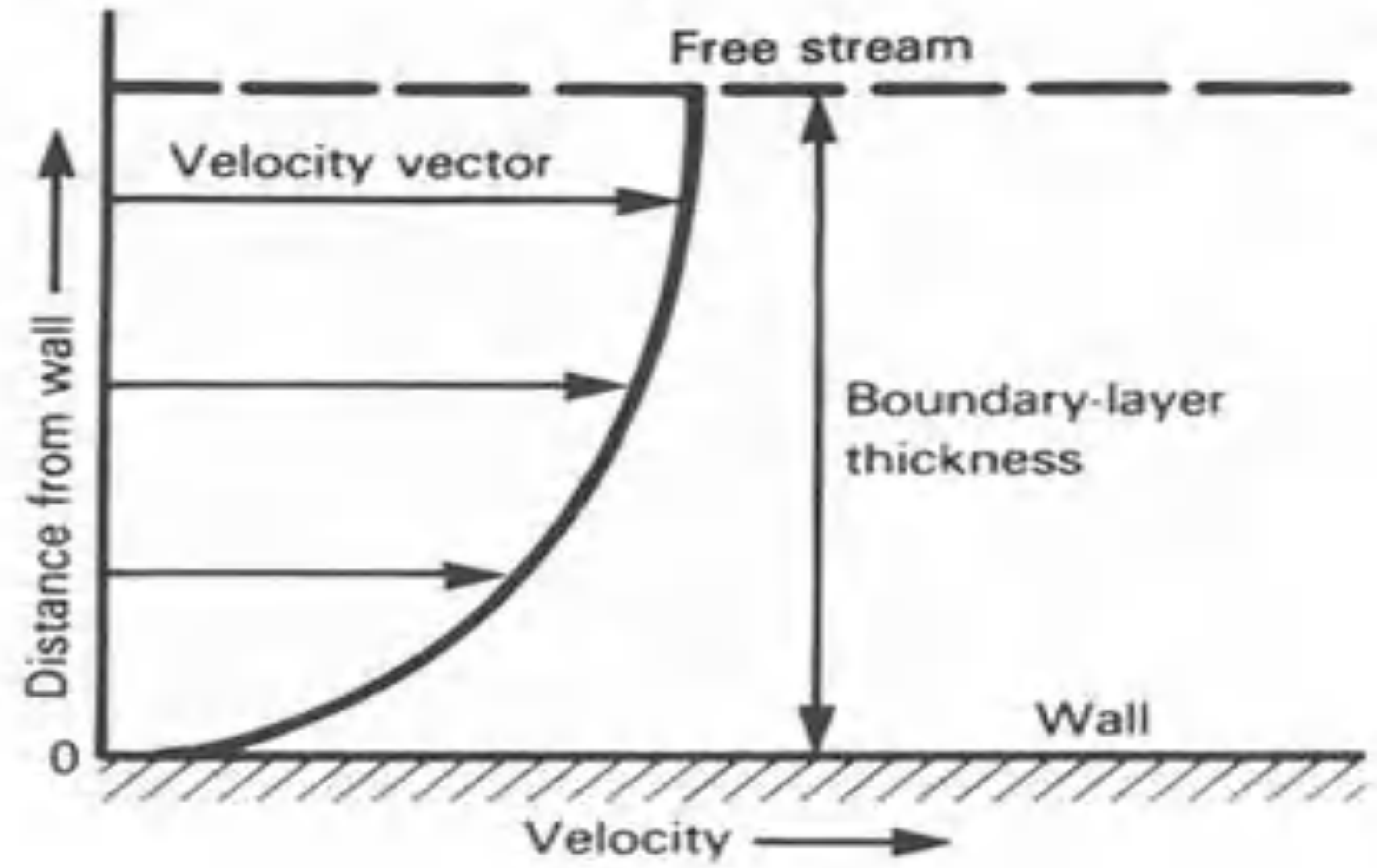


Figure 7.1.1 Boundary Layer Development (flat-plate)

BOUNDARY LAYER THICKNESS



entry length is about **0.04 Re × *D*** The flow beyond is said to be fully developed.

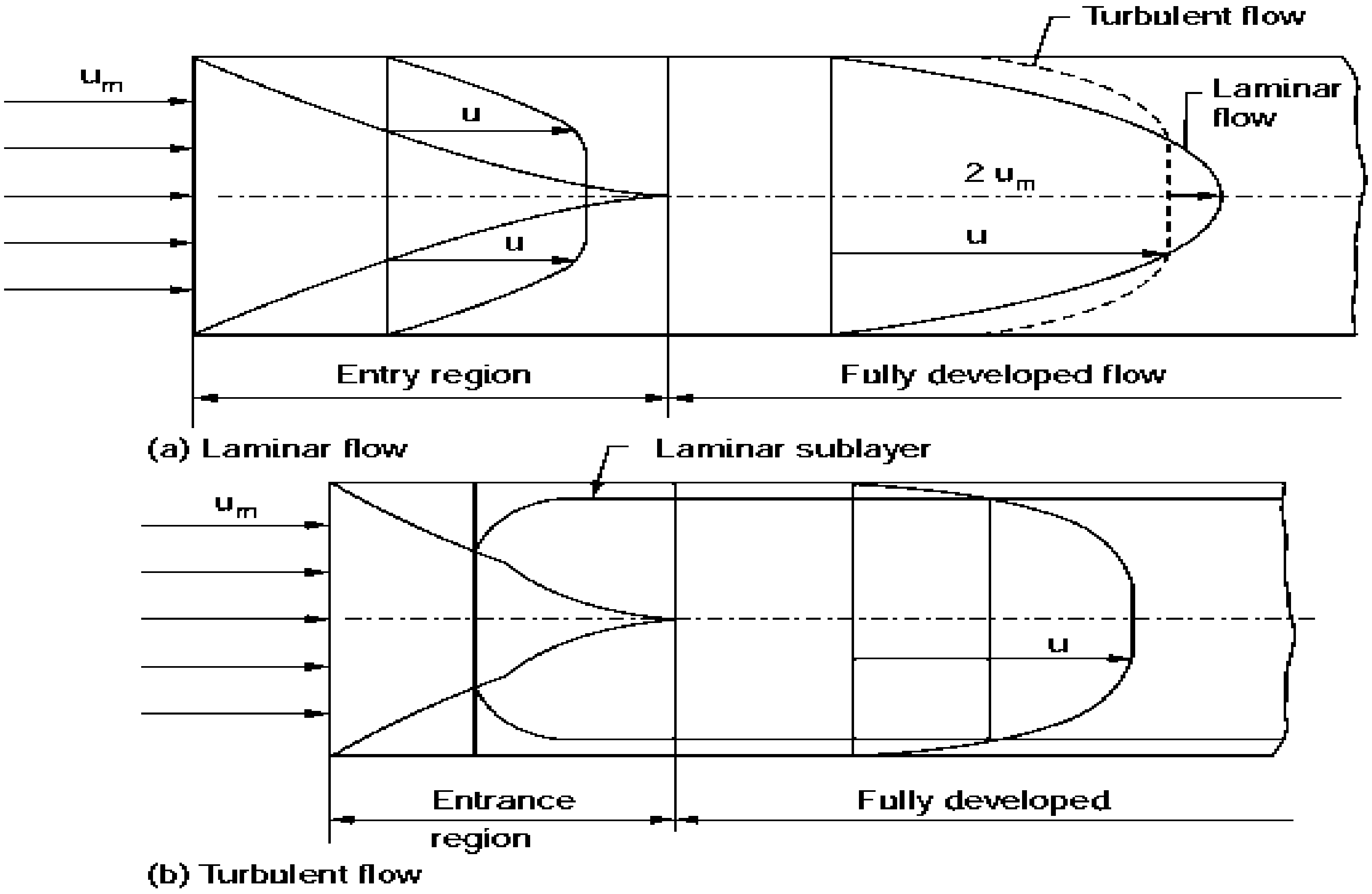
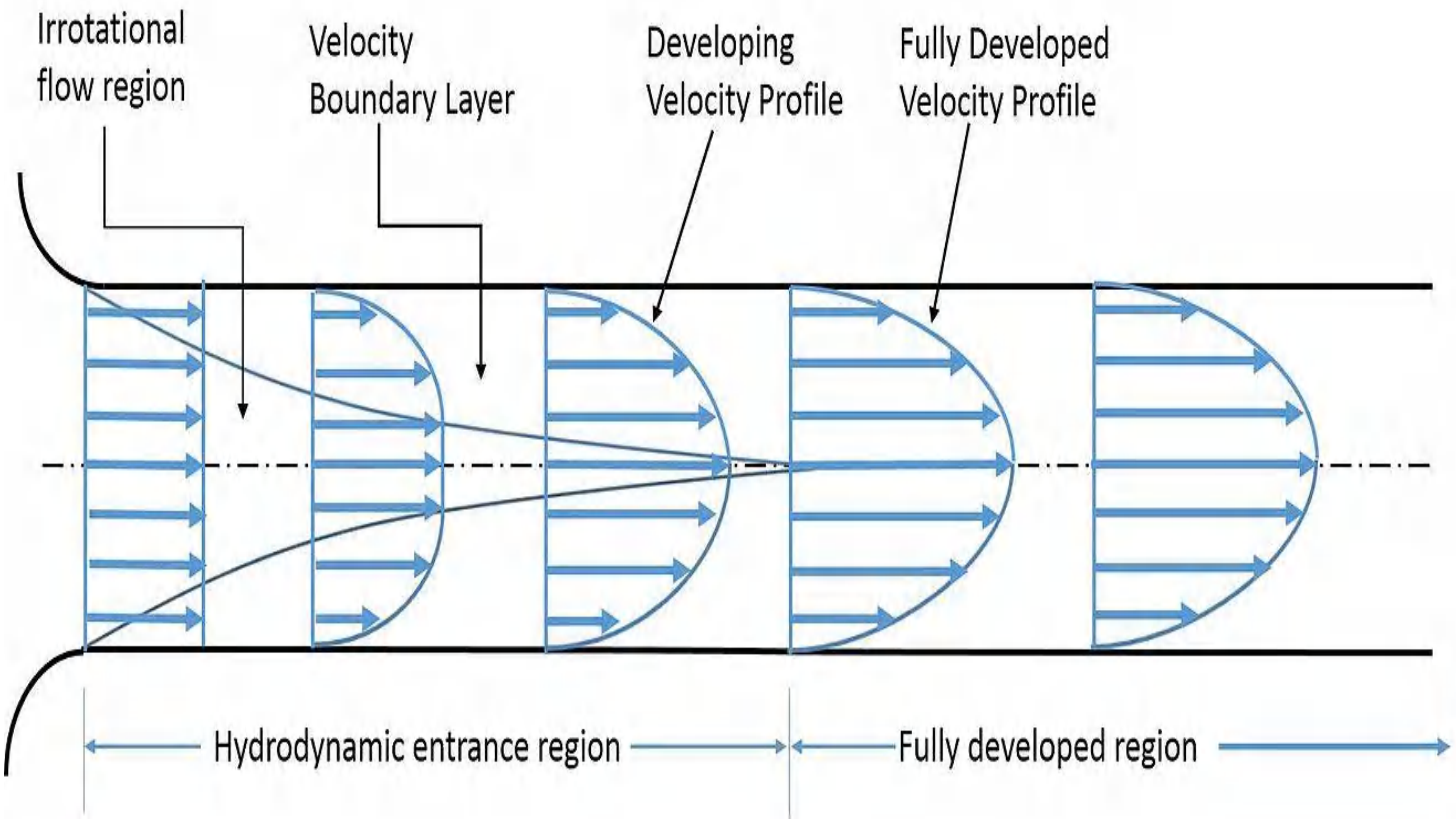


Figure 7.3.1 Boundary layer development (pipe flow)

Development of Boundary Layer



DEVELOPMENT OF BOUNDARY LAYER IN CLOSED CONDUITS (PIPES)

Consider an annular element of fluid in the flow as shown in Fig. 7.7.1*a*.

The dimensions are: inside radius = r; outside radius = $r + dr$, length = dx.

Surface area = $2\pi r dx$

Assuming steady fully developed flow, and using the relationship for force balance, the velocity being a function of radius only.

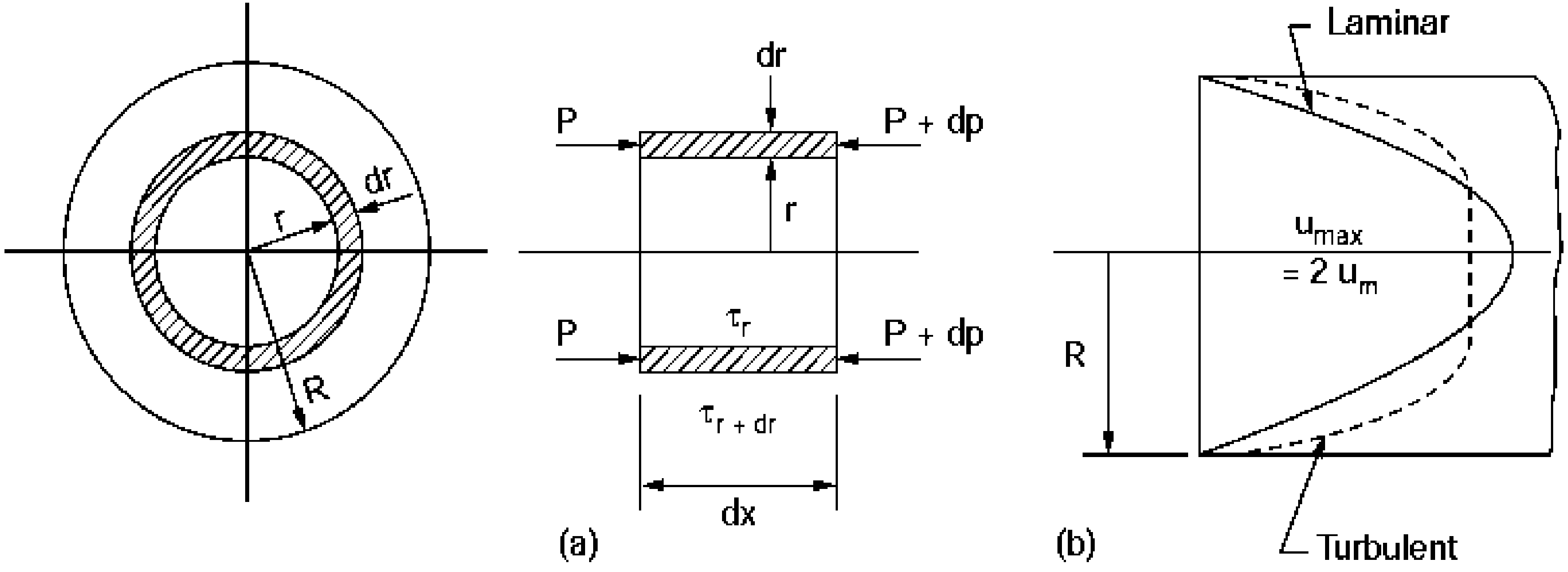


Figure 7.7.1

Net pressure force = $dp\ 2\pi\ rdr$

Net shear force $= \dfrac{d}{dr}\left(\mu \dfrac{du}{dr} 2\pi r dx\right) dr$, Equating the forces and reordering

$$\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{1}{\mu}\frac{dp}{dx}r$$

Integrating $\quad r\dfrac{du}{dr} = \dfrac{1}{\mu}\dfrac{dp}{dx}\dfrac{r^2}{2} + C$, at $r = 0 \quad \therefore \quad C = 0$

Integrating again and after simplification,

$$u = \frac{1}{\mu}\frac{dp}{dx}\frac{r^2}{4} + B$$

at $r = R$, $u = 0$ (at the wall)

$$\therefore \qquad B = -\frac{1}{\mu}\frac{dp}{dx}\frac{R^2}{4}$$

$$\therefore \qquad u = -\frac{1}{\mu}\frac{dp}{dx}\frac{R^2}{4}\left[1-\left(\frac{r}{R}\right)^2\right] \qquad (7.7.1)$$

The velocity is maximum at $r = 0$,

$$\therefore \qquad u_{max} = -\frac{1}{\mu}\frac{dp}{dx}\frac{R^2}{4} \qquad (7.72)$$

At a given radius, dividing 7.7.1 by (7.7.2), we get 7.7.3, which represents parabolic distribution.

$$\therefore \qquad \frac{u}{u_{max}} = 1-\left(\frac{r}{R}\right)^2 \qquad (7.7.3)$$

If the average velocity is u_{mean} then the flow is given by $Q = \pi R^2 u_{mean}$ (A)

HAGEN–POISEUILLE EQUATION FOR FRICTION DROP

In the case of laminar flow in pipes, calculation of pressure drop.
The equation is derived in this section

Using eqn (7.7.2), $u_{max} = -\frac{dP}{dL}\frac{1}{\mu}\cdot\frac{R^2}{4} = 2u_m$

$$\therefore \quad -\frac{dP}{dL} = \frac{8\,u_m\,\mu}{R^2}$$

$$\therefore \quad -\frac{dP}{dL} = \frac{8u_m\mu}{R^2} = \frac{32u_m\mu}{D^2}$$, Substituting for $-\frac{dP}{dL}$ as $\frac{\Delta P}{L}$

$$\Delta P = \frac{32\,\mu\,u_m L}{D^2} \qquad (7.9.1)$$

This can also be expressed in terms of volume flow rate Q as

$$Q = \frac{\pi D^2}{4} \cdot u_m$$

$$\therefore \quad u_m = 4Q/\pi D^2, \text{ substituting}$$

$$\Delta P = 128\, \mu\, L\, Q/\pi\, D^4 \qquad (7.9.2)$$

Converting ΔP as head of fluid

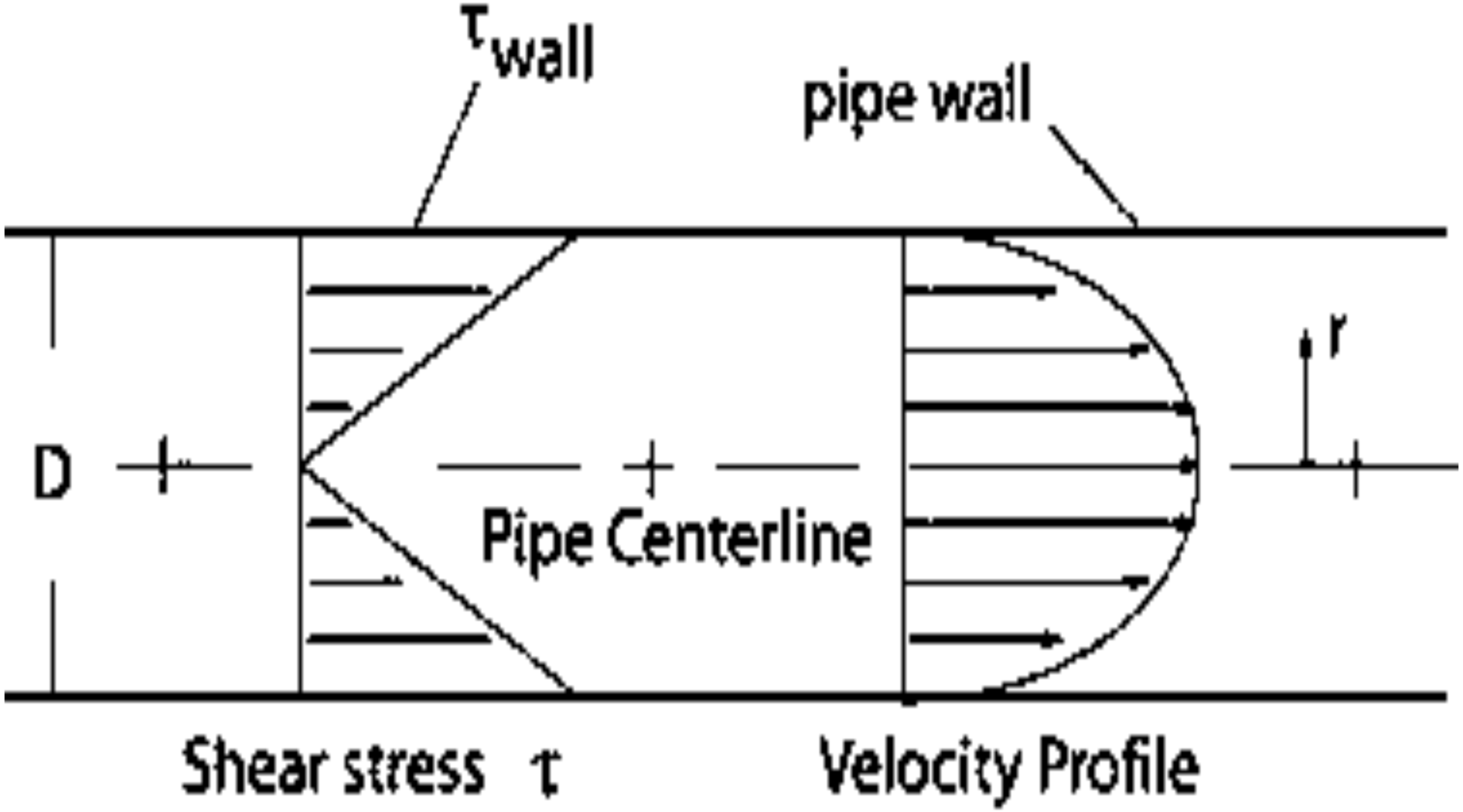
$$h_f = \frac{32\, \nu u_m\, L g_0}{g D^2} \qquad (7.9.3)$$

This equation is known as Hagen-Poiseuille equation

g_0 is the force conversion factor having a value of unity in the SI system of unit. Also $(\mu / \rho) = \nu$.

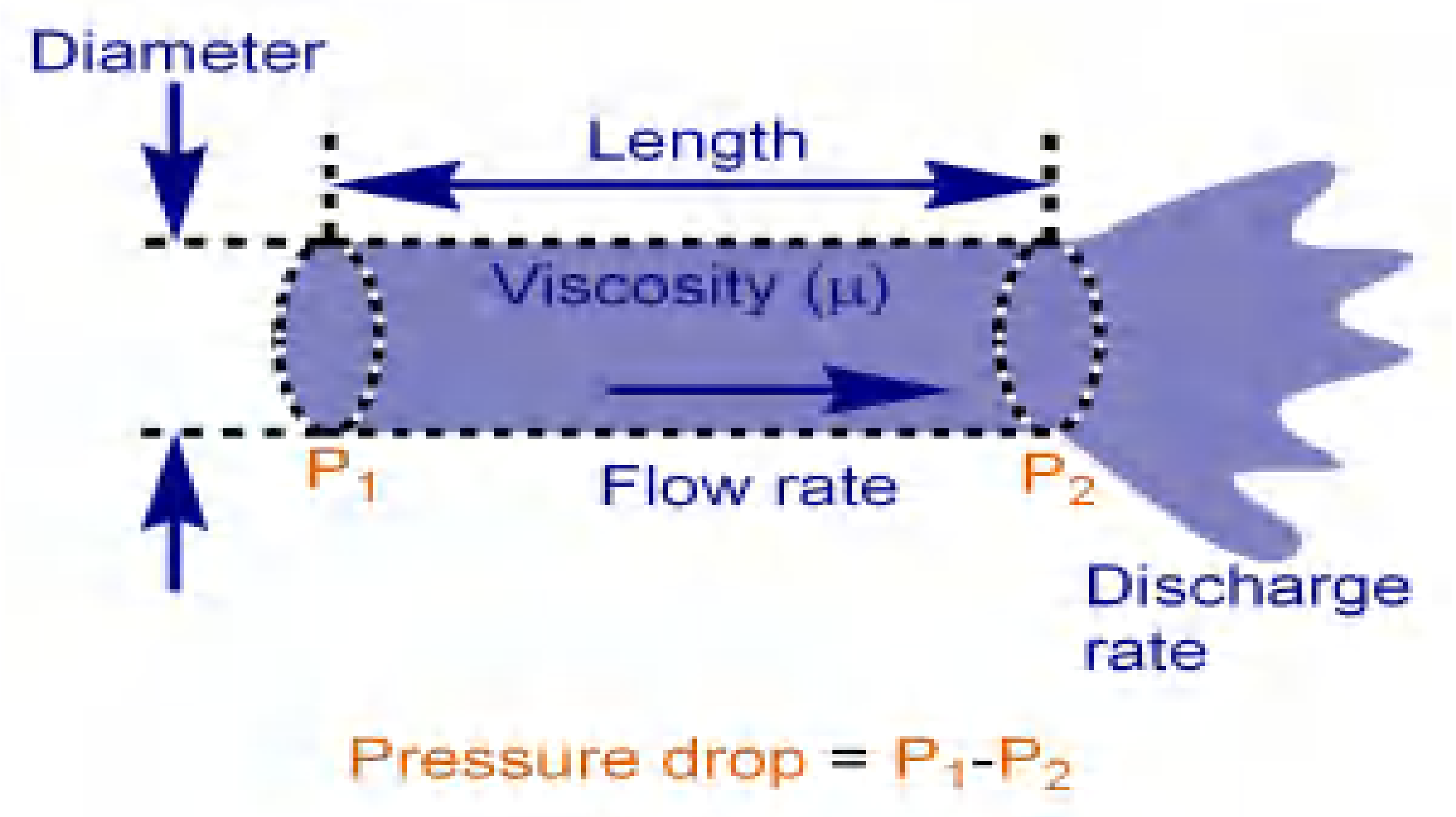
Hagen-Poiseuille equation is applicable **for laminar flow only** whereas **Darcy- Weisbach** equation is applicable **for all flows**

Velocity & Shear Stress distribution



HAGEN – POISEUILLE EQUATION

(Pressure drop for laminar flow in pipes)



$$\text{pressure drop } (p_1 - p_2) = \frac{32\,\mu \bar{u}\,L}{D^2}$$

Flow through pipes – Formulae Used

$$\text{pressure drop } (p_1 - p_2) = \frac{32\,\mu\bar{u}\,L}{D^2}$$

$$\bar{u} = \frac{Q}{area\left(\frac{\pi D^2}{4}\right)}$$

$$\text{shear stress at the pipe wall}, \quad \tau_o = \frac{-dP}{dx}\frac{R}{2}$$

Average velocity,

$$\bar{u} = \frac{1}{2}U_{max}$$

$$= \frac{1}{2}\left[\frac{-1}{4\mu}\frac{dP}{dx}R^2\right]$$

$$\text{Reynolds Number} = \frac{\rho D\bar{u}}{\mu}$$

1. A crude oil of viscosity 0.97 Poise and relative density 0.9 is flowing through a horizontal circular pipe of diameter 100 mm and of length 10 m. Calculate the difference of pressure at the two ends of the pipe, if 100 kg of the oil is collected in a tank in 30 seconds.

Given:

$\mu = 0.97 Poise = \frac{0.97}{10}$ = 0.097 Ns / m^2

Relative Density = 0.9

Density = 0.9 x 1000 = 900 kg / m^3

Dia. of pipe, D = 100 mm = 0.1 m

Length of pipe, L = 10 m

Mass of oil collected M = 100 kg

Time t = 30 seconds

To find out: Difference of pressure or (P_1 - P_2)

$$P_1 - P_2 = \frac{32\, \mu \bar{u} L}{D^2}$$

$$\text{Average velocity} = \frac{Q}{\text{Area}} \quad , \qquad m = \rho \times AV = \rho \times Q$$

$$\text{Now, mass of oil/ sec} \qquad m = \frac{100}{30}\ \text{kg/s} = \rho \times Q = 900 \times Q$$

$$\therefore \qquad \frac{100}{30} = 900 \times Q$$

$$\therefore \qquad Q = \frac{100}{30} \times \frac{1}{900} = 0.0037\ \ m^3/s$$

$$\therefore \qquad \bar{u} = \frac{Q}{Area} = \frac{0.0037}{\frac{\pi}{4} D^2} = \frac{0.0037}{\frac{\pi}{4}(0.1)^2} = 0.471\ m/s$$

Reynolds number, $R_e = \frac{\rho V D}{\mu}$

Where $\rho = 900\ kg/m^3$, $V = u = 0.471\ m/s$, $D = 0.1 m$,
$\mu = 0.097\ Ns/m^2$

$$R_e = 600 \text{ x } \frac{0.471 \text{ x } 0.1}{0.097} = 436.91$$

$$P_1 - P_2 = \frac{32\,\mu \bar{u} L}{D^2} = \frac{32 \text{ x } 0.097 \text{ x } 0.471 \text{ x } 10}{(0.1)^2}\ N/m^2$$

$$= 1462.28\ N/m^2$$

------***----

2. An oil of viscosity 0.1 Ns/m^2 and relative density 0.9 is flowing through a circular pipe of diameter 50 mm and of length 300 m. The rate of flow of fluid through the pipe is 3.5 litres/s. Find the pressure drop in a length of 300 m and also the shear stress at the pipe wall.

Given:

Viscosity, $\mu = 0.1\ Ns/m^2$

Relative Density = 0.9

∴ Density of oil = 0.9 x 1000 = 900 kg/m^3

D = 50 mm = 0.05 m

L = 300 m

$Q = 3.5 \text{ litres/s} = \frac{3.5}{1000} = 0.0035\ m^3/s$

To Find: (i) Pressure drop, $P_1 - P_2$

(ii) Shear stress at pipe wall, τ_o

(i) Pressure Drop (P_1 -P_2)

$$\bar{u} = \frac{Q}{\text{Area}} = \frac{0.0035}{\frac{\pi}{4}D^2} = \frac{0.0035}{\frac{\pi}{4}(0.05)^2} = 1782 \text{ m/s}$$

$$R_e = \frac{\rho V D}{\mu}$$

where ρ = 900 kg/ m^3, V= average velocity = u = 1.782 m/s

$$R_e = 900 \text{ x } \frac{1.782 \text{ x} 0.05}{0.1} = 801.9$$

As Reynold number is less than 2000, the flow is viscous or laminar

$$P_1 - P2 = \frac{32 \text{ x } 0.1 \text{ x } 1.782 \text{ x } 3000}{(0.05)^2} = 684288 \text{ N/m}^2$$

(ii) Shear Stress at the pipe wall (τ_0)

Shear stress at pipe wall,

$$\tau_0 = \frac{\partial p}{\partial x} \cdot \frac{R}{2}$$

$$\frac{\partial p}{\partial x} = \frac{(P_1 - P_2)}{x_2 - x_1} = \frac{(P_1 - P_2)}{x_2 - x_1} = \frac{P_1 - P_2}{L}$$

$$= \frac{684288}{300} = 2280.96 \text{ N/ m}^3$$

$$= \frac{D}{2} = \frac{0.05}{2} = 0.025 \text{ m}$$

$$\tau_0 = 2280.96 \text{ x } \frac{0.025}{2} = 28.512 \frac{\text{N}}{\text{m}^2}$$

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Problem 7. *Oil of specific gravity 0.92 flows at a rate of 4.5 litres/s through a pipe of 5 cm dia, the pressure drop over 100 m horizontal length being 15 N/cm².* ***Determine the dynamic viscosity of the oil.***

Using the equation 7.9.2 – Hagen-Poiseuille eqn. $\Delta p = 128\ \mu LQ/\pi D^4$

$\mu = \Delta p\ .\ \pi.D^4/128LQ$

$= 15 \times 10^4 \times \pi \times 0.054/128 \times 100 \times 0.0045 =$ **0.05113 Ns/m²** (Pa.s)

(**Note:** N/cm² → 104 N/m², litre = 0.001 m³)

Reynolds number = $uD\ \rho/\mu$, $u = Q \times 4/\pi D^2$

$\therefore$ Re = $(4Q/\pi\ D^2) \times (D\ \rho/\mu) = (0.0045 \times 920 \times 4)/(\pi \times 0.05 \times 0.05113)$

$= 2061.6$

$\therefore$ Flow is laminar but just on the verge of turning turbulent

(Note: Re = 4Q/π Dv)

Problem. 7.1. ***An oil of specific gravity 0.82 and kinematic viscosity 16×10^{-6} m²/s flows in a smooth pipe of 8 cm diameter at a rate of 2 l/s. Determine whether the flow is laminar or turbulent. Also calculate the velocity at the centre line and the velocity at a radius of 2.5 cm. What is head loss for a length of 10 m. What will be the entry length? Also determine the wall shear.***

Average flow velocity = volume flow/area (Q/A) = $4 \times 0.002/\pi \times 0.08^2$ =0.4 m/s

$$Re = \frac{uD}{v} = \frac{0.4 \times 0.08}{16 \times 10^{-6}} = 2000$$

This value is very close to transition value. However for smooth pipes the flow may be taken as laminar.

Centre line velocity = 2 × average velocity = 0.8 m/s

For velocity at 2.5 cm radius

$$\frac{u}{u_{max}} = 1 - \left(\frac{r}{R}\right)^2 \quad \therefore \quad \mathbf{u} = 0.8\left[1 - \left(\frac{2.5}{4}\right)^2\right] = \mathbf{0.4875\ m/s}$$

$$\mathbf{f} = 64/\text{Re} = 64/2000 = \mathbf{0.032}$$

$$\mathbf{h_f} = fLu^2/2gd = (0.032 \times 10 \times 0.4^4)/(2 \times 9.81 \times 0.08)$$

$$= \mathbf{0.03262\ m} \text{ of oil}$$

$$\mathbf{\Delta p} = h_f\gamma = 0.03262 \times 9810 \times 0.82 = \mathbf{262.4}\ \text{N/m}^2$$

Entry length = 0.058 Re.*D.* = 0.058 × 2000 × 0.08 = **9.28 m**

For highly viscous fluid entry length will be long. Wall shear is found from the definition of *f*.

$$\tau_o = \frac{f}{4}\frac{\rho}{g_o}\frac{u_m{}^2}{2} = \frac{0.032}{4}\times\frac{820}{1}\times\frac{0.4^2}{2} = \mathbf{0.5248\ N/m^2}$$

Wall shear can also be found using, $\tau_o = -\rho\, v\dfrac{du}{dr}$

$$u = u_{\max}\left[1 - \frac{r^2}{R^2}\right], \frac{du}{dr} = -\frac{U_{\max}\, 2r}{R^2}, \text{ at } r = R, \frac{du}{dr} = -u_{\max}\frac{2}{R}$$

Substituting, $\tau_o = 820 \times 16 \times 10^{-6} \times 0.8 \times 2/0.04 = 0.5248\ N/m^2$.

Minor Energy Losses

$$\text{loss of head due to sudden enlargement } h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$\text{loss of head due to sudden contraction } h_c = 0.5\ \frac{V_2^2}{2g}$$

$$\text{loss of head at the entrance of pipe } h_i = 0.5\frac{V^2}{2g}$$

$$\text{loss of head at the exit of the pipe } h_0 = \frac{V^2}{2g}$$

$$\text{loss of head due to an obstruction in a pipe } h_{obs} = \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]$$

A - Area of the pipe

a - area of obstruction

C_c – Coefficient of contraction

Loss of head due to bend in a pipe

$h_b = \frac{kV^2}{2g}$ k- Coefficient of bend

Loss of head in various in pipe fittings

$h = \frac{kV^2}{2g}$ k- Coefficient of pipe fittings

FLOW THROUGH PIPES IN SERIES

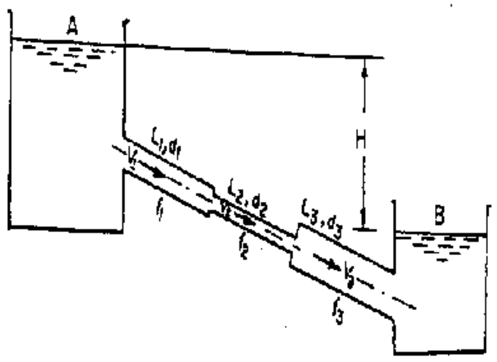
L_1, L_2, L_3 – Length of pipes 1, 2, and 3 respectively

d_1, d_2, d_3 – Diameter of pipes 1, 2, and 3 respectively

V_1, V2, V_3 – Velocity of flow through pipes 1, 2, 3

f_1, f_2, f_3 – Coefficient of friction for pipes 1,2,3

H—Difference of water level in the two tanks



The discharge passing through each pipe is same

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$$H = \frac{0.5{V_1}^2}{2g} + \frac{4f_1L_1{V_1}^2}{2gd_1} + \frac{0.5{V_2}^2}{2g} + \frac{4f_2L_2{V_2}^2}{2gd_2} + \frac{((V_2 - V_3))^2}{2g} + \frac{4f_3L_3{V_3}^2}{2gd_3} + \frac{V_3^2}{2g}$$

If minor losses are neglected,

$$H = \frac{4f_1 L_1 V_1^{\;2}}{2gd_1} + \frac{4f_2 L_2 V_2^{\;2}}{2gd_2} + \frac{4f_3 L_3 V_3^{\;2}}{2gd_3}$$

If $f_1 = f_2 = f_3 = f$

$$H = \frac{4f}{2g}\left[\frac{L_1 V_1^{\;2}}{d_1} + \frac{L_2 V_2^{\;2}}{d_2} + \frac{L_3 V_3^{\;2}}{d_3}\right]$$

FLOW THROUGH PARELLEL PIPES

The rate of flow in the main pipe is equal to the sum of rate of flow through branch pipes

$Q_1 = Q_1 + Q_2$

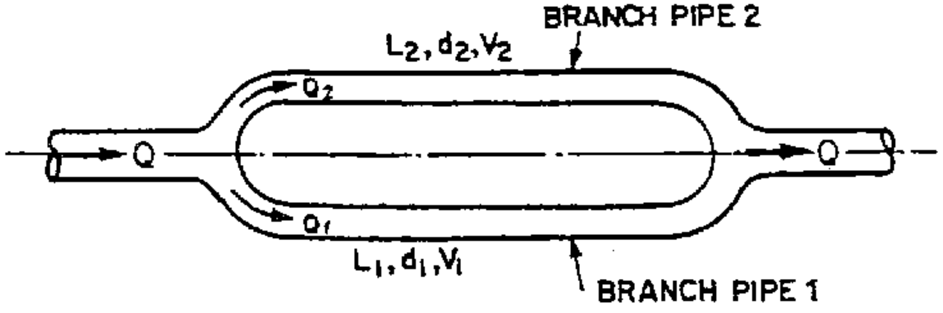
In this arrangement, the loss of head for each branch pipe is same

Loss of head for branch pipe 1 = Loss of head for branch pipe 2

$$\frac{4f_1 L_1 V_1{}^2}{2gd_1} = \frac{4f_2 L_2 V_2{}^2}{2gd_2}$$

if $f_1 = f_2$,

$$\frac{L_1 V_1{}^2}{d_1} = \frac{L_2 V_2{}^2}{d_2}$$



An oil of viscosity 0.1 Ns/m^2 and relative density 0.9 is flowing through a circular pipe of diameter 50mm and of length 300m. The rate of flow of fluid through the pipe is 3.5 litres/s. Find the pressure drop in a length of 300m and also the shear stress at the pipe wall.

Solution : given

Viscosity, $\mu = 0.1\ Ns/m^2$

Relative density = 0.9

$\therefore \rho_o$ or density of oil = 0.9 x 1000 = 900 kg/m^3

D = 50mm = 0.05 m

L= 300 m

Q = 3.5 litres/s = 0.0035 m^3/s

Find : 1. Pressure Drop, $P_1 - P_2$

2. Shear stress at pipe wall, τ_o

1. Pressure Drop, $(p_1 - p_2) = \frac{32\ \mu \bar{u}\ L}{D^2}$

Where

$$\bar{u} = \frac{Q}{\text{area}\left(\frac{\pi D^2}{4}\right)} = \frac{0.0035}{\text{area}\left(\frac{\pi\ 0.05^2}{4}\right)} = 1.782\ m/s$$

The Reynolds number (Re) is given by

$$Re = \frac{\rho v D}{\mu} = \frac{900 \text{ x } 1.782 \text{ x } 0.05}{0.1} = 801.9$$

As Reynolds number is less than 2000, the flow is viscous or laminar

$$(p_1 - p_2) = \frac{32 \text{ x } 0.1 \text{ x } 1.782 \text{ x } .05}{0.05^2}$$

$$= 684288 \text{ N/m}^2 = 68.43 \text{ N/cm}^2$$

2. shear stress at the pipe wall (τ_o)

The shear stress at any radius r is given by the equation

$$\tau_o = \frac{-dP}{dx}\frac{r}{2}$$

$\therefore$ Shear stress at pipe wall, where r = R is given by

$$\tau_o = \frac{-dP}{dx}\frac{R}{2}$$

$$-\frac{\partial p}{\partial x} = \frac{-(p_2 - p_1)}{x_2 - x_1} = \frac{(p_1 - p_2)}{x_2 - x_1} = \frac{(p_1 - p_2)}{L}$$

$$= \frac{684288}{300}\frac{N}{m^3} = 2280.96\frac{N}{m^3}$$

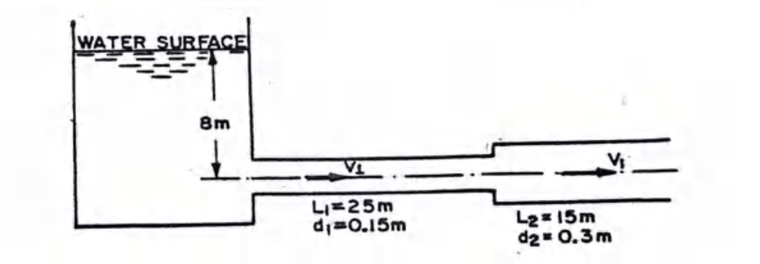
$$\tau_o = 2280.96\,\frac{0.025}{2}\,\text{N/m}^2$$

$$\tau_o = 28.512\frac{\text{N}}{\text{m}^2} \quad \text{Ans}$$

A horizontal pipe line 40m long is connected to a water tank at one end discharges likely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150mm diameter and its diameter is suddenly enlarged to 300mm. the height of water level in the tank is 8m above the centre of pipe. Considering all losses of head which occur, determine the rate of flow. Take f= 0.01 for the sections of the pipe

Solution: Given

Total length of pipe, L = 40m



Length of 1st pipe, L_1 = 25m

diameter of 1st pipe, d_1 = 150 mm = 0.15m

Length of 2nd pipe, L_2 = 40-25 = 15 m

Dia. of 2nd pipe, d_2 = 300mm = 0.3 m

Height of water, H = 8 m

Co-efficient of friction, f =0.01

Applying the Bernoulli's theorem to be free surface of water in the tank and outlet of pipe as shown in Fig. and taking reference line passing through the centre of pipe.

$$0 + 0 + 8 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + 0 + all\ losses$$

$$8 = 0 + \frac{v_2^2}{2g} + h_i + h_{f1} + h_e + h_{f2}$$

loss of head due to sudden enlargement $h_e = \frac{(V_1 - V_2)^2}{2g}$

loss of head at the entrance of pipe $h_i = 0.5 \frac{V_1^2}{2g}$

$$h_{f1} = head\ lost\ due\ to\ friction\ in\ pipe\ 1 = \frac{4fL_1{V_1}^2}{2gd_1}$$

$$h_{f2} = head\ lost\ due\ to\ friction\ in\ pipe\ 2 = \frac{4fL_2{V_2}^2}{2gd_2}$$

But from continuity equation, we have

$A_1 V_1 = A_2 V_2$

$$V_1 = \frac{A_2 v_2}{A_1} = \frac{d_2^2}{d_1^2} V_2 = \frac{0.03^2}{0.15^2} V_2 = 4V_2$$

Substituting the value V_1 in different head losses, we have

$$h_i = 0.5 \frac{V_1^2}{2g} = 0.5 \frac{4V_2^2}{2g} = \frac{8V_2^2}{2g}$$

$$h_{f1} = \frac{4 \text{ x } 0.01 \text{ x } 25x\ 4{V_2}^2}{2g \text{ x } d_1}$$

$$= \frac{4\ x\ .01\ x\ 25\ x\ 16}{0.15} \frac{V_2^2}{2g} = 106.67 \frac{V_2^2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

$$h_{f2} = \frac{4fL_2{V_2}^2}{2gd_2} = \frac{4 \text{ x } 0.01 \text{ x } 25x\ {V_2}^2}{2g \text{ x } d_2} = 2.0\frac{V_2^2}{2g}$$

Substituting the values in eqn (1), we get

$$8 = \frac{v_2^2}{2g} + \frac{8v_2^2}{2g} + 106.67\frac{v_2^2}{2g} + \frac{9v_2^2}{2g} + \frac{2v_2^2}{2g}$$

$$8 = 126.67\ \frac{v_2^2}{2}$$

$$v_2 = \sqrt{\frac{8\ x\ 2.0\ x\ g}{126.67}} = \sqrt{\frac{8\ x\ 2.0\ x\ 9.81}{126.67}} = 1.113\frac{m}{s}$$

$\therefore$ Rate of flow Q = A_2 x V_2 = π x $0.3^2/4$ x 1.113

= 0.07867 m^3/s = 78.67 litres/s---Ans

SVCE

Sri Venkateswara College of Engineering

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UNIT 3

DIMENSIONAL ANALYSIS

Introduction

- **Dimensional analysis** *is a mathematical technique which makes use of the study of the dimensions for solving several engineering problems.*
- Each physical phenomenon can be expressed by an equation giving relationship between different quantities, such quantities are dimensional and non-dimensional.
- Dimensional analysis helps in determining a systematic arrangement of the variables in the physical relationship, combining dimensional variables to form non-dimensional parameters.
- It is based on the *principle of dimensional homogeneity.*
- Dimensional analysis has become an important tool for analysing fluid flow problems.
- It is *specially useful in presenting experimental results in a concise form*.

Dimensions

- The various physical quantities used in fluid phenomenon can be expressed in terms of fundamental quantities or primary quantities.
- The fundamental quantities are ***mass, length, time*** **and** ***temperature***, designated by the letters, ***M, L, T, θ*** respectively.
- The quantities which are expressed in terms of the fundamental or primary quantities are called derived or secondary quantities, *(e.g.*. velocity, area, acceleration etc.).
- *The expression for a derived quantity in terms of the primary quantities is called the* dimension *of the physical quantity.*
- A quantity may either be expressed dimensionally in ***M-L-T*** **or** ***F-L-T*** system (some engineers prefer to use force instead of mass as fundamental quantity because the force is easy to measure).

	Quantity	Definition	Formula	Units	Dimensions
Basic machanicals	**Length or Distance**	*fundamental*	d	m (meter)	*L (Length)*
	Time	*fundamental*	t	s (second)	*T (Time)*
	Mass	*fundamental*	m	kg (kilogram)	*M (Mass)*
	Area	$distance^2$	$A = d^2$	m^2	L^2
	Volume	$distance^3$	$V = d^3$	m^3	L^3
	Density	mass / volume	d = m/V	kg/m^3	M/L^3
	Velocity	distance / time	v = d/t	m/s c (speed of light)	*L/T*
	Acceleration	velocity / time	a = v/t	m/s^2	L/T^2
	Momentum	mass × velocity	p = m·v	kg·m/s	*ML/T*
	Force Weight	mass × acceleration mass × acceleration of gravity	F = m·a W = m·g	N (newton) = $kg{\cdot}m/s^2$	ML/T^2
	Pressure or Stress	force / area	p = F/A	Pa (pascal) = N/m^2 = $kg/(m{\cdot}s^2)$	M/LT^2

Quantity	SI Unit		Dimension
velocity	m/s	ms^{-1}	LT^{1}
acceleration	m/s^2	ms^{-2}	LT^{2}
force	N $kg\ m/s^{2}$	$kg\ ms^{-2}$	$M\ LT^{2}$
energy (or work)	Joule J N m, $kg\ m^2/s^2$	$kg\ m^2s^{-2}$	ML^2T^{-2}
power	Watt W N m/s $kg\ m^2/s^3$	Nms^{-1} $kg\ m^2s^{-3}$	ML^2T^{-2}
pressure (or stress)	Pascal P, N/m^2, $kg/m/s^{2}$	Nm^{-2} $kg\ m^{-1}s^{-2}$	$ML^{-1}T^{-2}$
density	kg/m^3	$kg\ m^{-3}$	ML^{-3}
specific weight	N/m^3 $kg/m^2/s^2$	$kg\ m^{-2}s^{-2}$	$ML^{2}T^{-2}$
relative density	a ratio no units		1 no dimension
viscosity	$N\ s/m^2$ kg/m s	$N\ sm^{-2}$ $kg\ m^{-1}s^{-1}$	$M\ L^{-1}T^{-1}$
surface tension	N/m $kg\ /s^2$	Nm^{-1} $kg\ s^{-2}$	MT^{2}

Physical Quantity	Relation With Other Quantities	Dimensional Formula
Areas	$Length \times Breath$	$L \times L = L^2 = [M^0L^2T^0]$
Volume	$Length \times Breath \times Height$	$L \times L \times L = L^2 = [M^0L^3T^0]$
Density	$\frac{Mass}{Volume}$	$\frac{M}{L^3} = [ML^{-3}T^0]$
Speed or Velocity	$\frac{Distance}{Time}$	$\frac{L}{T} = [M^0LT^{-1}]$
Acceleration	$\frac{Velocity}{Time}$	$\frac{LT^{-1}}{T} = [M^0LT^{-2}]$
Momentum	$Mass \times Velocity$	$M \times LT^{-1} = [MLT^{-1}]$
Force	$Mass \times Acceleration$	$M \times LT^{-2} = [MLT^{-2}]$
Pressure	$\frac{Force}{Area}$	$\frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$
Work	$Force \times Distance$	$MLT^{-2} \times L = [ML^2T^{-2}]$
Energy	$Work$	$[ML^2T^{-2}]$
Power	$\frac{Work}{Time}$	$\frac{ML^2T^{-2}}{T} = [ML^2T^{-3}]$

Quantity	*Dim*
	M-L-T System
(*a*) **Fundamental Quantities**	
Mass, *M*	M
Length, *L*	L
Time, *T*	T
(*b*) **Geometric Quantities**	
Area, *A*	L^2
Volume, ∀	L^3
Moment of inertia	L^4
(*c*) **Kinematic Quantities**	
Linear velocity, *u*, *V*, *U*	LT^{-1}
Angular velocity, ω; rotational speed, *N*	T^{-1}
Acceleration, *a*	LT^{-2}
Angular acceleration, α	T^{-2}
Discharge, *Q*	L^3T^{-1}
Gravity, *g*	LT^{-2}

(d) Dynamic Quantities

Force, *F*	MLT^{-2}
Density, ρ	ML^{-3}
Specific weight, *w*	$ML^{-2}T^{-2}$
Dynamic viscosity, μ	$ML^{-1}T^{-1}$
Pressure, *p*; shear stress, τ	$ML^{-1}T^{-2}$
Modulus of elasticity, *E*, *K*	$ML^{-1}T^{-2}$
Momentum	MLT^{-1}
Angular momentum or moment of momentum	$ML^{2}T^{-1}$
Work, *W*; energy, *E*	$ML^{2}T^{-2}$
Torque, *T*	$ML^{2}T^{-2}$
Power, *P*	$ML^{2}T^{-3}$

Dimensional Homogeneity

- A physical equation is the relationship between two or more physical quantities. Eg. $Q = A.v$
- Any *correct equation* expressing a physical relationship between quantities, *must be dimensionally homogeneous.*
- *A* dimensionally homogeneous equation is applicable to all systems of units.
- Let us consider the equation: $P = \rho g h$

In a dimensionally homogeneous equation, only quantities having the same dimensions can be added, subtracted or equaled.

The *principle of homogeneity* proves useful in the following ways:

- It facilitates to determine the dimensions of a physical quantity.
- It helps to check whether an equation of any physical phenomenon is dimensionally homogeneous or not.
- It facilitates conversion of units from one system to another.
- It provides a step towards dimensional analysis which is effectively employed to plan experiments and to present the results meaningfully.

Methods of Dimensional Analysis

- With the help of dimensional analysis the equation of **a** physical phenomenon can be developed in terms of dimensionless groups or parameters and thus **reducing the number of variables.**
- *The methods of dimensional analysis are based on the Fourier*'s *principle of homogeneity.*
- The methods of dimensional analysis are:

1. Rayleigh's method
2. **Buckingham's pi-method**
3. Bridgman's method
4. Matrix-tensor method
5. By visual inspection of the variables involved
6. Rearrangement of differential equations

Buckingham's π-theorem states as follows:

"If there are n variables (dependent and independent variables) in a dimensionally homogeneous equation and if these variables contain m fundamental dimensions (such as M, L, T, etc.) then the variables are arranged into (n-m) dimensionless terms. These dimensionless terms are called π-terms."

If an equation involving *n* variables is dimensionally homogeneous, it can be reduced to a relationship among (n-m) independent dimensionless products, where *m* is the minimum number of reference dimensions required to describe the variables.

- **The dimensionless products are frequently referred to as "pi terms," and the theorem is called the Buckingham pi theorem.**

Procedure for solving problems by Buckingham's Pi-Theorem

1. List all the variables that are involved in the problem and find the total number of variables (n).
2. Express each of the variables in terms of basic dimension and find the number of basic dimension (m).
3. Determine the number of π-terms = n – m. Each π-terms contains (m +1) variables.
4. Select a number of repeating variables, which is equal to "m".
5. Form a π-terms by multiplying one of the non-repeating variable by the product of repeating variable and solve by equating the power of basic dimension.
6. Repeat step 5 for each of the remaining non-repeating variables.
7. Finally arranged in require dimensionless terms.

Selection of repeating variables

- The following points should be kept in view while selecting *m* repeating variables:

1. Repeating variables must contain *jointly* all the fundamental dimensions involved in the phenomenon. Usually the fundamental dimensions are *M*, *L* and *T*.
2. If only two fundamental dimensions are involved, there will be 2 repeating variables and they must contain *together* the two dimensions involved.
3. The repeating variables *must not* form the non-dimensional parameters among themselves.
4. The dependent variable *should not* be selected as repeating variable.
5. *No two repeating variables should have the same dimensions.*
6. The repeating variables should be chosen in such a way that one variable contains **geometric property** *(e g diameter, height, length* etc.), other variable contains **flow property** *(e g. Velocity, acceleration, speed* etc.) and third variable contains **fluid property** *(e.g. density, dynamic viscosity,* etc.).

1. The resistance R experienced by a partially submerged body depends upon the velocity V. length of the body l, viscosity of the fluid μ, density of the fluid ρ and gravitational acceleration g. Obtain a dimensionless expression for R.

Solution.

The resistance *R* is a function of:

(i) Velocity (ii) Length (iii) Viscosity (iv) Density,

(v) Gravitational acceleration

Mathematically, $R = f(V, l, \mu, \rho, g)$ ...(i)

or $f_1(R, V, l, \mu, \rho, g) = c$

Total number of variables, n = 6

Number of dimensionless π-terms = n – m = 6-3 = 3 π-terms

$f_1(\pi_1, \pi_2, \pi_3) = c$

m is obtained by writing dimensions of each variable as $R = MLT^{-2}$, $V = LT^{-1}$, $\mu = ML^{-1}T^{-1}$, $\rho = ML^{-3}$, $g = LT^{-2}$. Thus the fundamental dimensions in the problem are *M*, *L*, *T* and hence *m* = 3

- Each π-term = $m + 1$ variables

$$\left.\begin{aligned} \pi_1 &= l^{a_1} . V^{b_1} . \rho^{c_1} . R \\ \pi_2 &= l^{a_2} . V^{b_2} . \rho^{c_2} . \mu \\ \pi_3 &= l^{a_3} . V^{b_3} . \rho^{c_3} . g \end{aligned}\right\}$$

Each π-term is solved by the principle of dimensional homogeneity

$$\pi_1 = l^{a1} . V^{b1} . \rho^{c1} . R$$

$M^0L^0T^0 = L^{a1} . (LT^{-1})^{b1} . (ML^{-3})^{c1} . (MLT^{-2})$

Equating the exponents of M, L and T respectively, we get

For M : $0 = c_1 + 1$

For L : $0 = a_1 + b_1 - 3c_1 + 1$

For T : $0 = - b_1 - 2$

$c_1 = -1$, $b_1 = -2$,

$a_1 = - b_1 + 3c_1 - 1$

$a_1 = - 2$

substituting the value of a_1 , b_1 , c_1 in π_1

$$\pi_1 = l^{-2} . V^{-2} . \rho^{-1} . R = R / (l^2V^2\rho)$$

$$\pi_2 = l^{a2} \cdot V^{b2} \cdot \rho^{c2} \cdot \mu$$

$$M^0L^0T^0 = L^{a2} \cdot (LT^{-1})^{b2} \cdot (ML^{-3})^{c2} \cdot (ML^{-1}T^{-1})$$

Equating the exponents of M, L and T respectively, we get

For M : $0 = c_2 + 1$

For L : $0 = a_2 + b_2 - 3c_2 - 1$

For T : $0 = - b_2 - 1$

$c_2 = -1, b_2 = -1$

$a_2 = - b_2 + 3c_2 + 1$

$a_2 = - 1$

substituting the value of a_2 , b_2 , c_2 in π_2

$$\pi_2 = l^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu = \mu / (lV\rho)$$

$$\pi_3 = l^{a3} \cdot V^{b3} \cdot \rho^{c3} \cdot g$$

$$M^0L^0T^0 = L^{a3} \cdot (LT^{-1})^{b3} \cdot (ML^{-3})^{c3} \cdot (LT^{-2})$$

Equating the exponents of M, L and T respectively, we get

For M : $0 = c_3$

For L : $0 = a_2 + b_3 - 3c_3 + 1$

For T : $0 = - b_3 – 2$

$c_3 = 0$, $b_3 = - 2$,

$a_3 = - b_3 + 3c_3 - 1$

$a_3 = 1$

substituting the value of a_3 , b_3 , c_3 in π_3

$$\pi_3 = l^1 \cdot V^{-2} \cdot \rho^0 \cdot g = lg / V^2$$

$$f_1\,(\pi_1, \pi_2, \pi_3) = c$$

$$f_1\,\left(R / (l^2V^2\rho),\ \mu / (lV\rho),\ lg / V^2\right) = c$$

$$R / (l^2V^2\rho) = \varphi\,\left(\mu / (lV\rho),\ lg / V^2\right)$$

$$R = (l^2V^2\rho)\,.\,\varphi\,\left(\mu / (lV\rho),\ lg / V^2\right)$$

The resistance R is thus a function of Reynolds number $\left(\dfrac{\rho Vl}{\mu}\right)$ and Froude's number $\left(\dfrac{V}{\sqrt{lg}}\right)$.

2. Using Buckingham s π-theorem, show that the velocity through a circular orifice is given by

$$V = \sqrt{2gH}\,\phi\left[\frac{D}{H}, \frac{\mu}{\rho V H}\right]$$

Where: D = Diameter of the orifice, ρ = Mass density, H = Head causing flow, μ = Co-efficient of viscosity, g =Acceleration due to gravity.

Solution;

V is a function of: H, D, ρ, μ and g

Mathematically, $V = f(H, D, \rho, \mu, g)$

or $f_1(V, H, D, \rho, \mu, g) = c$

Total number of variables, $n = 6$

Writing dimensions of each variable, we have

$$V = LT^{-1}, H = L, D = L, \mu = ML^{-1}T^{-1}, \rho = ML^{-3}, g = LT^{-2}$$

Number of fundamental dimensions, m = 3

.'. Number of π-terms = $n - m = 6 - 3 = 3$

It can be written as:

$$f_1(\pi_1, \pi_2, \pi_3) = c$$

Each π-term contains (m + 1) variables, where *m* = 3 and is also equal to repeating variables.

Choosing *H, g,* ρ as *repeating variables* (*V* being a dependent variable should not be chosen as repeating variable), we get three π -terms as:

$$\pi_1 = H^{a_1} \cdot g^{b_1} \cdot \rho^{c_1} \cdot V$$
$$\pi_2 = H^{a_2} \cdot g^{b_2} \cdot \rho^{c_2} \cdot D$$
$$\pi_3 = H^{a_3} \cdot g^{b_3} \cdot \rho^{c_3} \cdot \mu$$

Each π-term is solved by the principle of dimensional homogeneity

$$\pi_1 = H^{a1} . g^{b1} . \rho^{c1} . V$$

$$M^0L^0T^0 = L^{a1} . (LT^{-2})^{b1} . (ML^{-3})^{c1} . (LT^{-1})$$

Equating the exponents of M, L and T respectively, we get

For M : $0 = c_1 + 0$

For L : $0 = a_1 + b_1 - 3c_1 + 1$

For T : $0 = - 2b_1 – 1$

$c_1 = 0$, $b_1 = -1/2$,

$a_1 = - b_1 + 3c_1 - 1$

$a_1 = - 1/2$

substituting the value of a_1 , b_1 , c_1 in π_1

$$\boldsymbol{\pi_1 = H^{-1/2} . g^{-1/2} . \rho^0 . V}$$

$$\boldsymbol{\pi_1 = V / \sqrt{gH}}$$

$\pi_2 = H^{a2} . g^{b2} . \rho^{c2} . D$

$M^0L^0T^0 = L^{a2} . (LT^{-2})^{b2} . (ML^{-3})^{c2} . L$

Equating the exponents of M, L and T respectively, we get

For M : $0 = c_2$

For L : $0 = a_2 + b_2 - 3c_2 + 1$

For T : $0 = - 2b_1$

$c_2 = 0, b_2 = 0$

$a_2 = - b_2 + 3c_2 - 1$

$a_2 = - 1$

substituting the value of a_2 , b_2 , c_2 in π_2

$\pi_2 = H^{-1} . g^0 . \rho^0 . D$

$\boldsymbol{\pi_2 = D / H}$

$\pi_3 = H^{a3} . g^{b3} . \rho^{c3} . \mu$

$M^0L^0T^0 = L^{a3} . (LT^{-2})^{b3} . (ML^{-3})^{c3} . ML^{-1}T^{-1}$

Equating the exponents of M, L and T respectively, we get

For M : $0 = c_3 + 1$

For L : $0 = a_3 + b_3 - 3c_3 - 1$

For T : $0 = - 2b_3 - 1$

$c_3 = -1, b_3 = -1/2$

$a_3 = - b_3 + 3c_3 + 1$

$a_3 = - 3/2$

substituting the value of a_3 , b_3 , c_3 in π_3

$$\pi_3 = H^{-3/2} . g^{-1/2} . \rho^{-1} . \mu$$

$$\pi_3 = \mu / (H^{3/2} . g^{1/2} . \rho)$$

$$\pi_3 = \mu / (H^{3/2} \cdot g^{1/2} \cdot \rho)$$

$$\pi_3 = \mu / (H \rho \cdot (Hg)^{1/2})$$

$$\pi_3 = \mu . V / (VH \rho \cdot (Hg)^{1/2})$$ *(Multiply and divide by V)*

$$\pi_3 = \mu . \pi_1 / (VH \rho)$$

$$f_1 (\pi_1, \pi_2, \pi_3) = c$$

$$f_1 (V / \sqrt{gH}, D / H, \mu . \pi_1 / (VH \rho)) = c$$

$$\frac{V}{\sqrt{gH}} = \phi\left[\frac{D}{H}, \frac{\mu}{H\rho V} . \pi_1\right]$$

$$V = \sqrt{2gH}\,\phi\left[\frac{D}{H}, \frac{\mu}{\rho V H}\right]$$

Multiplying and dividing by any constant does not change the character of π -terms

3. The pressure difference Δp in a pipe of diameter D and length l due to turbulent flow depends on the velocity V, viscosity μ, density ρ and roughness k. Using Buckingham's pi-theorem, obtain an expression for Δp.

Solution.

The pressure difference Δp is a function of: $D, l, V\ \mu, \rho, k$

Mathematically, $\Delta p = f(D, l, V\ \mu, \rho, k)$

or $f_1(\Delta p, D, l, V\ \mu, \rho, k) = c$

∴ Total number of variables, $n = 7$

Writing dimensions of each variable, we have

Δp (dimensions of pressure) $= ML^{-1}T^{-2}$, $D = L$, $l = L$, $V = LT^{-1}$,

$\mu = ML^{-1}T^{-1}$, $\rho = ML^{-3}$, $k = L$

Thus, number of fundamental dimensions, $m = 3$

Number of π -terms = n-m = 7-3 = 4 terms

It can be written as:

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4) = c$$

Each π-term contains (m + 1) variables, where *m* = 3 and is also equal to repeating variables.

Choosing *D, V,* ρ as *repeating variables* (being a dependent variable should not be chosen as repeating variable), we get four π -terms as:

$$\pi_1 = D^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot \Delta p$$

$$\pi_2 = D^{a_2} \cdot V^{b_2} \cdot \rho^{c_2} \cdot l$$

$$\pi_3 = D^{a_3} \cdot V^{b_3} \cdot \rho^{c_3} \cdot \mu$$

$$\pi_4 = D^{a_4} \cdot V^{b_4} \cdot \rho^{c_4} \cdot \mu$$

Each π-term is solved by the principle of dimensional homogeneity

$$\pi_1 = D^{a1} . V^{b1} . \rho^{c1} . \Delta p$$

$$M^0L^0T^0 = L^{a1} . (LT^{-1})^{b1} . (ML^{-3})^{c1} . (ML^{-1}T^{-2})$$

Equating the exponents of M, L and T respectively, we get

For M : $0 = c_1 + 1$

For L : $0 = a_1 + b_1 - 3c_1 - 1$

For T : $0 = - b_1 - 2$

$c_1 = -1, b_1 = -2,$

$a_1 = - b_1 + 3c_1 + 1$

$a_1 = 0$

substituting the value of a_1 , b_1 , c_1 in π_1

$$\pi_1 = D^0 . V^{-2} . \rho^{-1} . \Delta p$$

$$\pi_1 = \Delta p / (\rho V^2)$$

$$\pi_2 = D^{a2} . V^{b2} . \rho^{c2} . l$$

$$M^0L^0T^0 = L^{a2} . (LT^{-1})^{b2} . (ML^{-3})^{c2} . L$$

Equating the exponents of M, L and T respectively, we get

For M : $0 = c_2$

For L : $0 = a_2 + b_2 - 3c_2 + 1$

For T : $0 = - b_2$

$c_2 = 0, b_2 = 0,$

$a_2 = - b_2 + 3c_2 - 1$

$a_2 = - 1$

substituting the value of a_2 , b_2 , c_2 in π_2

$$\boldsymbol{\pi_2 = D^{-1} . V^0 . \rho^0 . l}$$

$$\boldsymbol{\pi_2 = l / D}$$

$$\pi_3 = D^{a3} \cdot V^{b3} \cdot \rho^{c3} \cdot \mu$$

$$M^0L^0T^0 = L^{a3} \cdot (LT^{-1})^{b3} \cdot (ML^{-3})^{c3} \cdot ML^{-1}T^{-1})$$

Equating the exponents of M, L and T respectively, we get

For M : $0 = c_3 + 1$

For L : $0 = a_3 + b_3 - 3c_3 - 1$

For T : $0 = - b_3 - 1$

$c_3 = -1$, $b_3 = -1$,

$a_3 = - b_3 + 3c_3 + 1$

$a_3 = - 1$

substituting the value of a_3 , b_3 , c_3 in π_3

$$\pi_3 = D^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu$$

$$\pi_3 = \mu / (DV\rho)$$

$$\pi_4 = D^{a4} \cdot V^{b4} \cdot \rho^{c4} \cdot k$$

$$M^0L^0T^0 = L^{a4} \cdot (LT^{-1})^{b4} \cdot (ML^{-3})^{c4} \cdot L$$

Equating the exponents of M, L and T respectively, we get

For M : $0 = c_4$

For L : $0 = a_4 + b_4 - 3c_4 + 1$

For T : $0 = - b_4$

$c_4 = 0, b_4 = 0,$

$a_4 = - b_4 + 3c_4 - 1$

$a_4 = - 1$

substituting the value of a_4 , b_4 , c_4 in π_4

$$\boldsymbol{\pi_4 = D^{-1} \cdot V^0 \cdot \rho^0 \cdot K}$$

$$\boldsymbol{\pi_4 = k / D}$$

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4) = c$$

$$f_1\left(\frac{\Delta p}{\rho V^2}, \frac{l}{D}, \frac{\mu}{DV\rho}, \frac{k}{D}\right) = 0$$

$$\frac{\Delta p}{\rho V^2} = \phi\left[\frac{l}{D}, \frac{\mu}{DV\rho}, \frac{k}{D}\right]$$

Expression for difference of pressure head (h_f)

Δp is a linear function of l/D, therefore taking this out of function

$$\frac{\Delta p}{\rho V^2} = \frac{l}{D}\phi\left[\frac{\mu}{DV\rho}, \frac{k}{D}\right]$$

$$\frac{\Delta p}{\rho} = V^2 \cdot \frac{l}{D}\phi\left[\frac{\mu}{DV\rho}, \frac{k}{D}\right]$$

- Dividing both sides by g, we get

$$\frac{\Delta p}{\rho g} = \frac{V^2}{g} \cdot \frac{l}{D} \phi \left[\frac{\mu}{DV\rho}, \frac{k}{D} \right]$$

Now $\phi \left[\frac{\mu}{DV\rho}, \frac{k}{D} \right]$ consists of following two terms:

(*i*) $\frac{\mu}{DV\rho}$ which is $\frac{1}{\text{Reynold number}}$ or $\frac{1}{Re}$

(*ii*) $\frac{k}{D}$

$\phi \left[\frac{1}{Re}, \frac{k}{D} \right]$ is put equal to f

where f is Co-efficient of friction (function of Re and k)

$$\frac{\Delta p}{\rho g} = \frac{4f}{2} \cdot \frac{V^2 l}{gD}$$

$$\frac{\Delta p}{\rho g} = h_f = \frac{4 f l V^2}{D \times 2g}$$

Rayleigh's Method.

- This method is used for determining the expression for a variable which depends upon maximum three or four variables only.

- If the number of independent variables becomes more than four, then it is very difficult to find the expression for the dependent variable

Uses of dimensional analysis

1. To test the dimensional homogeneity of any equation of fluid motion.
2. To derive rational formulae for a flow phenomenon.
3. To derive equations expressed in terms of non-dimensional parameters to show the relative significance of each parameter.
4. To plan model tests and present experimental results in a systematic manner, thus making it possible to analyse the complex fluid flow phenomenon.

Advantages of dimensional analysis

1. It expresses the functional relationship between the variables in dimensionless terms.
2. In hydraulic model studies it reduces the number of variables involved in a physical phenomenon, generally by *three*.
3. By the proper selection of variables, the dimensionless parameters can be used to make certain logical deductions about the problem.
4. It enables getting up a theoretical equation in a simplified dimensional form.
5. Dimensional analysis provides partial solutions to the problems that are too complex to be dealt with mathematically.
6. The conversion of units of quantities from one system to another is facilitated.

Limitations of Dimensional Analysis

- Dimensional analysis does not give any clue regarding the selection of variables. If the variables are wrongly taken, the resulting functional relationship is erroneous.
- It provides the information about the grouping of variables. In order to decide whether selected variables are pertinent or superfluous experiments have to be performed.
- The complete information is not provided by dimensional analysis; it only indicates that there is some relationship between parameters. It does not give the values of co-efficient in the functional relationship.
- The values of co-efficient and hence the nature of functions can be obtained only from experiments or from mathematical analysis.

Model Analysis—Introduction

- In order to know about the performance of the hydraulic structures (*eg.* dams, spillways etc.) or hydraulic machines (*e.g.* turbines, pumps etc.) before actually constructing or manufacturing them, their models are made and tested to get the required information.
- The model is the *small scale replica of the actual structure or machine.* The *actual structure or machine is* called **Prototype.**
- The models are not always smaller than the prototype, in some cases a model may be even larger or of the same size as prototype depending upon the need and purpose (*e.g.* the working of a wrist watch or a carburettor can be studied in a large scale model).

Applications of the model testing

Following are the important fields where applications of the model testing is of great use:

1. Civil engineering-structures such as *dams, spillways, canals* etc.
2. Flood control, investigation of silting, and scour in rivers, irrigation channels.
3. Turbines, pumps and compressors.
4. Design of harbours, ships and submarines.
5. Aeroplanes, rockets and missiles.
6. *Tall buildings* (to predict the wind loads on buildings, the stability characteristics of the buildings and airflow patterns in their vicinity).

Similitude or Principle of Similarity

- To find solutions to numerous complicated problems in hydraulic engineering and fluid mechanics model studies are usually conducted.
- In order that results obtained in the model studies represent the behaviour of prototype, the following three similarities must be ensured between the model and the prototype.

1. Geometric similarity
2. Kinematic similarity
3. Dynamic similarity

In the study of fluid mechanics, models are frequently used for testing and development purposes in laboratories before a full scale prototype is built. The model can be either smaller than the prototype (e.g., design of dam, airplane and automobiles) or larger than the prototype (e.g., study of interaction between red blood cells and the vessel wall).

Geometric Similarity

- For geometric similarity to exist between the model and the prototype, the ratios of corresponding lengths in the model and in the prototype must be same.
- Models which are not geometrically similar are known as *geometrically distorted models.*
- *Model must be the same shape as the prototype, but may be scaled by some constant scale factor.*

and, *L, B, H, D, A* and *V* Corresponding values of the prototype. Then, for *geometric similarity,* we must have the relation:

$$\frac{L_m}{L_p} = \frac{B_m}{B_p} = \frac{H_m}{H_p} = \frac{D_m}{D_p} = L_r$$

where L_r is called the *scale ratio* or the *scale factor.*

$$A_r = \text{area ratio} = \frac{A_m}{A_p} = L_r^2$$

$$V_r = \text{volume ratio} = \frac{V_m}{V_p} = L_r^3$$

Kinematic Similarity

- Kinematic similarity is the *similarity of motion*.
- If at the corresponding points in the model and in the prototype, the velocity or acceleration ratios are same and velocity or acceleration vectors point in the *same direction*, the two flows arc said to be *kinematically similar*.
- The *directions* of the velocities in the model and prototype *should be same*.
- *The* geometric similarity is a pre-requisite for kinematic similarity.

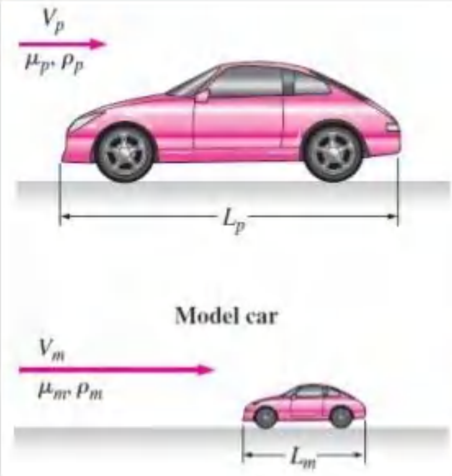
$$\frac{(V_1)_m}{(V_1)_p} = \frac{(V_2)_m}{(V_2)_p} = V_r \text{ velocity ratio}$$

$$\frac{(a_1)_m}{(a_1)_p} = \frac{(a_2)_m}{(a_2)_p} = a_r \text{ acceleration ratio}$$

Dynamic Similarity

- Dynamic similarity is the *similarity of forces.*
- The flows in the model and in prototype are dynamically similar if at all the corresponding points, identical types of forces are parallel and bear the same ratio.
-
- The directions of the corresponding forces at the corresponding points in the model and prototype should also be same.

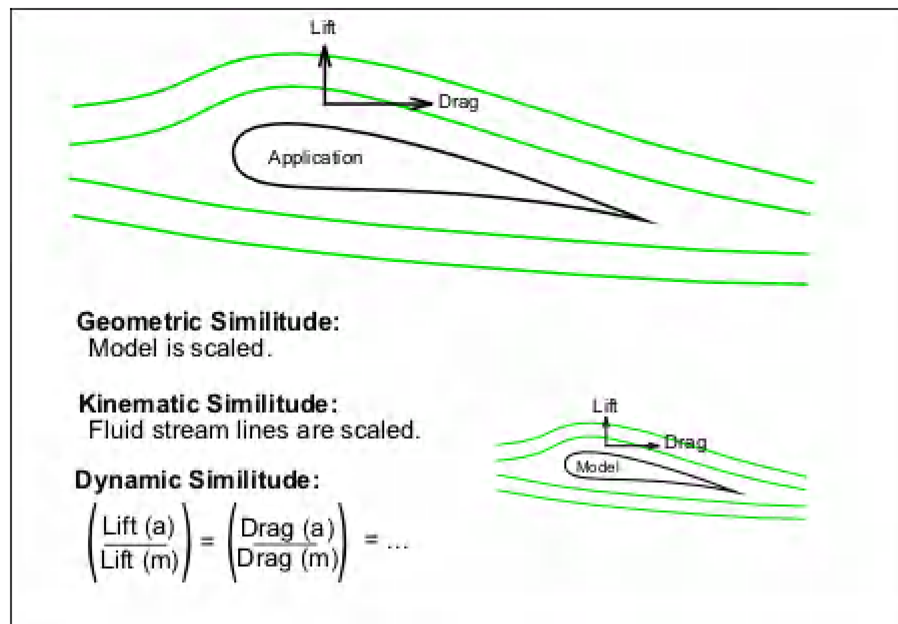
$$\frac{(F_i)_m}{(F_i)_p} = \frac{(F_v)_m}{(F_v)_p} = \frac{(F_g)_m}{(F_g)_p} \ldots\ldots = F_r \text{ (force ratio)}$$



Geometric similarity between a prototype car of length L_p and a model car of length L_m. In the case of aerodynamic drag on the automobile, there are only two Π's in the problem.

$$\Pi_1 = f(\Pi_2) \qquad \text{where} \qquad \Pi_1 = \frac{F_D}{\rho V^2 L^2} \qquad \text{and} \qquad \Pi_2 = \frac{\rho V L}{\mu}$$

F_D is the magnitude of the aerodynamic drag on the car, and so on forming drag coefficient equation.
The Reynolds number is the most well known and useful dimensionless parameter in all of fluid mechanics.



(3-c-ii) immersed bodies

Prototype

240 mph

$$\frac{F_D}{\frac{\rho V^2 L^2}{2}} = \phi\left(\frac{L_i}{L}, \frac{\varepsilon}{L}, \frac{\rho V L}{\mu}\right)$$

Model scale 1/10

240 mph

If model drag is 2 lb, determine drag force of the prototype?

Reynolds Number

$$\lambda_L = \frac{L_m}{L_p} = \frac{1}{10}$$

$$\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p$$

$$\text{so } \frac{\rho_m}{\rho_p} = \frac{\mu_m}{\mu_p}\frac{L_p}{L_m}\frac{V_p}{V_m} = 1$$

Forces Influencing Hydraulic Phenomena

- The forces which may affect/influence the flow characteristics of a problem are:

Inertia force(F_i)

- It *always* exists in the fluid flow problem (and hence it is customary to find out the force ratios with respect to inertia force).
- It is equal to the product of mass and acceleration of the flowing fluid and acts in the direction opposite to the direction of acceleration

Viscous force (F_v)

- It is present in fluid flow problems where viscosity is to play an important role.
- It is equal to the product of shear stress due to viscosity and surface area of the flow.

Gravity force (F_g)

- It is present in case of open surface flow.
- It is equal to the product of mass and acceleration due to gravity.

Pressure force (F_p)

- This type of force is present in case of pipe-flow.
- It is equal to the product of pressure intensity and cross-sectional area of the flowing fluid.

Surface tension force (F_s)

- It is equal to the product of surface tension and length of surface of the flowing fluid.

Elastic force (F_e)

- It is equal to the product of elastic stress and area of the flowing fluid.

Dimensionless Numbers and their Significance

The *dimensionless numbers* (also called non-dimensional parameters) are obtained by dividing the *inertia force* (which always exists when any mass in motion) by viscous force or gravity force or pressure force or surface tension force or elastic force. The important dimensionless numbers:

1. Reynolds number
2. Froude's number
3. Euler's number
4. Weber's number
5. Mach's number

1. Reynolds Number *(Re)*

- It is defined as the ratio of the inertia force to the viscous force.

This number assumes importance in the following flow situations

- (i) Motion of submarine completely under water,
- (ii) Low velocity motion around automobiles and airplanes,
- (iii) Incompressible flow through pipes of smaller sizes,
- (iv) Flow through low speed turbo-machines.

- Reynolds number signifies the relative predominance of the inertia to the viscous forces occurring in the flow systems.
- This number is taken as the criterion of dynamic similarity in the flow situations where the viscous forces predominate

1. Reynold's number: It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. The expression for Reynold's number is obtained as

Inertia force (F_i) = Mass X Acceleration of flowing fluid

$$\text{Inertia force } (F_i) = \rho \times \text{Volume } \times \frac{\text{Velocity}}{\text{Time}} = \rho \times \frac{\text{Volume}}{\text{Time}} \times \text{Velocity}$$

$$= \rho \times AV \times V \qquad \text{Volume per sec = Area X Velocity = A X V}$$

$$\text{Inertia force } (F_i) = \rho\, A\, V^2$$

Viscous force (F_v) = Shear stress X Area

$$\text{Visccous force } (F_v) = \tau \times A$$

$$= \left(\mu \frac{du}{dy}\right) \times A = \mu . \frac{V}{L} \times A \qquad \frac{du}{dy} = \frac{V}{L}$$

By definition, Reynold's number

$$R_e = \frac{F_i}{F_v} = \frac{\rho\, A\, V^2}{\mu \frac{V}{L} \times A} = \frac{\rho V L}{\mu} = \frac{V \times L}{(\mu/\rho)} = \frac{V \times L}{\nu} \qquad \mu/\rho = \nu$$

In case of pipe flow, the linear dimension L is taken as diameter, d. Hence Reynold's number for pipe flow,

$$R_e = \frac{V \times d}{\nu} \quad \text{or} \quad \frac{\rho V d}{\mu}$$

2. Froude Number (Fr)

- *It is defined as the* square root of the ratio of the inertia force and the gravity force.

$$Fr = \sqrt{\frac{F_i}{F_g}}$$

- Froude number governs the dynamic similarity of the flow situations; where gravitational force is most significant and all other forces are comparatively negligible.

This number assumes importance in the following flow situations

- (1) Flow over notches and weirs, spillway of a dam, etc.,
- (2) Flow through open channels
- (3) Motion of ship in rough sea.

2. Froude's Number (Fe): The Froud's Number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravitational force. Mathematically, it is expressed as

$$F_e = \sqrt{\frac{F_i}{F_g}}$$

Inertia force $(F_i) = \rho\, A\, V^2$

F_g = Force due to gravity = Mass X Acceleration due to gravity = $\rho \times L^3 \times g$

$$= \rho \times L^2 \times L \times g = \rho \times A \times L \times g$$

$$F_e = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho A V^2}{\rho A L g}} = \sqrt{\frac{V^2}{Lg}} = \frac{V}{\sqrt{Lg}}$$

3. Euler's Number (Eu)

- It is defined as the square root of the ratio of the inertia force to the pressure force.
- The Euler number is important in the flow problems/situations in which a pressure gradient exists.

This number assumes importance in the following flow situations

(i) Discharge through orifices

(ii) (ii) Pressure rise due to sudden closure of valves

(iii) (iii) Flow through pipes

(iv) (iv) Water hammer created in penstocks.

3. Euler's number(Eu): It is deflned as the sqoare root of the ratio of inertia force of a flowing fluid to the surface tension force. Mathematically, it is expressed as

$$\text{Euler's number } (E_u) = \sqrt{\frac{F_i}{F_p}}$$

F_p = Intensity of pressure X Area = ρ X A

Inertia force (F_i) = ρ A V^2

$$E_u = \sqrt{\frac{F_i}{F_p}} = \sqrt{\frac{\rho A V^2}{p \times A}} = \sqrt{\frac{V^2}{p/\rho}} = \frac{V}{\sqrt{p/\rho}}$$

4. Weber's number (We): It is defined as the square root of the ratio of inertia force of a flowing fluid to the surface tension force. Mathematically, it is expressed as

$$\text{Weber's number } (W_e) = \sqrt{\frac{F_i}{F_g}}$$

Inertia force (F_i) = ρ A V^2

F_s = Surface tension force = Surface tension per unit length X Length = σ X L

$$W_e = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho A V^2}{\sigma \times L}} = \sqrt{\frac{\rho \times L^2 V^2}{\sigma \times L}}$$

4. Weber Number (We)

- It is defined as the square root of the ratio of the inertia force to the surface tension force.

  - We $= \sqrt{\dfrac{F_i}{F_s}}$

- This number assumes importance in the following flow situations:
  (i) Capillary movement of water in soils
- (ii) Flow of blood in veins and arteries
- (iii) Liquid atomization
- (iv) formation of bubbles or droplets.

5. Mach Number (M)

- It is defined as the square root of the ratio of the inertia force to the elastic force. Mathematically, M

- The Mach number is important in
- compressible flow problems at high velocities,
- such as high velocity flow in pipes or motion of high-speed projectiles and missiles.

$$M = \sqrt{\frac{F_i}{F_e}}$$

5. Mach nnmber(M): Mach number is defined as the square root of the ratio of inertia force of a flowing floid to the elastic force. Mathematically, it is expressed as

$$\text{Mach numher (M)} = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{F_i}{F_e}}$$

$$F_i = \rho A V^2$$

Fe = Eiastic force = Elastic stress X Area = K X A = K X L^2

$$M = \sqrt{\frac{\rho A V^2}{K \times L^2}} = \sqrt{\frac{\rho \times L^2 V^2}{K \times L^2}} = \sqrt{\frac{V^2}{K/\rho}} = \frac{V}{\sqrt{K/\rho}}$$

$\sqrt{\frac{K}{P}}$ = C = Velocity of sound in the fluid $M = \frac{V}{C}$.

Sl. No.	Dimensionless number	Aspects			
		Symbol	Group of variables	Significance	Field of application
1.	*Reynolds number*	*Re*	$\frac{\rho VL}{\mu}$	$\frac{\text{Inertia force}}{\text{Viscous force}}$	Laminar viscous flow in confined passages (where *viscous effects are significant*)
2.	*Froude's number*	*Fr*	$\frac{V}{\sqrt{Lg}}$	$\frac{\text{Inertia force}}{\text{Gravity force}}$	Free surface flows (where *gravity effects are important*)
3.	*Euler's number*	*Eu*	$\frac{V}{\sqrt{p/\rho}}$	$\frac{\text{Inertia force}}{\text{Pressure force}}$	Conduit flow (where *pressure variations are significant*)
4.	*Weber's number*	*We*	$\frac{V}{\sqrt{\sigma/\rho L}}$	$\frac{\text{Inertia force}}{\text{Surface tension}}$	Small surface waves, capillary and sheet flow (where *surface tension is important*)
5.	*Mach's number*	*M*	$\sqrt{\frac{V}{K/\rho}}$	$\frac{\text{Inertia force}}{\text{Elastic Force}}$	High speed flow (whcre *compressibility effects* are *significant*).

	pressure	dynamic viscosity	velocity	characteristic length	mass density
pressure	None	$\left\{\frac{p}{v^2 \rho}, \frac{\mu}{d v \rho}\right\}$	$\left\{\frac{d^2 p \rho}{\mu^2}, \frac{d v \rho}{\mu}\right\}$	$\left\{\frac{p}{v^2 \rho}, \frac{d v \rho}{\mu}\right\}$	$\left\{\frac{d p}{v \mu}, \frac{d v \rho}{\mu}\right\}$
dynamic viscosity	$\left\{\frac{\mu}{d v \rho}, \frac{p}{v^2 \rho}\right\}$	None	$\left\{\frac{\mu}{d \sqrt{p} \sqrt{\rho}}, \frac{v \sqrt{\rho}}{\sqrt{p}}\right\}$	None	$\left\{\frac{v \mu}{d p}, \frac{v^2 \rho}{p}\right\}$
velocity	$\left\{\frac{d v \rho}{\mu}, \frac{d^2 p \rho}{\mu^2}\right\}$	$\left\{\frac{v \sqrt{\rho}}{\sqrt{p}}, \frac{\mu}{d \sqrt{p} \sqrt{\rho}}\right\}$	None	$\left\{\frac{v \sqrt{\rho}}{\sqrt{p}}, \frac{d \sqrt{p} \sqrt{\rho}}{\mu}\right\}$	$\left\{\frac{v \mu}{d p}, \frac{d^2 p \rho}{\mu^2}\right\}$
characteristic length	$\left\{\frac{d v \rho}{\mu}, \frac{p}{v^2 \rho}\right\}$	None	$\left\{\frac{d \sqrt{p} \sqrt{\rho}}{\mu}, \frac{v \sqrt{\rho}}{\sqrt{p}}\right\}$	None	$\left\{\frac{d p}{v \mu}, \frac{v^2 \rho}{p}\right\}$
mass density	$\left\{\frac{d v \rho}{\mu}, \frac{d p}{v \mu}\right\}$	$\left\{\frac{v^2 \rho}{p}, \frac{v \mu}{d p}\right\}$	$\left\{\frac{d^2 p \rho}{\mu^2}, \frac{v \mu}{d p}\right\}$	$\left\{\frac{v^2 \rho}{p}, \frac{d p}{v \mu}\right\}$	None

Types of Model

- Undistorted Models

  Models that are geometrically similar to their prototypes (ie scale ratios for the all directions of linear dimensions in model and prototype are same)

- Distorted Models

  Models in which the different scale ratios are used for linear dimensions

eg river model, harbor model.

Scale Effect

If complete similarity does not exist in model and its prototype there will be some discrepancy between the results obtained from model when compared with results in prototype. This effect is called scale effect.

Scale Ratio

(a) Scale ratio for time

$$\text{Time} = \frac{\text{Length}}{\text{Velocity}}$$

then ratio of time for prototype and model is

$$T_r = \frac{T_p}{T_m} = \frac{(\frac{L}{V})_p}{(\frac{L}{V})_m} = \frac{\frac{L_p}{V_p}}{\frac{L_m}{V_m}} = \frac{L_p}{L_m} \times \frac{V_m}{V_p} = L_r \times \frac{1}{\sqrt{L_r}} = \sqrt{L_r} \qquad \frac{V_p}{V_m} = \sqrt{L_r}$$

(b) Scale ratio for accelcration

$$\text{Accelaration} = \frac{V}{T}$$

$$a_r = \frac{a_p}{a_m} = \frac{(\frac{V}{T})_p}{(\frac{V}{T})_m} = \frac{V_p}{T_p} \times \frac{T_m}{V_m} = \frac{V_p}{V_m} \times \frac{T_m}{T_p} = \sqrt{L_r} \times \frac{1}{\sqrt{L_r}} = 1$$

$$\frac{V_p}{V_m} = \sqrt{L_r}, \qquad \frac{T_p}{T_m} = \sqrt{L_r}$$

(c) Scale ratio for discharge

$$Q = A \times V = L^2 \times \frac{L}{T} = \frac{L^3}{T}$$

$$Q_r = \frac{Q_p}{Q_m} = \frac{(\frac{L^3}{T})p}{(\frac{L^3}{T})m} = (\frac{L_p}{L_m})^3 \times \left(\frac{T_m}{T_p}\right) = \times \frac{1}{\sqrt{L_r}} = L_r^{2.5}$$

(d) Scale ratio for force

$$\text{Force} = \text{Mass X Acceleration} = \rho\, L^3 \times \frac{V}{T} = \rho\, L^2 \times \frac{L}{T}.V = \rho\, L^2V^2$$

Ratio for foree, $F_r = \frac{F_p}{F_m} = \frac{\rho_p L_p^2 V_p^2}{\rho_m L_m^2\, V_m^2} = \frac{\rho_p}{\rho_m} \times (\frac{L_p}{L_m})^2 \times (\frac{V_p}{V_m})^2$

If the fluid used in model and prototype is same, then

$$\frac{P_p}{\rho_m} = 1 \ \text{(or)} \rho_p = \rho_m L_r^3$$

$$F_r = (\frac{L_p}{L_m})^2 \times (\frac{V_p}{V_m})^2 = L_r^2 \times (\sqrt{L_r})^2 = L_r^2 . L_r = L_r^3$$

(e) Scale ratio for pressure intensity

$$p = \frac{\text{Force}}{\text{Area}} = \frac{\rho\, L^2\, V^2}{L^2} = \rho\, V^2$$

Pressure ratio, $$p_r = \frac{p_p}{p_m} = \frac{\rho_p V_p^2}{\rho_m V_m^2}$$

If fluid is same, then $\rho_p = \rho_m$

$$P_r = \frac{V_p^2}{V_m^2} = (\frac{V_p}{V_m})^2 = L_r$$

(f) Scale ratio for work, energy, torque, moment etc.

Torque = Force X Distance = F X L

Torque ratio, $$T_r^* = \frac{T_p^*}{T_m^*} = \frac{(F\times L)_p}{(F\times L)_m} = F_r \times L_r = L_r^3 \times L_r = L_r^4$$

(g) Scale ratio for power

Power = Work per unit time

$$\text{Power} = \frac{F \times L}{T}$$

$$\text{Power ratio, } P_r = \frac{P_p}{P_m} = \frac{\frac{F_p \times L_p}{T_p}}{\frac{F_m \times L_m}{T_m}} = \frac{F_p}{F_m} \times \frac{L_p}{L_m} \times \frac{1}{\frac{T_p}{T_m}}$$

$$P_r = F_r . L_r . \frac{1}{T_r} = L_r^3 \times L_r \times \frac{1}{\sqrt{L_r}} = L_r^{3.5}$$

Reynold's Number - problem

- Water is flowing through a pipe of diameter 30cm at a velocity of 4 m/s . Find the velocity of oil flowing in another pipe of diameter 10 cm, if the condition for dynamic similarity is satisfied. The viscosity of water and oil is given by 0.01 Poise and 0.025 Poise. (Sp. Gravity of oil = 0.8).

Pipe 1 - Water

Diameter d1 = 30 cm = 0.3 m

Velocity v_1 = 4 m/s

Density ρ_1 = 1000 kg/m^3

μ_1 = 0.001 N-s/m^2

Pipe 2 - Oil

Diameter d_2 = 10 cm = 0.1 m

Velocity v_2 = **?** m/s

Density ρ_2 = 1000 x 0.8 kg/m^3

μ_2 = 0.0025 N-s/m^2

$$\frac{\rho_1 \times V_1 \times D_1}{\mu_1} = \frac{\rho_2 \times V_2 \times D_2}{\mu_2}$$

Find V_2

Ans: V_2 = 37.5 m/s

Problem related to Froude No.

- A spillway model is to be built to a geometrically similar scale of 1/50 across a flume of 600 mm width. The prototype is 15m high and maximum head on it is expected to be 1.5m.

  (i) What is the height of model and head to be used

  (ii) If the flow over the model at a particular head is 12 litres per second, what flow per metre length of the prototype is expected?

  (iii) If the negative pressure in the model is 200 mm, what is the negative pressure in the prototype.

Solution : Scale = (1/50)

1 unit in model = 50 units in prototype

$(L_p/L_m) = L_r = 50$

- Model
  - Width B_m = 0.6m
  - Height H_m = ?m
  - Maxi head H_{m*} = ?m
  - Q_m = 12 lit/sec
  - -ve pr. h_m = **- 0.2 m**
- Prototype
  - Width B_p = ?
  - Height H_p = 15m
  - Maxi head H_{p*} = 1.5m
  - Q_p = ? m^3/sec
  - -ve pr. h_p = ? m

Width of prototype = 0.6 * 50 = 30 m

Height of model = 15/50 = 0.3m

Maxi. Head of model = 1.5 / 50 = 0.03m = 3 cm

Scale ratio for $Q_p / Q_m = L_r^{2.5} = 50^{2.5}$ = 17677.67

So discharge in prototype $Q_p = 12 \times 10^{-3} \times 17677$ = 212 m^3/s

Discharge per unit width Q_p = 212 / 30 = 7.07 m^3/s

Negative head on prototype = - 0.2 x 50 = - 10 m

Model Testing In Partially Sub-merged Bodies

34. Resistance R, to the motion of a completely sub-merged body is given by

$R = \rho V^2 l^2 \varphi\left(\frac{Vl}{\nu}\right)$ where ρ and ν are density and kinematic viscosity of the fluid while l is the length of the body and V is the velocity of flow. If the resistance of a one-eight scale air-ship model when tested in water at 12 m/s is 22 N, what will be the resistance in air of the air-ship at the corresponding speed? Kinematic viscosity of air is 13 times that of water and density of water is 810 times of air.

- Model- water
- $V_m = 12$ m/s
- $R_m = 22$N

- Prototype – Air
- $V_p = ?$
- $R_p = ?$

$$v_p = 13 \times v_m$$

$$(v_p / v_m) = 13$$

$$\rho_m = 810\rho_p$$

$$(\rho_p / \rho_m) = 1/810$$

- Step 1 Find V_p

Using relation $(VL/\nu)_p = (VL/\nu)_m$

- Step 2: Find R_p

Using Relation $(R / \rho V^2 L^2)_p = (R / \rho V^2 L^2)_m$

Ans:

$V_p = 19.5$ m/s

$R_p = 4.59$N

Distorted Model Problem

- The discharge through the weir is 1.5 m^3/s. Find the discharge through the model of the weir if the horizontal dimension scale is 1/50 and the vertical dimension scale is 1/10.

Ans : $Q_m = 9.48 \times 10^{-4}\ m^3/s$

12. 15. A pipe of diameter 1.5 m is required to transport an oil of sp. gr. 0.90 and viscosity 3 X 10^{-2} poise at the rate of 3000 litter/sec Tests were conducted on a 15 cm diameter pipe using water at 20°C. Find the velocity and rate of flow in the model. Viscosity of water at 20°C = 0.01 poise.

Solution. Given:

Dia. of prototype, D_p = 1.5 m

Viscosity of fluid, μ_p = 3 X 10-2 poise

Q for prototype, Q_p = 3000 litis = 3.0 m^3/s

Sp. gr. of oil, S_p = 0.9

:. Density of oil, $P_p = S_p$ x 1000 = 0.9 x 1000 = 900 kg/m3

Dia. of the model, D_m = 15 cm = 0.15 m

Viscosity of water at 20°C = .01 poise = 1 x 10-2 poise or 11", = 1 X 10-2 poise

Density of water or ρ_m = 1000 kg/m^3

For pipe flow, the dynamic similarity will be obtained if the Reynold's number in the model and prototype are equal

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_P V_P D_P}{\mu_P}$$

$$\frac{V_m}{V_P} = \frac{\rho_P}{\rho_m} \cdot \frac{D_P}{D_m} \cdot \frac{\mu_m}{\mu_P}$$

$$= \frac{900}{1000} \times \frac{1.5}{0.15} \times \frac{1 \times 10^{-2}}{3 \times 10^{-2}} = \frac{900}{1000} \times 10 \times \frac{1}{3} = 3.0$$

$$V_P = \frac{\text{Rate of flow in prototype}}{\text{Area of prototype}} = \frac{3.0}{\frac{\pi}{4}(D_P)^2} = \frac{3.0}{\frac{\pi}{4}(1.5)^2}$$

$$= \frac{3.0 \times 4}{\pi \times 2.25} = 1.697 \text{ m/s}$$

$$V_m = 3.0 \times V_P = 3.0 \times 1.697 = \mathbf{5.091 \text{ m/s. Ans.}}$$

Rate of flow through model

$$Q_m = A_m \times V_m = \frac{\pi}{4}(D_m)^2 \times V_m = \frac{\pi}{4}(0.15)^2 \times 5.091 \text{ m}^3\text{/s}$$

$$= 0.0899 \text{ m}^3\text{/s} = 0.0899 \times 1000 \text{ lit/s} = \mathbf{89.9 \text{ lit/s. Ans.}}$$

Reynold's Number - problem

- Water is flowing through a pipe of diameter 30cm at a velocity of 4 m/s . Find the velocity of oil flowing in another pipe of diameter 10 cm, if the condition for dynamic similarity is satisfied. The viscosity of water and oil is given by 0.01 Poise and 0.025 Poise. (Sp. Gravity of oil = 0.8).

Pipe 1 - Water

Diameter d1 = 30 cm = 0.3 m

Velocity v_1 = 4 m/s

Density ρ_1 = 1000 kg/m^3

μ_1 = 0.001 N-s/m^2

Pipe 2 - Oil

Diameter d_2 = 10 cm = 0.1 m

Velocity v_2 = ? m/s

Density ρ_2 = 1000 x 0.8 kg/m^3

μ_2 = 0.0025 N-s/m^2

$$\frac{\rho_1 \times V_1 \times D_1}{\mu_1} = \frac{\rho_2 \times V_2 \times D_2}{\mu_2}$$

Find V_2

Ans: V_2 = 35.2 m/s

Problem related to Froude No.

12.23 A spillway model is to be built to a geometrically similar scale of 1/50 across a flume of 600 mm width. The prototype is 15m high and maximum head on it is expected to be 1.5m.

- (i) What is the height of model and head to be used
- (ii) If the flow over the model at a particular head is 12 litres per second, what flow per metre length of the prototype is expected?
- (iii) If the negative pressure in the model is 200 mm, what is the negative pressure in the prototype.

Solution : Scale = (1/50)

1 unit in model = 50 units in prototype

$(L_p/L_m) = L_r = 50$

- Model
  - Width $B_m = 0.6m$
  - Height $H_m = ?m$
  - Maxi head $H_{m*} = ?m$
  - $Q_m = 12$ lit/sec
  - -ve pr. h_m = **- 0.2 m**
- Prototype
  - Width $B_p = ?$
  - Height $H_p = 15m$
  - Maxi head $H_{p*} = 1.5m$
  - $Q_p = ?\ m^3/sec$
  - -ve pr. $h_p = ?$ m

Width of prototype = 0.6 * 50 = 30 m

Height of model = 15/50 = 0.3m

Maxi. Head of model = 1.5 / 50 = 0.03m = 3 cm

Scale ratio for $Q_p / Q_m = L_r^{2.5} = 50^{2.5} = 17677.67$

So discharge in prototype $Q_p = 12 \times 10^{-3} \times 17677 = 212\ m^3/s$

Discharge per unit width $Q_p = 212 / 30 = 7.07\ m^3/s$

Negative head on prototype = - 0.2 x 50 = - 10 m

12.20

A ship model of scale 1/50 *is towed through sea water at a speed of 1 mls. A force of 2 N is required to tow the model. Determine the speed of ship and the propulsive force on the ship, if prototype is subjected to wave resistance only.*

Solution. Given:

Scale ratio of length, $Lr = 50$

Speed of model, $V_m = I\ mls$

Force required for model, $F_m = 2$ N

Let the speed of ship $= Vp$

and the propulsive force for ship $= F_{p.}$

As prototype is subjected to wave resistance only for dynamic similarity, the Froude number should be same for model and prototype. Hence for velocity ratio, for Froude model law using equation

Force scale ratio is given by equation

Model Testing In Partially Sub-merged Bodies

34. Resistance R, to the motion of a completely sub-merged body is given by

$R = \rho V^2 l^2 \varphi\left(\frac{Vl}{v}\right)$ where ρ and v are density and kinematic viscosity of the fluid while l is the length of the body and V is the velocity of flow. If the resistance of a one-eight scale air-ship model when tested in water at 12 m/s is 22 N, what will be the resistance in air of the air-ship at the corresponding speed? Kinematic viscosity of air is 13 times that of water and density of water is 810 times of air.

- Model- water
- $V_m = 12$ m/s
- $R_m = 22N$

- Prototype – Air
- $V_p = ?$
- $R_p = ?$

$v_p = 13 \times v_m$

$(v_p / v_m) = 13$

$\rho_m = 810\rho_p$

$(\rho_p / \rho_m) = 1/810$

- Step 1 Find V_p

Using relation $(VL/\nu)_p = (VL/\nu)_m$

- Step 2: Find R_p

Using Relation $(R / \rho V^2L^2)_p = (R / \rho V^2L^2)_m$

Ans:

$V_p = 19.5$ m/s

Rp = 4.59N

Distorted Models

Advantages and Disadvantages of Distorted Models

A distorted model has the following advantages and disadvantages:

Advantages

1. The model size can be sufficiently reduced by its distortion. As a result or this, the cost or the model is considerably reduced and its operation is simplified.

2. The vertical exaggeration results in steeper slopes of water surface. which can be easilyand accurately measured.

3. The Reynold's number or a model is considerably increased and surface resistance is decreased due to exaggerated water slopes, This helps in' simulation of the flow conditions in the model and its prototype.

Disadvantages

1. There is an unfavorable psychological effect on the observer.
2. The behavior or now or a model differs in action from that or the prototype
3. The magnitude and direction of the pressures is not correctly reproduced.
4. The velocities are not correctly reproduced. as the vertical exaggeration causes distortion of lateral velocity and kinetic energy

1. Scale ratio for velocity

Let V_P = Velocity in prototype

V_m = Velocity in model.

Then $$\frac{V_P}{V_m} = \frac{\sqrt{2gh_P}}{\sqrt{2gh_m}} = \sqrt{\frac{h_P}{h_m}} = \sqrt{(L_r)_V} \qquad \left(\because \frac{h_P}{h_m} = (L_r)_V\right)$$

2. Scale ratio for area of flow

Let A_P = Area of flow in prototype = $B_P \times h_P$

A_m = Area of flow in model = $B_m \times h_m$

$$\therefore \quad \frac{A_P}{A_m} = \frac{B_P \times h_P}{B_m \times h_m} = \frac{B_P}{B_m} \times \frac{h_P}{h_m} = (L_r)_H \times (L_r)_V$$

3. Scale ratio for discharge

Let Q_P = Discharge through prototype = $A_P \times V_P$

Q_m = Discharge through model = $A_m \times V_m$

$$\therefore \quad \frac{Q_P}{Q_m} = \frac{A_P \times V_P}{A_m \times V_m} = (L_r)_H \times (L_r)_V \times \sqrt{(L_r)_V} = (L_r)_H \times [(L_r)_V]^{3/2} \quad \text{...(12.43)}$$

Note: in the following problems,

$S_V = (Lr)_V$

$S_H = (Lr)_H$

Example 27·3. *A model of weir is made to a horizontal scale of 1/40 and vertical scale 1/9. Find the discharge of the prototype, if the model is discharging 1 litre/s.*

Solution. Given : $\frac{1}{s_H} = \frac{1}{40}$ or $s_H = 40$; $\frac{1}{s_V} = \frac{1}{9}$ or $s_V = 9$ and $q = 1$ litre/s

We know that discharge of the prototype,

$$Q = q \times s_H \times s_V^{1.5} = 1 \times 40 \times (9)^{1.5} = 1080 \text{ litres/s} \quad \textbf{Ans.}$$

Example 27·4. *The discharges of a model and prototype were found to be 0·02 m³/s and 150 m³/s respectively. If vertical scale ratio of the model is 1:25, determine the horizontal scale ratio of the model.*

Solution. Given : $q = 0{\cdot}02\ \text{m}^3/\text{s}$; $Q = 150\ \text{m}^3/\text{s}$ and $\frac{1}{s_V} = \frac{1}{25}$ or $s_V = 25$.

Let s_H = Horizontal scale ratio of the model.

We know that discharge of the prototype (Q),

$$150 = q \times s_H \times s_V^{1\cdot5} = 0{\cdot}02 \times s_H \times (25)^{1\cdot5} = 2{\cdot}5\ s_H$$

$$\therefore \quad s_H = 150/2{\cdot}5 = {\cdot}60 \quad \textbf{Ans.}$$

Example 27·5. *A diversion weir 240 m long has discharging capacity of 250 m³/s under a head of 1·2 m. A model of this weir is to be constructed in laboratory where the available channel is 3 m wide and 500 m deep. Design the suitable model for the weir, if the water available in the laboratory is 25 litres/s.*

Solution. Given : $L = 240$ m; $Q = 250\ m^3/s$; $H = 1{\cdot}2$ m; $l = 3$ m; Depth of channel = 500 mm = 0·5 m and $q = 25$ litres/s = $0{\cdot}025\ m^3/s$.

First of all, let us design an undistorted model. From given data, we find that the scale model,

$$\frac{1}{s_H} = \frac{l}{L} = \frac{3}{240} = \frac{1}{80} \quad \text{or} \quad s_H = 80$$

and head of water in the model.

$$h = \frac{H}{s_H} = \frac{1{\cdot}2}{80} = 0{\cdot}015 \text{ m} = 15 \text{ mm}$$

We know that with 15 mm head of water, it will be difficult to take observations as the flow will be predominated by the surface tension force. Moreover, the flow with such a small head will be streamline in nature, whereas in case of the prototype the flow will be turbulent. As a result of this, we will have to exaggerate the vertical scale ratio.

Now let s_V = Vertical scale ratio of the model.

We know that discharging capacity of the weir (Q),

$$250 = q \times s_H \times s_V^{1.5} = 0.025 \times 80 \times s_V^{1.5} = 2\, s_V^{1.5}$$

$$\therefore \quad s_V^{1.5} = 250/2 = 125 \text{ or } s_V = 25$$

and height of water in the model,

$$h = \frac{H}{s_V} = \frac{1.2}{25} = 0.048 \text{ m} = 48 \text{ mm} \quad \textbf{Ans.}$$

Distorted Model Problem

- The discharge through the weir is 1.5 m^3/s. Find the discharge through the model of the weir if the horizontal dimension scale is 1/50 and the vertical dimension scale is 1/10.

Ans : $Q_m = 9.48 \times 10^{-4}\ m^3/s$

Reference

- Bansal, R.K., "Fluid Mechanics and Hydraulics Machines", 5th edition, Laxmi Publications Pvt. Ltd, New Delhi, 2008
- Modi P.N and Seth "Hydraulics and Fluid Mechanics including Hydraulic Machines", Standard Book House New Delhi. 2003.

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UNIT 4

PUMPS

PUMPS

UNIT IV PUMPS 10

Impact of jets - Euler's equation - Theory of rotodynamic machines – various efficiencies– velocity components at entry and exit of the rotor- velocity triangles - Centrifugal pumps– working principle - work done by the impeller - performance curves - Reciprocating pump- working principle – Rotary pumps –classification.

Centrifugal Pump

- Working principle
- Velocity Triangle
- Efficiency
- Minimum speed
- Net Positive Suction Head(NPSH)
- MultiStage Pumps

Reciprocating Pumps

- Working principle
- Indicator Diagrams
- Air Vessels
- Negative slip
- Savings in work done

Impact of Jets

i) Force exerted on a Flat Plate

- Flat Vertical Fixed Plate and Flat Vertical Moving Plate
- Flat Inclined Fixed Plate and Flat Inclined Moving Plate

IMPULSE MOMENTUM PRINCIPLE

When applied to a single body Newton's second law can be started as
"The sum of forces on the body equals the rate of change of momentum of the body in the direction of the force".

In equation from (F and V are in the same direction)

$$\Sigma F = d(mv)/dt$$

This can also be written as $\Sigma F dt = d(mV)$

Where, m is the mass of the body and V is the velocity of the body and t is the time. This also means the impulse **Fdt equals the change in momentum of the body during the time dt.**

Force acting on a straight Plate

Consider a jet of water coming out from the nozzle, strikes a flat vertical plate as shown in Fig. 17.1

Let V = velocity of the jet, d = diameter of the jet,

$$a = \text{area of cross-section of the jet} = \frac{\pi}{4} d^2.$$

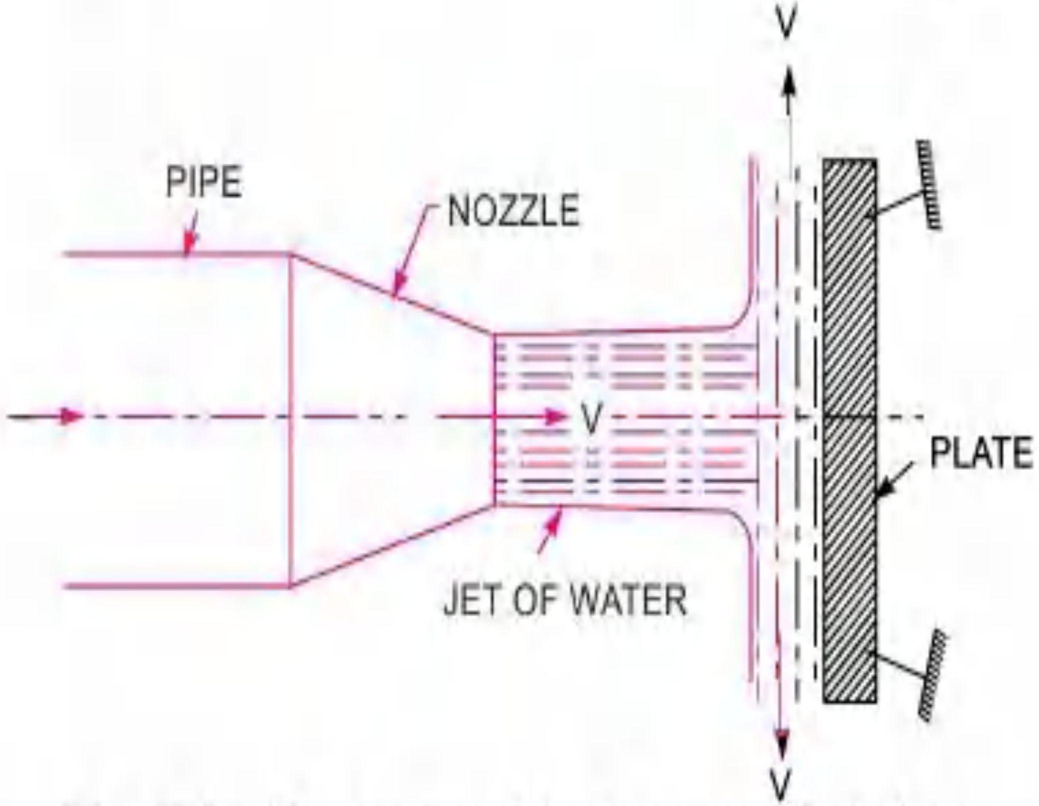
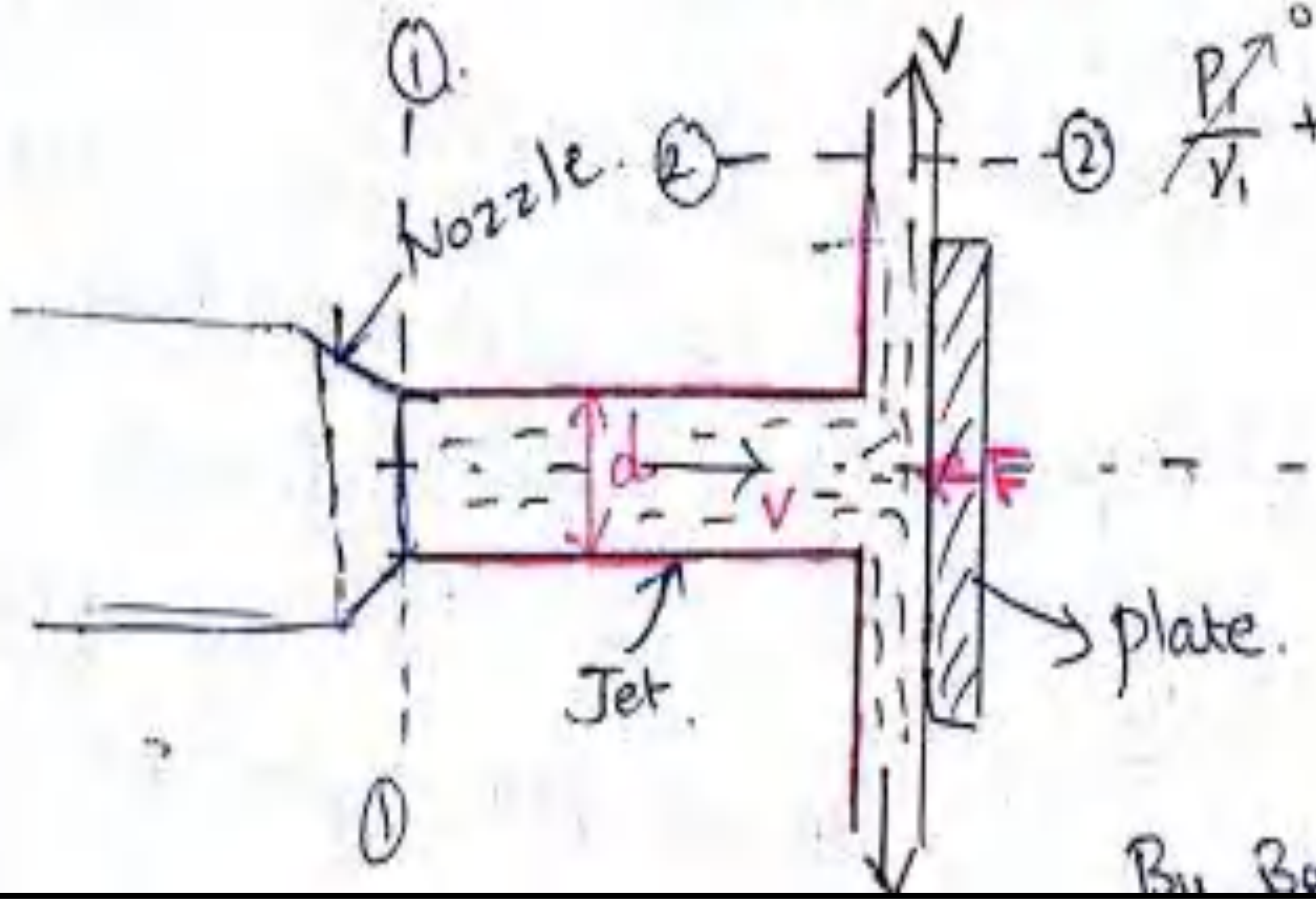


Fig. 17.1 *Force exerted by jet on vertical plate.*

Force Exerted by Fluid Jet on Stationary Flat Plate.



$$\frac{P_1}{\gamma_1} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma_2} + \frac{V_2^2}{2g} + z_2$$

$V_1 = V_2$.

By Bernoulli Eqn

The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking, will get deflected through 90°. Hence the component of the velocity of jet, in the direction of jet, after striking will be zero.

The force exerted by the jet on the plate in the direction of jet,

$$F_x = \text{Rate of change of momentum in the direction of force}$$

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} [\text{Initial velocity} - \text{Final velocity}]$$

$$= (\text{Mass/sec}) \times (\text{velocity of jet before striking} - \text{velocity of jet after striking})$$

$$= \rho a V[V - 0] \qquad (\because \text{mass/sec} = \rho \times a V)$$

$$= \rho a V^2 \qquad \ldots(17.1)$$

Force acting on a inclined Plate

Let V = Velocity of jet in the direction of x,

θ = Angle between the jet and plate,

a = Area of cross-section of the jet.

Then mass of water per sec striking the plate = $\rho \times aV$.

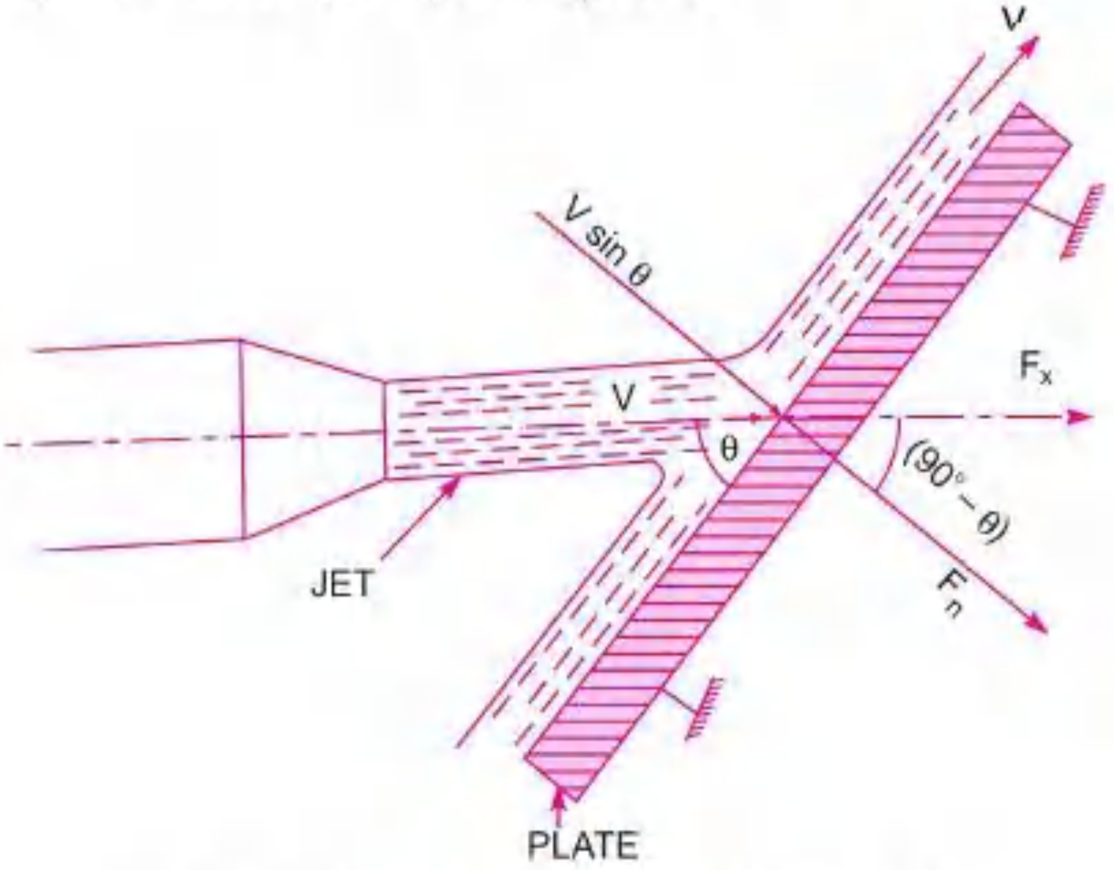


Fig. 17.2 *Jet striking stationary inclined plate.*

If the plate is smooth and if it is assumed that there is no loss of energy due to impact of the jet, then jet will move over the plate after striking with a velocity equal to initial velocity *i.e.*, with a velocity V. Let us find the force exerted by the jet on the plate in the direction normal to the plate. Let this force is represented by F_n

Then F_n = mass of jet striking per second

× [Initial velocity of jet before striking in the direction of n

– Final velocity of jet after striking in the direction of n]

$$= \rho a V [V \sin\theta - 0] = \rho a V^2 \sin 0 \quad \text{...(17.2)}$$

This force can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of flow. Then we have,

F_x = component of F_n in the direction of flow

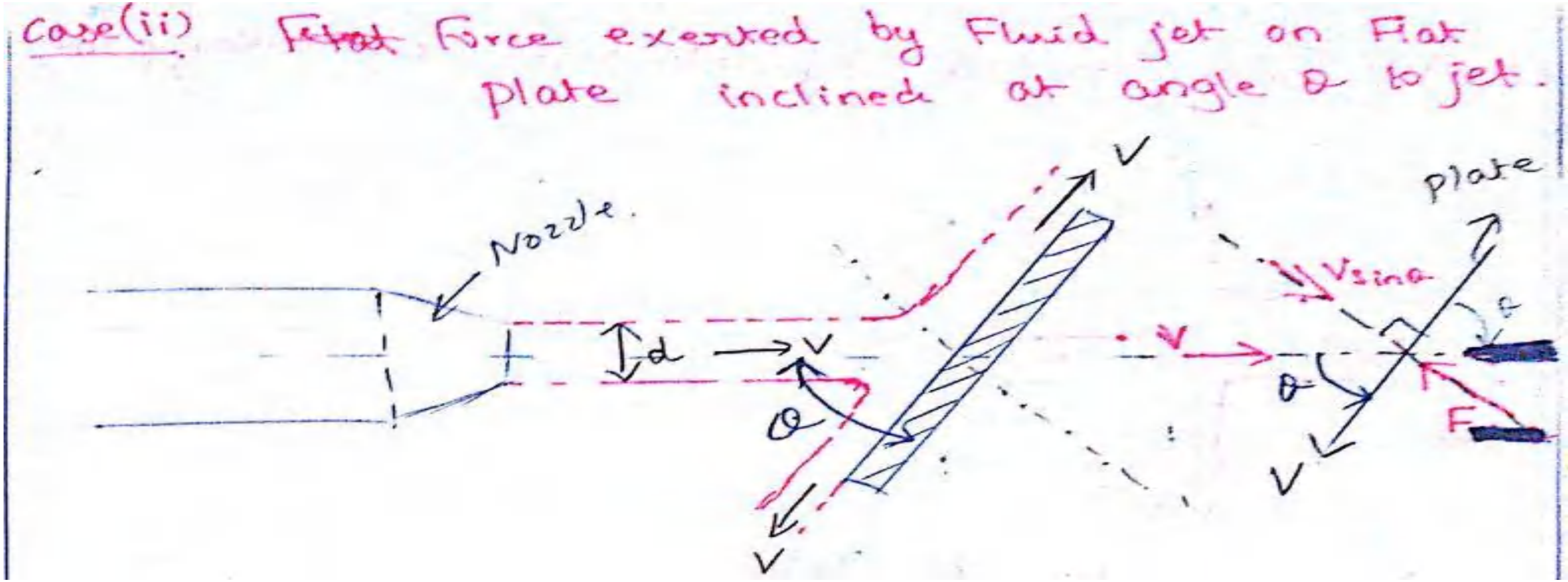
$$= F_n \cos(90° - \theta) = F_n \sin 0 = \rho A V^2 \sin\theta \times \sin\theta \quad (\therefore F_n = \rho a V^2 \sin\theta)$$

$$= \rho A V^2 \sin^2\theta \quad \text{...(17.3)}$$

And, F_y = component of F_n, perpendicular to flow

$$= F_n \sin(9\theta° - \theta) = F_n \cos\theta = \rho A V^2 \sin\theta \cos\theta. \quad \text{...(17.4)}$$

Inclined Plate

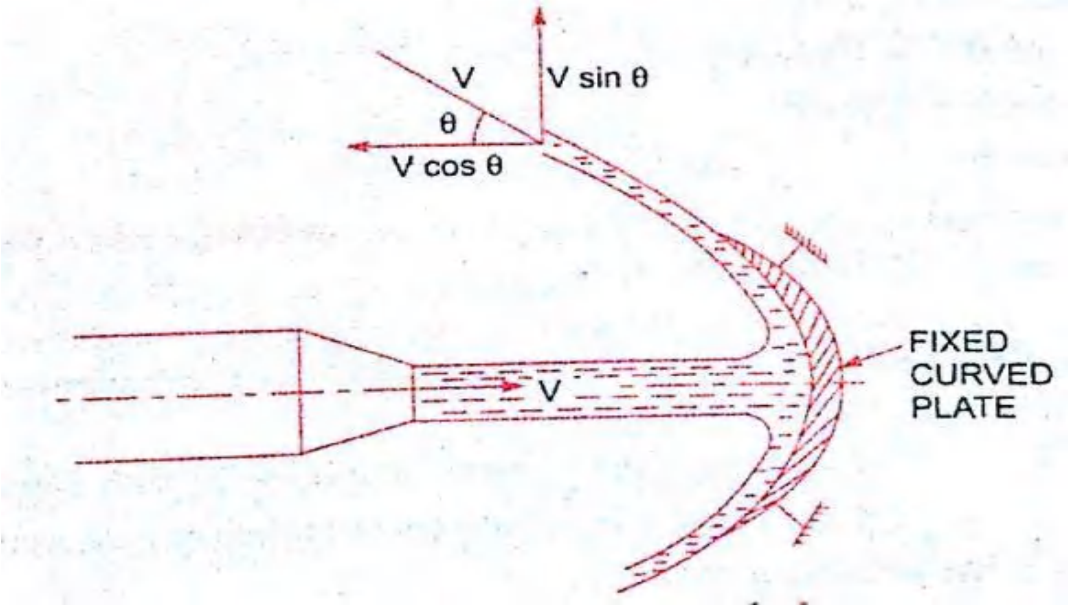


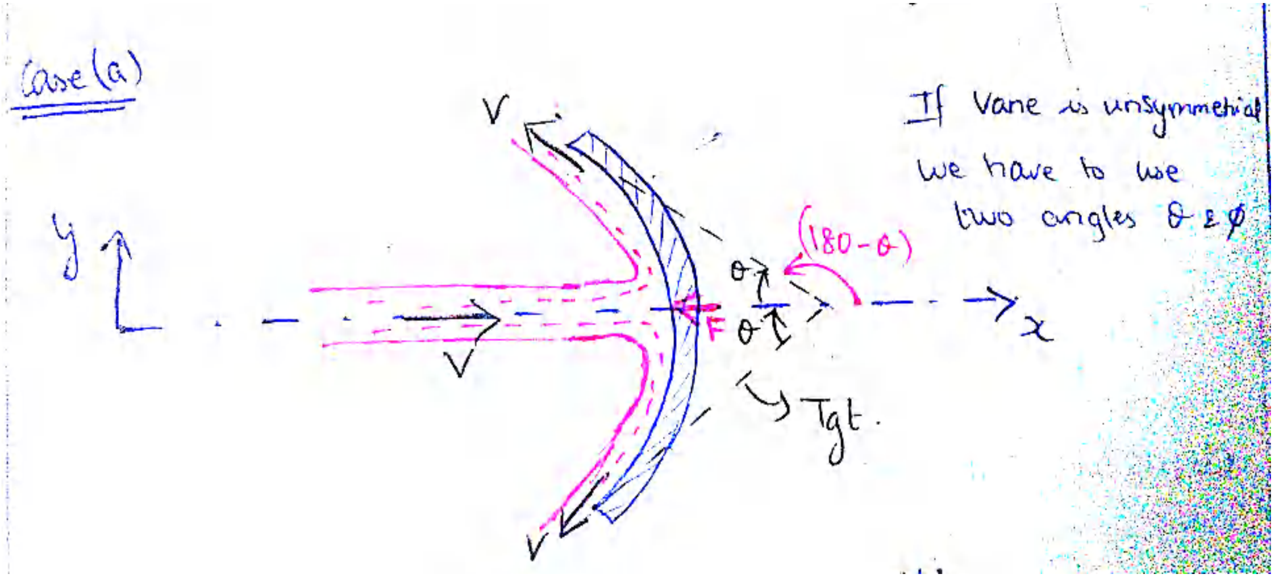
Impact of Jets

i) Curved Vane

- Symmetrical Unsymmetrical Vane
- Stationery and Moving Vane
- Direction of Flow- Jet entering at centre and Jet entering Tangentially

Curved Plate – Symmetrical Stationary Vanes- jet entering at centre





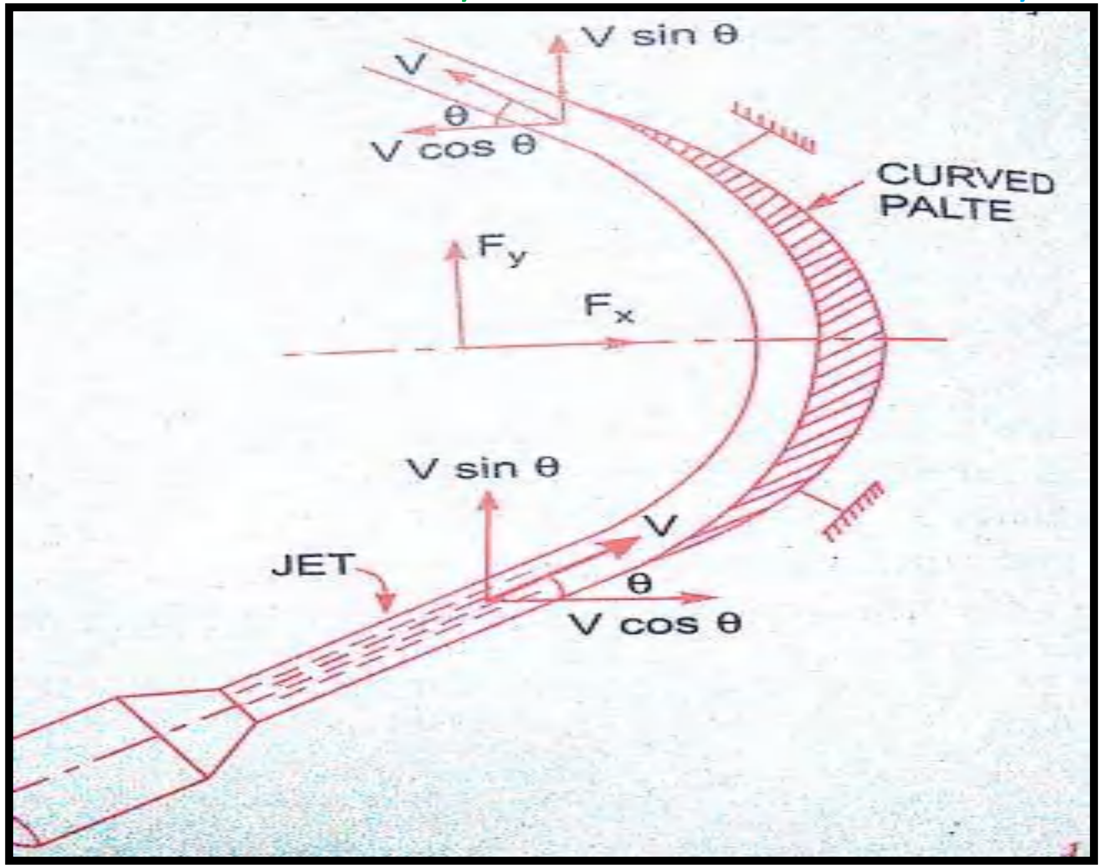
To calculate Force exerted by jet along the direction of jet

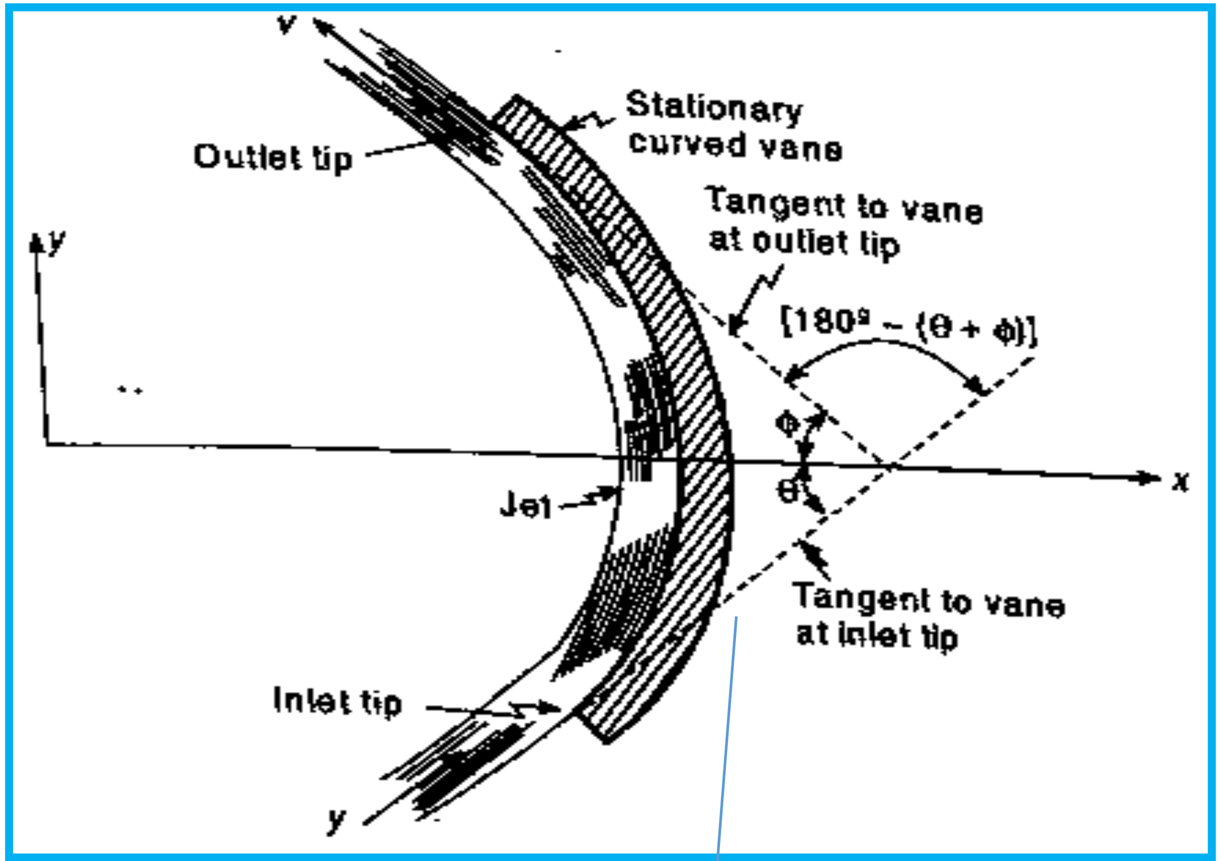
- F = (mass/second) * (Initial Velocity in direction of jet – Final velocity in direction of jet)
- F=ρQ * (Initial Velocity – Final velocity)
- $F = \rho * A * V * (V - (-V \cos\theta))$
- $F = \rho * A * V^2 (1+\cos\theta)$………(i)

But for Flat Plate, Force exerted by jet on Plate $F_p = \rho * A * V^2$ ….(F_{plate} or F_{vane} ? has high Force)

- In above equation(i) when θ =90 then it will become flat plate and θ =0 vane will become semi circular plate

Curved Plate – UnSymmetrical Stationary Vanes- jet entering tangentially



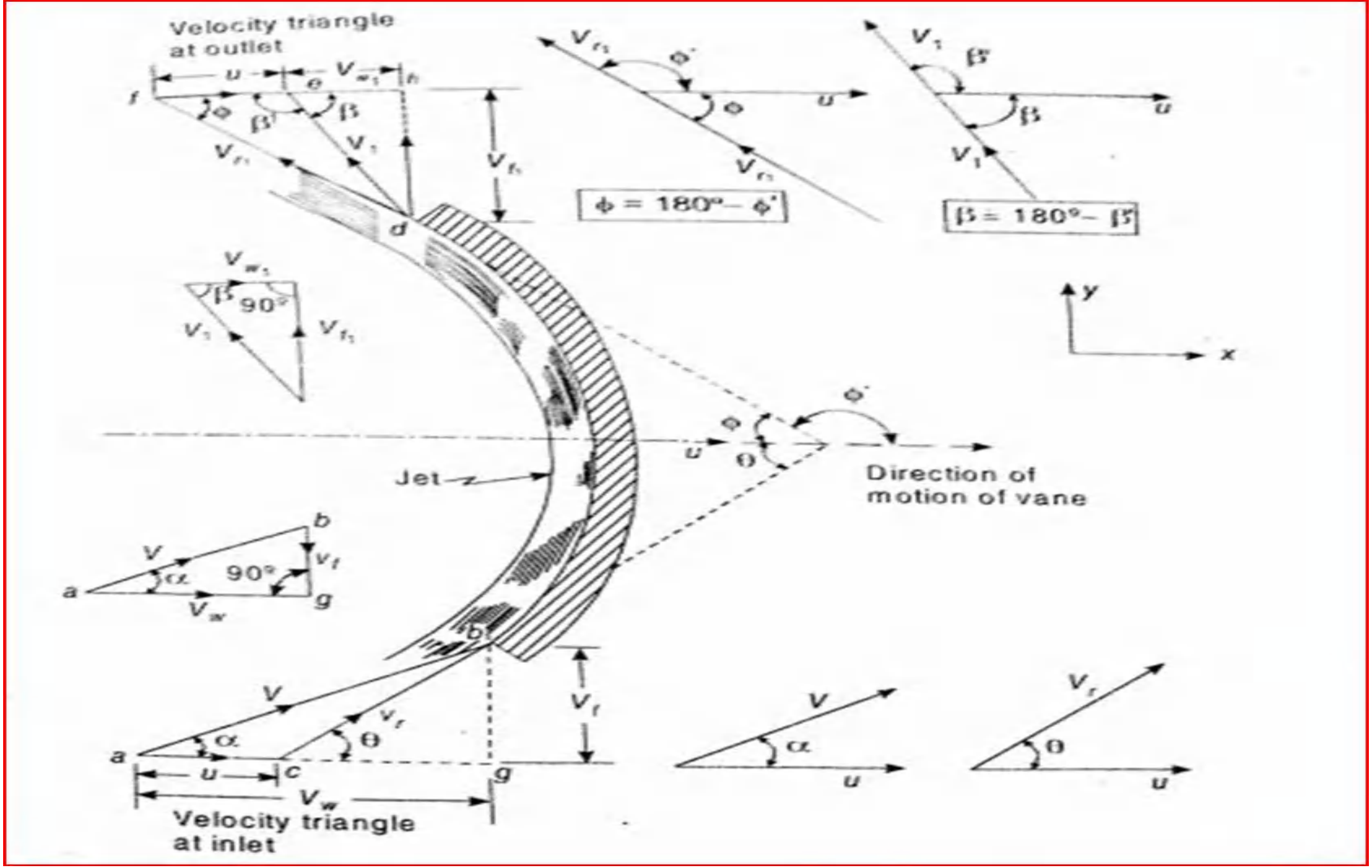


- F= ρQ * (Initial Velocity – Final velocity) along *x*- direction
- Case i- Symmetrical
- F_x= ρ*A*V* (V cos θ - (- V cosθ))
- F= ρ*A*V^2 (2cosθ)………(i)

Case ii- UnSymmetrical

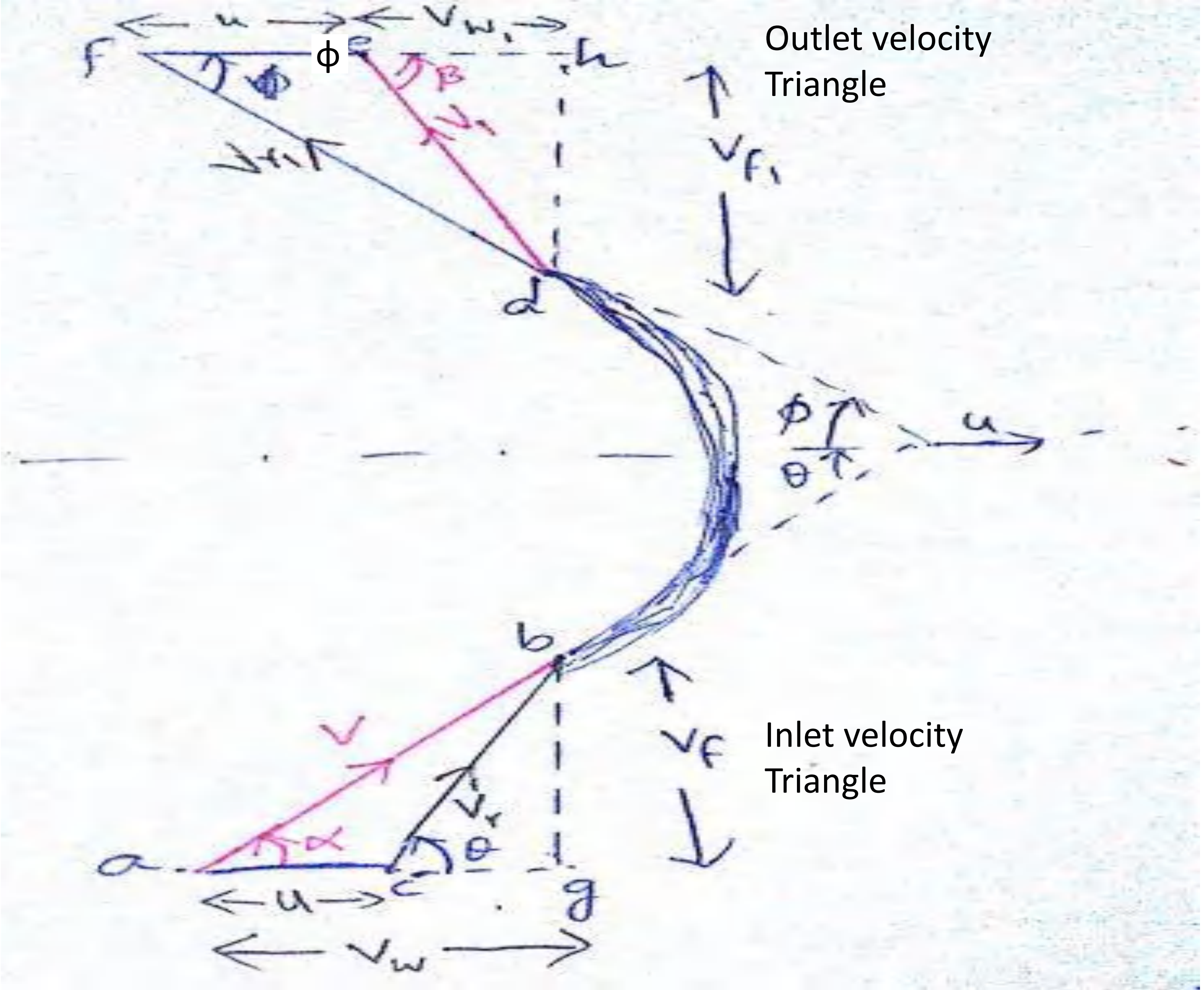
F_x= ρ*A*V* (V cos θ - (-V cos ϕ))

F= ρ*A*V^2 (cos θ+cos ϕ)………(i)



Velocity triangle at outlet
Velocity triangle at inlet
Jet
Direction of motion of vane
φ = 180° − φ'
β = 180° − β'

Curved Plate – Unsymmetrical Moving Vanes- jet entering tangentially



(A) Jet strikes the curved plate at the centre.

Component of velocity in the direction of jet = $-V \cos \theta$.

(–ve sign is taken as the velocity at outlet is in the opposite direction of the jet of water coming out from nozzle).

Component of velocity perpendicular to the jet = $V \sin \theta$

Force exerted by the jet in the direction of jet,

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}]$$

where V_{1x} = Initial velocity in the direction of jet = V

V_{2x} = Final velocity in the direction of jet = $-V \cos \theta$

$$\therefore \quad F_x = \rho a V[V - (-V \cos \theta)] = \rho a V[V + V \cos \theta]$$

$$= \rho a V^2[1 + \cos \theta] \quad \text{...(17.5)}$$

Similarly,

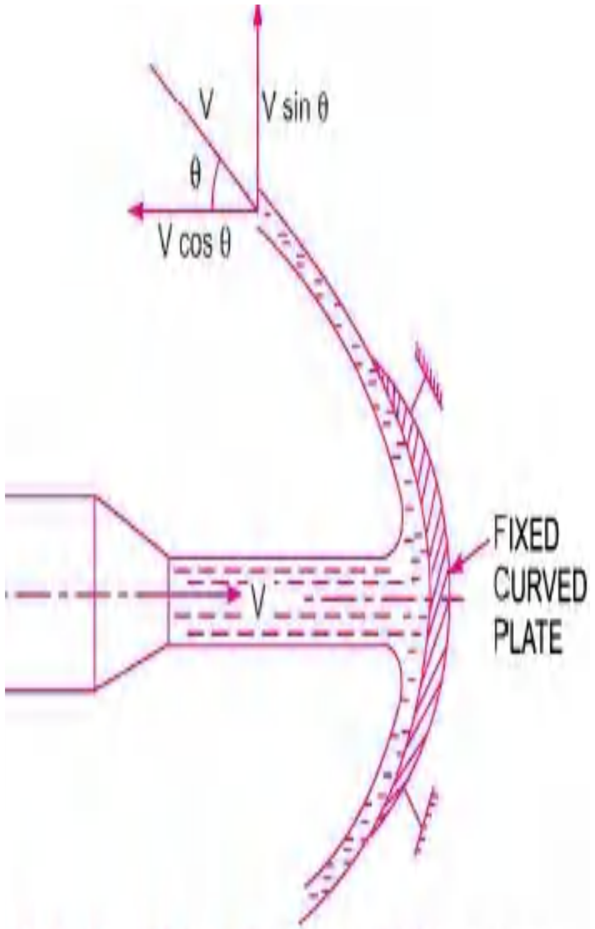
$$F_y = \text{Mass per sec} \times [V_{1y} - V_{2y}]$$

where V_{1y} = Initial velocity in the direction of $y = 0$

V_{2y} = Final velocity in the direction of $y = V \sin \theta$

$$\therefore \quad F_y = \rho a V[0 - V \sin \theta] = -\rho a V^2 \sin \theta \quad \text{...(17.6)}$$

–ve sign means that force is acting in the downward direction. In this case the angle of deflection of the jet $= (180° - \theta)$ ...[17.6 (A)]



17.3 Jet striking a fixed curved plate at centre.

(B) Jet strikes the curved plate at one end tangentially when the plate is symmetrical. Let the jet strikes the curved fixed plate at one end tangentially as shown in Fig. 17.4. Let the curved plate is symmetrical about x-axis. Then the angle made by the tangents at the two ends of the plate will be same.

Let V = Velocity of jet of water,

θ = Angle made by jet with x-axis at inlet tip of the curved plate.

If the plate is smooth and loss of energy due to impact is zero, then the velocity of water at the outlet tip of the curved plate will be equal to V. The forces exerted by the jet of water in the directions of x and y are

$$F_x = (\text{mass/sec}) \times [V_{1x} - V_{2x}]$$

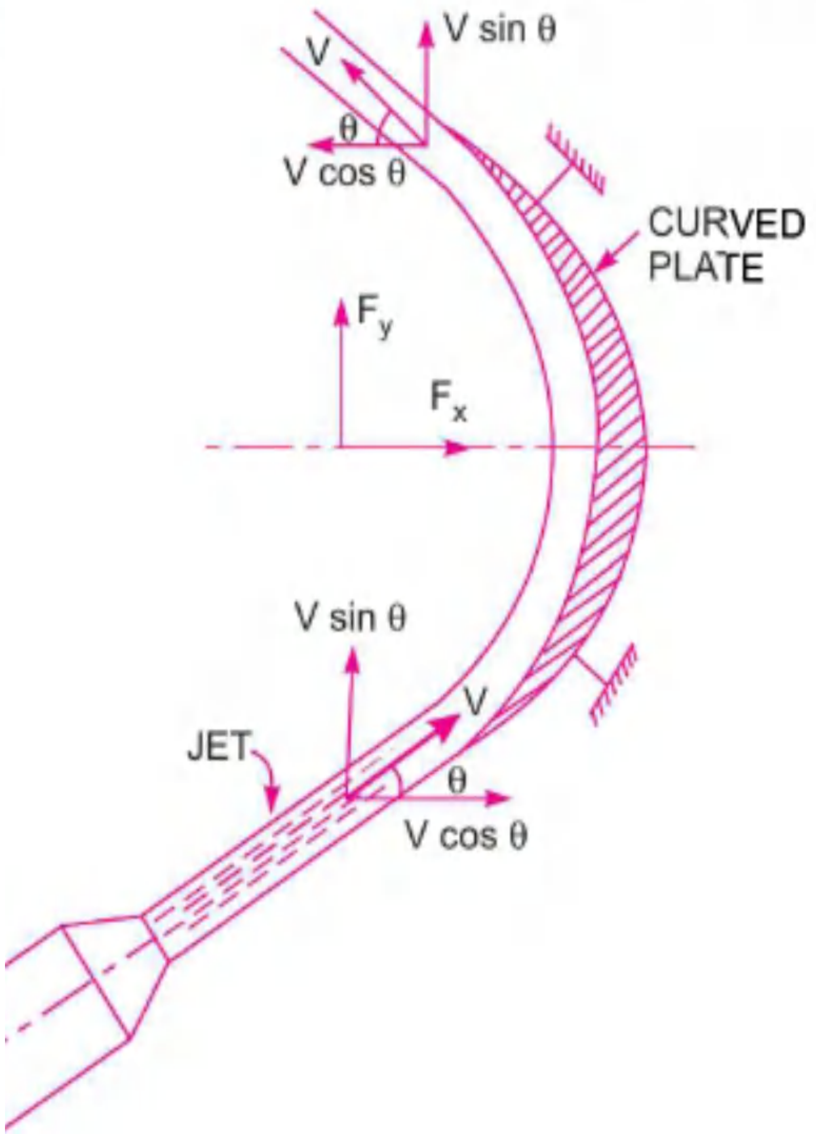
$$= \rho a V[V \cos \theta - (-V \cos \theta)]$$

$$= \rho a V[V \cos \theta + V \cos \theta]$$

$$= 2\rho a V^2 \cos \theta \qquad ...(17.7)$$

$$F_y = \rho a V[V_{1y} - V_{2y}]$$

$$= \rho a V[\, V \sin \theta - V \sin \theta \,] = 0$$



(C) Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical. When the curved plate is unsymmetrical about x-axis, then angle made by the tangents drawn at the inlet and outlet tips of the plate with x-axis will be different.

Let θ = angie made by tangent at iniet tip with x-axis,

ϕ = angie made by tangent at outlet tip with x-axis.

The two components of the velocity at inlet are

$$V_{1x} = V \cos \theta \text{ and } V_{1y} = V \sin \theta$$

The two components of the velocity at outlet are

$$V_{2x} = -V \cos \phi \text{ and } V_{2y} = V \sin \phi$$

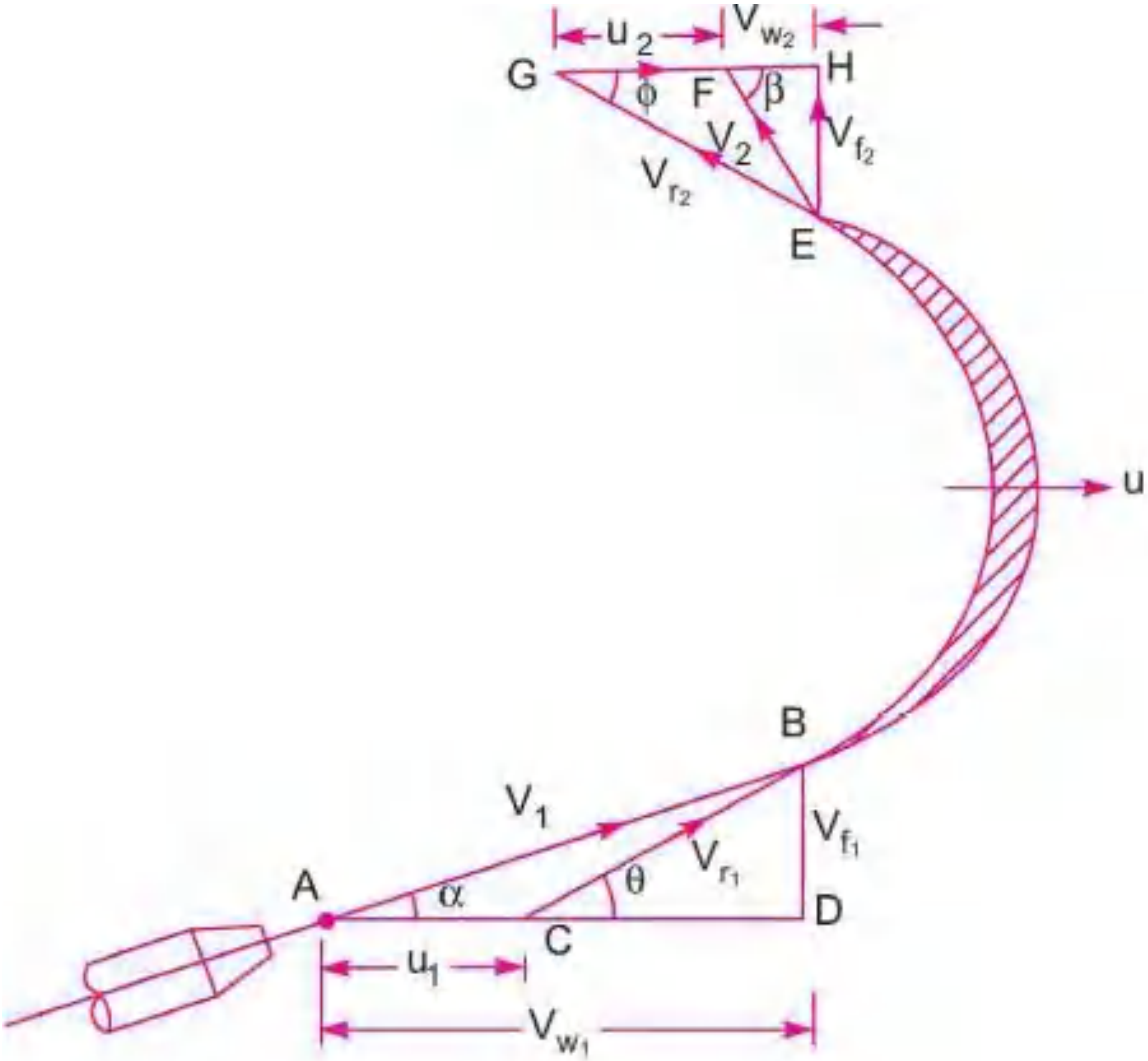
∴ The forces exerted by the jet of water in the directions of x and y are

$$F_x = \rho aV[V_{1x} - V_{2x}] = \rho aV[V \cos \theta - (-V \cos \phi)]$$

$$= \rho aV[V \cos \theta + V \cos \phi] = \rho aV^2 [\cos \theta + \cos \phi] \quad \text{...(17.8)}$$

$$F_y = \rho aV[V_{1y} - V_{2y}] = \rho aV[V \sin \theta - V \sin \phi]$$

$$= \rho aV^2 [\sin \theta - \sin \phi]. \quad \text{...(17.9)}$$



u 2
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G
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Vr2
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u
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θ
D
C
u1
Vw1

V_i = Velocity of the jet at inlet.
u_i = Velocity of the plate (vane) at inlet.
V_{r1}= Relative velocity of jet and plate at inlet.
α = Angle between the direction of the jet and direction of motion of the plate, also called guide blade angle.
ϴ = Angle made by the relative velocity (v") with the direction of motion at inlet also called vane angle at inlet.

V_{w_1} and V_{f_1} = The components of the velocity of the jet V_1, in the direction of motion and perpendicular to the direction of motion of the vane respectively.
V_{w_1} = It is also known as velocity of whirl at inlet.
V_{f_1} = It is also known as velocity of flow at inlet.
V_2 = Velocity of the jet, leaving the vane or velocity of jet at outlet of the vane.
u_2 = Velocity of the vane at outlet.
V_{r_2} = Relative velocity of the jet with respect to the vane at outlet.

β = Angle made by the velocity V_2 with the direction of motion of the vane at outlet.

ϕ = Angle made by the relative velocity V_{r_2} with the direction of motion of the vane at outlet and also called vane angle at outlet.

V_{w_1} and V_{f_1} = Components of the velocity V_2, in the direction of motion of vane and perpendicular to the direction of motion of vane at outlet.

V_{w_2} = It is also called the velocity of whirl at outlet.

V_{f_2} = Velocity of flow at outlet.

i) Force Calculation

Force striking on Vane along x direction

$F_x = \rho Q * (\text{Initial Velocity} - \text{Final Velocity})$

$F_x = \rho*(A*V_{r1})*(V_{r1}\cos\theta - (-V_{r2}\cos\phi)$

$F_x = \rho*(A*V_{r1})*((V_{w1}-u) + (V_{w2}+u))$

$F_x = \rho*(A*V_{r1})*(V_{w1} + V_{w2})$

This equation is valid if β is acute

If angle $\beta = 90^o$ then $V_{w2}=0$ then,

$F_x = \rho*(A*V_{r1})*(V_{w1})$

If angle β is obtuse angle then,

$F_x = \rho*(A*V_{r1})*(V_{w1} - V_{w2})$

ii) Work done per second= $F_x * u$

iii) Efficiency $= \dfrac{\text{Work done per second}}{\text{Kinetic Energy}}$

1. *A jet 30 mm diameter with velocity of 10 m/s strikes a vertical plate in the normal direction.* ***Determine the force on the plate if*** *(i) The plate is stationary (ii) If it moves with a velocity of 4 m/s towards the jet and (iii) If the plate moves away from the plate at a velocity of 4 m/s.*

Case (*i*) The total *x* directional velocity is lost.

$\therefore F = m\,V,\ m = \rho AV$

$$\mathbf{F} = \frac{\pi \times 0.03^2}{4} \times 10 \times 10 \times 1000 = \mathbf{70.7\ N}$$

Case (*ii*) $\dot{m} = \rho A(V_r),\quad V_r = V + u = 14$

$$\therefore \quad \mathbf{F} = \frac{\pi \times 0.032}{4} \times 14 \times 1000 \times 10 = \mathbf{99\ N}$$

Case (*iii*) $F\ V_r = V - u = 6\text{ m/s}$

$$\mathbf{F} = \frac{\pi \times 0.03^2}{4} \times 6 \times 1000 \times 10 = \mathbf{42.4\ N}$$

Problem 17.3 *A jet of water of diameter 75 mm moving with a velocity of 25 m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60°. Find the force exerted by the jet on the plate (i) in the direction normal to the plate and (ii) in the direction of the jet.*

Solution. Given :

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

∴ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.075)^2 = 0.004417 \text{ m}^2$

Velocity of jet, $V = 25 \text{ m/s}$.

Angle between jet and plate $\theta = 60°$

(*i*) The force exerted by the jet of water in the direction normal to the plate is given by equation (17.2) as

$$F_n = \rho a V^2 \sin \theta$$

$$= 1000 \times .004417 \times 25^2 \times \sin 60° = \mathbf{2390.7 \text{ N. Ans.}}$$

(*ii*) The force in the direction of the jet is given by equation (17.3),

$$F_x = \rho a V^2 \sin^2 \theta$$

$$= 1000 \times .004417 \times 25^2 \times \sin^2 60° = \mathbf{2070.4 \text{ N. Ans.}}$$

Problem 17.4 *A jet of water of diameter 50 mm strikes a fixed plate in such a way that the angle between the plate and the jet is 30°. The force exerted in the direction of the jet is 1471.5 N. Determine the rate of flow of water.*

Solution. Given :

Diameter of jet, $d = 50 \text{ mm} = 0.05 \text{ m}$

$\therefore$ Area, $a = \dfrac{\pi}{4}(.05)^2 = .001963 \text{ m}^2$

Angle, $\theta = 30°$

Force in the direction of jet, $F_x = 1471.5$ N

Force in the direction of jet is given by equation (17.3) as $F_x = \rho a V^2 \sin^2 \theta$

As the force is given in Newton, the value of ρ should be taken equal to 1000 kg/m^3

$$\therefore \quad 1471.5 = 1000 \times .001963 \times V^2 \times \sin^2 30° = .05\ V^2$$

$$\therefore \quad V^2 = \frac{150}{.05} = 3000.0$$

$$V = 54.77 \text{ m/s}$$

$\therefore$ Discharge, $Q = \text{Area} \times \text{Velocity}$

$= .001963 \times 54.77 = 0.1075 \text{ m}^3/\text{s} =$ **107.5 liters/s. Ans.**

Problem 17.5 *A jet of water of diameter 50 mm moving with a velocity of 40 m/s, strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate.*

Solution. Given :

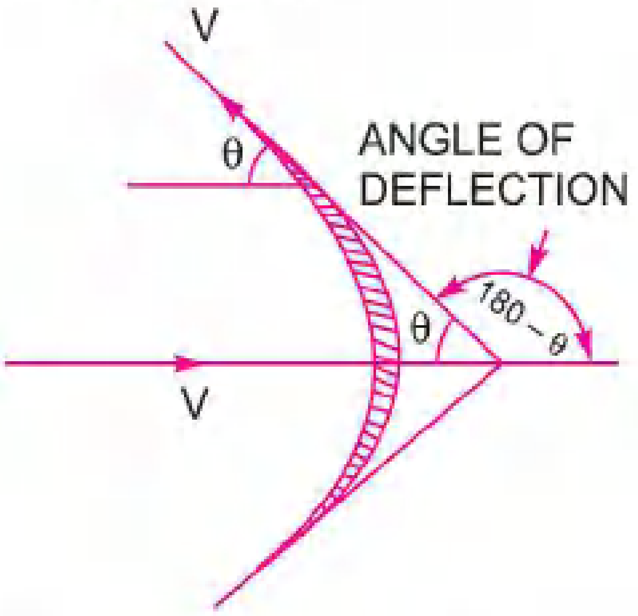


Fig. 17.5

Diameter of the jet, $d = 50 \text{ mm} = 0.05 \text{ m}$

∴ Area, $a = \frac{\pi}{4} (.05)^2 = 0.001963 \text{ m}^2$

Velocity of jet, $V = 40 \text{ m/s}$

Angle of deflection $= 120°$

From equation [17.6 (*A*)], the angle of deflection $= 180° - \theta$

∴ $180° - \theta = 120°$ or $\theta = 180° - 120° = 60°$

Force exerted by the jet on the curved plate in the direction of the jet is given by equation (17.5) as

$$F_x = \rho a V^2 [1 + \cos \theta]$$

$$= 1000 \times .001963 \times 40^2 \times [1 + \cos 60°] = \mathbf{4711.15 \text{ N. Ans.}}$$

Problem 17.6 *A jet of water of diameter 75 mm moving with a velocity of 30 m/s, strikes a curved fixed plate tangentially at one end at an angle of 30° to the horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by the jet on the plate in the horizontal and vertical direction.*

Solution. Given :

Diameter of the jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} (.075)^2 = .004417 \text{ m}^2$

Velocity of jet, $V = 30 \text{ m/s}$

Angle made by the jet at inlet tip with horizontal, $\theta = 30°$

Angle made by the jet at outlet tip with horizontal, $\phi = 20°$

The force exerted by the jet of water in the direction of x is given by equation (17.8) and in the direction of y by equation (17.9),

$$\therefore \quad F_x = \rho a V^2 [\cos\theta + \cos\phi]$$

$$= 1000 \times .004417 [\cos 30° + \cos 20°] \times 30^2 = \mathbf{7178.2 \text{ N. Ans.}}$$

and

$$F_y = \rho a V^2 [\sin\theta - \sin\phi]$$

$$= 100\theta \times .004417 [\sin 30° - \sin 20°] \times 30^2 = \mathbf{628.13 \text{ N. Ans.}}$$

***3.** A jet of water at a velocity of 100 m/s strikes a series of moving vanes fixed at the periphery of a wheel, at the rate of 5 kg/s. The jet is inclined at 20° to the direction of motion of the vane. The blade speed is 50 m/s. The water leaves the blades at an angle of 130° to the direction of motion.*

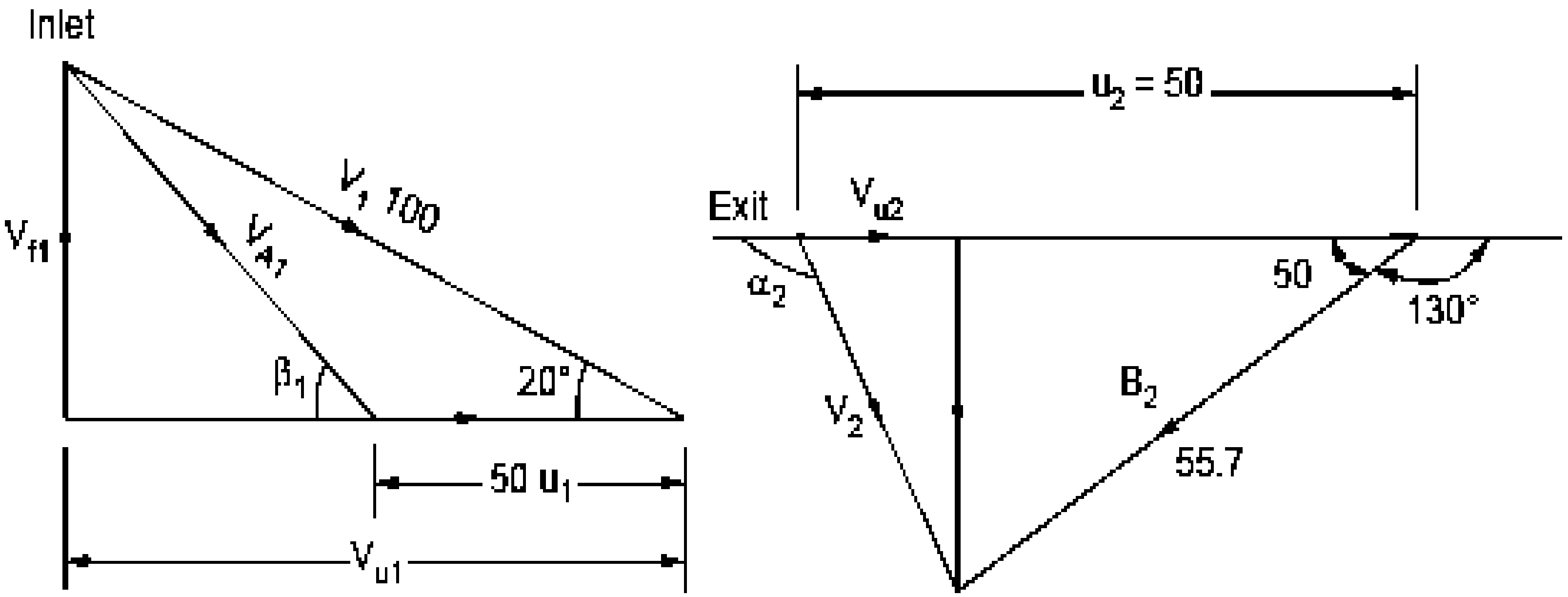
Calculate the blade angles at the forces on the wheel in the axial and tangential direction.

V = u + v

V= Velocity of jet

u = Velocity of blade

v = relative velocity



$$\tan\beta_1 = \frac{V_1 \sin\alpha_1}{V_1 \cos\alpha_1 - u} = \frac{100 \times \sin 20}{100 \cos 20 - 50}$$

Blade angle at inlet $\therefore \quad \beta_1 = 37.88°$

$$\sin \beta_1 = \frac{V_1 \sin \alpha_1}{V_{r_1}}$$

$$\therefore \qquad V_{r_1} = \frac{100 \sin 20}{\sin 37.88} = 55.7 \text{ m/s}$$

In this type of blade fixing

$$V_{r_2} = V_{r_1} \quad \text{and} \quad u_2 = u_1$$

Referring to the exit triangle

$$V_{r_2} \cos 50 < u = 50$$

Hence this shape

$$V_{r_2} \cos 50 = 35.8. \quad \therefore \quad Vu_2 = 50 - 35.8$$

$$= \mathbf{14.2\ m/s\ in\ the\ same\ direction\ as}\ V_{u_1}$$

$$\therefore \text{ Tangential force} = 500 \times (V_{u_1} - V_{u_2})$$

$$V_{u_1} = 100 \cos 20 = 93.97 \text{ m/s}$$

$$\therefore \textbf{ Tangential force} = 5\,(93.97 - 14.2) = \mathbf{3488\ N}$$

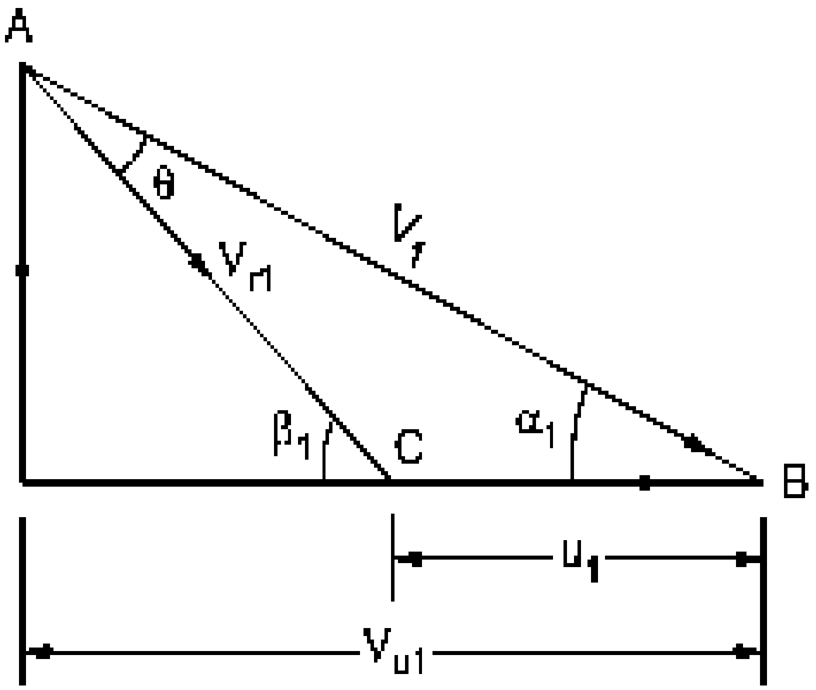
Axial force $\mathbf{F} = \dot{m}\,[V_1 \sin \alpha - V_{r_1} \sin \beta_2]$

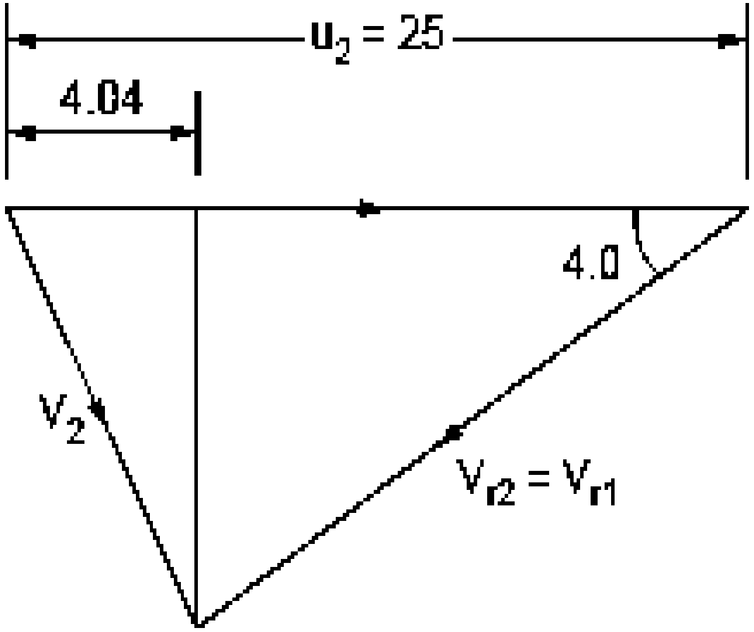
$$= 5\,[100 \sin 20 - 55.7 \,.\, \sin 50] = \mathbf{-8.5\ N}$$

***7p)**. Water jet at the rate of 10 kg/s strikes the series of moving blades at a velocity of 50 m/s. The blade angles with respect to the direction of motion are 35° and 140°. If the peripheral speed is 25 m/s, **determine the inclination of the jet** so that water enters the blades without shock. Also **calculate the power developed and the efficiency** of the system.*

Assume blades are mounted on the periphery of the wheel.

In this type of mounting *u* remains the same so also relative velocity. β1, *V*1 and *u* are known :





$$\frac{V_1}{\sin(180-\beta_1)} = \frac{u}{\sin\theta}$$

$$\therefore \quad \frac{50}{\sin(180-35)} = \frac{25}{\sin\theta}$$

Solving $\theta = 16.7°$. $\therefore$ $\alpha_1 = 180 - (180 - 35) - 16.7 = \mathbf{18.3°}$

Direction of the jet is 18.3° to the direction of motion.

$$V_{u_1} = 50 \times \cos 18.3 = 47.47 \text{ m/s},$$

$$V_{r_1} = \frac{50 \sin 18.3}{\sin 35} = 27.37 \text{ m/s}$$

$$\beta_2 = (180 - 140) = 40°, \quad V_{r_2} \cos 40 = 20.96 < 25\ (u)$$

$\therefore$ The shape of the exit triangle will be as in figure

$$V_{u_2} = u - V_{r_2} \cos \beta_2 = 25 - 20.96 = 4.04 \text{ m/s}$$

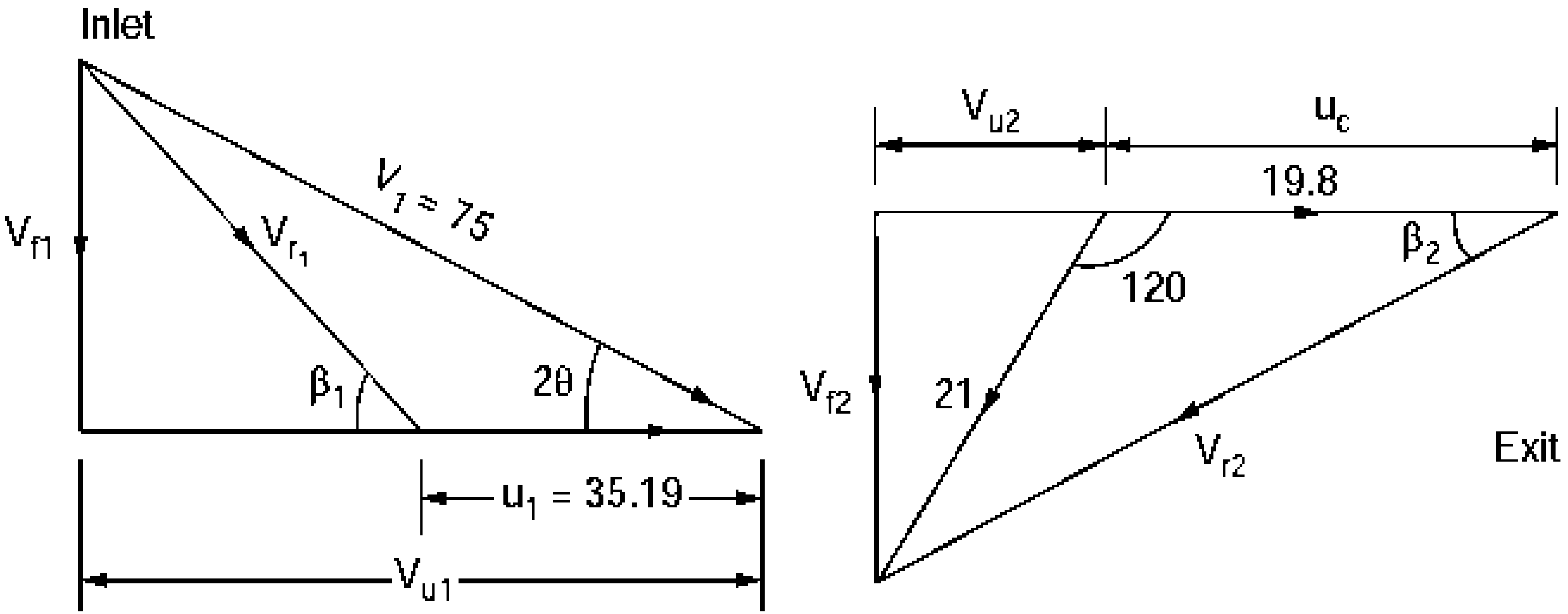
Tangential force $= m\,(V_{u_1} - V_{u_2}) = 10\,(47.47 - 4.04) = \mathbf{434.3\ N}$

Power $= F \times u = 434.3 \times 25 = \mathbf{10.86 \times 10^3\ W}$

Energy in jet $= \dfrac{10 \times 50^2}{2} = 12.5 \times 10^3$ W

$$\therefore \quad \eta = \frac{10.86 \times 10^3}{12.5 \times 10^3} = 0.8686 \text{ or } \mathbf{86.86\ \%}$$

8p). *Curved vanes fixed on a wheel on the surface receive water at angle of 20° to the tangent of the wheel. The inner and outer diameter of the wheel are 0.9 and 1.6 m respectively. The speed of rotation of the wheel is 7 revolutions per second. The velocity of water at entry is 75 m/s. The water leaves the blades with an absolute velocity of 21 m/s at an angle of 120° with the wheel tangent at outlet. The flow rate is 400 kg/s.* ***Determine the blade angles*** *for shockless entry and exit.* ***Determine the torque and power.*** *A also determine the radial force.*



Blade velocity $u_1 = \pi dN = \pi \times 1.6 \times 7 = 35.19$ m/s

$$u_2 = \frac{9}{16} \times 35.19 = 19.8 \text{ m/s}$$

$$\tan \beta_1 = \frac{V_1 \sin \alpha_1}{V_1 \cos \alpha_1 - u} = \frac{75 \times \sin 20}{75 \times \cos 20 - 35.19}$$

Solving $\beta_1 = \mathbf{36°}$

$$\tan \beta_2 = \frac{21 \sin 60}{19.8 + 21 \cos 60}$$

Solving $\beta_2 = \mathbf{30.97°}$

$$\mathbf{T} = \dot{m}\,[V_{u_1} r_1 + V_{u_2} r_2] \quad \text{(in this case, } V_{u_2} \text{ is in the opposite direction)}$$

$$\therefore \qquad \Delta V_w = V_{u_1} + V_{u_2}$$

$$= 400\,[0.8 \times 75 \cos 20 + 0.45 \times 21 \cos 60] = \mathbf{24443\ Nm}$$

$$\textbf{Power} = 24443 \times \omega = 24443 \times 2\pi \times 7 = 1075042\ W$$

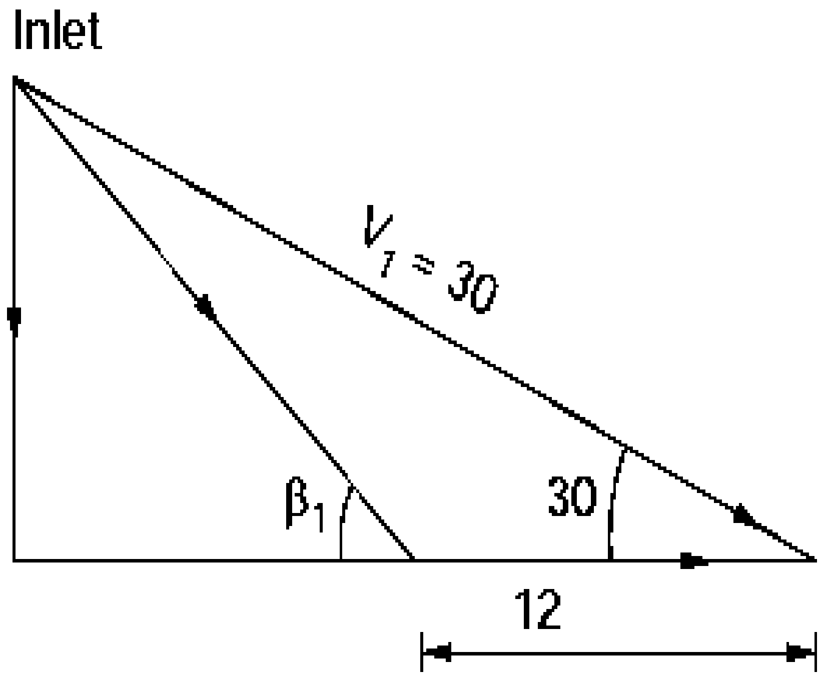
or $\underline{\Omega}$ **1075 kW.**

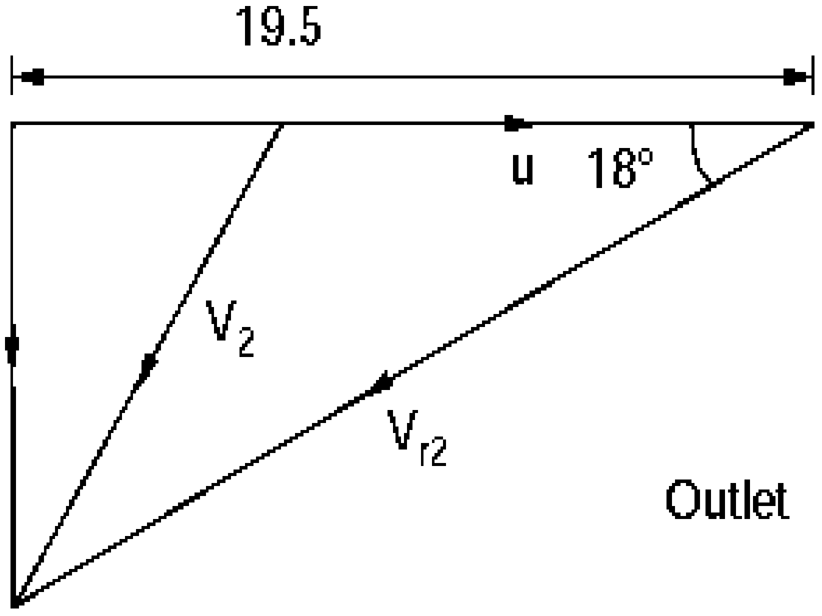
$$\text{Power in the jet} = \frac{75^2}{2} \times 400 = 1125000 \text{ W or } \mathbf{1125\ kW}$$

$$\eta = \frac{1075}{1125} = 0.955 \text{ or } \mathbf{95.5\%}$$

$$\textbf{Radial}\ \text{force} = 400\,(75 \sin 20 - 21 \sin 60) = \mathbf{2986\ N.}$$

Problem 6. *A jet of water with a velocity of 30 m/s impinges on a series of vanes moving at 12 m/s at 30° to the direction of motion. The vane angle at outlet is 162° to the direction of motion.* ***Complete (i) the vane angle at inlet*** *for shockless entry and (ii)* ***the efficiency of power transmission.***





$$\tan \beta_1 = \frac{V_1 \sin \alpha_1}{V_1 \cos \alpha_1 - u} = \frac{30 \sin 30}{30 \cos 30 - 12} = 1.073$$

$$\therefore \quad \beta_1 = \mathbf{47°}$$

$$\sin \beta_1 = \frac{30 \sin 30}{V_{r_1}}$$

$$\therefore \quad V_{r_1} = \frac{30 \sin 30}{\sin \beta_1} = 20.5 \text{ m/s} = V_{r_2}$$

$V_{r_2} \cos \beta_2 > u_1 \quad \therefore$ hence the shape of the triangle.

$$V_{u_1} = 30 \cos 30 = 25.98 \text{ m/s}$$

$$V_{u_2} = 20.5 \cos 18 - 12 = 7.5 \text{ m/s}$$

Assuming unit mass flow rate :

$$\mathbf{P} = u\,[V_{w_1} + V_{w_2}] = 12\,[25.98 + 7.5] = \mathbf{401.76\ W/kg/s}$$

Energy in the jet $= \dfrac{30^2}{2} = 450\ W.$

$\therefore$ $$\eta = \frac{401.76}{450} = \mathbf{0.893} \text{ or } 89.3\%$$

7. A jet of water having a velocity of 15m/s strikes a curved vane which is moving with a velocity of 5 m/s. The vane is symmetrical and is so shaped that jet is deflected through 120°.

- Find angle of jet at inlet so that there is no shock (i.e no energy loss)
- What is absolute velocity of jet at outlet. (Both Magnitude and Direction)
- Work Done per second in direction of jet

Given data:

Inlet:

Absolute Velocity of jet of water V_1=15m/s

Velocity of Vane u = 5 m/s

Deflected Angle = 120°

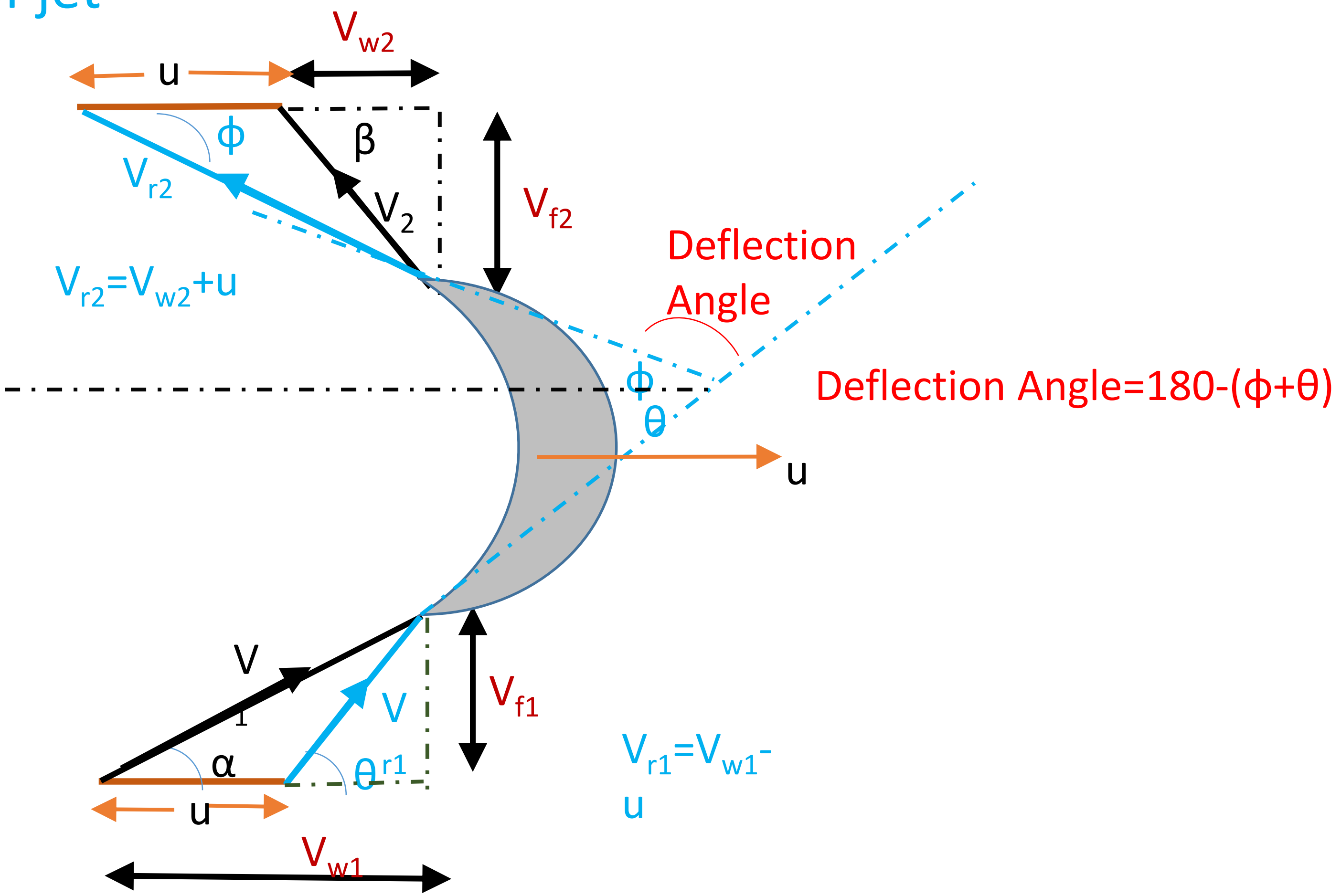
So Vane angle θ=ϕ= (180-120)/2 = 30°

To find:

i) Angle of Jet at inlet α

ii) Absolute Velocity V_2 (Magnitude)

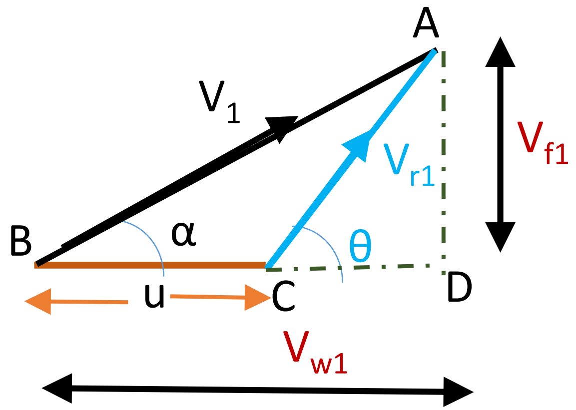
Direction β



Given data:
Inlet:
Absolute Velocity of jet of water
V_1=15m/s
Velocity of Vane u = 5 m/s
Deflected Angle = 120°
So Vane angle θ=ϕ= (180-120)/2 = 30°

To find:
i) Angle of Jet at inlet α
ii) Absolute Velocity V_2=?
iii) Direction of Absolute velocity β

Inlet velocity triangle-To find α & V_{r1}



Sine rule for Triangle ABC

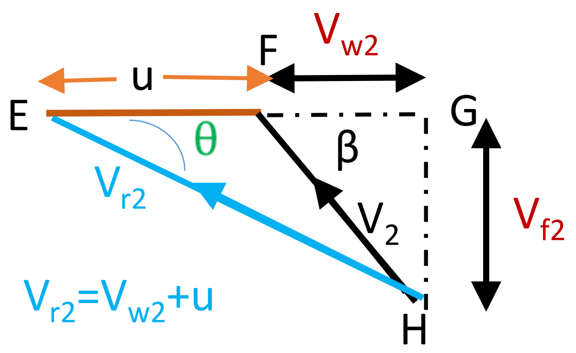
$$\frac{AB}{\sin(180-\theta)} = \frac{BC}{\sin(180-(180-\theta+\alpha))} = \frac{AC}{\sin\alpha}$$

Sine rule for Triangle ABC

$$\frac{15}{\sin(150)} = \frac{5}{\sin(30-\alpha)} = \frac{V_{r1}}{\sin\alpha}$$

$\alpha = 20.4^0$ AC= V_{r1}=10.46 m/s = V_{r2}

Outlet velocity triangle- to find V_2 and β



$V_{r2}=V_{w2}+u$

In Triangle EHG
$u+V_{w2}= V_{r2} * \cos\theta$ $V_{f2}= V_{r2} * \sin\theta$
Find V_{w2} and V_{f2}

So $\tan\beta = (V_{f2}/V_{w2})$ Find β=??????

From triangle FGH find V_2=???????

Answers
i) α= 20.4°
ii) Absolute Velocity V_2=6.62 m/s
iii) Direction is 52^0

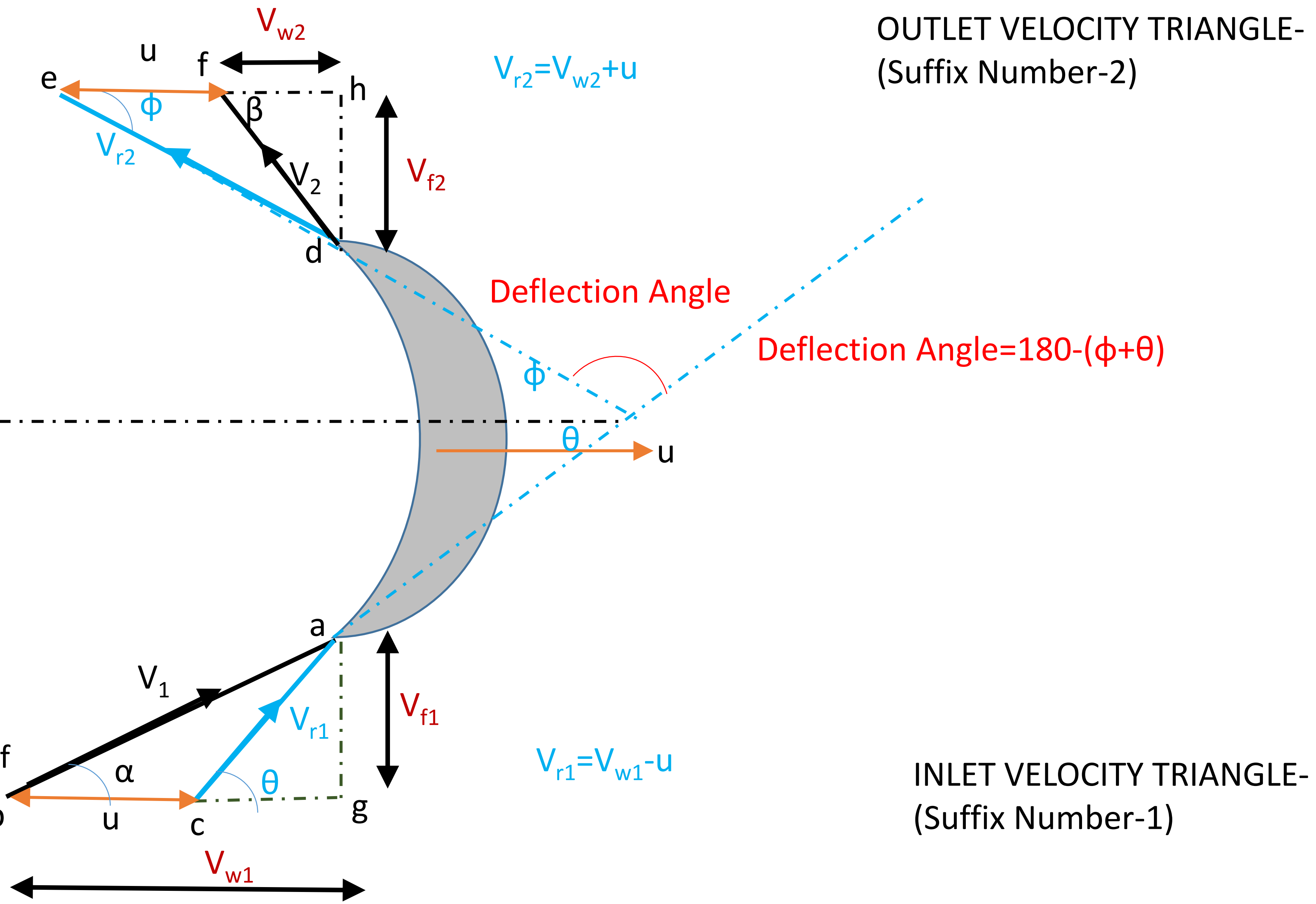
Notations:
V- Absolute velocity
u- Speed of vane
V_r-Relative Velocity

(while resolving Absolute velocity)
V_w-Whril velocity (Horizontal)
V_f- Flow Velocity (vertical)

Angle Symbols
θ- angle b/w Vr1 @ inlet with direction of motion of plate (V_{r1})
(Vane angle)

Φ- angle b/w V_{r2} @ outlet with direction of motion of plate (V_{r2})

α- Guide Blade angle
.β- Angle b/w Abs. Velocity & Motion of plate



Jet tangentially Striking a Moving (u) Unsymmetrical (θ&ϕ) Vane

Pumps

- Pump is a hydraulic machine which converts the mechanical energy to hydraulic energy which mainly in the form of pressure energy.
- Two types of pump
  - **Roto Dynamic Pump**
    - Centrifugal Pumps-

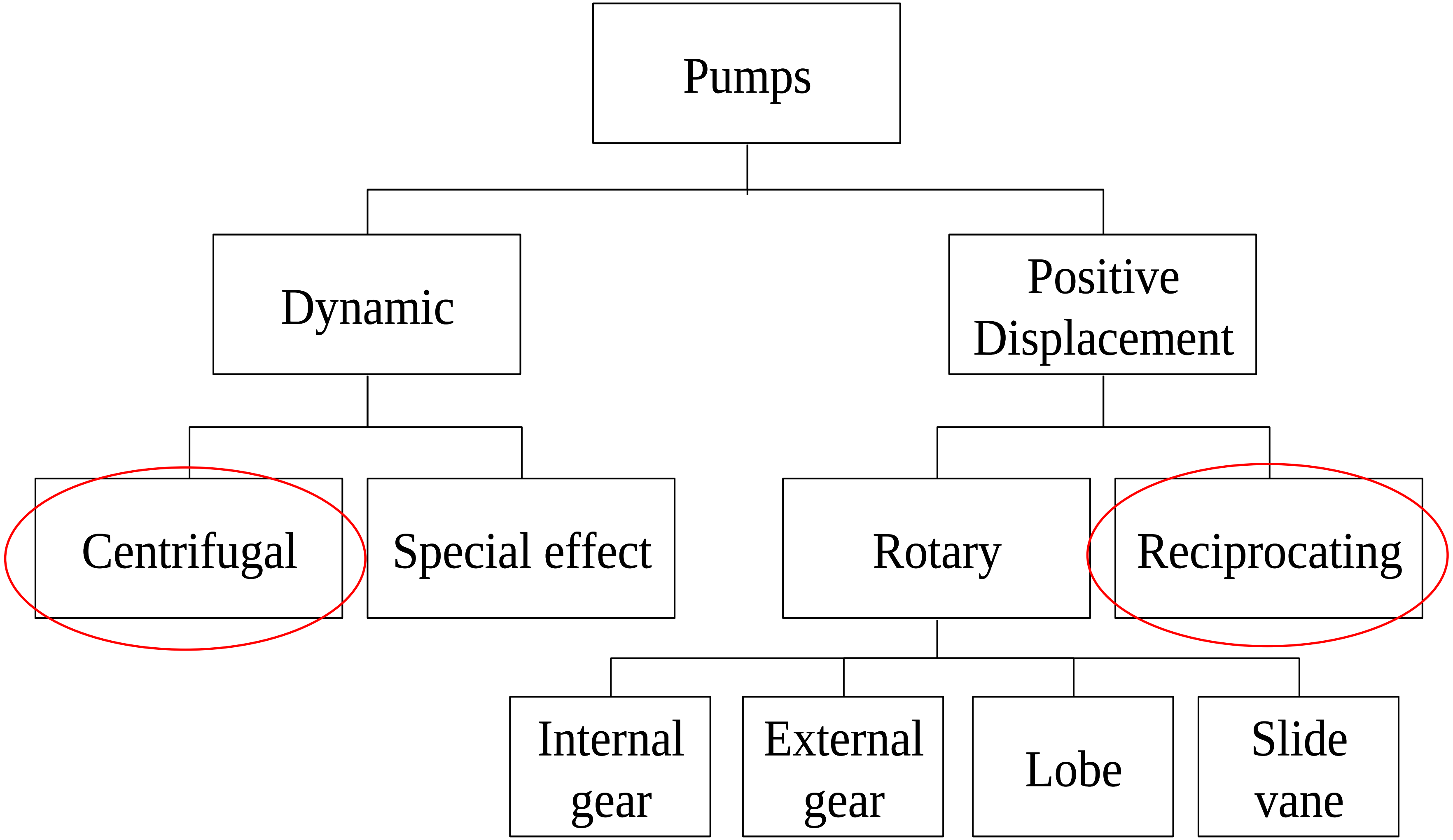
      Mechanical energy is converted to hydraulic energy by centrifugal action (rotary motion)
  - **Positive Displacement Pump**
    - Reciprocating Pumps

      Mechanical energy is converted to hydraulic energy by sucking the liquid into a cylinder in which the piston exerts thrust on the liquid and increases the pressure energy.

Type of Pumps

Pump Classification

Classified by operating principle



Classification

1) **Rotodynamic pumps** which move the fluid by dynamic action of imparting momentum to the fluid using mechanical energy.

2) **Reciprocating pumps** which first trap the liquid in a cylinder by suction and then push the liquid against pressure.

3) **Rotary positive displacement pumps** which also trap the liquid in a volume and push the same out against pressure.

Rotodynamic pumps can be operated at **high speeds**

Classification- Direction of flow

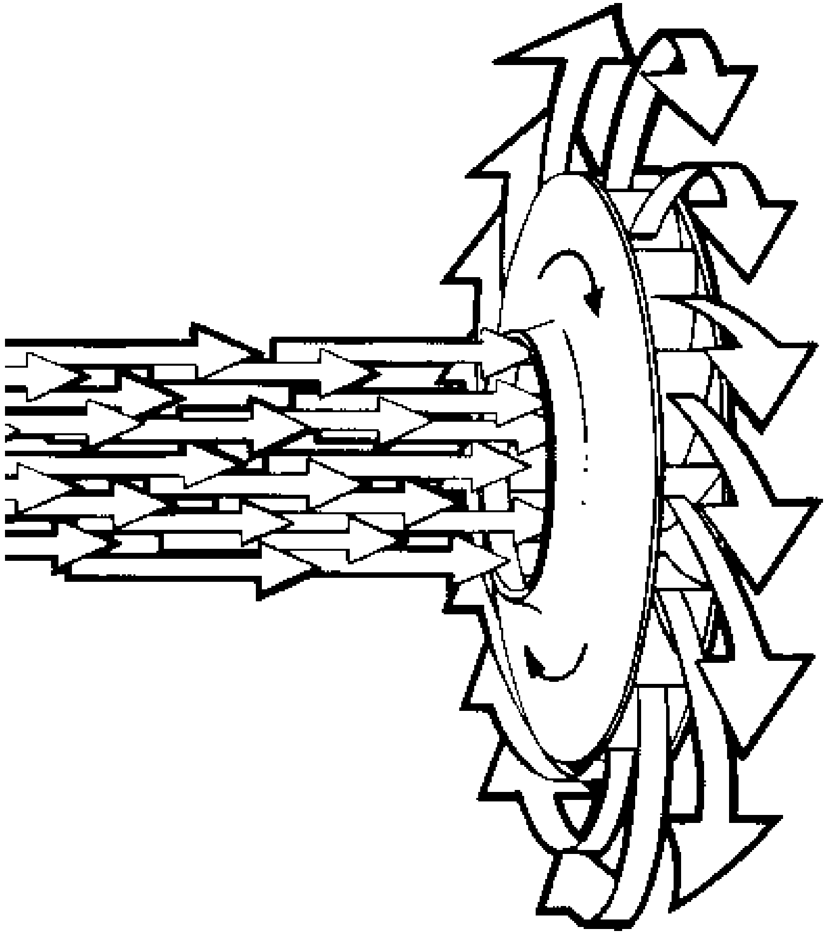
- **Radial flow** or purely centrifugal pumps generally handle **lower volumes at higher pressures.**
- **Mixed flow** pumps handle comparatively **larger volumes** at **medium range of pressures.**
- **Axial flow pumps** can handle **very large volumes,** but the **pressure** against which these pumps operate is **limited**.
- The overall efficiency of the three types are nearly the same.

Based on Head- low head (10 m and below), **medium head** (10-50 m) **and high head** pumps

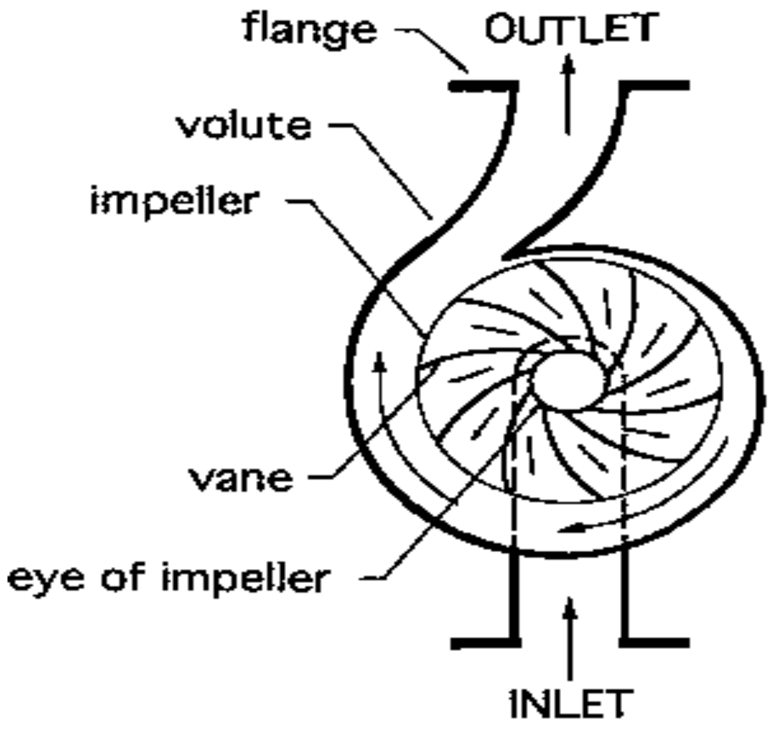
Centrifugal Pumps

- **This machine consists of an *IMPELLER* rotating within a case (diffuser)**

- **Liquid directed into the center of the rotating impeller is picked up by the impeller's vanes and accelerated to a higher velocity by the rotation of the impeller and discharged by centrifugal force into the case (diffuser).**



Centrifugal Pumps



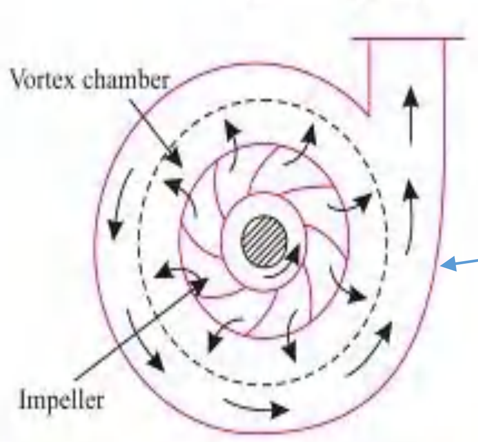


Fig. 3.3. Vortex casing.

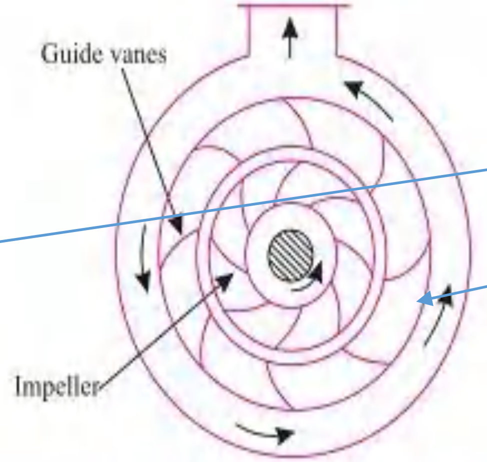


Fig. 3.4. Casing with guide blades.

Main Components:

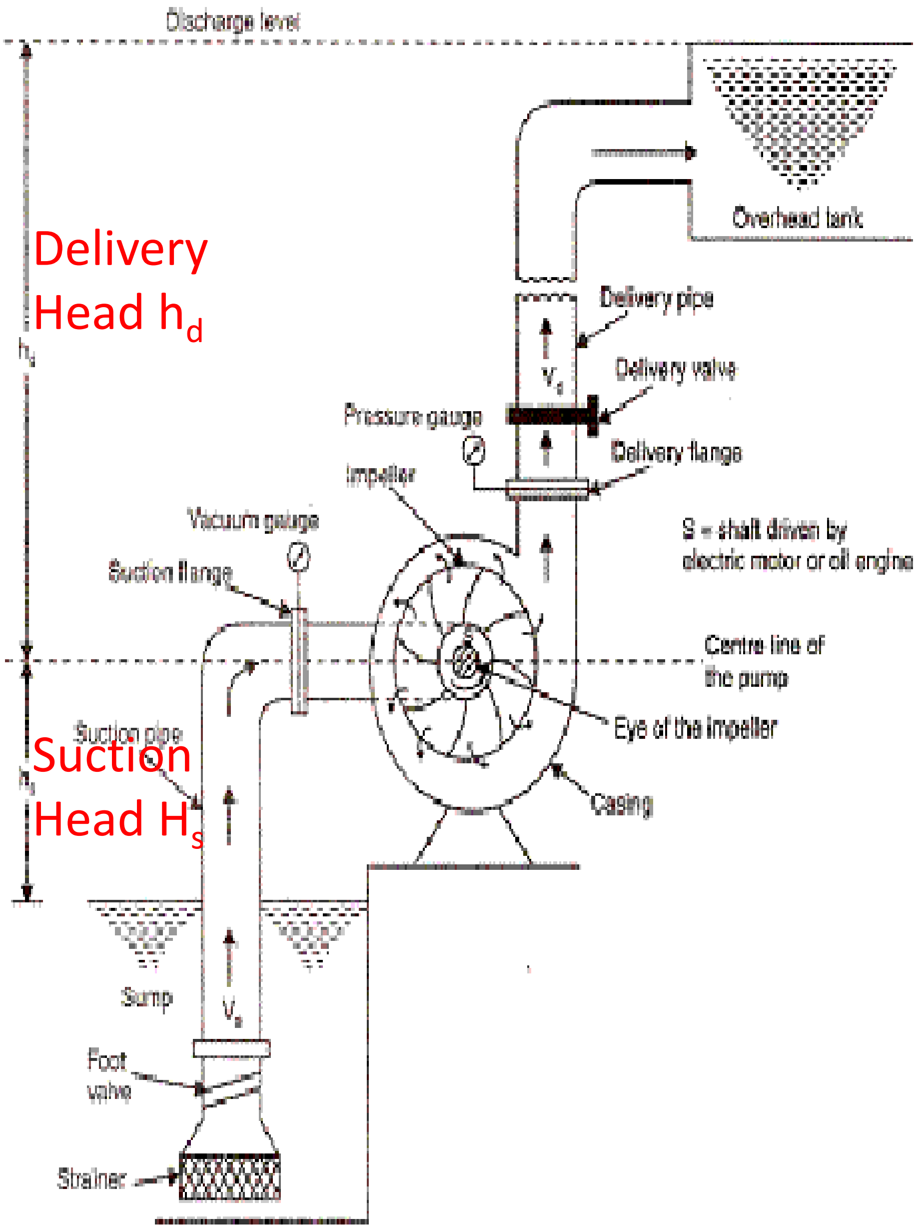
1. Impeller – Rotating part of the centrifugal pump. Series of Backward Curved Vanes

2. Casing- Air tight Chamber with increasing area.
    1. Volute Casing
    2. Vortex Casing
    3. Diffuser Casing

    To reduce the losses in the formation of eddies

3. Suction Pipe with foot value

4. Delivery pipe



Delivery Head H_d= Vertical distance between center line of pump and Water surface in tank in which water is delivered

Suction Head H_s= Vertical distance between center line of pump and Water surface in tank from which water is lifted

Static Head =The sum of suction head and delivery head

Static Head = $H_d + H_s$

Manometric Head H_m= Head against which centrifugal pump has to work (Gross head)

i) $H_m = \frac{V_{w2}u_2}{g} - Loss\ of\ head\ in\ impeller$

ii) $H_m = H_s + H_d + H_{fs} + H_{fd} + \frac{V_d^2}{2g}$

4. Manometric Head (H_m). The manometric head is defined as the head against which a centrifugal pump has to work. It is denoted by 'H_m'. It is given by the following expressions :

(*a*) H_m = Head imparted by the impeller to the water – Loss of head in the pump

$$= \frac{V_{w_2} u_2}{g} - \text{Loss of head in impeller and casing} \quad ...(19.4)$$

$$= \frac{V_{w_2} u_2}{g} \text{ ...if loss of pump is zero} \quad ...(19.5)$$

(*b*) H_m = Total head at outlet of the pump – Total head at the inlet of the pump

$$= \left(\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right) \quad ...(19.6)$$

where $\frac{p_o}{\rho g}$ = Pressure head at outlet of the pump = h_d

(*c*)
$$H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g} \quad ...(19.7)$$

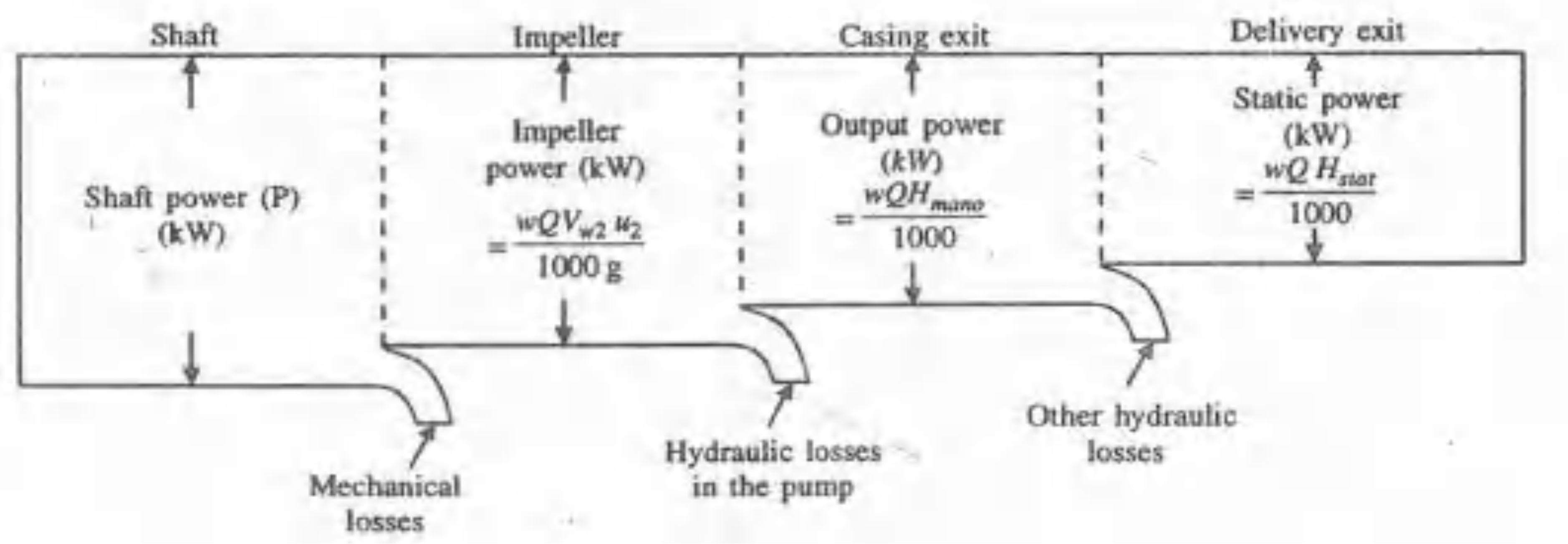
where h_s = Suction head, h_d = Delivery head,
h_{f_s} = Frictional head loss in suction pipe, h_{f_d} = Frictional head loss in delivery pipe, and
V_d = Velocity of water in delivery pipe.

3. Total or gross or effective head

- It is equal to the static head plus all the head losses occurring in flow before, through and after the impeller.

Losses in centrifugal pump

1. Mechanical losses
2. Hydraulic losses
3. Leakage loss



Efficiencies of a Centrifugal Pump.

power is transmitted from the shaft of the electric motor to the shaft of the pump and then to the impeller. From the impeller, the power is given to the water. Thus power is decreasing from the shaft of the pump to the impeller and then to the water.

The following are the important efficiencies of a centrifugal pump:

(a) Manometric efficiency, η_{man}

(b) Mechanical efficiency, η_m and

(c) Overall efficiency, η_0

Manometric Efficiency η_{man}

The ratio of the manometric head to the head imparted by the impeller to the water

$$\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}$$

$$= \frac{H_m}{\left(\frac{V_{w_2} u_2}{g}\right)} = \frac{gH_m}{V_{w_2} u_2}$$

The power at the impeller of the pump is more than the power given to the water at outlet of the pump. The ratio of the power given to water at outlet of the pump to the power available at the impeller, is known as manometric efficiency

$$\text{The power given to water at outlet of the pump} = \frac{WH_m}{1000} \text{ kW}$$

$$\text{The power at the impeller} = \frac{\text{Work done by impeller per second}}{1000} \text{kW}$$

$$= \frac{W}{g} \times \frac{V_{w_2} \times u_2}{1000} \text{kW}$$

$$\eta_{man} = \frac{\frac{W \times H_m}{1000}}{\frac{W}{g} \times \frac{V_{w_2} \times u_2}{1000}} = \frac{g \times H_m}{V_{w_2} \times u_2}.$$

Mechanical Efficiency (η_m),

The ratio of the power available at the impeller to the power at the shaft of the centrifugal pump is known as mechanical efficiency.

$$\eta_m = \frac{\text{Power at the impeller}}{\text{Power at the shaft}}$$

The power at the impeller in kW = work done by Impeller per sec/ 1000

$$= \frac{W}{g} \times \frac{V_{w_2} u_2}{1000}$$

$$\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} u_2}{1000}\right)}{\text{S.P.}}$$

Overall efficiency, η_0

It is defined as ratio of power output of the pump to the power input to the pump. The power output of the pump in kW

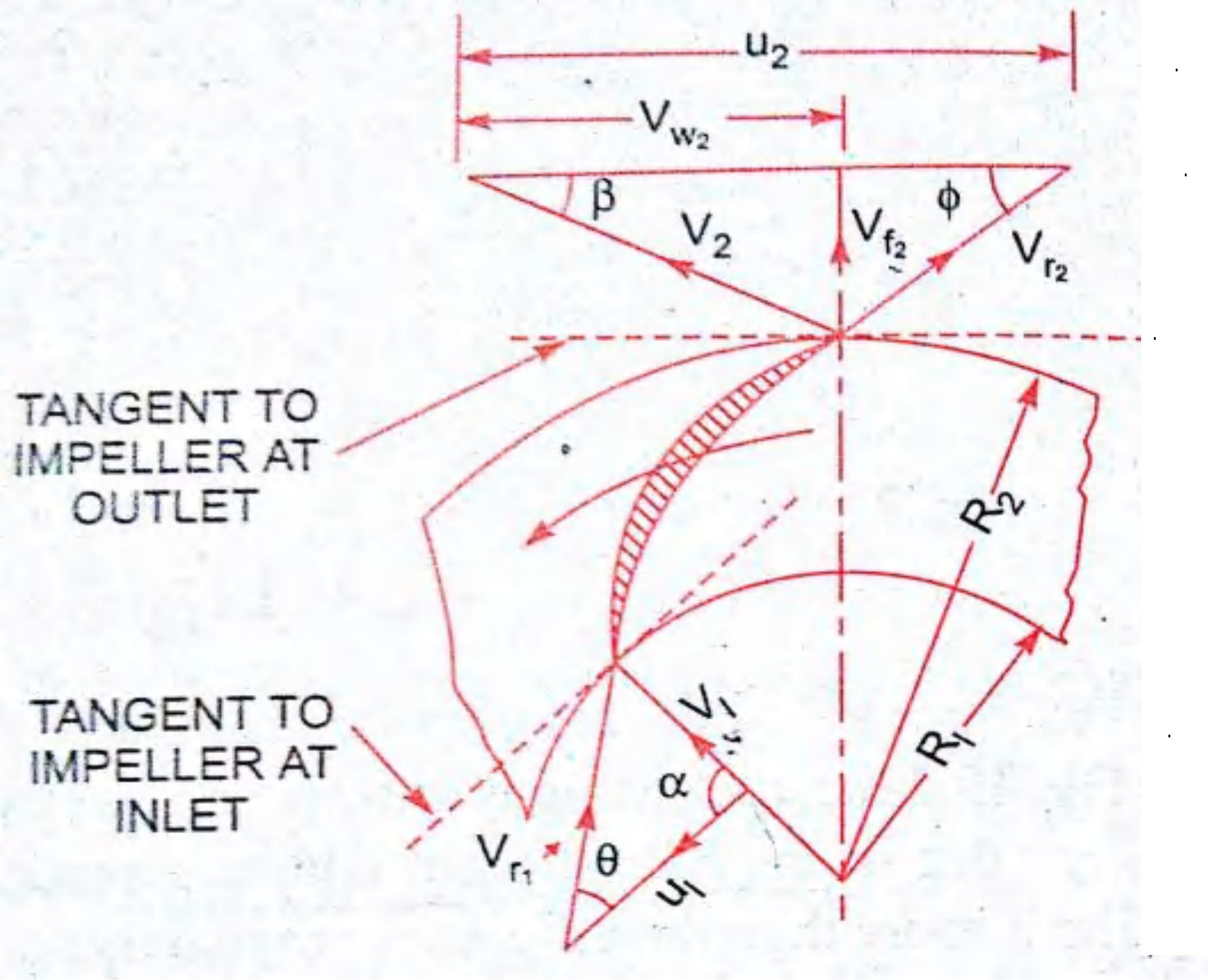
$$= \frac{\text{Weight of water lifted} \times H_m}{1000} = \frac{WH_m}{1000}$$

Power input to the pump = Power supplied by the electric motor

$$\therefore \qquad \eta_o = \frac{\left(\frac{WH_m}{1000}\right)}{\text{S.P.}}$$

Also $\qquad \eta_o = \eta_{man} \times \eta_m.$

Velocity Diagram-Centrifugal Pump



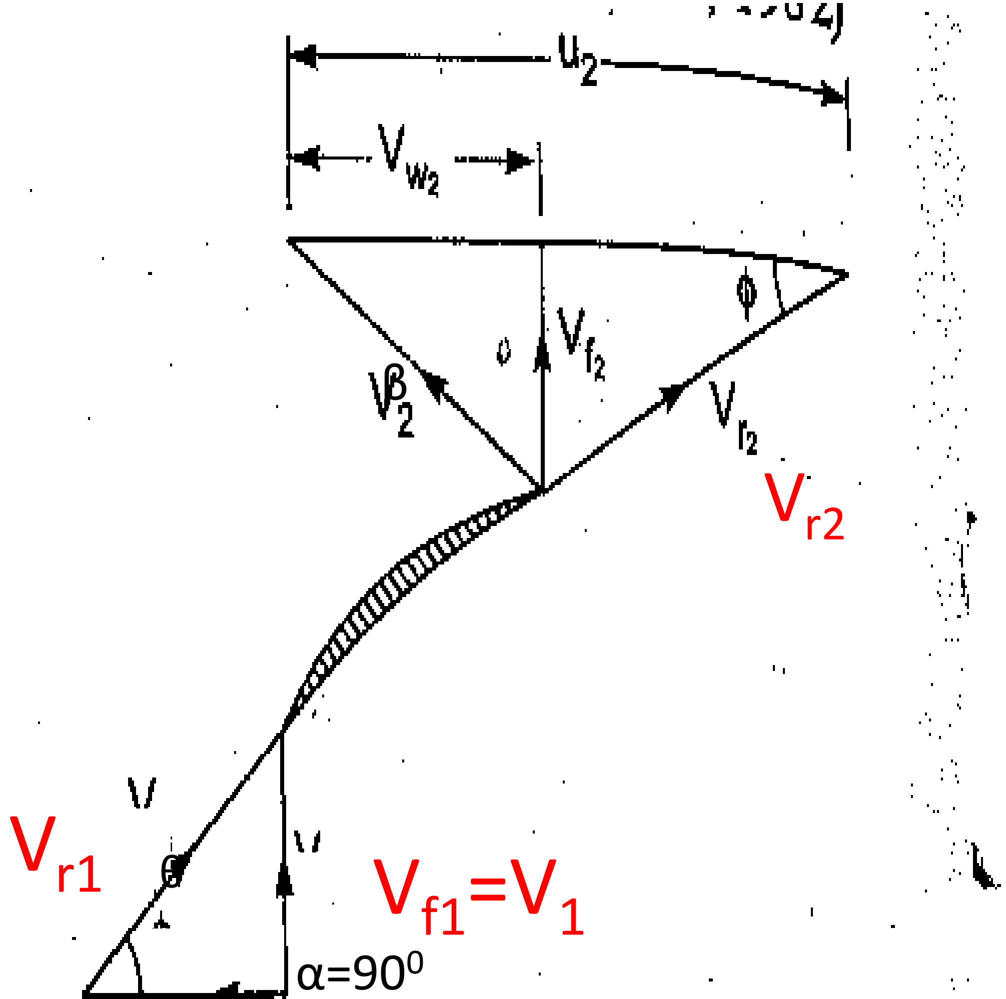


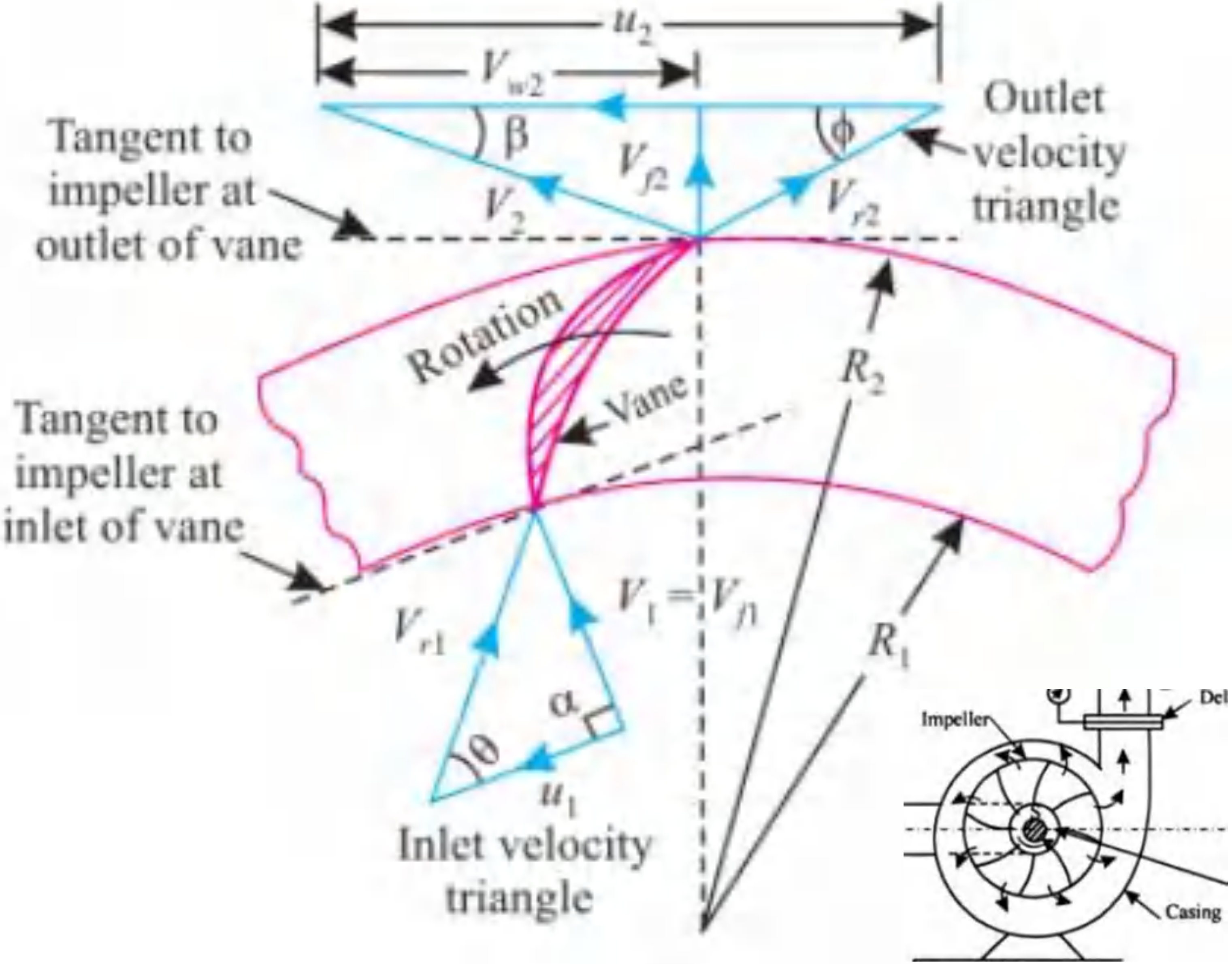
Fig. 19.5

Work done by the impeller on liquid

- The following assumptions are made for derive expression for work done by the impeller of a centrifugal pump on the liquid
  1. Liquid enters the impeller eye in *radial direction*, the whirl component $\mathbf{V_{w1}}$ (of the inlet absolute velocity V_1) is *zero* and the flow component equals the absolute velocity itself *(i.e* $V_{f1} = V_1$) **$\alpha = 90°$**
  2. No energy loss in the impeller due to friction and eddy formation
  3. No loss due to shock at entry
  4. There is uniform velocity distribution in the narrow passages formed between two adjacent vanes ($V_{f1} = V_{f2}$)

Portion of impeller of a pump with the one vane and velocity triangles at the inlet and outlet tips of the vane

Velocity triangles for an impeller vane



- At inlet of the blade at radius R_1 moves at tangential velocity $\mathbf{u_1 = \omega R_1}$
- At outlet of the blade at radius R_2 moves at tangential velocity $\mathbf{u_2 = \omega R_2}$

$\mathbf{V_1}$ - Absolute velocity of water at *inlet*

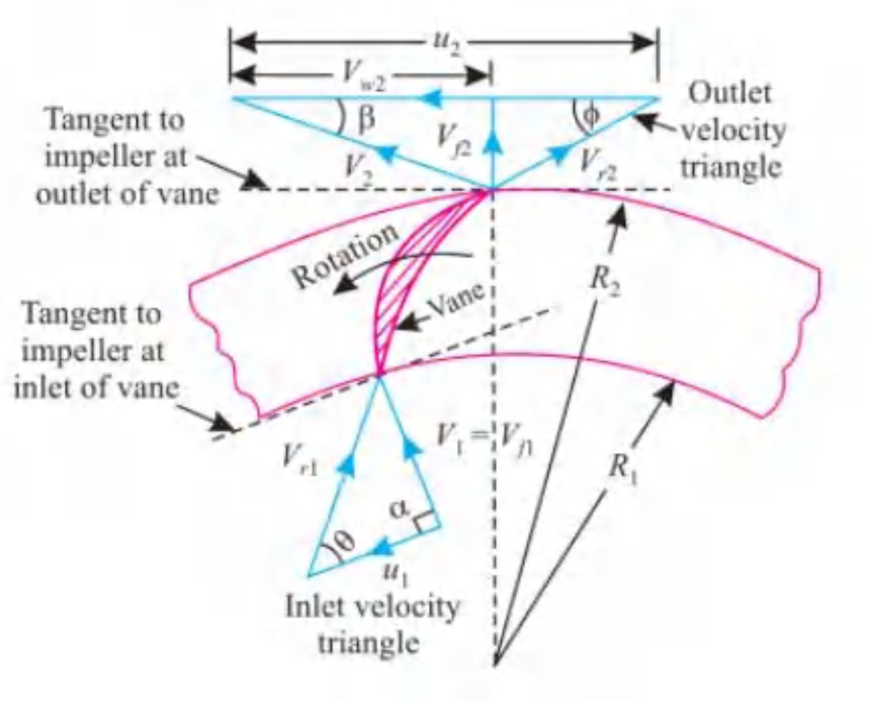
$\mathbf{V_{w1}}$ - Velocity of whirl at inlet

$\mathbf{V_{r1}}$- Relative velocity of liquid at inlet

$\mathbf{V_{f1}}$- Velocity of flow at inlet

α- Angle **made** by absolute velocity

at **inlet** with the direction of motion of vane



θ - Angle made by the relative velocity **(V_{r1})** at inlet with the direction of motion of vane

V_2 , V_{w2},V_{r2}, V_{f2}, β and φ are the corresponding values at *outlet of the blade*

Liquid passing through the impeller, the velocity of whirl changes and there is a change of moment of momentum

Torque on the impeller = Rate of change of moment of momentum

- Moment of momentum at inlet = 0
- Moment of momentum at outlet = $W/g \cdot (V_{w2}R_2)$
- Torque = $W/g \cdot (V_{w2}R_2) - 0$ = **$W/g \cdot (V_{w2}R_2)$**
- **Work done per second = Torque * Angular velocity**

$= W/g \cdot (V_{w2}R_2) * \omega = W/g \cdot (V_{w2}u_2)$

(Note: flow at inlet is radial, $V_{w1} = 0$)

Work done per second per weight of liquid

$= (V_{w2}u_2)/g$

Work done per second = W/g . ($V_{w2}u_2 – V_{w1}u_1$)
(Note: flow at inlet is not radial V_{w1})

Work done per second per weight of liquid = $(V_{w2}u_2 - V_{w1}u_1)/g$

This is known as the Euler Momentum equation for centrifugal pumps and also called as Euler Head H_e (theoretical head)

Weight of liquid W = w x Q

- **Volume of liquid $Q = \pi\ D_1\ B_1 \times V_{f1} = \pi\ D_2\ B_2 \times V_{f2}$**
- **B_1 and B_2 are blade (impeller) widths at inlet and out let**

WORK DONE BY THE CENTRIFUGAL PUMP

. The water enters the impeller radially at inlet for hest efficiency of the pump, which means the absolute velocity of water at inlet makes an angle of 90° with the direction of motion of the impeller at inlet. Hence angle $\alpha = 90°$ and $V_{w_1} = 0$. For drawing the velocity triangles, the same notations are nsed as that for turbines. Fig. 19.3 shows the velocity triangles at the inlet and outlet tips of the vanes fixed to an impeller.

Let N = Speed of the impeller in r.p.m.,

D_1 = Diameter of impeller at inlet,

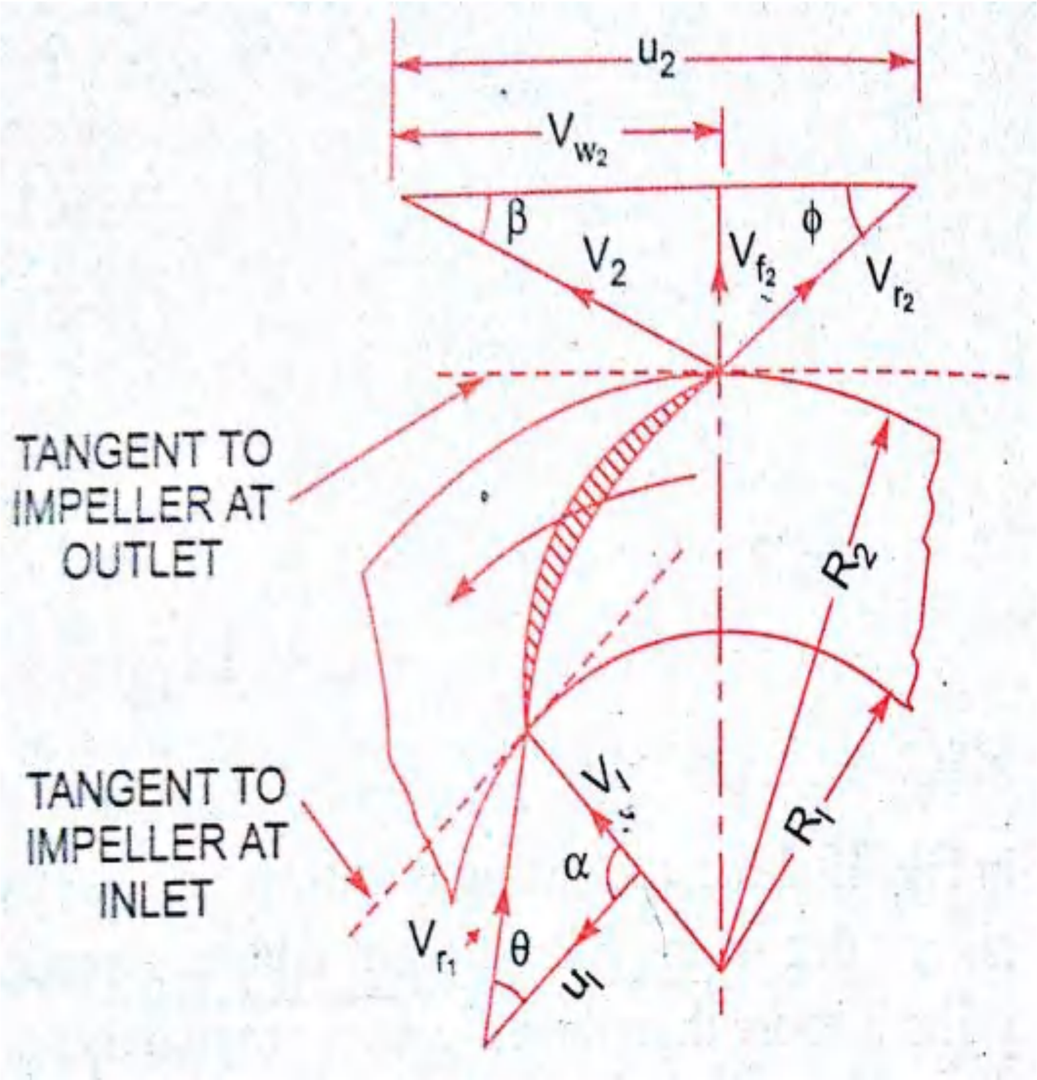
u_1 = Tangential velocity of impeller at inle.,

$$u_1 = \frac{\pi D_1 N}{60}$$

D_2 = Diameter of impeller at outlet,

u_2 = Tangential velocity of impeller at outlet

$$u_2 = \frac{\pi D_2 N}{60}$$



V_1 = Absolute velocity of water at inlet,

V_{r_1} = Relative velocity of water at inlet,

α = Angle made by absolute velocity (V_1) at inlet with the direction of motion of vane,

θ = Angle made by relative velocity (V_{r_1}) at inlet with the direction of motion of vane, and V_2, V_{r_2}, β and ϕ are the corresponding values at outlet.

As the water enters the impeller radially which means the absolute velocity of water at inlet is in the radial direction and hence angle $\alpha = 90°$ and $V_{w_1} = 0$.

A centrifugal pump is the reverse of a radially inward flow reaction turbine. But in case of a radially inward flow reaction turbine, the work done by the water on the runner per second per unit weight of the water striking per second is given by equation (18.19) as

$$= \frac{1}{g}[V_{w_1}u_1 - V_{w_2}u_2]$$

$\therefore$ Work done by the impeller on the water per second per unit weight of water striking per second

$$= -\text{ [Work done in case of turbine]}$$

$$= -\left[\frac{1}{g}\left(V_{w_1}u_1 - V_{w_2}u_2\right)\right] = \frac{1}{g}\left[V_{w_2}u_2 - V_{w_1}u_1\right]$$

$$= \frac{1}{g}V_{w_2}u_2 \qquad (\because \quad V_{w_1} = 0 \text{ here}) \quad ...(19.1)$$

Work done by impeller on water per second

$$= \frac{W}{g} . V_{w_2} u_2 \qquad \text{...(19.2)}$$

where W = Weight of water = $\rho \times g \times Q$

where Q = Volume of water

and Q = Area × Velocity of flow = $\pi D_1 B_1 \times V_{f_1}$

$$= \pi D_2 B_2 \times V_{f_2} \qquad \text{...(19.2A)}$$

where B_1 and B_2 are width of impeller at inlet and outlet and V_{f_1} and V_{f_2} are velocities of flow at inlet and outlet.

Equation (19.1) gives the head imparted to the water by the impeller or energy given by impeller to water per unit weight per second.

$$V_2 = \sqrt{V_{f_2}{}^2 + V_{w_1}{}^2}$$

For Best efficiency,

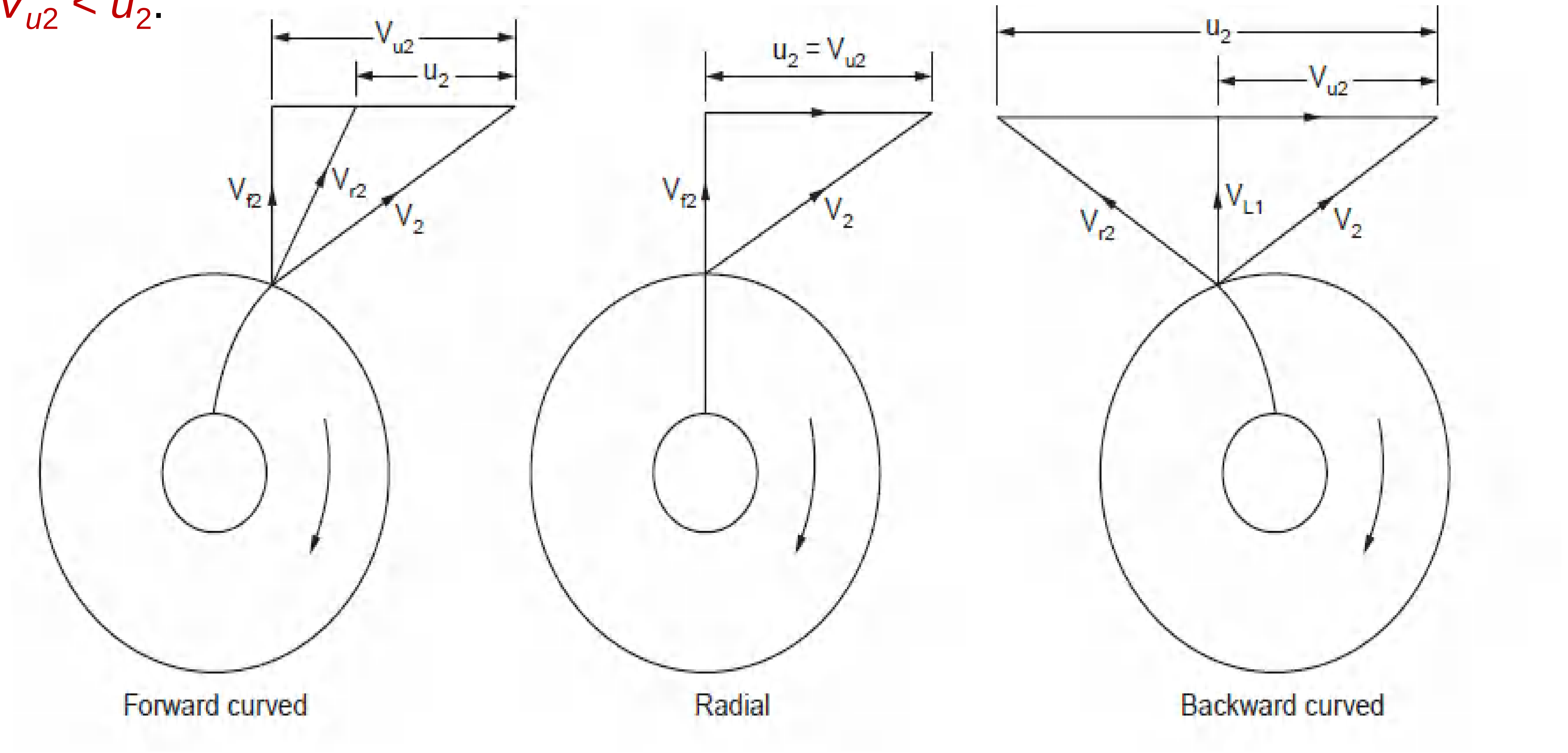
Water enters radially at Inlet

Angle $\alpha = 90^o$

So $V_{w1} = 0$

Therefore $V_{f1} = V_1$

In the case of forward curved blading $V_{u2} > u_2$ and V_2 is larger comparatively. In the case of radial blades $V_{u2} = u_2$. In the case of backward curved blading, $V_{u2} < u_2$.



The rising characteristics of the forward curved blading leads to increase of power input with increase of Q.
The power curve is not self limiting and damage to motor is possible. The forward curved blading is rarely used.

The backward curved blading leads to self limiting power characteristics and reduced losses in the exit kinetic energy.

So the backward curved blading is almost universally used.

The radial blading also leads to rising power characteristics and it is used only in small sizes.

EFFICIENCY

Turbines-

Overall Efficiency= $\frac{S.P}{W.P}$

Water Power (W.P) → (Hydraulic Efficiency) → Runner Power (R.P) → (Mechanical Efficiency) → Shaft Power (S.P)

Pumps-

Water Power measured in terms of Manometric Head. ← (Manometric Efficiency) ← Work done per second by Impeller on water ← (Mechanical Efficiency) ← Shaft Power (S.P)

W.P= $\gamma * Q * H_m$

Where H_m is the Manometric head

Overall Efficiency= $\frac{W.P}{S.P}$

$\gamma = \rho \times g$

1.Tangential or peripheral velocity

$$inlet\ u_1 = \frac{\pi D_1 N}{60}\ \ or\ \ \omega R_1$$

$$outlet\ \ u_2 = \frac{\pi D_2 N}{60}\ \ or\ \ \omega R_1$$

2.Discharge

$$Q = \pi D_1 B_1 V f_1 = \pi D_2 B_2 V f_2$$

W=Unit Weight of water per second

W= $\gamma * \frac{Volume}{time} = \gamma * Q$

3.Workdone by the pump per second

$$Power = \frac{Work\ done}{Time} = \frac{\gamma}{g} * Q * (Vw_2 u_2)$$

$$Workdone\ per\ unit\ weight\ of\ water = \frac{1}{g} * (Vw_2 u_2)$$

EFFICIENCY

1.Manometric efficiency:

$$\eta_{man} = \frac{manometric\ head}{head\ imparted\ by\ impeller} = \frac{\gamma * Q * Hm}{\frac{\gamma}{g} * Q * Vw_2 u_2} = \frac{H_m}{\left(\frac{V_{w_2} u_2}{g}\right)} = \frac{gH_m}{V_{w_2} u_2}$$

2.Mechanical efficiency:

$$\eta_M = \frac{Power\ at\ the\ impeller}{shaft\ Power}$$

$$\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} u_2}{1000}\right)}{S.P.}$$

3.Overall efficiency:

$$\eta_0 = \frac{\left[\frac{WH_m}{1000}\right]}{P}$$

$$\eta_o = \eta_{man} \times \eta_m.$$

4.Specific speed for pump

$$N_s = \frac{N\sqrt{Q}}{H_m^{\frac{3}{4}}}$$

Where W -Weight of Water lifted per second

W= $\gamma * Q$

Shaft Power P in KW

19,2. A centrifugal pump is to discharge 0.118m3/s at a speed of 1450rpm against a head of 25m. The impeller Dia is 250mm. Its width at outlet is 50mm &manometer efficiency is 75%.Find vane angle at the Outer periphery of impeller.

Give Data:

$$Q = 0.118m^3/s$$

$$N = 1200rpm$$

$$H_m = 25m$$

$$D_2 = 0.25m$$

$$B_2 = 0.05, \eta_{man} = 0.75$$

To find:

$$\phi$$

Solution:

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} \quad so \ \ find \ u_2 = \frac{\pi D_2 N}{60}$$

$$\eta_{man} = \frac{gH}{V_{w_2} u_2} \qquad V_{w_2} = \frac{gH}{\eta_{man} u_2}$$

$$Q = \pi D_2 B_2 Vf_2 \qquad Vf_2 = \frac{Q}{\pi D_2 B_2}$$

$$u_2 = 18.98m/s$$

$$V_{w_2} = 17.23m/s \qquad Vf_2 = 3m/s$$

$$\phi = 59.75^0$$

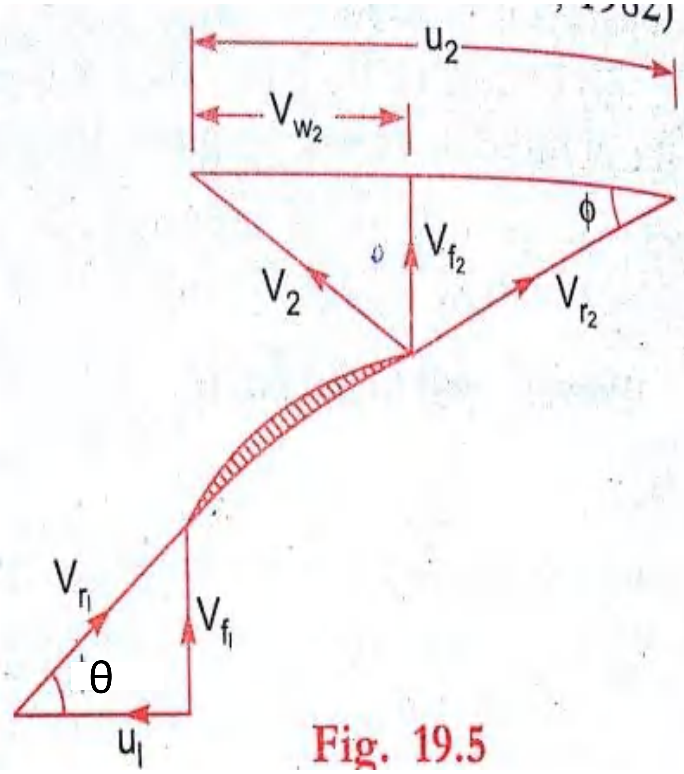


Fig. 19.5

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1450}{60} = 18.98 \text{ m/s}$$

Discharge is given by $Q = \pi D_2 B_2 \times V_{f_2}$

$$\therefore \quad V_{f_2} = \frac{Q}{\pi D_2 B_2} = \frac{0.118}{\pi \times 0.25 \times .05} = 3.0 \text{ m/s.}$$

Using equation (19.8), $\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 25}{V_{w_2} \times 18.98}$

$$\therefore \quad V_{w_2} = \frac{9.81 \times 25}{\eta_{man} \times 18.98} = \frac{9.81 \times 25}{0.75 \times 18.98} = 17.23.$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{\left(u_2 - V_{w_2}\right)} = \frac{3.0}{(18.98 - 17.23)} = 1.7143$$

$\therefore \quad \phi = \tan^{-1} 1.7143 = 59.74°$ or **59° 44′. Ans.**

19.4. A centrifugal pump having outer diameter equal to 2 times the inner dia &running at 1000rpm. Works against a total head of 40m.The velocity of flow through the impeller is constant (at inlet = outlet) & equal to 2.5m/s. The vanes are set back at an angle of 40^0 at outlet and outer dia and width are 500mm,50mm.
Find 1.Vane angle at inlet 2.Workdone by impeller on water per second 3.manomtric efficiency.

Give Data:

$$\phi = 40$$
$$N = 1000 rpm$$
$$D_2 = 2D_1$$
$$D_2 = 0.5m, D_1 = 0.25m$$
$$V_{f_1} = V_{f_2} = 2.5$$
$$B_2 = 0.05$$

To find:

Vane Angle, Work Done, Manometric Efficiency

Solution:

1.Vane angle at inlet:

$$\tan\theta = \frac{V_{f_1}}{u_1} \qquad u_1 = \frac{\pi D_1 N}{60}$$

2.Workdone

$$= \rho Q V_{w_2} u_2 \qquad Q = \pi D_2 B_2 V f_2$$

$$u_2 - V_{w_2} = \frac{V_{f_2}}{\tan\phi} \qquad u_2 = \frac{\pi D_2 N}{60}$$

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

3.Manometric efficiency

$$\eta_{man} = \frac{gH}{V_{w_2} u_2}$$

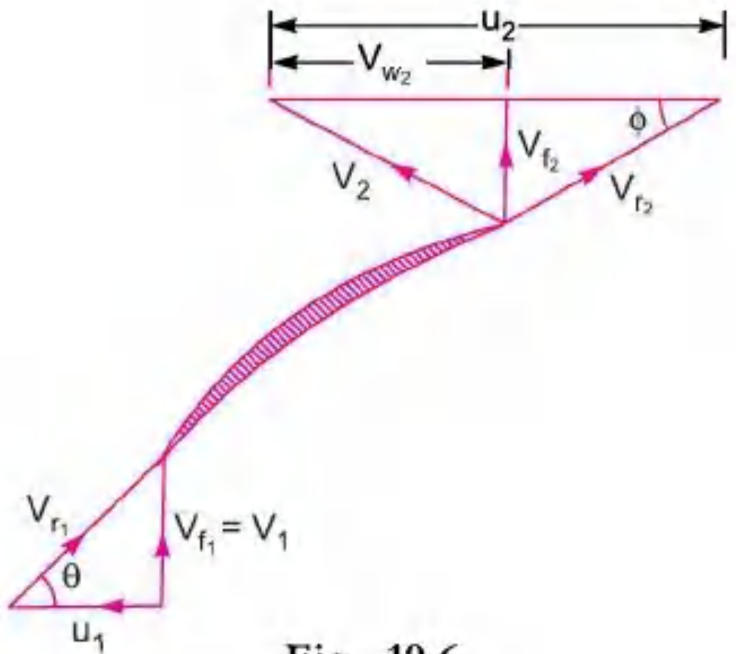


Fig. 19.6

Answers-
u_1=13.09 m/s θ=10.18°
u_2=26.18 m/s Q=0.198 m^3/s
V_{w2}= 23.2 m/s
Work Done= 119 KW
Manometric Efficiency= 64.4%

(*i*) **Vane angle at inlet (θ).**

From inlet velocity triangle $\tan\theta = \dfrac{V_{f_1}}{u_1} = \dfrac{2.5}{13.09} = 0.191$

$\therefore \qquad \theta = \tan^{-1} .191 = 10.81°$ or **10° 48′. Ans.**

(*ii*) **Work done by impeller on water per second** is given by equation (19.2) as

$$= \frac{W}{g} \times V_{w_2} u_2 = \frac{\rho \times g \times Q}{g} \times V_{w_2} \times u_2$$

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times V_{w_2} \times 26.18 \qquad \ldots(i)$$

But from outlet velocity triangle, we have

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{2.5}{(26.18 - V_{w_2})}$$

$$\therefore \qquad 26.18 - V_{w_2} = \frac{2.5}{\tan\phi} = \frac{2.5}{\tan 40°} = 2.979$$

$$\therefore \qquad V_{w_2} = 26.18 - 2.979 = 23.2 \text{ m/s.}$$

Substituting this value of V_{w_2} in equation (*i*), we get the work done by impeller as

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times 23.2 \times 26.18$$

$= \mathbf{119227.9}$ **Nm/s. Ans.**

(*iii*) **Manometric efficiency (η_{man}).** Using equation (19.8), we have

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = 0.646 = \mathbf{64.4\%}. \textbf{ Ans.}$$

19.1. **The internal and external dia** of the impeller of a centrifugal pump are **200mm &400mm** respectively The pump is running **at 1200rpm**. The vane angles of the impeller inlet and outlet are **20^0&30^0** respectively .The water enters the impeller radially & velocity of flow is constant. Find work done by the impeller per unit weight of water.

Give Data:

$$D_1 = 0.2m$$

$$D_2 = 0.4m$$

$$N = 1200rpm$$

$$\theta = 20^0$$

$$\phi = 30^0$$

$$V_{f_1} = V_{f_2}$$

To find:

$$W$$

Solution:

Work done by water per second per unit weight of water

$$= \frac{w}{g} * V_{w_2} * u_2$$

$$= \frac{\frac{w}{g} * V_{w_2} * u_2}{w}$$

$$workdone = \frac{1}{g} * V_{w_2} * u_2$$

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$V_{f_2} = V_{f_1}$

So find V_{f_1}

$$\tan\theta = \frac{V_{f_1}}{u_1}$$

$$u_1 = \frac{\pi D_1 N}{60}$$

$$u_1 = 12.56 m/s$$

$$V_{f_1} = \tan 20 * 12.6$$

$$V_{f_1} = 4.57$$

$$u_2 = \frac{\pi D_2 N}{60}$$

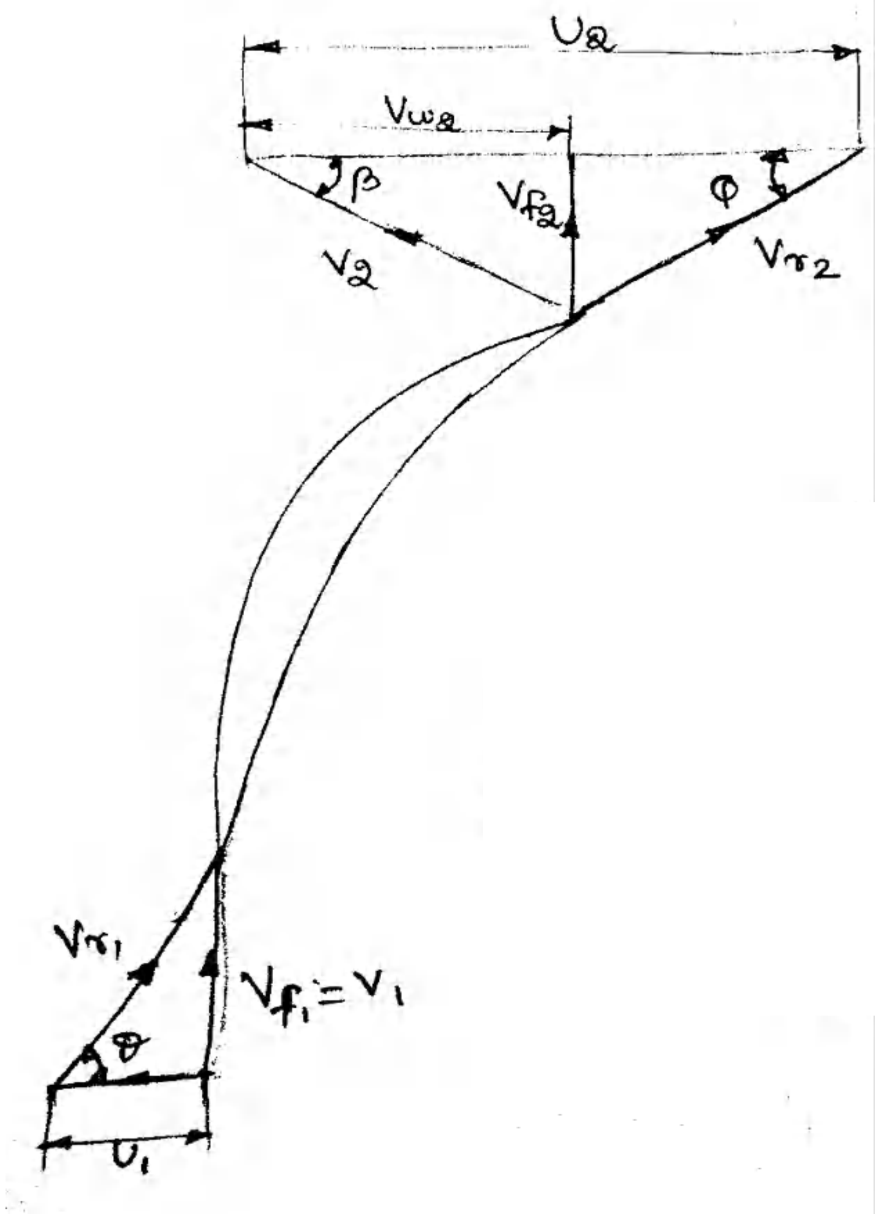
$$u_2 = 25.13 m/s$$

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$u_2 - V_{w_2} = \frac{V_{f_2}}{\tan\phi}$$

$$V_{w_2} = 17.214 m/s$$

$$W = 44.1 Nm/s$$



Problem 19.3 *A centrifugal pump delivers water against a net head of 14.5 metres and a design speed of 1000 r.p.m. The vanes are curved back to an angle of 30° with the periphery. The impeller diameter is 300 mm and outlet width is 50 mm. Determine the discharge of the pump if manometric efficiency is 95%.*

Solution. Given :

Net head, $H_m = 14.5$ m

Speed, $N = 1000$ r.p.m.

Vane angle at outlet, $\phi = 30°$

Impeller diameter means the diameter of the impeller at outlet

∴ Diameter, $D_2 = 300$ mm $= 0.30$ m

Outlet width, $B_2 = 50$ mm $= 0.05$ m

Manometric efficiency, $\eta_{man} = 95\% = 0.95$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1000}{60} = 15.70 \text{ m/s.}$$

Now using equation (19.8), $\eta_{man} = \dfrac{gH_m}{V_{w_2} \times u_2}$

$$\therefore \quad 0.95 = \frac{9.81 \times 14.5}{V_{w_2} \times 15.70}$$

$$\therefore \quad V_{w_2} = \frac{0.95 \times 14.5}{0.95 \times 15.70} = 9.54 \text{ m/s.}$$

Refer to Fig. 19.5. From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} \text{ or } \tan 30° = \frac{V_{f_2}}{(15.70 - 9.54)} = \frac{V_{f_2}}{6.16}$$

$$\therefore \quad V_{f_2} = 6.16 \times \tan 30° = 3.556 \text{ m/s.}$$

$\therefore$ Discharge, $Q = \pi D_2 B_2 \times V_{f_2}$

$$= \pi \times 0.30 \times 0.05 \times 3.556 \text{ m}^3\text{/s} = \mathbf{0.1675 \text{ m}^3\text{/s.}} \textbf{ Ans.}$$

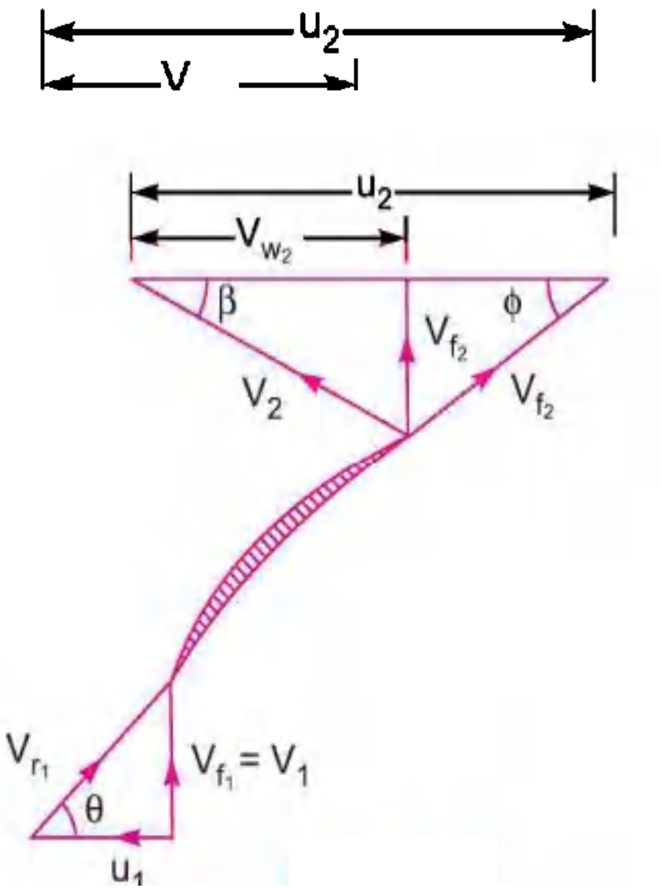
Problem 19.6 *The outer diameter of an impeller of a centrifugal pump is 400 mm and outlet width is 50 mm. The pump is running at 800 r.p.m. and is working against a total head of 15 m. The vanes angle at outlet is 40° and manometric efficiency is 75%. Determine :*

(i) velocity of flow at outlet, *(ii) velocity of water leaving the vane,*

(iii) angle made by the absolute velocity at outlet with the direction of motion at outlet, and

(iv) discharge.

Solution. Given :



Outer diameter, $D_2 = 400 \text{ mm} = 0.4 \text{ m}$

Width at outlet, $B_2 = 50 \text{ mm} = 0.05 \text{ m}$

Speed, $N = 800 \text{ r.p.m.}$

Head, $H_m = 15 \text{ m}$

Vane angle at outlet, $\phi = 40°$

Manometric efficiency, $\eta_{man} = 75\% = 0.75$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 800}{60} = 16.75 \text{ m/s.}$$

Using equation (19.8), $\eta_{man} = \dfrac{gH_m}{V_{w_2}\ u_2}$

$$0.75 = \frac{9.81 \times 15}{V_{w_2} \times 16.75}$$

$$\therefore \qquad V_{w_2} = \frac{9.81 \times 15}{0.75 \times 16.75} = 11.71 \text{ m/s.}$$

From the outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{V_{f_2}}{(16.75 - 11.71)} = \frac{V_{f_2}}{5.04}$$

(*i*) $\therefore$ $V_{f_2} = 5.04 \tan \phi = 5.04 \times \tan 40° =$ **4.23 m/s. Ans.**

(*ii*) **Velocity of water leavlng the vane (V_2).**

$$V_2 = \sqrt{V_{f_2}^2 + V_{w_2}^2} = \sqrt{4.23^2 + 11.71^2}$$

$$= \sqrt{17.89 + 137.12} = \mathbf{12.45\ m/s.} \quad \textbf{Ans.}$$

(*iii*) **Angle made by absolute velocity at outlet (β),**

$$\tan \beta = \frac{V_{f_2}}{V_{w_2}} = \frac{4.23}{11.71} = 0.36$$

$\therefore$ $\beta = \tan^{-1} 0.36 = 19.80°$ or **19° 48′. Ans.**

(*iv*) **Discharge through pump is given by,**

$$Q = \pi D_2 B_2 \times V_{f_2} = \pi \times 0.4 \times 0.05 \times 4.23 = \mathbf{0.265\ m^3/s.} \quad \textbf{Ans.}$$

Problem 19.7 *A centrifugal pump is running at 1000 r.p.m. The outlet vane angle of the impeller is 45° and velocity of flow at outlet is 2.5 m/s. The discharge through the pump is 200 litres/s when the pump is working against a total head of 20 m. If the manometric efficiency of the pump is 80%, determine :*

(*i*) *the diameter* of the impeller, and* (*ii*) *the width of the impeller at outlet.*

Solution. Given :

Speed, $N = 1000$ r.p.m.

Outlet vane angle, $\phi = 45°$

Velocity of flow at outlet, $V_{f_2} = 2.5$ m/s

Discharge, $Q = 200$ litres/s $= 0.2$ m^2/s

Head, $H_m = 20$ m

Manometric efficiency, $\eta_{man} = 80\% = 0.80$

From outlet velocity triangle, we have

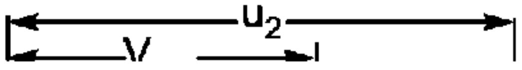


Fig. 19.8

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

or $$u_2 - V_{w_2} = \frac{V_{f_2}}{\tan \phi} = \frac{2.5}{\tan 45} = 2.5$$

$\therefore$ $$V_{w_2} = (u_2 - 2.5) \quad \text{...(}i\text{)}$$

Using equation (19.8), $\eta_{man} = \frac{gH_m}{V_{w_2} u_2}$

$$0.80 = \frac{9.81 \times 20}{V_{w_2} u_2}$$

$$\therefore \quad V_{w_2} u_2 = \frac{9.81 \times 20}{0.80} = 245.25 \quad \text{...}(ii)$$

Substituting the value of V_{w_2} from equation (*i*) in (*ii*), we get

$$(u_2 - 2.5)u_2 = 245.25$$

$$u_2^2 - 2.5u_2 - 245.25 = 0$$

which is a quadratic equation in u_2 and its solution is

$$u_2 = \frac{2.5 \pm \sqrt{(2.5)^2 + 4 \times 245.25}}{2} = \frac{2.5 + \sqrt{6.25 + 981}}{2}$$

$$= \frac{2.5 \pm 31.42}{2} = 16.96 \text{ or } - 14.46$$

$$\therefore \quad u_2 = 16.96 \quad (\because \text{ –ve value is not possible})$$

(*i*) *Diameter of impeller* (D_2).

Using, $$u_2 = \frac{\pi D_2 N}{60}$$

$$\therefore \quad 16.96 = \frac{\pi D_2 N}{60} = \frac{\pi \times D_2 \times 1000}{60}$$

$$\therefore \quad D_2 = \frac{16.96 \times 60}{\pi \times 1000} = 0.324 \text{ m} = \textbf{324 mm. Ans.}$$

(*ii*) *Width of impeller at outlet* (B_2).

Discharge, $$Q = \pi D_2 B_2 V_{f_2}$$

$$0.2 = \pi \times .324 \times B_2 \times 2.5$$

$$\therefore \quad B_2 = \frac{0.2}{\pi \times .324 \times 2.5} = 0.0786 \text{ m} = \textbf{78.6 mm. Ans.}$$

Net Positive Suction Head (NPSH)

NPSH can be defined as two parts:

NPSH Available ($NPSH_A$): The absolute pressure at the suction port of the pump.

AND

NPSH Required ($NPSH_R$): The minimum pressure required at the suction port of the pump to keep the pump from cavitating.

$NPSH_A$ is a function of your system and must be calculated, whereas $NPSH_R$ is a function of the pump and must be provided by the pump manufacturer.

$NPSH_A$ MUST be greater than $NPSH_R$ for the pump system to operate without cavitating.

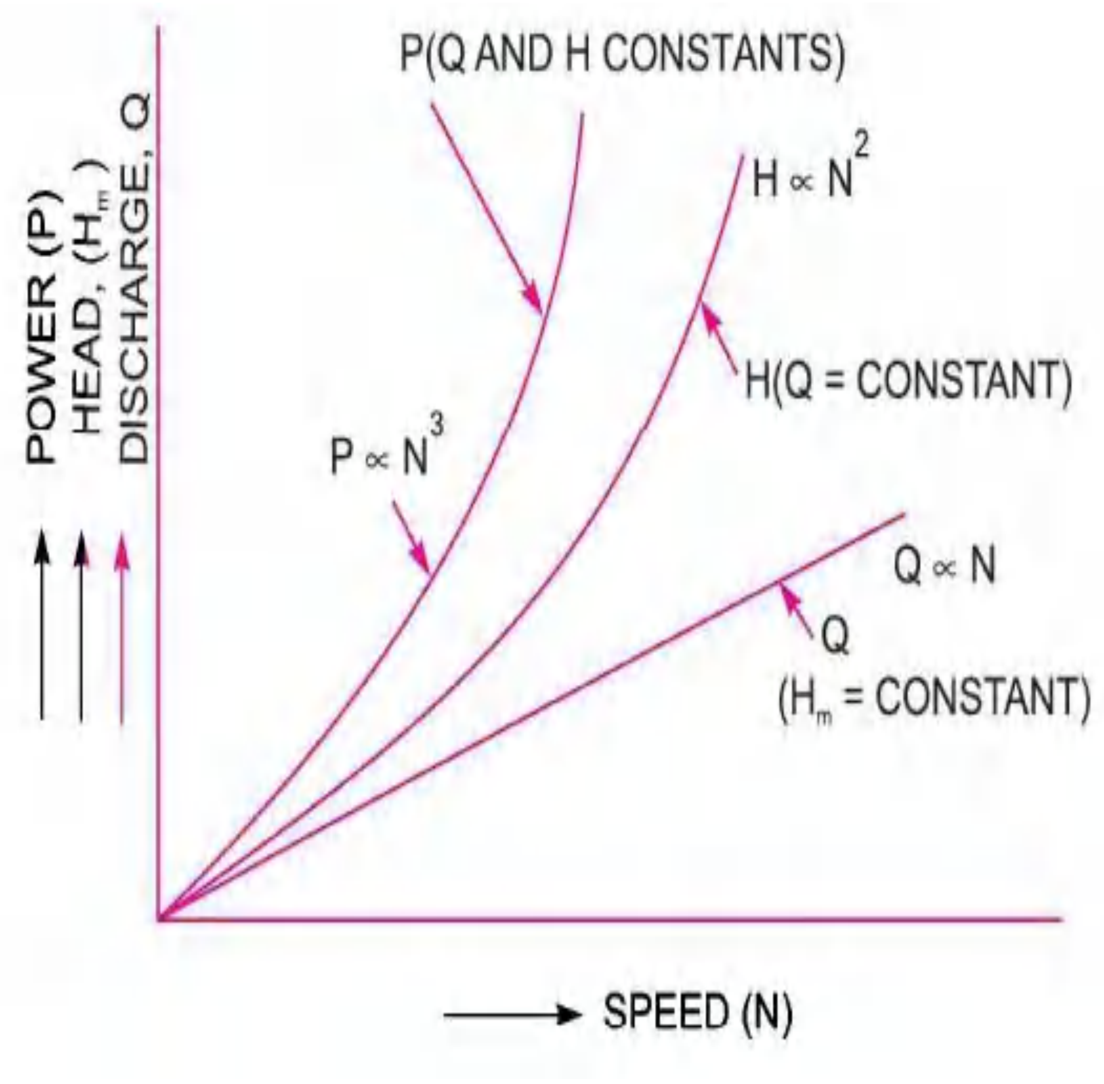
Put another way, you must have more suction side pressure *available* than the pump *requires*.

Main Characteristic Curves

The main characteristic curves with respect to speed, of a centrifugal pump consists of

- variation of head (manometric head, *Hm*),
- power
- and discharge.

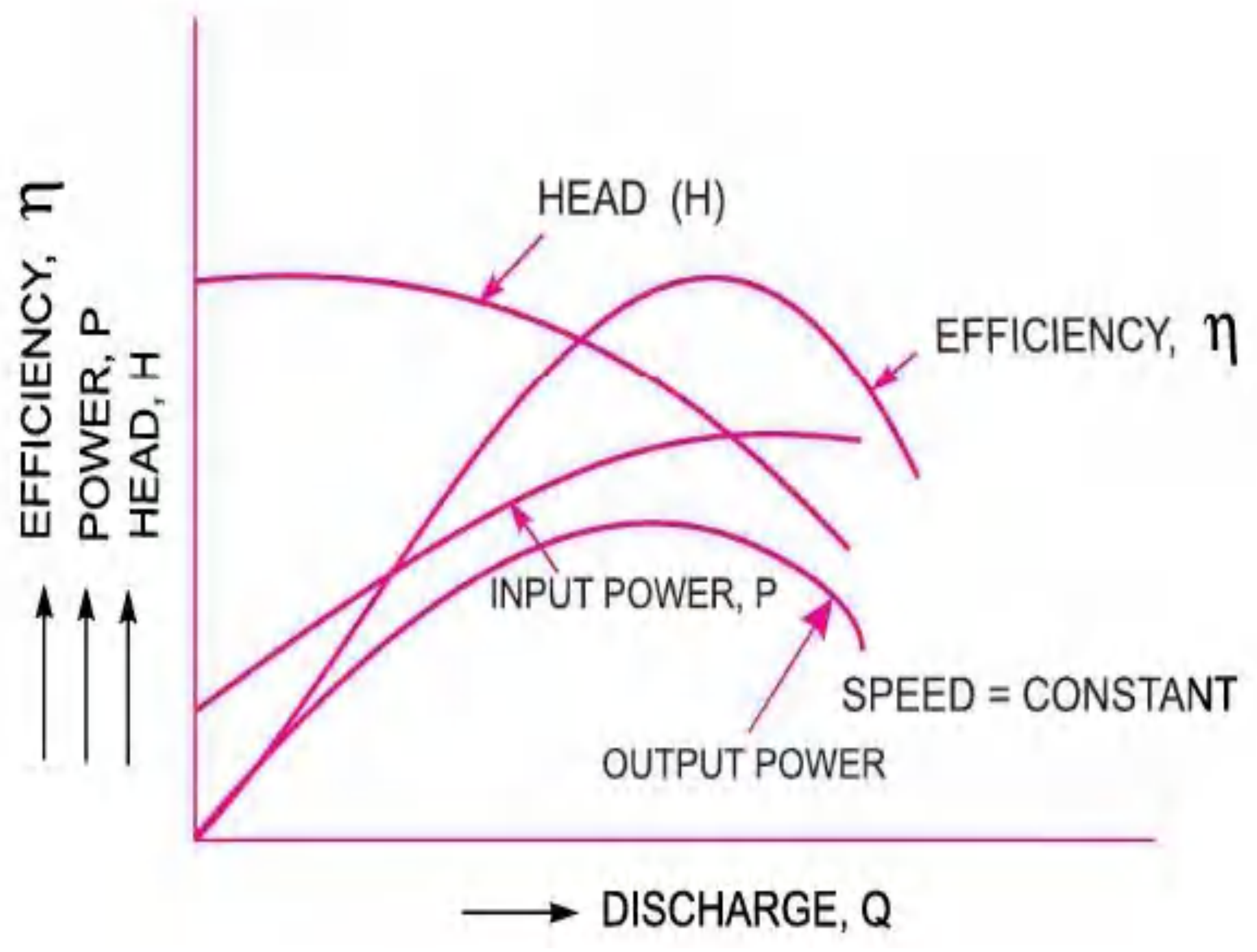
- For plotting curves of manometric head *versus* speed, discharge is kept constant.
- For plotting curves of discharge *versus* speed, manometric head *(Hm)* is kept constant.
- And for plotting curves of power *versus* speed, the manometric head and discharge are kept constant.



Operating Characteristic Curves. If the speed is kept constant, the variation of manometric head, power and efficiency with respect to discharge gives the operating characteristics of the pump. The input power curve for pumps shall not pass through the origin. It will be slightly away from the origin on the *y-axis*, as even at zero discharge some power is needed to overcome mechanical losses.

The head curve will have maximum value of head when discharge is zero.

The output power curve will start from origin as at $Q = 0$, output power (ρQgH) will be zero.



CAVITATION

Pump cavitation occurs when the Suction pressure drops below the vapor pressure of the liquid. Vapor bubbles form at the inlet of the pump and are moved to the discharge of the pump where they collapse, often damage pump.

Cavitation is often characterized by:

- Loud noise often described as a grinding or “marbles” in the pump
- Loss of capacity (bubbles are now taking up space where liquid should be)
- Pitting damage to parts as material is removed by the collapsing bubbles

MULTI STAGE PUMP

Pump Capacity can be increased by
1.) Diameter of impeller D or
2.) Speed of the impeller N or
3.) Both the cases

Classification Based on Head

- Low head – pump can able to lift up to 15m
- Medium Head- pump can able to lift from 15m to 40m
- High head- above 40m

So, IF we want

- to deliver the water to a greater head (example – In Oil Pipe Lines, boiler feed pump)
  – Pumps connected in series
- to deliver more discharge of water (example – Pumping stations to discharge for a city)-
  Pumps connected in parallel

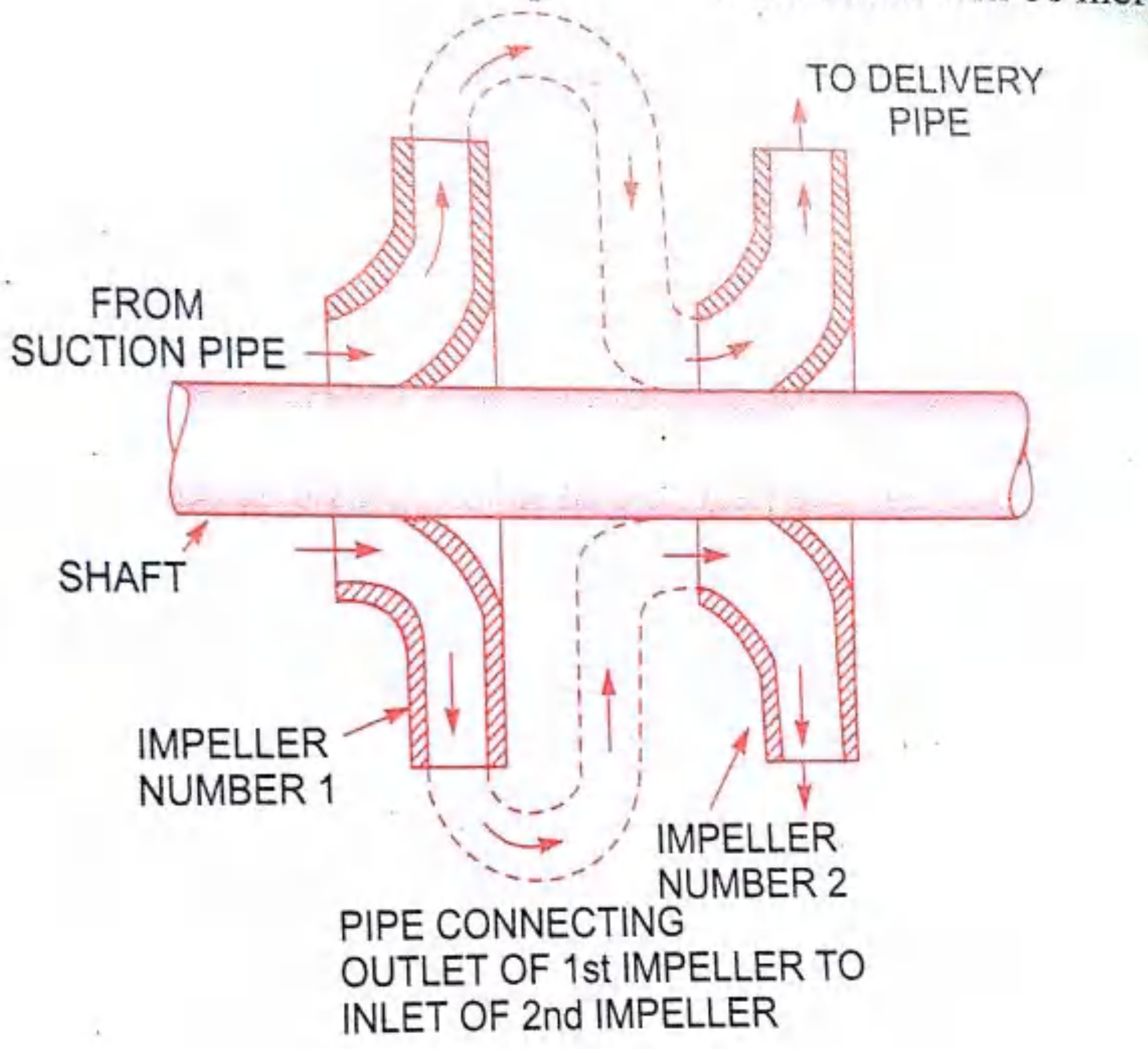
Multi Stage Pump- Pumps are connected Series

For Series Connection,

Conditions-
Discharge Q is same in all pumps

But the pressure head is increasing from one impeller to another impeller.

So , Total Head = $n*H_m$



Multi Stage Pump- Pumps are connected Parallel

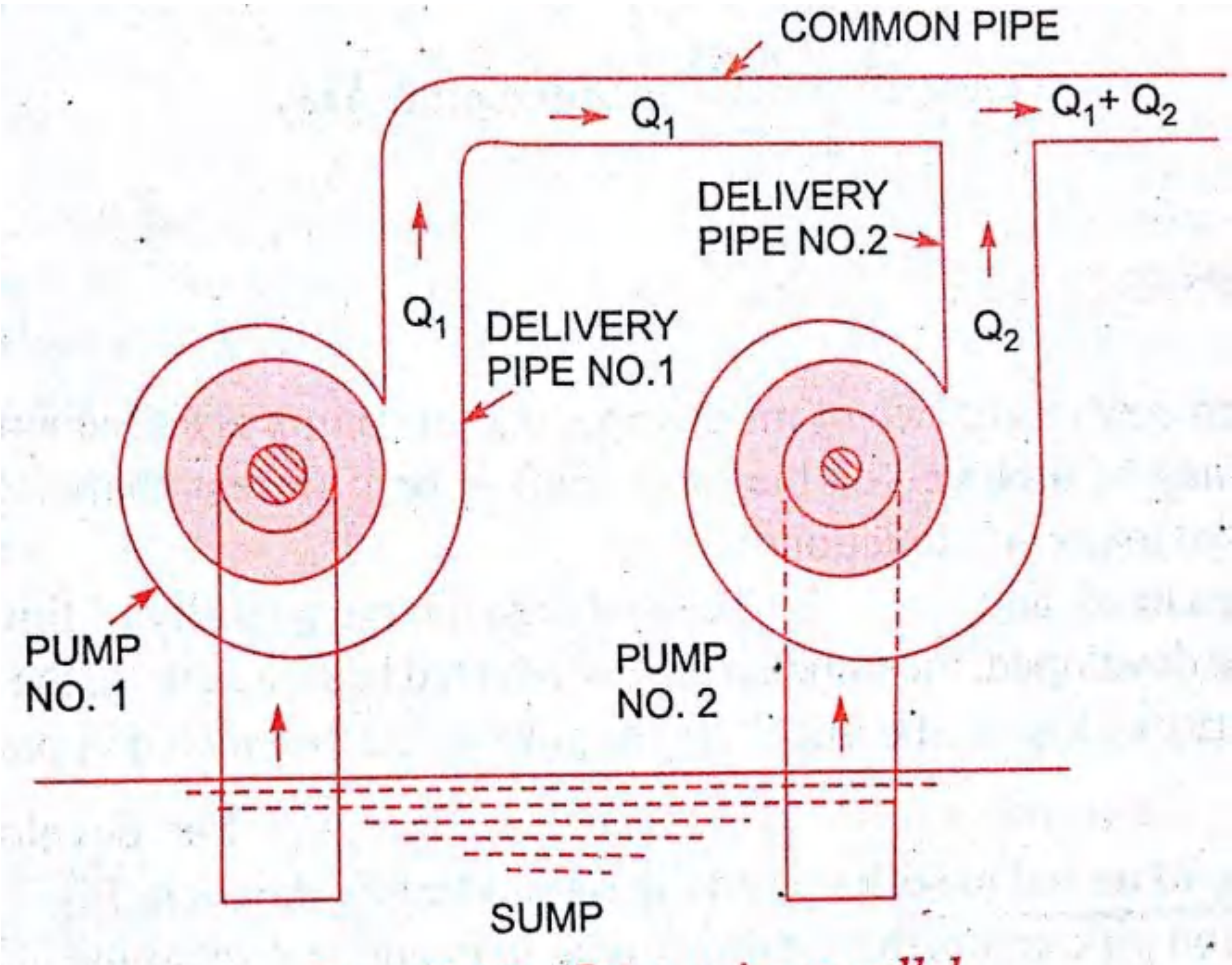


Fig. 19.13 *Pumps in parallel.*

Head Imparted By both the pump will same.

IF n number of pumps are connected in parallel.

The Discharge=

$Q = Q_1 + Q_2 + Q_3 \ldots\ldots\ldots + Q_n$

If same pumps are installed then

Total Discharge multi stage pump = n*Q

1. A **three stage** centrifugal pump has impellers 400 mm in diameter and 20 mm width at outlet. The vanes are curved backward at the outlet at 45^0 and the circumferential area at the outlet is reduced by 10%. The manometric efficiency is 90% and overall efficiency is 80%. Determine the head generated by the three stage pump when running at 1000 rpm delivering 50 litres per second. What is the shaft power? Anna University Question – 2016

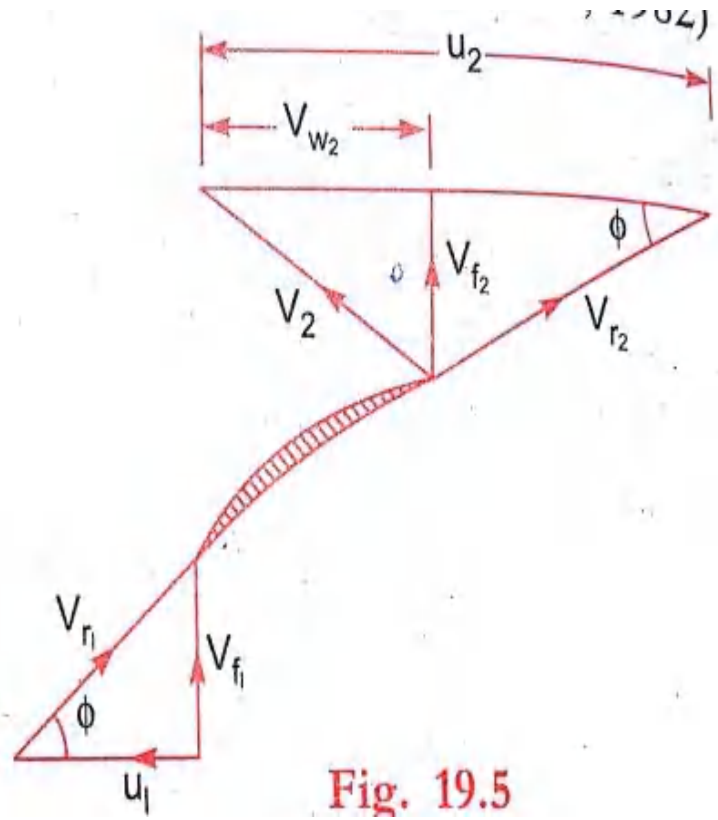


Fig. 19.5

Given Data :

No of stages = 3

D2 = 0.4m

B2 = 0.02m

$\phi = 45^0$

Area at outlet = Reduces by 10%

= $0.9 * \pi * D_2 * B_2$

N = 1000 rpm

Q = 0.05 m^3/s

Manometric Efficiency = 90%

Overall Efficiency = 80%.

To find-

1. Head Imparted By three stage pump = n* H
2. Shaft Power for 3 stage pump.

Q= $(0.9 * \pi * D_2 * B_2) * Vf2$ V_{f2}= 2.21 m/s

$$u_2 = \frac{\pi D_2 N}{60}$$ u_2= 20.94 m/s

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$ V_{w2}= 18.73 m/s

Manometric efficiency= 0.9

$$\eta_{man} = \frac{gH}{V_{w_2} u_2}$$ H= 35.98m

Total Head = 36.38 * 3 = 107.94 m

Shaft Power = (ρ*g*Q*Total Head)/0.8 = 66KW

Minimum speed for starting a centrifugal pump

For Minimum speed

The condition is

$$\frac{u_2^{\,2}}{2g} - \frac{u_1^{\,2}}{2g} = H_m$$

$$N = \frac{120 * \eta_{mean} * V_{w_2} * D_2}{\pi\left(D_2^{\,2} - D_1^{\,2}\right)}$$

2. The diameters of a centrifugal pump at inlet & outlet are 30cm &60cm respectively. Find the minimum starting speed of the pump if it works against a head of 30m.

Give Data:

$$H_m = 30m$$

$$D_1 = 0.3m$$

$$D_2 = 0.6m$$

To find:

Minimum speed

Solution:

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m$$

$$u_1 = \frac{\pi D_1 N}{60}$$

$$u_1 = 0.0157 * N$$

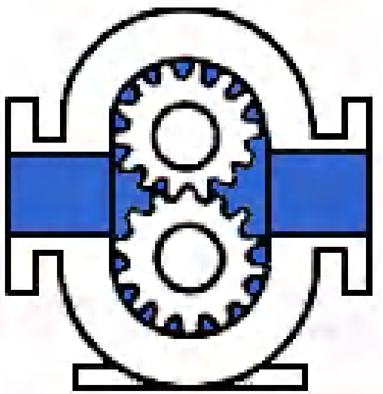
$$u_2 = \frac{\pi D_2 N}{60}$$

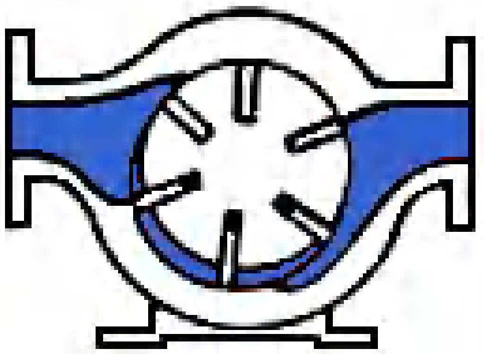
$$u_2 = 0.03141 * N$$

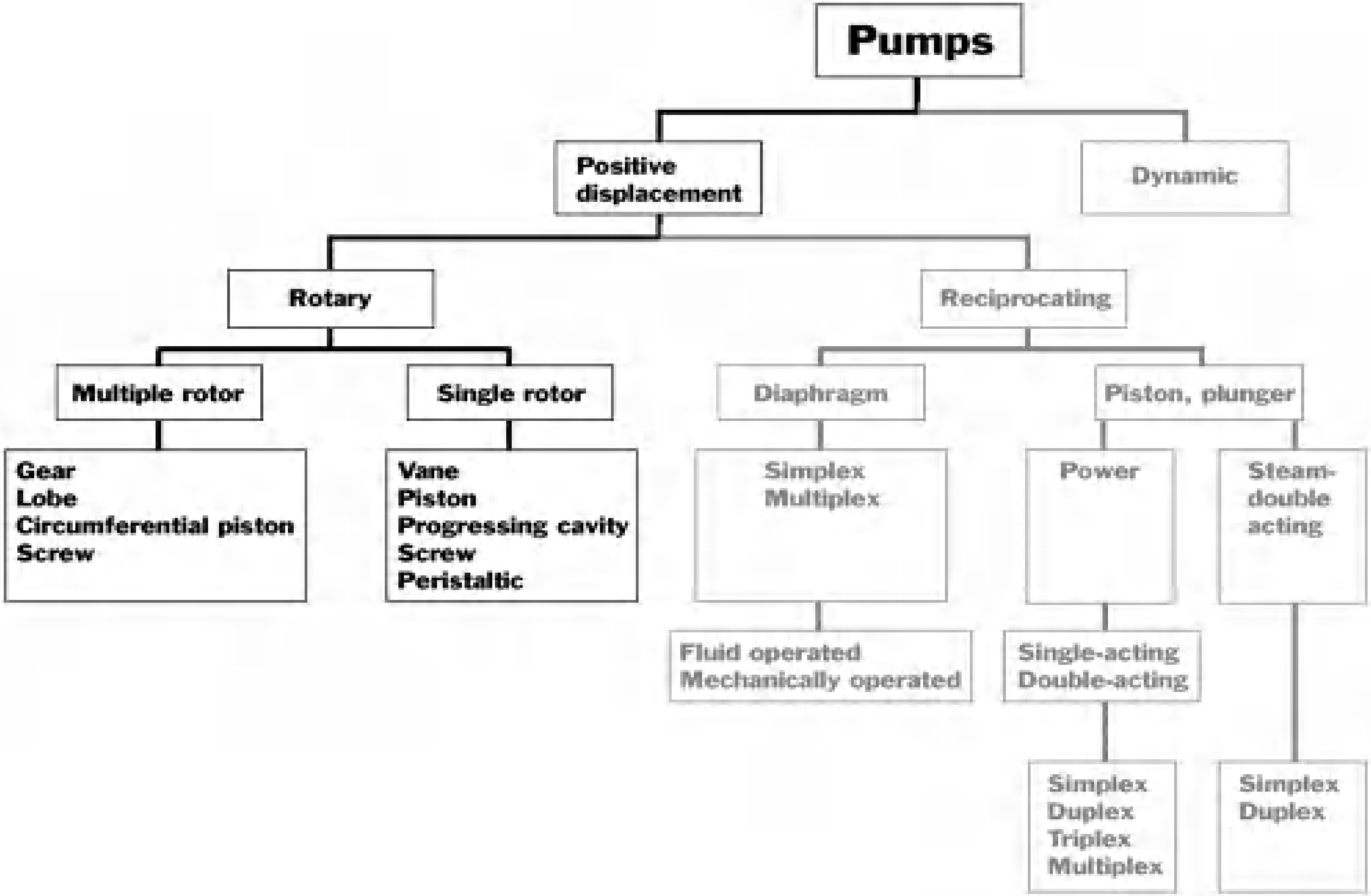
$$\frac{(0.03141 * N)^2}{2g} - \frac{(0.0157 * N)^2}{2g} = 30$$

$$N = 891.8 rpm$$

POSITIVE DISPLACEMENT PUMPS- ROTARY
3 types

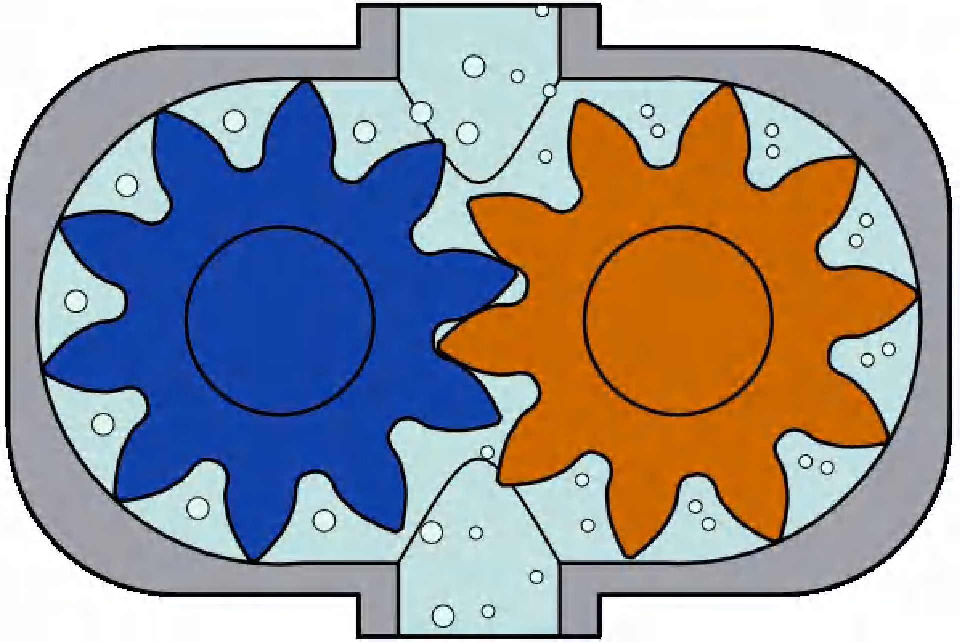


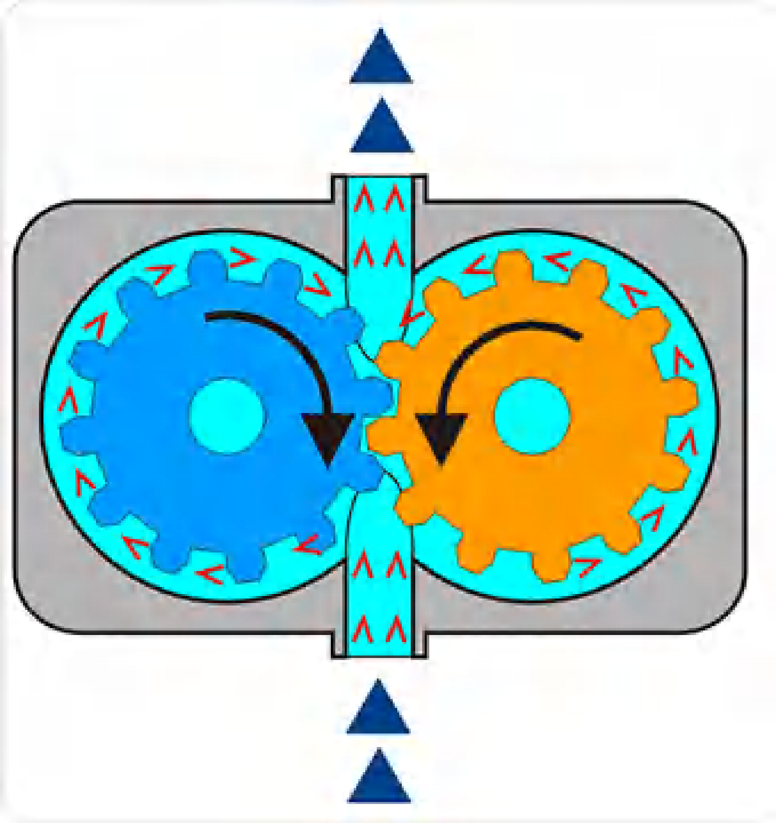


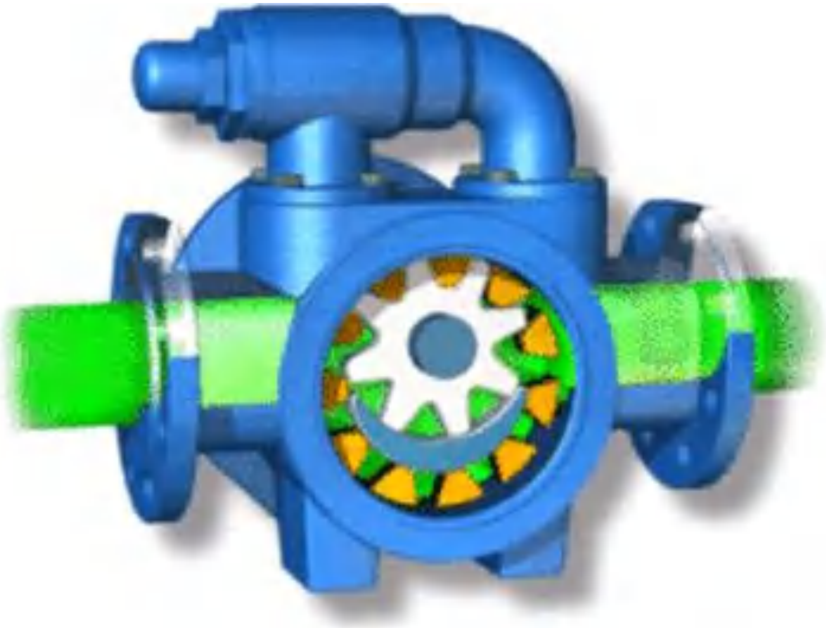

Pumps
Positive displacement
Dynamic
Rotary
Reciprocating
Multiple rotor
Single rotor
Diaphragm
Piston, plunger
Gear
Lobe
Circumferential piston
Screw
Vane
Piston
Progressing cavity
Screw
Peristaltic
Simplex
Multiplex
Power
Steam-double acting
Fluid operated
Mechanically operated
Single-acting
Double-acting
Simplex
Duplex
Triplex
Multiplex
Simplex
Duplex

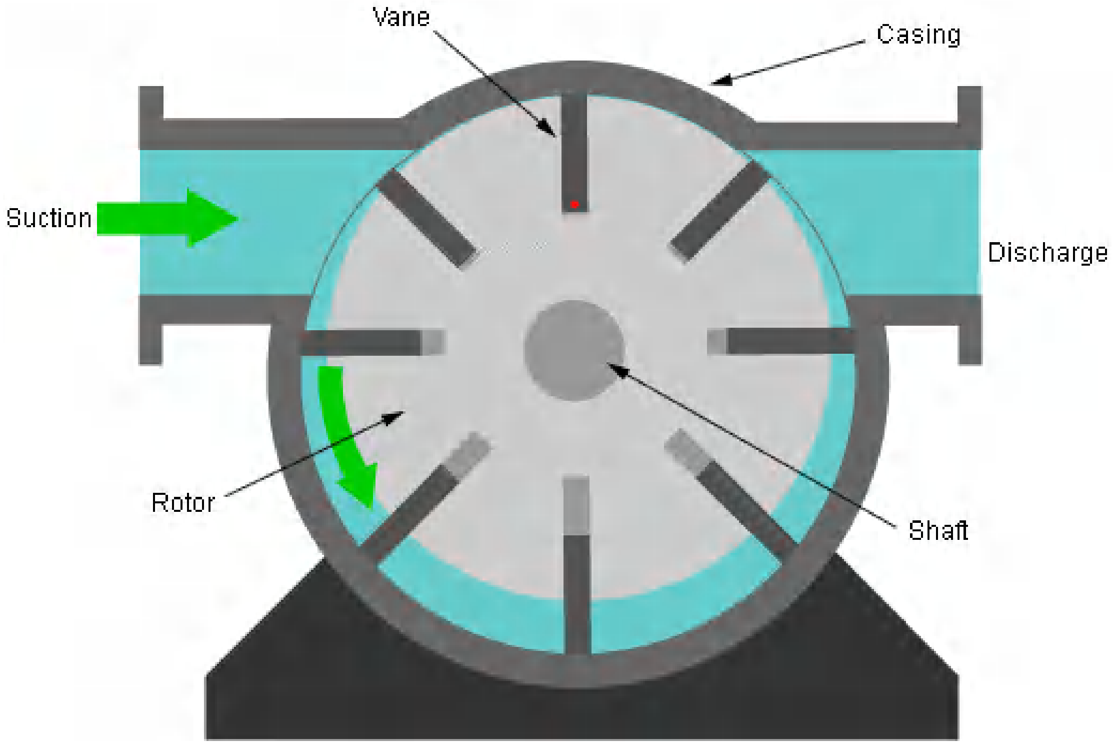
- ROTARY PUMPS

1. EXTERNAL GEAR PUMP
2. INTERNAL GEAR PUMP
3. TWO OR THREE LOBE PUMP
4. VANE PUMP
5. SCREW PUMP








Vane
Casing
Suction
Discharge
Rotor
Shaft

Main parts of Reciprocating Pump

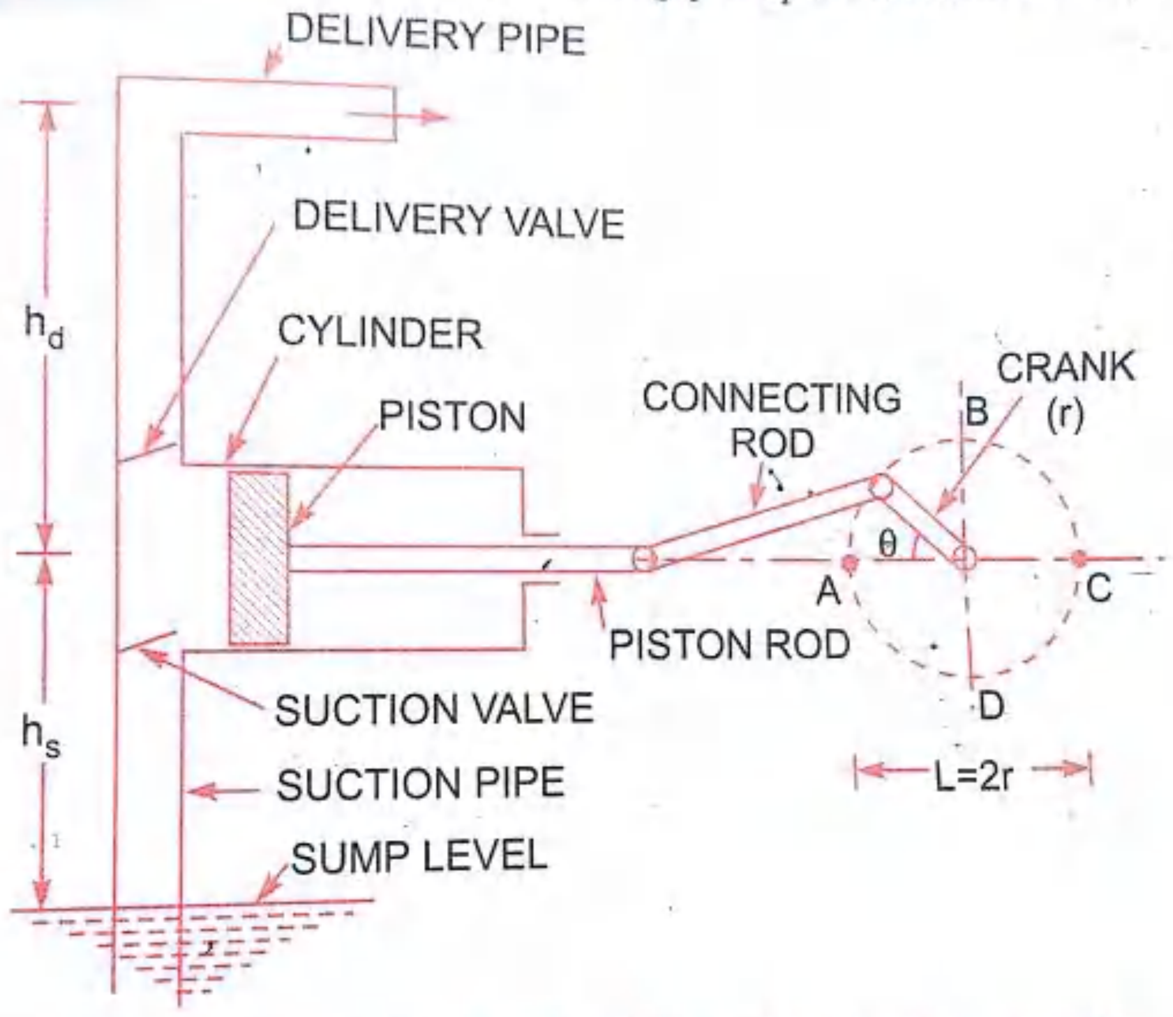
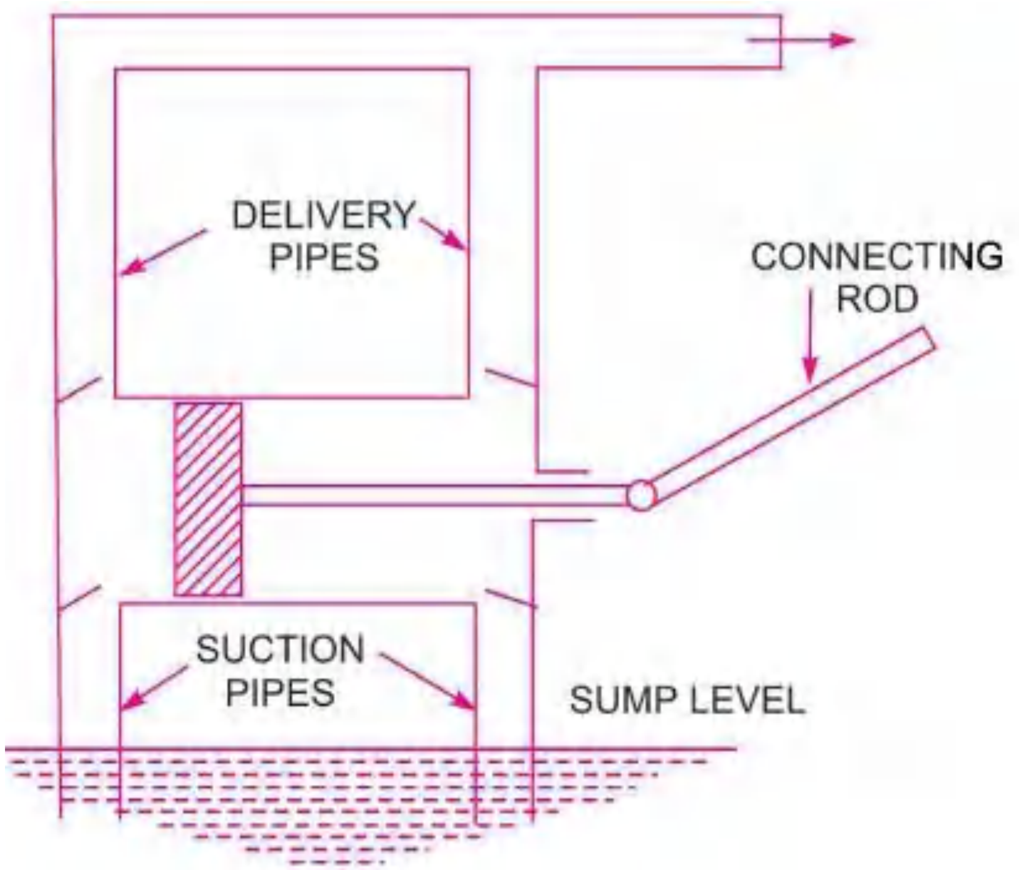


Fig. 20.1 *Main parts of a reciprocating pump.*

- A cylinder with piston
- Crank
- Connecting rod
- Suction Pipe
- Delivery Pipe
- Suction Valve and
- Delivery Valve

Single acting Reciprocating Pump



Double acting Reciprocating Pump

Discharge, Weight of water delivered and Power required to drive a reciprocating pump

1. Discharge

- Discharge of the pump = Discharge in one revolution x No of revolution
- Discharge of pump = Volume / Time

$Q=(A*L*N/60)$ where N is number of revolution per minute.

Where A- Cross sectional Area of piston A or cylinder and

L – Length of Stroke = 2* radius of crank.

2. Weight of water delivered per second:

$$= \frac{Weight\ of\ water\ delivered}{second} = \frac{Wt\ density * Volume}{Time} = \gamma * Q = \rho * g * Q$$

3. Power = Work done / time

Work Done per second = Weight of water delivered per second* Height of water lifted

$$= (\rho * g * Q) * (h_s + h_d)$$

Double Acting Reciprocating pump

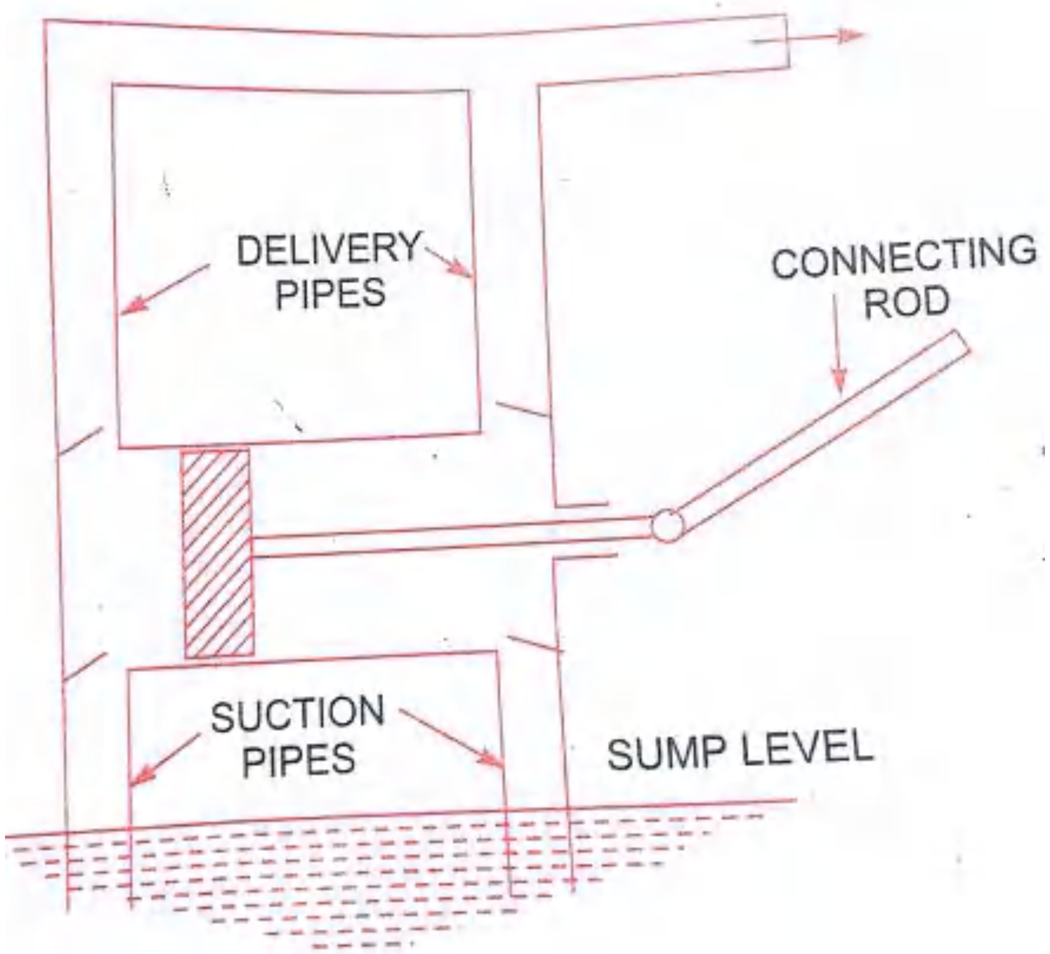


Fig. 20.2

Discharge for a double acting reciprocating pump

= 2 * Discharge for a single acting Pump
= 2* (ALN/60)

Similarly Power required to drive a pump

Power =2* (ρ*g*Q) *(h_s+h_d)

Slip of a Reciprocating Pump

- Slip of a pump $= Q_{th} - Q_{act}$
- % of slip $= \frac{Q_{th} - Q_{act}}{Q_{th}} = 1 - \frac{Q_{act}}{Q_{th}} = 1 - \mathrm{Cd}$

Negative Slip

Slip is defined as the difference between theoretical discharge and actual discharge. When actual discharge is more than theoretical discharge , then slip will be negative. In that case it is called as negative slip.

When negative slip will occur?

When the suction valve remains open during the delivery stroke of the piston and some quantity of water goes directly from the suction pipe to the delivery pipe, which leads to, actual discharge more than the theoretical discharge. Suction pipe is long and delivery pipe is short

Indicator Diagram

- The indicator diagram for a reciprocating pump is defined as the graph between the pressure head in cylinder and the distance travelled by the piston from inner dead center for one complete revolution of the crank.
- It defines the work done by the reciprocating pump during one complete cycle.
- Pressure is plotted on vertical ordinate while stroke length is plotted on horizontal abscissa as shown in Figure below

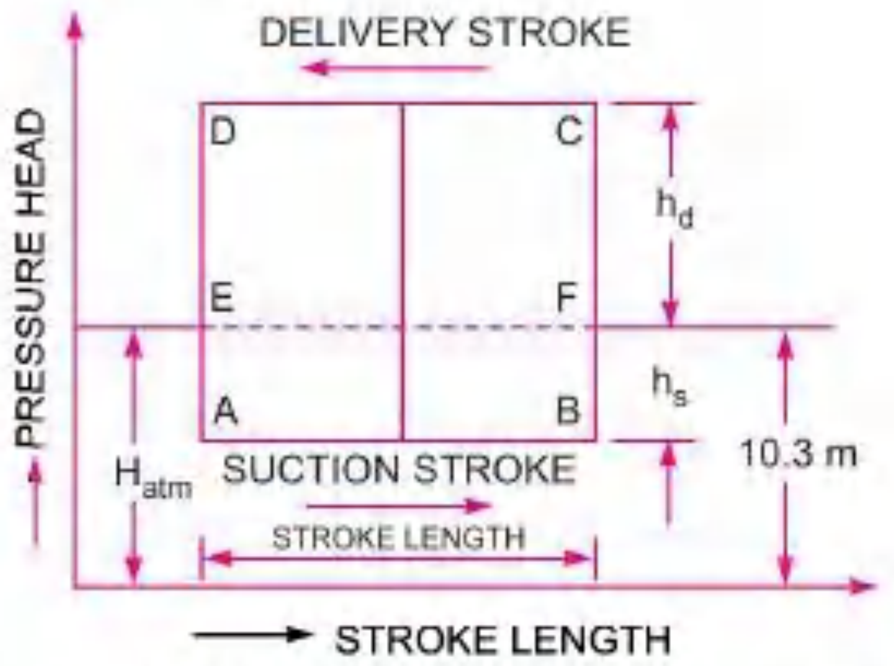


Fig. 20.4 *Ideal indicator diagram.*

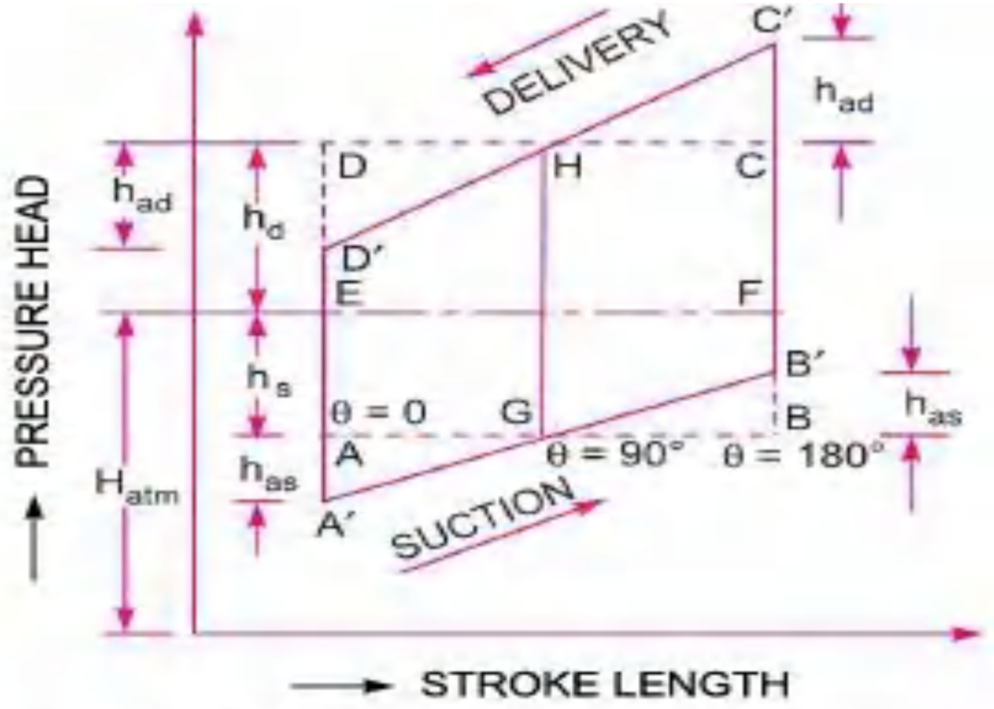


Fig. 20.5 *Effect of acceleration on indicator diagram.*

Variation of acceleration in Suction and Delivery pipe due to acceleration in piston

- In Suction Pipe $h_{as} = \frac{l_s}{g} X \frac{A}{a_s} \omega^2 r \cos\theta$

- In delivery pipe $h_{ad} = \frac{l_d}{g} X \frac{A}{a_d} \omega^2 r \cos\theta$

Where, l_s and l_d = Length of suction and delivery pipe

Angular Speed $\omega = \frac{2\pi N}{60}$

Crank Radius r= L /2, L is Stroke length, A = Cylinder (Bore) area

1. When $\theta = 0°$, $h_a = \frac{l}{g} \times \frac{A}{a} \omega^2 r$ as $\cos 0° = 1$

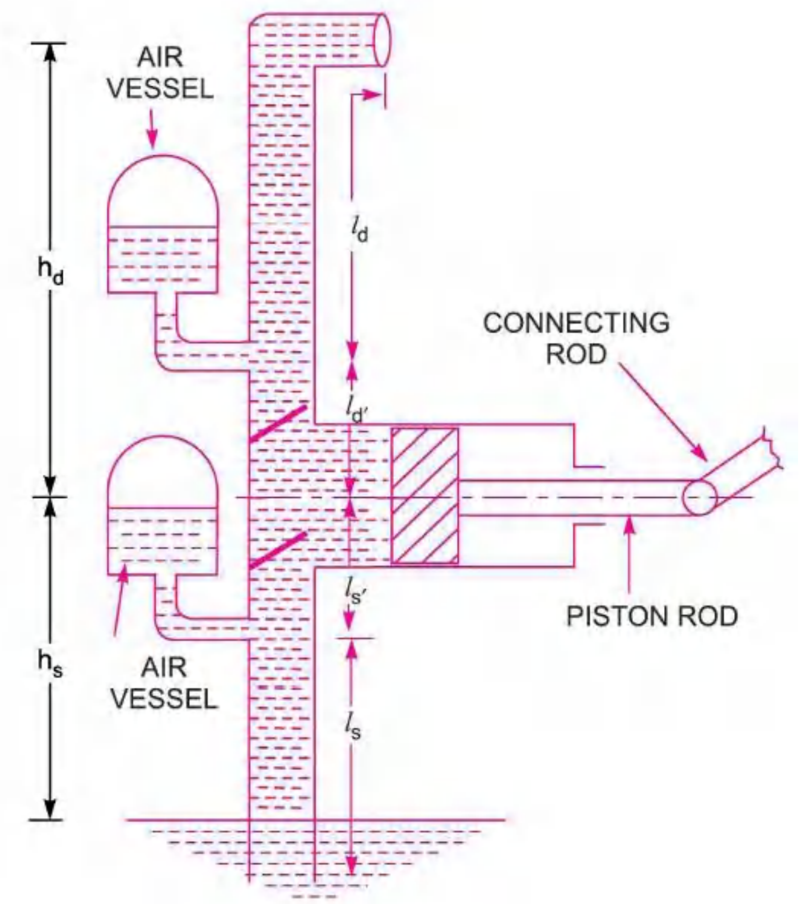
2. When $\theta = 90°$, $h_a = 0$ as $\cos 90° = 0$

3. When $\theta = 180°$, $h_a = -\frac{l}{g} \times \frac{A}{a} \omega^2 r$ as $\cos 180° = -1$

AIR VESSELS

An air vessel is a closed chamber containing compressed air in the top portion and liquid (or water) at the bottom of the chamber.

An air vessel is fitted to the suction pipe and to the delivery pipe at a point close to the cylinder of a single-acting reciprocating pump :



(i) to obtain a continuous supply of liquid at a uniform rate,

(ii) to save a considerable amount of work in overcoming the frictional resistance in the suction and delivery pipes, and

(iii) to run the pump at a high speed without separation.

Centrifugal pumps	*Reciprocating pumps*
1. The discharge is continuous and smooth.	1. The discharge is fluctuating and pulsating.
2. It can handle large quantity of liquid.	2. It handles small quantity of liquid only.
3. It can be used for lifting highly viscous liquids.	3. It is used only for lifting pure water or less viscous liquids.
4. It is used for large discharge through smaller heads.	4. It is meant for small discharge and high heads.
5. Cost of centrifugal pump is less as compared to reciprocating pump.	5. Cost of reciprocating pump is approximately four times the cost of centrifugal pump.
6. Centrifugal pump runs at high speed. They can be coupled to electric motor.	6. Reciprocating pump runs at low speed. Speed is limited due to consideration of separation and cavitation.
7. The operation of centrifugal pump is smooth and without much noise. The maintenance cost is low.	7. The operation of reciprocating pump is complicated and with much noise. The maintenance cost is high.
8. Centrifugal pump needs smaller floor area and installation cost is low.	8. Reciprocating pump requires large floor area and installation cost is high.
9. Efficiency is high.	9. Efficiency is low.

1. A double acting reciprocating Pump running at 50 rpm is discharging 1.75 m^3 of water per minute. The pump has a stroke length of 400 mm. The diameter of piston is 250 mm. The delivery and suction head are 25m and 4m respectively.
Find % slip of pump and theoretical power required to drive the pump. (AU – 2015)

Given Data :

Speed of Pump N= 50 r.p.m

Actual Discharge,

$Q = 1.75 m^3/min$

$= 0.029 m^3/s$

L = 0.4 m

D = 0.25 m

$H_d = 25$ m

$H_s = 4$m

To find –

1. % Slip
2. Power required

Formula Used :

$Slip = Q_{th} – Q_{act}$

$Q_{th} = 2 * (A*L*N)/60$

$A = (\pi D^2/4) = 0.049 m^2$

Power = weight of water delivered per second* Height of water lifted

$= 2*(\rho*g*Q_{th}) *(h_s+h_d)$

Answers:-

$Q_{th} = 0.03267 m^3/s$

$Slip = 0.03267 – 0.029 = 0.00367 m^3/s$

% slip = (0.00367/0.03267)*100 = 11%

Power required = 18.5 kW

Problem 20.3 *The cylinder bore diameter of a single-acting reciprocating pump is 150 mm and its stroke is 300 mm. The pump runs at 50 r.p.m. and lifts water through a height of 25 m. The delivery pipe is 22 m long and 100 mm in diameter. Find the theoretical discharge and the theoretical power required to run the pump. If the actual discharge is 4.2 litres/s, find the percentage slip. Also determine the acceleration head at the beginning and middle of the delivery stroke.*

Solution. Given :

Dia. of cylinder, $D = 150 \text{ mm} = 0.15 \text{ m}$

$\therefore$ Area, $A = \left(\frac{\pi}{4}\right) \times 0.15^2 = 0.01767 \text{ m}^2$

Stroke, $L = 300 \text{ mm} = 0.3 \text{ m}$

Speed of pump, $N = 50 \text{ r.p.m.}$

Total height through which water is lifted,

$H = 25 \text{ m}$

Length of delivery pipe, $l_d = 22 \text{ m}$

Diameter of delivery pipe, $d_d = 100 \text{ mm} = 0.1 \text{ m}$

Actual discharge, $Q_{act} = 4.2 \text{ litres/s} = \frac{4.2}{1000} \text{ m}^3\text{/s} = 0.0042 \text{ m}^3\text{/s}$.

(*i*) *Theoretical discharge* (Q_{th})

Theoretical discharge for a single-acting reciprocating pump is given by equation (20.1), as

$$Q_{th} = \frac{A \times L \times N}{60} = \frac{0.01767 \times 0.3 \times 50}{60} = 0.0044175 \text{ m}^3\text{/s}$$

$$= 0.0044175 \times 1000 \text{ litres/s} = \mathbf{4.4175 \text{ litres/s. Ans.}}$$

(*ii*) *Theoretical power* (P_t)

Theoretical power is given by, $P_t = \dfrac{\text{(Theoretical weight of water lifted/s)} \times \text{Total height}}{1000}$

$$= \frac{\rho \times g \times Q_{th} \times H}{1000}$$

$$= \frac{1000 \times 9.81 \times 0.0044175 \times 25}{1000} \qquad (\because Q_{th} = 0.0044175 \text{ m}^3/\text{s})$$

$$= \mathbf{1.0833 \text{ kW. Ans.}}$$

(*iii*) *The percentage slip*

The percentage slip is given by,

$$\% \text{ slip} = \left(\frac{Q_{th} - Q_{act}}{Q_{th}}\right) \times 100 = \left(\frac{4.4175 - 4.2}{4.4175}\right) \times 100 = \mathbf{4.92\%. \text{ Ans.}}$$

(*iv*) *Acceleration head at the beginning of delivery stroke.*

The acceleration head in the delivery pipe is given by equation (20.15) as :

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \times \cos\theta$$

where a_d = Area of delivery pipe = $\frac{\pi}{4} \times (0.1)^2 = 0.007854$

$$\omega = \text{Angular speed} = \frac{2\pi N}{60} = \frac{2\pi \times 50}{60} = 5.236$$

$$r = \text{Crank radius} = \frac{L}{2} = \frac{0.3}{2} = 0.15 \text{ m}$$

$\therefore$ $$h_{ad} = \frac{22}{9.81} \times \frac{0.01767}{0.007854} \times 5.236^2 \times 0.15 \times \cos\theta = 20.75 \times \cos\theta$$

At the beginning of delivery stroke, $\theta = 0°$ and hence $\cos\theta = 1$

$\therefore$ h_{ad} = **20.75 m. Ans.**

(*v*) *Acceleration head at the middle of delivery stroke.*

At the middle of delivery stroke, $\theta = 90°$ and hence $\cos\theta = 0$.

$\therefore$ $h_{ad} = 20.75 \times 0 =$ **0. Ans.**

Problem 20.4 *The length and diameter of a suction pipe of a single-acting reciprocating pump are 5 m and 10 cm respectively. The pump has a plunger of diameter 15 cm and a stroke length of 35 cm. The centre of the pump is 3 m above the water surface in the pump. The atmospheric pressure head is 10.3 m of water and pump is running at 35 r.p.m. Determine :*

(i) Pressure head due to acceleration at the beginning of the suction stroke,

(ii) Maximum pressure head due to acceleration, and

(iii) Pressure head in the cylinder at the beginning and at the end of the stroke.

Solution. Given :

Length of suction pipe, $l_s = 5 \text{ m}$

Diameter of suction pipe, $d_s = 10 \text{ cm} = 0.1 \text{ m}$

$\therefore$ Area, $a_s = \frac{\pi}{4} d_s^2 = \frac{\pi}{4} \times 0.1^2 = .007854 \text{ m}^2$

Diameter of plunger, $D = 15 \text{ cm} = 0.15 \text{ m}$

$\therefore$ Area of plunger, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times .15^2 = .01767 \text{ m}^2$

Stroke length, $L = 35 \text{ cm} = 0.35 \text{ m}$

$\therefore$ Crank radius, $r = \frac{L}{2} = \frac{0.35}{2} = 0.175 \text{ m}$

Suction head, $h_s = 3$ m

Atmospheric pressure head, $H_{atm} = 10.3$ m of water

Speed of pump, $N = 35$ r.p.m.

Anguiar speed of the crank is given by,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 35}{60} = 3.665 \text{ rad/s.}$$

(*i*) The pressure head due to acceleration in the suction pipe is given hy equation (20.14) as

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r \cos\theta$$

At the beginning of stroke, $\theta = 0°$ and hence $\cos\theta = 1$

$\therefore$ $$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r = \frac{5}{9.81} \times \frac{.01767}{.007854} \times 3.665^2 \times .175 = \textbf{2.695 m. Ans.}$$

(*ii*) Maximum pressure head due to acceleration in suction pipe is given by equation (20.16), as

$$(h_{as})_{\max} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r = \textbf{2.695 m. Ans.}$$

(*iii*) Pressure head in the cyiinder at the heginning of the suction stroke (Refer to Fig. 20.5)

$$= h_s + h_{as} = 3.0 + 2.695 = 5.695.$$

This pressure head in the cylinder is below the atmospheric pressure head.

This pressure head in the cylinder is below the atmospheric pressure head.

∴ Absolute pressure head in the cylinder at the beginning of suction stroke

= Atmospheric pressure head – 5.695

= 10.3 – 5.695 = **4.605 m of water (abs.) Ans.**

Similarly, pressure head in the cylinder at the end of suction stroke

= $h_s - h_{as}$ = 3.0 – 2.695 = 0.305 m below atmospheric pressure head

= 10.3 – 0.305 = **9.995 m of water (abs.) Ans.**

The cylinder bore diameter of single acting reciprocating is 150 mm and its stroke is 300mm. Pump running at 50 rpm lifts water through height of 25m. The delivery pipe is 22m long and 100mm in diameter. Find the theoretical discharge. If the actual discharge is 4.2 litres/sec, find the %slip. Also find the acceleration head at the beginning and middle of delivery stroke.

Formula Used :

(AU – 2015)

Slip = $Q_{th} - Q_{act}$

$Q_{th} = (A*L*N)/60$

Area of cylinder = $(\pi*(0.15)^2)/4 = 0.01767\ m^2$

Area of Delivery pipe $a_d = (\pi*(0.1)^2)/4 = 0.00785\ m^2$

Power = weight of water delivered per second* Height of water lifted

= $(\rho*g*Q_{th}) *(h_s+h_d)$

Pressure head due to acceleration

In Suction Pipe $h_{as} = \frac{l_s}{g} * \frac{A}{a_s} * \omega^2 * r\cos\theta$

In delivery pipe $h_{ad} = \frac{l_d}{g} * \frac{A}{a_d} * \omega^2 * r\cos\theta$

Answers:-

$Q_{th} = 0.0044175\ m^3/s = 4.4175$ lit/s

% slip = ((4.4175-4.2)/4.4175)*100 = 4.9%

Power= 1.08kW

$$h_{ad} = \frac{22}{9.81} * \frac{0.01767}{0.00785} * \left(\frac{2*3.14*50}{60}\right)^2 * \frac{0.3}{2}\cos\theta$$

Delivery Stroke

At beginning θ=0, cos 0=1

h_{ad} = 20.75m.

At middle θ=90, cos 90=0

h_{ad} = 0.

- 1) A single acting reciprocating pump, running at 50 rpm, delivers 0.01 m^3/s of water. The diameter of the piston is 200 mm and stroke length 400 mm. Determine: (a) the theoretical discharge of the pump, (b) Co-efficient of discharge, and (c) Slip and the percentage slip of the pumps.

Problem 20.6 *A single-acting reciprocating pump has piston diameter 12.5 cm and stroke length 30 cm. The centre of the pump is 4 m above the water level in the sump. The diameter and length of suction pipe are 7.5 cm and 7 m respectively. The separation occurs if the absolute pressure head in the cylinder during suction stroke falls below 2.5 m of water. Calculate the maximum speed at which the pump can run without separation. Take atmospheric pressure head = 10.3 m of water.*

Solution. Given :

Diameter of piston, $D = 12.5$ cm $= 0.125$ m

$\therefore$ Area, $A = \frac{\pi}{4}(.125)^2 = .01227\ m^2$

Stroke length, $L = 30$ cm $= 0.30$ m

$\therefore$ Crank radius, $r = \frac{L}{2} = \frac{0.30}{2} = 0.15$ m

Suction head, $h_s = 4.0$ m

Diameter of suction pipe, $d_s = 7.5$ cm $= 0.075$ m

$\therefore$ Area of suction pipe, $a_s = \frac{\pi}{4}(.075)^2 = .004418\ m^2$

Length of suction pipe, $l_s = 7.0$ m

Separation pressure head, $h_{sep} = 2.5$ m (absolute)

Atmospheric pressure head, $H_{atm} = 10.3$ m

From the indicator diagram, drawn in Fig. 20.5, it is clear that the absolute pressure bead during suction stroke is minimum at the beginning of the stroke. Thus, the separation can take place at the beginning of the stroke oniy. In that case the pressure head in the cyiinder at the beginning of stroke becomes = h_{sep}.

But pressure head in the cylinder at the beginning of suction stroke

$$= (h_s + h_{as}) \text{ m below atmospheric pressure head}$$

$$= \text{Atmospheric pressure head} - (h_s + h_{as}) \text{ m absolute}$$

$$= H_{atm} - (h_s + h_{as}) \text{ m (abs.)}$$

$$= 10.3 - (4.0 + h_{as})$$

$\therefore$

$$h_{sep} = 10.3 - (4.0 + h_{as})$$

$$2.5 = 10.3 - 4.0 - h_{as}$$

or

$$h_{as} = 10.3 - 4.0 - 2.5 = 3.80 \text{ m.} \quad \text{...}(i)$$

But from equation (20.14), h_{as} at the beginning of suction stroke is given by the relation

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \quad (\because \theta = 0^\circ, \because \cos\theta = 1) \text{ ...}(ii)$$

Equating equations (*i*) and (*ii*), we get

$$3.80 = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r = \frac{7.0}{9.81} \times \frac{.01227}{.004418} \times \omega^2 \times .15$$

$\therefore$
$$\omega^2 = \frac{3.80 \times 9.81 \times .004418}{7.0 \times .01227 \times .15} = 12.783$$

or
$$\omega = \sqrt{12.783} = 3.575 \text{ radian/s.}$$

But
$$\omega = \frac{2\pi N}{60}$$

$\therefore$
$$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 3.575}{2\pi} = \mathbf{34.14 \text{ r.p.m. Ans.}}$$

Thus, the maximum speed at which the pump can run without separation is 34.14 r.p.m.

Summary of Pump`s Problems, Causes & Solution

Problem	Possible Cause	Solution
Pump is cavitating	• High suction temperature • Viscosity • Low suction pressure • Restriction in line • Too much horsepower • Pump speed too high • Too high NPSH • Small suction	• Lower temperature • Agitate suction tank • Increase level or pressure • Shut down and clear • Decrease horsepower • Lower RPM • Lower NPSH requirements • Increase diameter suction line
Pump vapor locked	• Pump not vented before startup • Variable-speed pumps	• Shut down and bleed off • Use a turbine or variable-speed motor drive
Specific gravity of product changes	• Different product composition	• Leave alone; will not affect pump capacity
Excessive vibration	• Starved suction • Bearings worn • Caused by the formation of vapor pockets • Pump misaligned • Rotor out of balance; usually intermittent • Shaft bent • Loose foundation bolts • Driver vibrating • Instrument malfunctions	• Pinch down discharge valve on pump • Shut down and replace • Vent pump to reestablish full capacity • Shut down and have realigned • Remove rotating element, check impeller; if passages clogged, remove foreign material; if impeller is damaged, a new one will need to be installed • Shut down and repair • Secure pump to foundation • Disconnect coupling and check driver • Locate and correct
Fails to deliver liquid	• Pump not primed • Wrong rotation • Suction line not filled with liquid • Air/vapor in suction line • NPSH insufficient • Low level in suction tank	• Prime it • Reverse • Fill suction line • Bleed off • Increase NPSH • Increase level

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UNIT 4
TURBINES

HYDRAULIC TURBINES

- Classification of turbines – heads and efficiencies – velocity triangles. Axial, radial and mixed flow turbines. Pelton wheel, Francis turbine and Kaplan turbines-working principles - work done by water on the runner – draft tube. Specific speed - unit quantities – performance curves for turbines – governing of turbines

Introduction

- A hydraulic turbine is a *prime mover that uses the* energy of flowing water and converts it into the mechanical energy.
- This mechanical energy is used in running an electric generator which is directly coupled the shaft of the hydraulic turbine.
- It is also known as *water turbine* since the fluid medium used in them is water.
- First hydro-electric station was probably started in America in 1882.
- In India, the first major hydroelectric power plant of 4.5 MW capacity named as Sivasamudram Scheme in Mysore was commissioned in 1902.
- Hydro (water) power is a conventional renewable source of energy which is clean, free from pollution and generally has a good environment effect

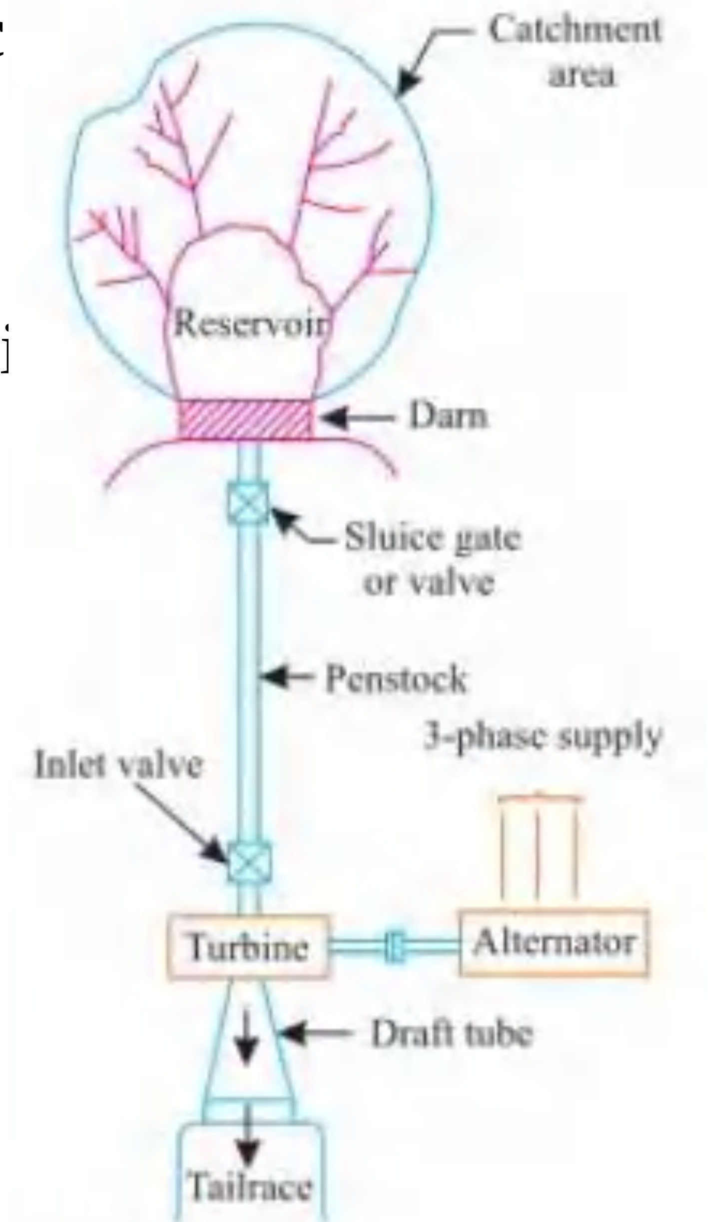
The following factor are major constraints in the utilization of hydropower resourc

i. Large investments

ii. Long gestation period

iii. Increased cost of power transmi

Hydroelectric power plant



CLASSIFICATION OF HYDRAULIC TURBINES

1. **According to the head and quantity of water available**
2. **According to the name of the originator**
3. **According to the action of water on moving blades**
4. **According to the direction of flow of water in the runner**
5. **According to the disposition of the turbine shaft**
6. **According to the specific speed N**

1. According to the head and quantity of water available

i. **Impulse turbine *requires* high head *and* small quantity of flow**

ii. **Reaction turbine requires low head and high rate of flow**

Actually there are two types of reaction turbines, one for medium head and medium flow and the other flow low head and large flow

2.According the name of the inventor

Pelton turbine named after Lester Allen Pelton.

***Francis turbine named after* James Bichens Francis.**

***Kaplan turbine* named after Dr. Victor Kaplan.**

3. According to direction of flow of water in the runner

i. Tangential flow turbines (Pelton turbine)- *Water strikes the runner tangential to the path of rotation*

ii. Radial flow for turbine (no more used)

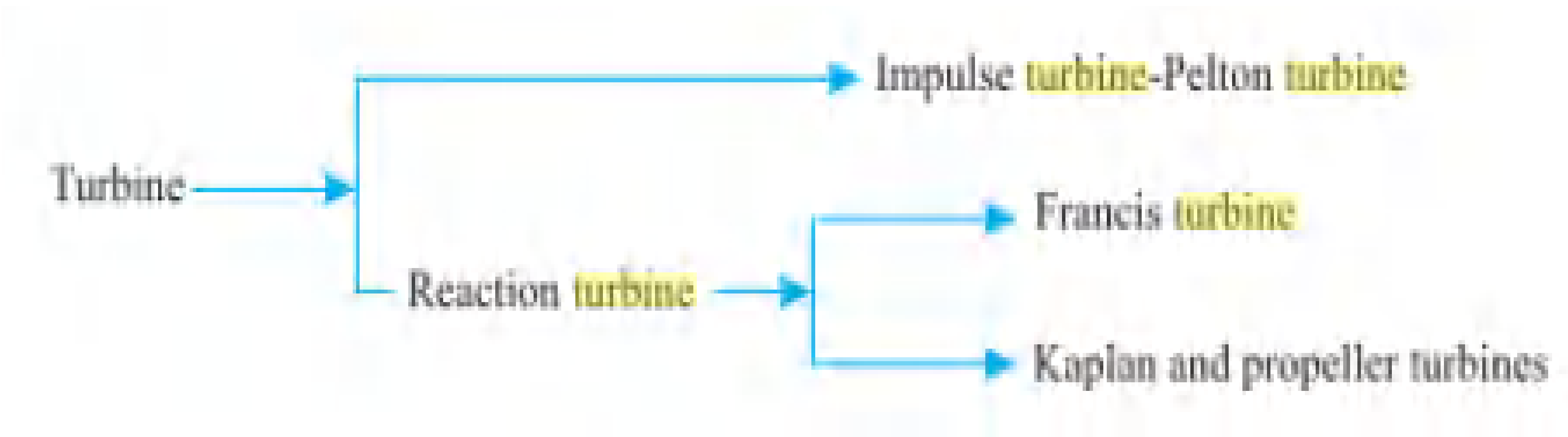
iii. Axial flow turbine (Kaplan turbine)- *Water flows parallel to the axis of the turbine shaft*

iv. Mixed (radial and axial) flow turbine (Francis turbine)- Water enters the blades radially m and comes out axially, parallel to the turbine shaft\

4. According to the disposition of the turbine shaft

- Turbine shaft may be either vertical or horizontal. In modern practice Pelton turbines usually have horizontal shafts whereas the rest, especially the large units, have vertical shafts

5. According to action of water on the moving blades



According to specific speed

- It is defined as the speed of a geometrically similar turbine that would develop *l kW under 1 m head*.
- All geometrically similar turbine will have same specific speeds when operating under the same head

$$\text{Specific speed, } N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

N **=** working speed, *P* = Power output of the turbine, and

H = The net or effective head in metres

- Turbine with low specific speeds work under high head and low discharge conditions, while high *specific* speed turbines work under low head and high discharge conditions

COMPARISON BETWEEN IMPLUSE AND REACTION TURBINES

S. No.	Aspects	Impulse turbine	Reaction turbine
1.	*Conversion of fluid energy*	The available fluid energy is converted into K.E. by a nozzle.	The energy of the fluid is partly transformed into K.E. before it (fluid) enters the runner of the turbine.
2.	*Changes in pressure and velocity*	The pressure remains same (atmospheric) throughout the action of water on the runner.	After entering the runner with an excess pressure, water undergoes changes both in velocity and pressure while passing through the runner.
3.	*Admittance of water over the wheel*	Water may be allowed to enter a part or whole of the wheel circumference.	Water is admitted over the circumference of the wheel.
4.	*Water-tight causing*	Required	Not necessary.
5.	*Extent to which the water fills the wheel/ turbine*	The wheel/turbine does not run full and air has a free access to the buckets.	Water completely fills all the passages between the blades and while flowing between inlet and outlet sections does work on the blades.
6.	*Installation of unit*	Always installed above the tail race. No draft tube is used.	Unit may be installed above or below the tail race, use of a draft tube is made.
7.	*Relative velocity of water*	Either remaining constant or reduces slightly due to friction.	Due to continuous drop in pressure during flow through the blade, the relative velocity increases.
8.	*Flow regulation*	— By means of a needle valve fitted into the nozzle. — Impossible without loss.	— By means of a guide-vane assembly. — Always accompanied by loss.

IMPULSE TURBINES-PELTON WHEEL

- In an impulse turbine the ***pressure energy of*** wate***r is converted into kinetic energy*** When passed through the nozzle and forms the high velocity jet of water.
- The Pelton wheel or Pelton turbine is a *tangential flow impulse turbine.*

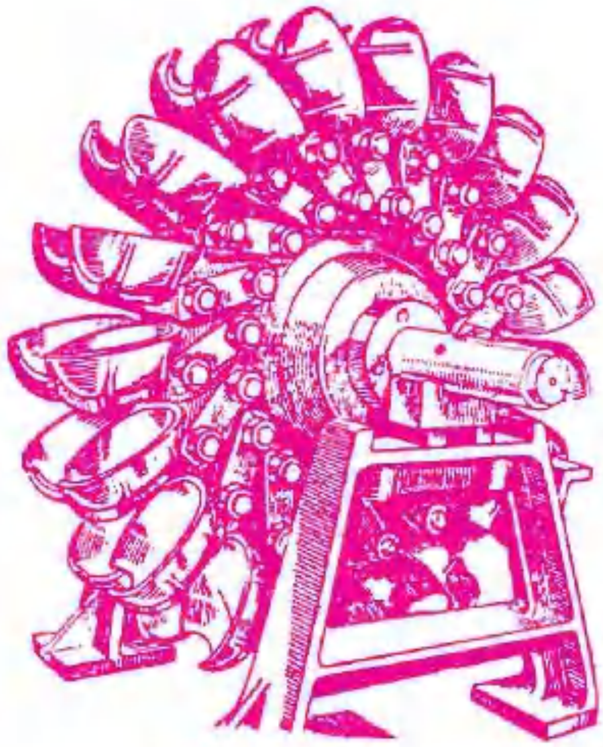
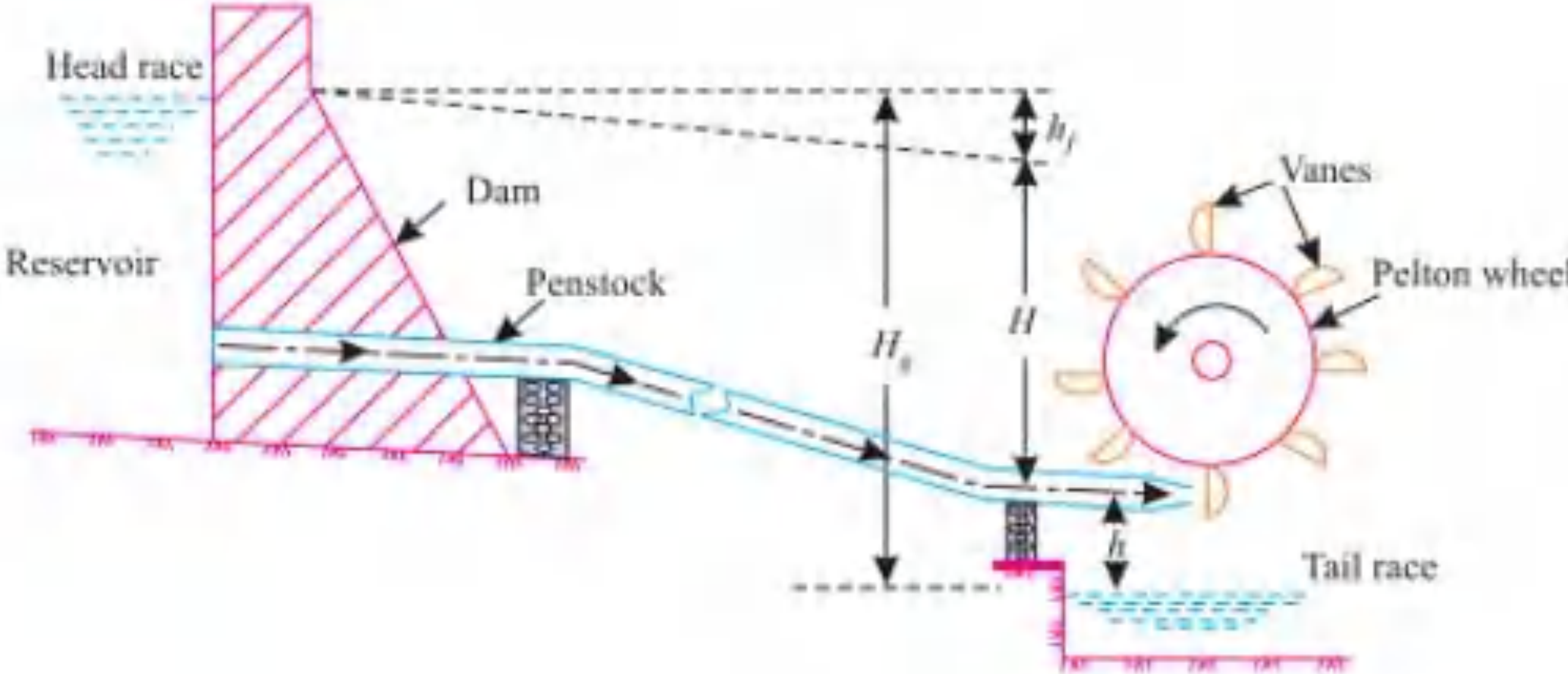


Fig. 18.3 *Runner of a pelton wheel.*



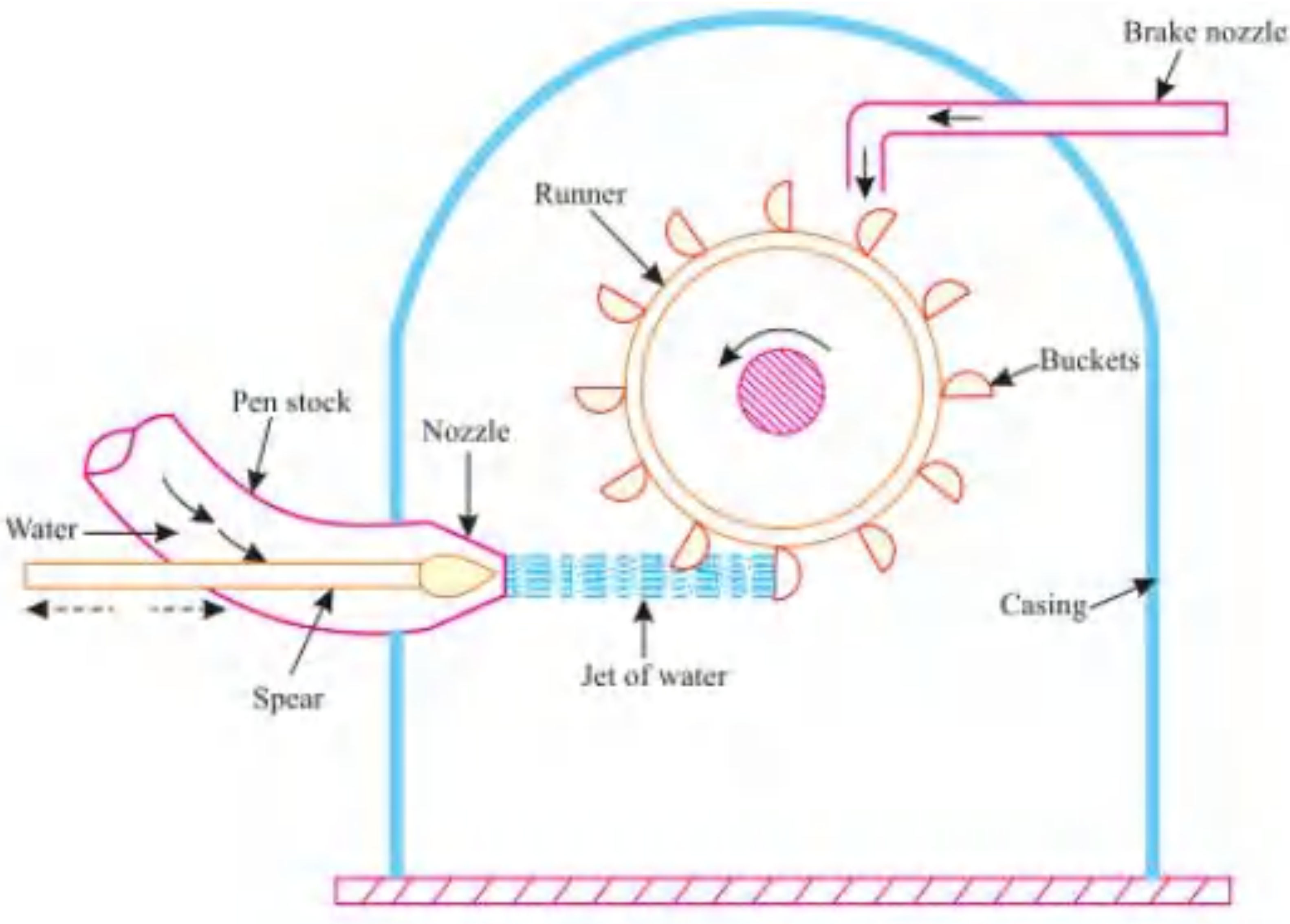
1. Gross head. The gross (total) head is the difference between the water level at the reservoir (also known as the *head race*) and the water level at the tail race. It is denoted by H_g.

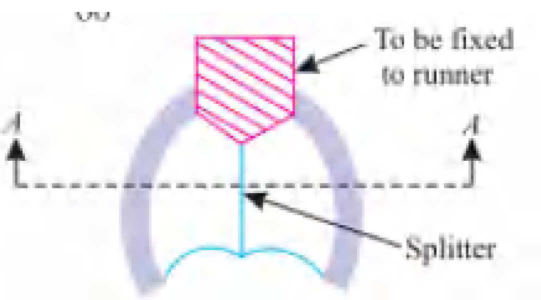
2. Net or effective head. The head available at the inlet of the turbine is known as net or effective head. It is denoted by H and is given by:

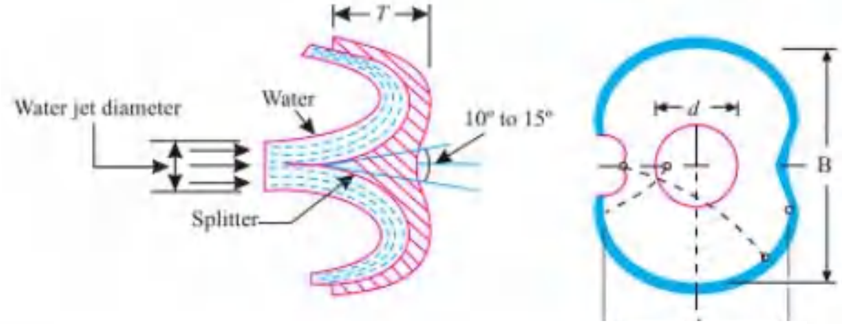
$$H = H_g - h_f - h$$

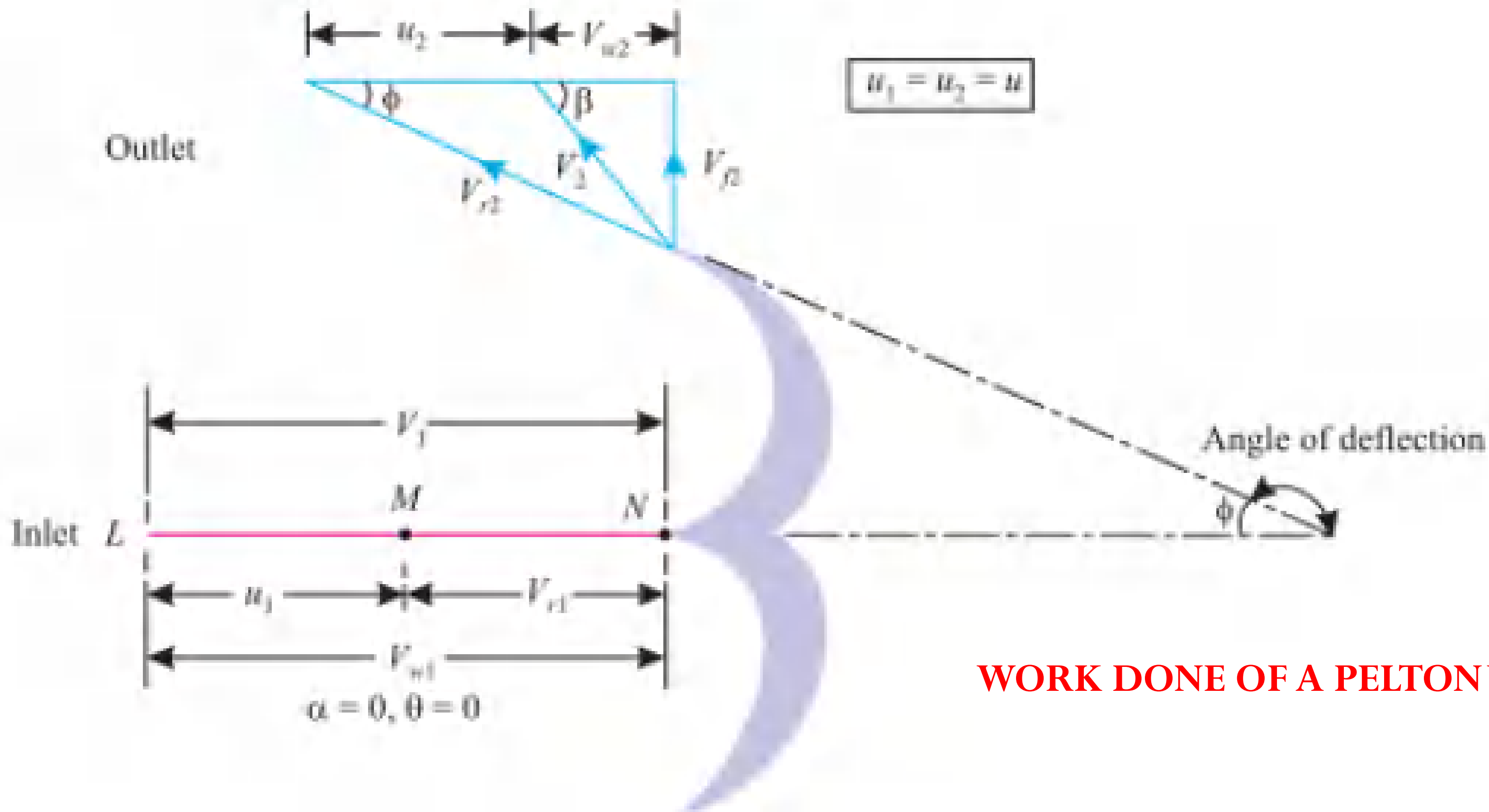
where, h_f = Total loss of head between the head race and entrance of the turbine

Construction and working of Pelton wheel/turbine

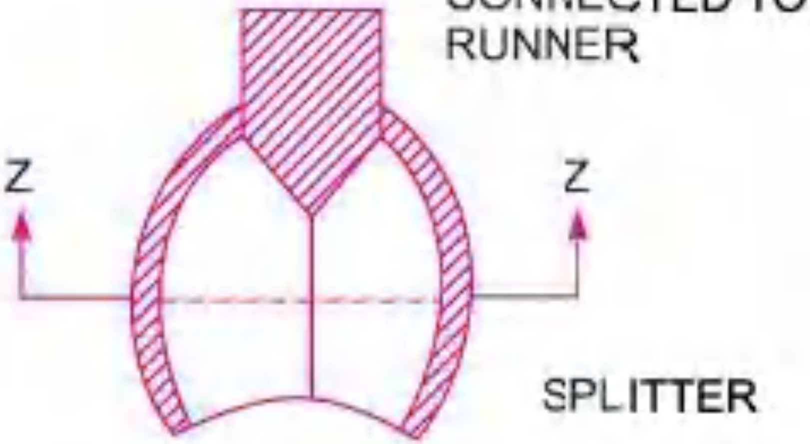


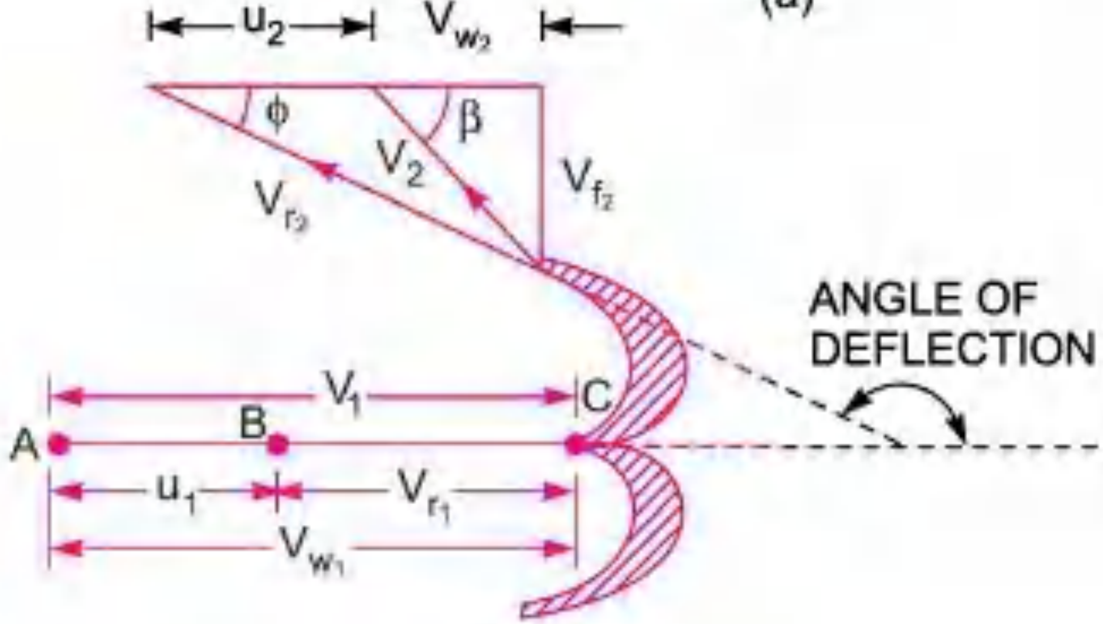






WORK DONE OF A PELTON WHEEL


THIS IS
CONNECTED TO
RUNNER
Z
Z
SPLITTER
(a)


u_2
V_{w_2}
ϕ
β
V_2
V_{r_2}
V_{f_2}
ANGLE OF
DEFLECTION
V_1
C
A
B
u_1
V_{r_1}
V_{w_1}
(b)

Outlet velocity triangle

$V_{r1} = V_{r2}$

To find V_{w2}

$$\cos\phi = \frac{V_{w2} + u_2}{V_{r2}}$$

$$u_1 = u_2 = \frac{\pi DN}{60}$$

Inlet velocity triangle

$V_1 = V_{w1}$

$$V_1 = \sqrt{2gH}$$

$V_{r1} = V_1 - u_1$

$$u_1 = \frac{\pi DN}{60}$$

To calculate number of jets required

$$No. of\ jets = \frac{Total\ Discharge\ (Q)}{Discharge\ of\ one\ jet\ (q)}$$

$$Overall\ Efficiency = \frac{shaft\ power}{Water\ power}$$

PELTON TURBINE

Let H = Net head acting on the Pelton wheel

$= H_g - h_f$

where H_g = Gross head and $h_f = \dfrac{4fLV^2}{D^* \times 2g}$

where D^* = Dia. of Penstock, N = Speed of the wheel in r.p.m.,

D = Diameter of the wheel, d = Diameter of the jet.

Then V_1 = Velocity of jet at inlet = $\sqrt{2gH}$

$$u = u_1 = u_2 = \frac{\pi DN}{60}.$$

The velocity triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{w_1} = V_1$$

$$\alpha = 0^\circ \quad \text{and } \theta = 0^\circ$$

From the velocity triangle at outlet, we have

$$V_{r_2} = V_{r_1} \text{ and } V_{w_2} = V_{r_2} \cos\phi - u_2.$$

The force exerted by the jet of water in the direction of motion is given by equation (17.19) as

$$F_x = \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \quad \text{...(18.8)}$$

As the angle β is an acute angle, +ve sign should be taken. Also this is the case of series of vanes, the mass of water striking is $\rho a V_1$ and not $\rho a V_{r_1}$. In equation (18.8), 'a' is the area of the jet which is given as

$$a = \text{Area of jet} = \frac{\pi}{4}d^2.$$

Now work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u \text{ Nm/s} \quad \text{...(18.9)}$$

Power given to the runner by the jet

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{1000} \text{ kW} \quad \text{...(18.10)}$$

- Work done by the jet per second on the runner

$W = \rho a V_1 (V_{w1} + V_{w2}) u$

$W = \rho Q (V_{w1} + V_{w2}) u$, a = area of the jet = $\frac{\pi d^2}{4}$

$\rho = 1000 \text{ kg/m}^3$

V_1 = velocity of the jet at inlet $V_1 = c_v \sqrt{2gH}$

H = Head of water

C_v = Coefficient of velocity

V_{w1} = Velocity of whirl at inlet

V_{w2} = Velocity of whirl at outlet

u = Velocity of wheel or speed of bucket $u = \frac{\pi D N}{60}$

D = Wheel diameter

N = Speed of wheel

- Work done /sec per unit weight of water striking /sec

$$= \frac{\rho a V_1 [V_{w1} + V_{w2}]\, u}{Weight\ of\ water\ striking/sec}$$

$$= \frac{\rho a V_1 [V_{w1} + V_{w2}]\, u}{\rho a V_1 g}$$

Hydraulic efficiency

$$\eta_h = \frac{Workdone/sec}{K.E\ of\ jet\ /sec}$$

$$= \frac{\rho a V_1 [V_{w1} + V_{w2}]\, u}{\frac{1}{2}(\rho a V_1) V_1^2}$$

$$= \frac{2\,[V_{w1} + V_{w2}]\, u}{V_1^2}$$

Efficiencies of a Turbine. The following are the important efficiencies of a turbine.

(*a*) Hydraulic Efficiency, η_h (*b*) Mechanical Efficiency, η_m

(*c*) Volumetric Efficiency, η_v and (*d*) Overall Efficiency, η_o

EFFICIENCIES

1) Hydraulic Efficiency

$$\eta_h = \frac{\text{Power developed by the runner}}{\text{Power supplied at the inlet of turbine}}$$

The energy supplied to the jet at inlet is in the form of kinetic energy and is equal to $\frac{1}{2}mV^2$

$\therefore$ K.E. of jet per second $= \frac{1}{2}(\rho a V_1) \times V_1^2$

$\therefore$ **Hydraulic efficiency,** $\eta_h = \frac{\text{Work done per second}}{\text{K.E. of jet per second}}$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\frac{1}{2}(\rho a V_1) \times V_1^2} = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} \quad \text{...(18.12)}$$

Now $V_{w_1} = V_1,\ V_{r_1} = V_1 - u_1 = (V_1 - u)$

$\therefore$ $V_{r_2} = (V_1 - u)$

and $V_{w_2} = V_{r_2} \cos\phi - u_2 = V_{r_2} \cos\phi - u = (V_1 - u)\cos\phi - u$

Substituting the values of V_{w_1} and V_{w_2} in equation (18.12),

$$\eta_h = \frac{2\left[V_1 + (V_1 - u)\cos\phi - u\right] \times u}{V_1^2}$$

$$= \frac{2\left[V_1 - u + (V_1 - u)\cos\phi\right] \times u}{V_1^2} = \frac{2(V_1 - u)\,[1 + \cos\phi]\,u}{V_1^2}. \quad \text{...(18.13)}$$

The efficiency will be maximum for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2u(V_1 - u)(1 + \cos\phi)}{V_1^2}\right] = 0$$

$$\text{or} \quad \frac{(1 + \cos\phi)}{V_1^2}\frac{d}{du}(2uV_1 - 2u^2) = 0 \quad \text{or} \quad \frac{d}{du}[2uV_1 - 2u^2] = 0 \quad \left(\because \frac{1 + \cos\phi}{V_1^2} \neq 0\right)$$

$$\text{or} \quad 2V_1 - 4u = 0 \quad \text{or} \quad u = \frac{V_1}{2} \quad \text{...(18.14)}$$

Equation (18.14) states that hydraulic efficiency of a Pelton wheel will be maximum when the velocity of the wheel is half the velocity of the jet of water at inlet. The expression for maximum efficiency will be obtained by substituting the value of $u = \frac{V_1}{2}$ in equation (18.13).

$$\therefore \qquad \text{Max. } \eta_h = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1 + \cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

$$= \frac{2 \times \frac{V_1}{2}(1 + \cos\phi)\,\frac{V_1}{2}}{V_1^2} = \frac{(1 + \cos\phi)}{2}. \qquad \text{...(18.15)}$$

2) Mechanical Efficiency

$$\eta_h = \frac{\text{Power available at the turbine shaft}}{\text{Power developed by turbine runner}} = \frac{\text{Shaft power}}{\text{Bucket power}}$$

$$= \frac{P}{wQ_a\left(\frac{V_{w1} + V_{w2}}{g}\right)u} = \frac{P}{wQ_a H_r}$$

3) Volumetric Efficiency

$$\eta_v = \frac{\text{Volume of water actually striking the runner } (Q_a)}{\text{Total water supplied by the jet to the turbine } (Q)}$$

- **Overall Efficiency**

$$\eta_o = \frac{\text{Volume available at the shaft of the turbine}}{\text{Power supplied at the inlet of the turbine}} = \frac{\text{Shaft power}}{\text{Water power}}$$

$$= \frac{\text{S.P.}}{\text{W.P.}}$$

$$= \frac{\text{S.P.}}{\text{W.P.}} \times \frac{\text{R.P.}}{\text{R.P.}}$$

(where R.P. = Power delivered to runner)

$$= \frac{\text{S.P.}}{\text{R.P.}} \times \frac{\text{R.P.}}{\text{W.P.}}$$

$$= \eta_m \times \eta_h$$

$$\because \frac{\text{S.P.}}{\text{R.P.}} = \eta_m \quad \frac{\text{R.P.}}{\text{W.P.}} = \eta_h$$

Points to be Remembered for Pelton Wheel

(i) The velocity of the jet at inlet is given by $V_1 = C_v\sqrt{2gH}$

where *Cv* = Co-efficient of velocity = 0.98 or 0.99
H = Net head on turbine

(ii) The velocity of wheel *(u)* is given by $u = \phi\sqrt{2gH}$

where ϕ = Speed ratio. The value of speed ratio varies from 0.43 to 0.48.

(iii) The angle of deflection of the jet through buckets is taken at **165°** if no angle of deflection is given.

(iv) The mean diameter or the pitch diameter *D* of the Pelton wheel is given by

$$u = \frac{\pi DN}{60} \text{ or } D = \frac{60u}{\pi N}.$$

(v) **Jet Ratio.** It is defined as the ratio of the pitch diameter *(D)* of the Pelton wheel to the diameter of the jet *(d).* It is denoted by *'m'* and is given as

$$m = \frac{D}{d} \ (= 12 \text{ for most cases})$$

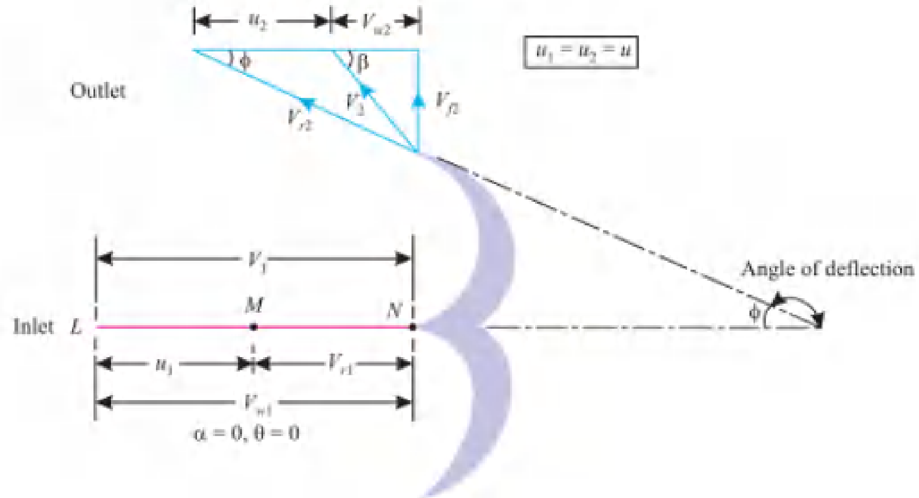
(vi) **Number of buckets** on a runner is given by

$$Z = 15 + \frac{D}{2d} = 15 + 0.5\ m$$ where m = Jet ratio

(vii) **Number of Jets.** It is obtained by dividing the total rate of flow through the turbine by the rate of flow of water through a single jet.

1. **A Pelton wheel is receiving water from a penstock with a gross head of 510 m. One-third of gross head is lost in friction in the penstock. The rate of flow through the nozzle is 2.2 m^3/s. the angle of deflection of the jet is 165 deg. Determine i) The power given by water to the runner ii) Hydraulic efficiency of the pelton wheel.**

Take Cv = 1 and Ku = 0.45



Solution. Gross head, $H_g = 510$ m

Head lost in friction, $h_f = \frac{H_g}{3} = \frac{510}{3} = 170$ m

$\therefore$ Net head, $H = H_g - h_f = 510 - 170 = 340$ m

Discharge, $Q = 2{\cdot}2$ m^3/s

Angle of deflection $= 165°$

Angle, $\phi = 180° - 165° = 15°$

Co-efficient of velocity, $C_v = 1.0$

Speed ratio, $K_u = 0.45$

The power given by water to the runner :

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 1.0\sqrt{2\times 9.81\times 340} = 81.67$ m/s

Velocity of wheel, $u = K_u\sqrt{2gH} = 0.45\sqrt{2\times 9.81\times 340} = 36.75$ m/s

Refer to fig. 2.7. $V_{r1} = V_1 - u_1 = V_1 - u = 81.67 - 36.75 = 44.92$ m/s $\quad (\because u_1 = u_2 = u)$

Also, $V_{w1} = V_1 = 81.67$ m/s

From outlet velocity triangle, we have:

$$V_{r2} = V_{r1} = 44.92 \text{ m/s}$$

Also, $V_{r2}\cos\phi = u_2 + V_{w2} = u + V_{w2}$

or, $V_{w2} = V_{r2}\cos\phi - u = 44.92\cos 15^\circ - 36.75 = 6.64$ m/s

Work done by the jet on the runner per second

$$= \rho Q\,(V_{w1} + V_{w2}) \times u$$

$$= 1000 \times 2.2\,(81.67 + 6.64) \times 36.75 = 7139863 \text{ Nm/s}$$

∴ Power given by water to the runner = 7139863 J/s

or, $W \simeq$ **7139.8 kW** **(Ans.)**

ii) Hydraulic efficiency of the pelton wheel.

$$\eta_h = \frac{2\,(V_{w1} + V_{w2}) \times u}{V_1^2}$$

$$= \frac{2\,(81.67 + 6.64) \times 36.75}{(81.67)^2} = 0.973$$

2.Thefollowing data relates to the Pelton wheel

Head: 72 m, Speed of the wheel: 240 rp.m.
Shaft power of the wheel: 115 kW, Speed ratio: 0.45
Co-efficient of velocity: 0.98, Overall efficiency: 85%
Design Pelton wheel.

Diameter of the wheel:

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 72} = 36.8$ m/s

Bucket velocity, $u\ (= u_1 = u_2) = K_u \times V_1 = 0.45 \times 36.8 = 16.56$ m/s

$$u = \frac{\pi DN}{60}, \text{ or, } D = \frac{60u}{\pi N} = \frac{60 \times 16.56}{\pi \times 240} = \mathbf{1.32\ m}$$

Diameter of Jet:

$$\text{Overall efficiency, } \eta_0 = \frac{\text{Shaft power}}{\text{Water power}} = \frac{P}{wQH}$$

$$0.85 = \frac{115}{9.81 \times Q \times 72}$$

$$Q = \frac{115}{0.85 \times 9.81 \times 72} = 0.1915 \text{ m}^3/\text{s}$$

$$Q = \text{Area of jet} \times \text{velocity of jet}$$

$$0.1915 = \frac{\pi}{4} \times d^2 \times V_1 = \frac{\pi}{4} d^2 \times 36.8$$

$$d = \left(\frac{0.1915 \times 4}{\pi \times 36.8}\right)^{1/2} = 0.0814 \text{ m}$$

- **Size of buckets:**

$$\text{Width of the bucket, } B = 3 \text{ to } 4 \text{ times jet diameter } (d)$$
$$= 3.5\,d = 3.5 \times 81.4 = \mathbf{285\ mm}\ \textbf{(Ans.)}$$

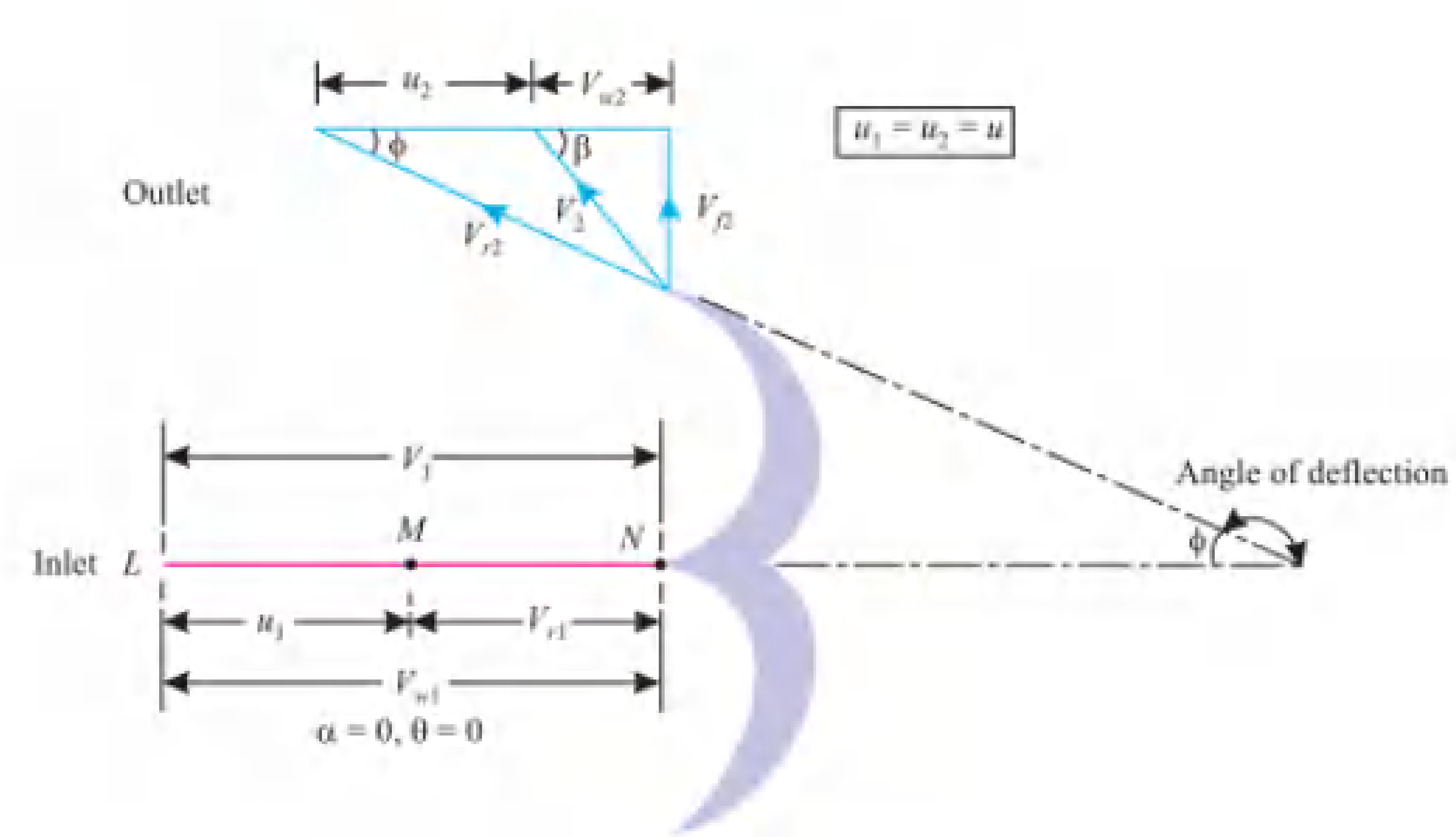
$$\text{Radial length of bucket, } L = 2 \text{ to } 3 \text{ times jet diameter } (d)$$
$$= 2.5\,d = 2.5 \times 81.4 = \mathbf{203.5\ mm}\ \textbf{(Ans.)}$$

$$\text{Depth of bucket, } T = 0.8 \text{ to } 1.2 \text{ times jet diameter } (d)$$
$$= 1.0\,d = \mathbf{81.4\ mm}\ \textbf{(Ans.)}$$

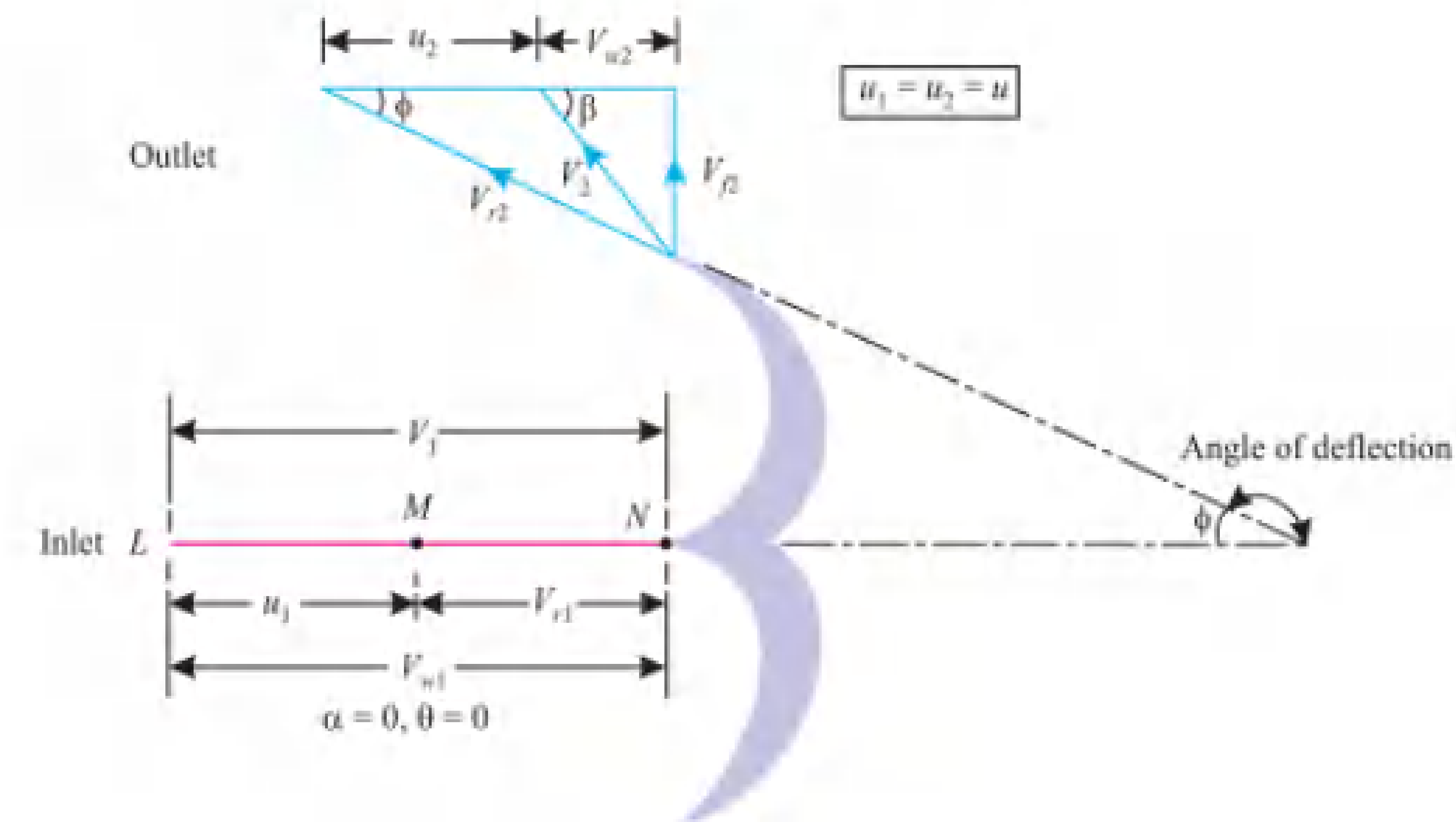
- **Number of buckets on the wheel:**

$$Z = 15 + \frac{D}{2d} = 15 + \frac{1.32 \times 1000}{2 \times 81.4} = \mathbf{23}$$

3. A single jet Pelton turbine is required to drive a generator to develop 10000 kW. The available head at the nozzle is 760 m. Assuming electric generation efficiency 95%. Pelton wheel efficiency 87%, co-efficient of velocity for nozzle = 0.97, mean bucket velocity 0.46 of jet velocity, outlet angle of bucket = 15º and relative velocity of the water leaving the buckets 0.85 of that inlet. Determine 1) The flow rate 2) Diameter of jet(d) 3) Bucket pitch circle diameter(D)



The force exerted by the jet on the buckets 4) The best synchronous speed for generation of 50 Hz and the corresponding mean diameter if the ratio of the mean bucket circle diameter to the jet diameter is not to be less than 10



Problem 18.1

A Pelton wheel has mean bucket speed of 10 meters per second with a jet of water flowing at vthe rate of 700 liters/s under a head of 30 metres. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume the Co-efficient of velocity as 0.98.

Solution: Given

Speed of bucket, $u = u_1 = u_2 = 10$ m/s

Discharge, $Q = 700$ litres/s $= 0.7\ m^3/s$, Head of water, $H = 30$ m

Angle of deflection = 160°

$\therefore$ Angle, $\phi = 180 - 160 = 20°$

Co-efficient of velocity, $C_v = 0.98$

The velocity of jet,

$$V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2\ x\ 9.8\ x\ 30} = 23.77\frac{m}{s}$$

$$v_{r1} = v_1 - u_1 = 23.77 - 10 = 13.77 m/s$$

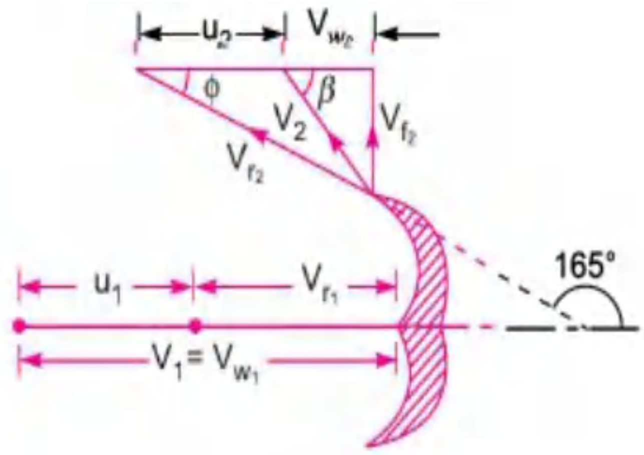
$v_{w1} = v_1 = 23.77$ m/s

From outlet velocity triangle

$v_{r2} = v_{r1} = 13.77$ m/s

$v_{w2} = v_{r2} \cos \phi - u_2$

$= 13.77 \cos 20 - 10 = 2.94$ m/s



Workdone by the jet per second on the runner ıs

$$= \rho a V_1 [V_{w1} + V_{w2}]\, u$$

$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10$

$= 186970$ Nm/s

∴ Power given to turbine = 186970 / 1000 = 186.97 kW.

The hydraulic efficiency of the turbine is

$$\eta_h = \frac{2\,[V_{w1} + V_{w2}]\,u}{V_1^2} = \frac{2\,[23.77 + 2.94]\,10}{23.77\ x\ 23.77} = 0.9454$$

$$\eta_h = 94.54\% \qquad Ans$$

Problem 18.2 *A Pelton wheel is to be designed for the following specifications: Shaft power = 11,772 kW; Head = 380 meters; Speed = 750 rpm. ; Overall efficiency = 86% ; Jet diameter is not to exceed one-sixth of the wheel diameter. Determine: (i) The wheel diameter, (ii) The number of iets required, and (iii) Diameter of the jet. Take Kv = 0.985 and Ku = 0.4*

Solution. Given: Shaft power, S.P. = 11,772 kW Head, H= 380 m Speed, N = 750 r.p.m.

Overall efficiency, $\eta_0 = 86\%$ or 0.86

Ratio of jet dia. to wheel dia. $= \frac{d}{D} = \frac{1}{6}$

Co-efficient of velocity, $K_{v_1} = C_v = 0.985$

Speed ratio, $K_{u_1} = 0.45$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.985\sqrt{2 \times 9.81 \times 380} = 85.05$ m/s

The velocity of wheel, $u = u_1 = u_2$

$$= \text{Speed ratio} \times \sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 380} = 38.85 \text{ m/s}$$

But $u = \frac{\pi DN}{60}$ $\therefore$ $38.85 = \frac{\pi DN}{60}$

or $D = \frac{60 \times 38.85}{\pi \times N} = \frac{60 \times 38.85}{\pi \times 750} =$ **0.989 m. Ans.**

But $$\frac{d}{D} = \frac{1}{6}$$

$\therefore$ Dia. of jet, $$d = \frac{1}{6} \times \text{D} = \frac{0.989}{6} = \mathbf{0.165 \text{ m. Ans.}}$$

Discharge of one jet, $$q = \text{Area of jet} \times \text{Velocity of jet}$$

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4}(.165) \times 85.05 \text{ m}^3/\text{s} = 1.818 \text{ m}^3/\text{s} \quad ...(i)$$

Now $$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{11772}{\frac{\rho g \times Q \times H}{1000}}$$

$$0.86 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 380}, \text{ where } Q = \text{Total discharge}$$

$\therefore$ Total discharge, $$Q = \frac{11772 \times 1000}{1000 \times 9.81 \times 380 \times 0.86} = 3.672 \text{ m}^3/\text{s}$$

$\therefore$ Number of jets $$= \frac{\text{Total discharge}}{\text{Discharge of one jet}} = \frac{Q}{q} = \frac{3.672}{1.818} = \mathbf{2 \text{ jets. Ans.}}$$

Problem 18.3 *The penstock supplies water from a reservoir to the Pelton wheel with a gross head of 500 m. One third of the gross head is lost in friction in the penstock. The rate of flow of water through the nozzle fitted at the end of the penstock is 2.0 m%. The angle of deflection of the jet is 165°.*
Determine the power given by the water to the runner and also hydraulic efficiency of the Pelton wheel. Take speed ratio = 0.45 and Cv = 1.0.

Solution. Given:

Gross head, Hg = 500m. Head lost in friction, $hf = Hg/3 = 500/3 = 166.6$ m

∴ Net head, $H = H_g - h_f = 500 - 166.7 = 333.30$ m

Discharge, $Q = 2.0\ m^3/s$

Angle of deflection $= 165°$

∴ Angle, $\phi = 180° - 165° = 15°$

Speed ratio $= 0.45$

Co-efficient of velocity, $C_v = 1.0$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 1.0 \times \sqrt{2 \times 9.81 \times 333.3} = 80.86$ m/s

Velocity of wheel, $u = \text{Speed ratio} \times \sqrt{2gH}$

or $u = u_1 = u_2 = 0.45 \times \sqrt{2 \times 9.81 \times 333.3} = 36.387$ m/s

$\therefore \quad V_{r_1} = V_1 - u_1 = 80.86 - 36.387$

$= 44.473$ m/s

Also $\quad V_{w_1} = V_1 = 80.86$ m/s

From outlet velocity triangle, we have

$$V_{r_2} = V_{r_1} = 44.473$$

$$V_{r_2} \cos \phi = u_2 + V_{w_2}$$

or $$44.473 \cos 15° = 36.387 + V_{w_2}$$

or $$V_{w_2} = 44.473 \cos 15° - 36.387 = 6.57 \text{ m/s.}$$

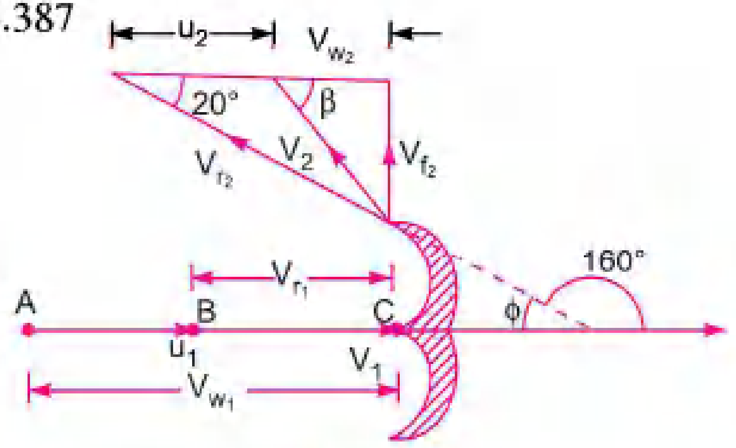


Fig. 18.7

Work done by the jet on the runner per second is given by equation (18.9) as

$$\rho a V_1 [V_{w_1} + V_{w_2}] \times u = \rho Q [V_{w_1} + V_{w_2}] \times u \qquad (\because \quad aV_1 = Q)$$

$$= 1000 \times 2.0 \times [80.86 + 6.57] \times 36.387 = 6362630 \text{ Nm/s}$$

$\therefore$ Power given by the water to the runner in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{6362630}{1000} = \textbf{6362.63 kW. Ans.}$$

Hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[80.86 + 6.57] \times 36.387}{80.86 \times 80.86}$$

$= \textbf{0.9731}$ or **97.31%. Ans.**

Problem 18.5 *A Pelton wheel is working under a gross head of 400 m. The water is supplied through penstock of diameter 1 m and length 4 km from reservoir to the Pelton wheel. The co-efficient of friction for the penstock is given as .008. The jet of water of diameter 150 mm strikes the buckets of the wheel and gets deflected through an angle of 165°. The relative velocity of water at outlet is reduced by 15% due to friction between inside surface of the bucket and water. If the velocity of the buckets is 0.45 times the jet velocity at inlet and mechanical efficiency as 85% determine:*
(i) Power given to the runner, (ii) Shaft power,
(iii) Hydraulic efficiency and overall efficiency.

Solution. Given :

Gross head, $H_g = 400$ m

Diameter of penstock, $D = 1.0$ m

Length of penstock, $L = 4$ km $= 4 \times 1000 = 4000$ m

Co-efficient of friction, $f = .008$

Diameter of jet, $d = 150$ mm $= 0.15$ m

Angle of deflection $= 165°$

$\therefore$ Angle, $\phi = 180° - 165° = 15°$

Relative velocity at outlet, $V_{r_2} = 0.85\, V_{r_1}$

Velocity of bucket, $u = 0.45 \times$ Jet velocity

Mechanical efficiency, $\eta_m = 85\% = 0.85$

Let V^* = Velocity of water in penstock, and

V_1 = Velocity of jet of water.

Using continuity equation, we have

Area of penstock × V^* = Area of jet × V_1

or
$$\frac{\pi}{4}D^2 \times V^* = \frac{\pi}{4}d^2 \times V_1$$

∴
$$V^* = \frac{d^2}{D^2} \times V_1 = \frac{0.15^2}{1.0^2} \times V_1 = .0225\ V_1 \qquad \text{...}(i)$$

Applying Bernoulli's equation to the free surface of water in the reservoir and outlet of the nozzle, we get

$$H_g = \text{Head lost due to friction} + \frac{V_1^2}{2g}$$

or
$$400 = \frac{4fLV^{*2}}{D \times 2g} + \frac{V_1^2}{2g} = \frac{4 \times .008 \times 4000 \times V^{*2}}{1.0 \times 2 \times 9.81} + \frac{V_1^2}{2g}$$

Substituting the value of V^* from equation (i), we get

$$400 = \frac{4 \times .008 \times 4000}{2 \times 9.81} \times (0.0225\ V_1)^2 + \frac{V_1^2}{2g}$$

$$= .0033\ V_1^2 + .051\ V_1^2 \text{ or } 400 = .0543\ V_1^2$$

∴
$$V_1 = \sqrt{\frac{400}{.0543}} = 85.83 \text{ m/s.}$$

Now velocity of bucket, $u_1 = 0.45\ V_1 = 0.45 \times 85.83 = 38.62$ m/s

From inlet velocity triangle, $V_{r_1} = V_1 - u_1 = 85.83 - 38.62 = 47.21$ m/s

$$V_{w_1} = V_1 = 85.83 \text{ m/s}$$

From outlet velocity triangle, $V_{r_2} = 0.85 \times V_{r_1} = 0.85 \times 47.21 = 40.13$ m/s

$$V_{w_2} = V_{r_2} \cos\phi - u_2 = 40.13 \cos 15^\circ - 38.62$$

$$= 0.143 \text{ m/s} \qquad (\because \quad u = u_1 = u_2 = 38.62)$$

Discharge through nozzle is given as

$$Q = \text{Area of jet} \times \text{Velocity of jet} = a \times V_1$$

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} (.15)^2 \times 85.83 = 1.516 \text{ m}^3\text{/s}$$

Work done on the wheel per second is given by equation (18.9) as

$$= \rho a V_1 \, [V_{w_1} + V_{w_2}] \times u = \rho Q \left[V_{w_1} + V_{w_2}\right] \times u$$

$$= 1000 \times 1.516 \, [85.83 + .143] \times 38.62 = 5033540 \text{ Nm/s}$$

(*i*) Power given to the runner in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{5033540}{1000} = \mathbf{5033.54 \text{ kW. Ans.}}$$

(*ii*) Using equation (18.4) for mechanical efficiency,

$$\eta_m = \frac{\text{Power at the shaft}}{\text{Power given to the runner}} = \frac{\text{S.P.}}{5033.54}$$

$\therefore$ $$\text{S.P.} = \eta_m \times 5033.54 = 0.85 \times 5033.54 = \mathbf{4278.5\ kW.\ Ans.}$$

(*iii*) Hydraulic efficiency is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2}$$

$$= \frac{2\,[85.83 + .143] \times 38.62}{85.83 \times 85.83} = 0.9014 = \mathbf{90.14\%.\ Ans.}$$

Overall efficiency is given by equation (18.6) as

$$\eta_0 = \eta_m \times \eta_h = 0.85 \times .9014 = 0.7662 \text{ or } \mathbf{76.62\%.\ Ans.}$$

Design of Pelton Wheel.

1. Diameter of the jet (d),
2. Diameter of wheel (D),
3. Width of the buckets which is $= 5 \times d$,
4. Depth of the buckets which is $= 1.2 \times d$, and
5. Number of buckets on the wheel.

Size of buckets means the width and depth of the buckets.

Problem 18.11 *A Pelton wheel* is *to be designed for a head of 60 m when running at 200 r.p.m. The Pelton wheel develops* 95.6475 *kW shaft power. The velocity of the buckets = 0.45 times the velocity of the jet, overall efficiency = 0.85 and co-efficient of the velocity* is *equal to 0.98*

Head, $H = 60m$

Speed N = 200 r.p.m

Shaft power, S.P. = 95.6475 kW

Velocity of bucket, u = 0.45 x Velocity of jet

Overall efficiency, η = 0.85

Co-efficient of velocity, C_v = 0.98

Design of Pelton wheel means to find diameter of jet (d), diameter of wheel (D), Width and depth of buckets and number of buckets on the wheel.

(*i*) *Velocity of jet*, $V_1 = C_v \times \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 60} = 33.62$ m/s

$\therefore$ Bucket velocity, $u = u_1 = u_2 = 0.45 \times V_1 = 0.45 \times 33.62 = 15.13$ m/s

But $u = \dfrac{\pi D N}{60}$, where D = Diameter of wheel

$$\therefore \quad 15.13 = \frac{\pi \times D \times 200}{60} \quad \text{or} \quad D = \frac{60 \times 15.13}{\pi \times 200} = \mathbf{1.44\ m.\ Ans.}$$

(*ii*) *Diameter of the jet* (d)

Overall efficiency $\eta_o = 0.85$

But $\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{95.6475}{\left(\frac{\text{W.P.}}{1000}\right)} = \frac{95.6475 \times 1000}{\rho \times g \times Q \times H}$ $(\because \quad \text{W.P.} = \rho g Q H)$

$$= \frac{95.6475 \times 1000}{1000 \times 9.81 \times Q \times 60}$$

$\therefore$ $Q = \frac{95.6475 \times 1000}{\eta_o \times 1000 \times 9.81 \times 60} = \frac{95.6475 \times 1000}{0.85 \times 1000 \times 9.81 \times 60} = 0.1912 \text{ m}^3/\text{s}.$

But the discharge, Q = Area of jet × Velocity of jet

$\therefore$ $0.1912 = \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} d^2 \times 33.62$

$\therefore$ $d = \sqrt{\frac{4 \times 0.1912}{\pi \times 33.62}} = 0.085 \text{ m} = $ **85 mm. Ans.**

(*iii*) *Size of buckets*

Width of buckets $= 5 \times d = 5 \times 85 = 425$ mm

Depth of buckets $= 1.2 \times d = 1.2 \times 85 =$ **102 mm. Ans.**

(*iv*) *Number of buckets on the wheel* is given by equation (18.17) as

$Z = 15 + \frac{D}{2d} = 15 + \frac{1.44}{2 \times .085} = 15 + 8.5 =$ **23.5 say 24. Ans.**

Problem 18.12 *Determine the power given by the jet of water to the runner of a Pelton wheel which is having tangential velocity as 20 m/s. The net head on the turbine is 50 m and discharge through the jet water is 0.03 m³/s. The side clearance angle is 15° and take $C_v = 0.975$.*

Solution. Given :

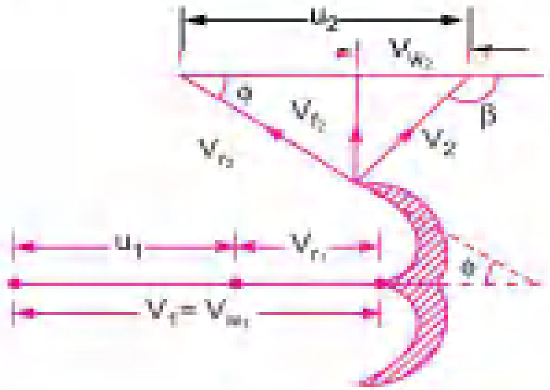


Fig. 18.9

Tangential velocity of wheel, $u = u_1 = u_2 = 20$ m/s

Net head, $H = 50$ m

Discharge, $Q = 0.03 \text{ m}^3/\text{s}$

Side clearance angle, $\phi = 15°$

Co-efficient of velocity, $C_v = 0.975$

Velocity of the jet, $V_1 = C_v \times \sqrt{2gH}$

$= 0.975 \times \sqrt{2 \times 9.81 \times 50}$

$= 30.54$ m/s

From inlet triangle, $V_{w_1} = V_1 = 30.54$ m/s

$V_{r_1} = V_{w_1} - u_1 = 30.54 - 20.0 = 10.54$ m/s

From outlet velocity triangle, we have

$V_{r_2} = V_{r_1} = 10.54$ m/s

$V_{r_2} \cos\phi = 10.54 \cos 15° = 10.18$ m/s

As $V_{r_2} \cos\phi$ is less than u_2, the velocity triangle at outlet will be as shown in Fig. 18.9.

$\therefore$ $V_{w_2} = u_2 - V_{r_2} \cos\phi = 20 - 10.18 = 9.82$ m/s.

Also as β is an obtuse angle, the work done per second on the runner,

$$= \rho a V_1 \left[V_{w_1} - V_{w_2}\right] \times u = \rho Q \left[V_{w_1} - V_{w_2}\right] \times u$$

$$= 1000 \times .03 \times [30.54 - 9.82] \times 20 = 12432 \text{ Nm/s}$$

$\therefore$ Power given to the runner in kW $= \dfrac{\text{Work done per second}}{1000} = \dfrac{12432}{1000} =$ **12.432 kW. Ans.**

Problem 18.13 *The three-jet Pelton turbine is required to generate 10,000 kW under a net head of 400 m. The blade angle at outlet is 15° and the reduction in the relative velocity while passing over the blade is 5%. If the overall efficiency of the wheel is 80%, C_v = 0.98 and speed ratio = 0.46, then find: (i) the diameter of the jet, (ii) total flow in m^3/s and (iii) the force exerted by a jet on the buckets.*

If the jet ratio is not to be less than 10, find the speed of the wheel for a frequency of 50 hertz/sec and the corresponding wheel diameter.

Solution. Given :

No. of jets $= 3$

Total power, $P = 10000$ kW

Net head, $H = 400$ m

Blade angle at outlet, $\phi = 15°$

Relative velocity at outlet = 0.95 of relative velocity at inlet

or $V_{r_2} = 0.95\ V_{r_1}$

Overall efficiency, $\eta_o = 0.80$

Value of $C_v = 0.98$

Speed ratio $= 0.46$

Frequency, $f = 50$ hertz/sec

Now using equation (18.6 A), $\eta_o = \dfrac{P}{\left(\dfrac{\rho \times g \times Q \times H}{1000}\right)}$

where Q = Total discharge through three nozzles and $\rho = 1000 \text{ kg/m}^3$

$$\therefore \quad 0.80 = \frac{10000}{\left(\dfrac{1000 \times 9.81 \times Q \times 400}{1000}\right)}$$

$$\therefore \quad Q = \frac{10000}{0.8 \times 9.81 \times 400} = \mathbf{3.18\ m^3/s.\ Ans.}$$

Discharge through one nozzle $= \dfrac{3.18}{3} = 1.06 \text{ m}^3/\text{s}.$

(*i*) Diameter of the jet (d).

Discharge through one nozzle = Area of one jet × Velocity

But velocity of jet, $V_1 = C_v \times \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 400} = 87$ m/s

$$\therefore \quad 1.06 = \frac{\pi}{4} d^2 \times 87$$

$$\therefore \quad d = \sqrt{\frac{4 \times 1.06}{\pi \times 87}} = 0.125 \text{ m} = \mathbf{125\ mm.\ Ans.}$$

(*ii*) Total flow in m³/s $= 3.18 \text{ m}^3/\text{s}.$

(iii) **Force exerted by a jet on the wheel.**

$$\text{Speed ratio} = \frac{u_1}{\sqrt{2gH}}$$

$\therefore$ $u_1 = \text{Speed ratio} \times \sqrt{2gH} = 0.46 \times \sqrt{2 \times 9.81 \times 400} = 40.75 \text{ m/s}$

Now $V_{r_1} = V_1 - u_1 = 87 - 40.75 = 46.25 \text{ m/s}$

and $V_{r_2} = 0.95\, V_{r_1} = 0.95 \times 46.25 = 44.0 \text{ m/s}$

$V_{w_1} = V_1 = 87 \text{ m/s}$

$V_{w_2} = V_{r_2} \cos\phi - u_2 = 44 \times \cos 15° - 40.75$ $(\because\ u_1 = u_2 = 40.75 \text{ m/s})$

$= 1.75 \text{ m/s}$

Force exerted by a single jet on the buckets

$= \rho \times \text{discharge through one jet} \times (V_{w_1} + V_{w_2})$

$= 1000 \times 1.06\,(87 + 1.75) = 94075 \text{ N} =$ **94.075 kN. Ans.**

(iv) **Jet ratio** $= 10$ or $\dfrac{D}{d} = 10$

$\therefore$ Dia. of wheel, $D = 10 \times d = 10 \times 0.125 = 1.25 \text{ m}$

But, $u_1 = \dfrac{\pi D N}{60}$

$\therefore$ $N = \dfrac{60 \times u_1}{\pi \times D} = \dfrac{60 \times 40.75}{\pi \times 1.25} = 620 \text{ r.p.m.}$

Now using the relation, $N = \dfrac{60 \times f}{p}$

where f = frequency in hertz per second,

p = pairs of poles, and N = speed.

$$\therefore \quad p = \frac{60 \times f}{N} = \frac{60 \times 50}{620} = 4.85$$

Take the next whole number *i.e.*, 5. Hence, pairs of poles are 5.

Now corresponding to five pairs of poles, the speed of the turbine will become as given below :

$$N = \frac{60 \times f}{p} = \frac{60 \times 50}{5} = 600 \text{ r.p.m.}$$

But

$$u = \frac{\pi D N}{60}$$

As the peripheral velocity is constant. Hence with the change of speed, diameter of wheel will change.

$$\therefore \quad D = \frac{60 \times u}{\pi \times N} = \frac{60 \times 40.75}{\pi \times 600} = 1.3 \text{ m}$$

$$\therefore \quad \text{Jet ratio becomes} = \frac{D}{d} = \frac{1.30}{0.125} > 10$$

Hence the given condition is satisfied.

Specific speed.

$$N_s = \frac{N\sqrt{p}}{H^{5/4}}$$

where *N* is in rps, *P* in *W* and *H* in *m*.

Significance of specific speed.

- Specific speed does not indicate the speed of the machine.
- It can be considered to indicate the flow area and shape of the runner.
- When the head is large, the velocity when potential energy is converted to kinetic energy will be high.
- The flow area required will be just the nozzle diameter. This cannot be arranged in a fully flowing type of turbine. Hence the best suited will be the impulse turbine.
- When the flow increases, still the area required will be unsuitable for a reaction turbine. So multi jet unit is chosen in such a case. As the head reduces and flow increases purely radial flow reaction turbines of smaller diameter can be chosen. As the head decreases still further and the flow increases, wider rotors with mixed flow are found suitable.
- The diameter can be reduced further and the speed increased up to the limit set by mechanical design. As the head drops further for the same power, the flow rate has to be higher. Hence axial flow units are found suitable in this situation. Keeping the power constant, the specific speed increases with *N* and decreases with head.
- The speed variation is not as high as the head variation. Hence specific speed value increases with the drop in available head

REACTION TURBINE

In reaction turbines the available potential energy is progressively converted in the turbines rotors and the reaction of the accelerating water causes the turning of the wheel.

(Both pressure energy and kinetic energy will available at the runner part of the turbine)

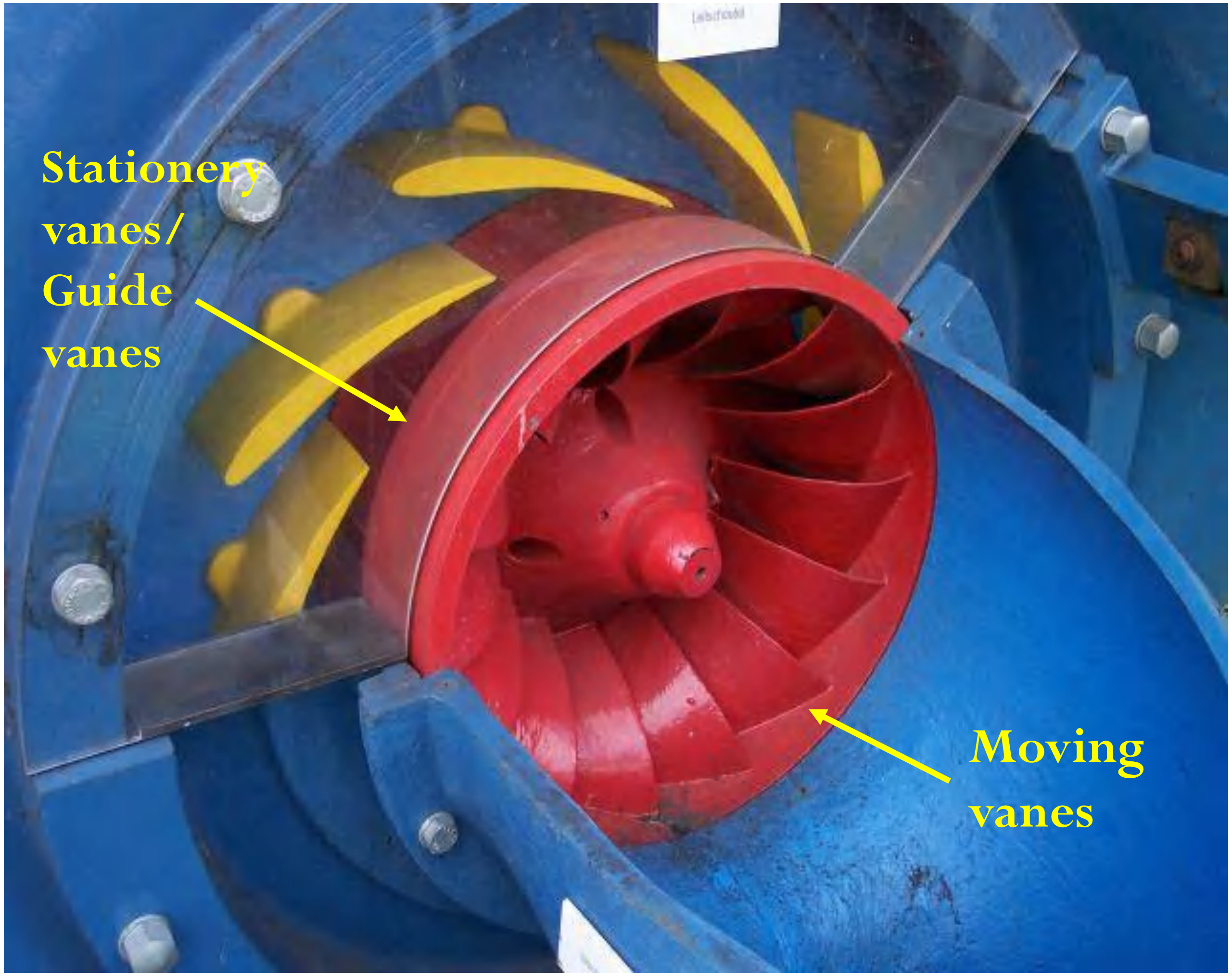
Pressure Energy- Water will be flowing under pressure to hit the runner. Entire runner is surrounded by water.

(in Kaplan also entire runner and hub is surrounded by water-so Kaplan is also one example of reaction turbine)

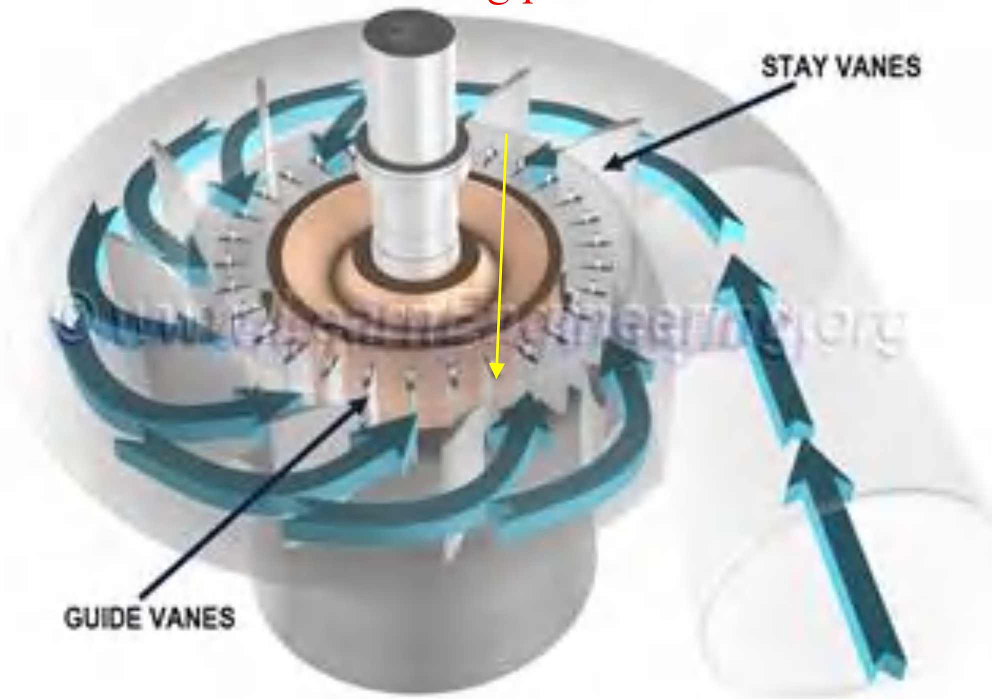
Kinetic Energy- When water hits the runner a part of pressure energy is converted to kinetic energy by rotating the runner.

REACTION TURBINE

- Energy of fluid partly transferred into kinetic energy before it enters the runner
- It enters the runner with excess pressure.
- Pressure energy is converted into kinetic energy as water passes through runner.
- The difference in pressure between inlet and outlet of runner (reaction pressure) is responsible for motion of runner.
- Eg: Francis turbine, Kaplan Turbine


Stationery
vanes/
Guide
vanes
Moving
vanes

Runner- The only moving part.



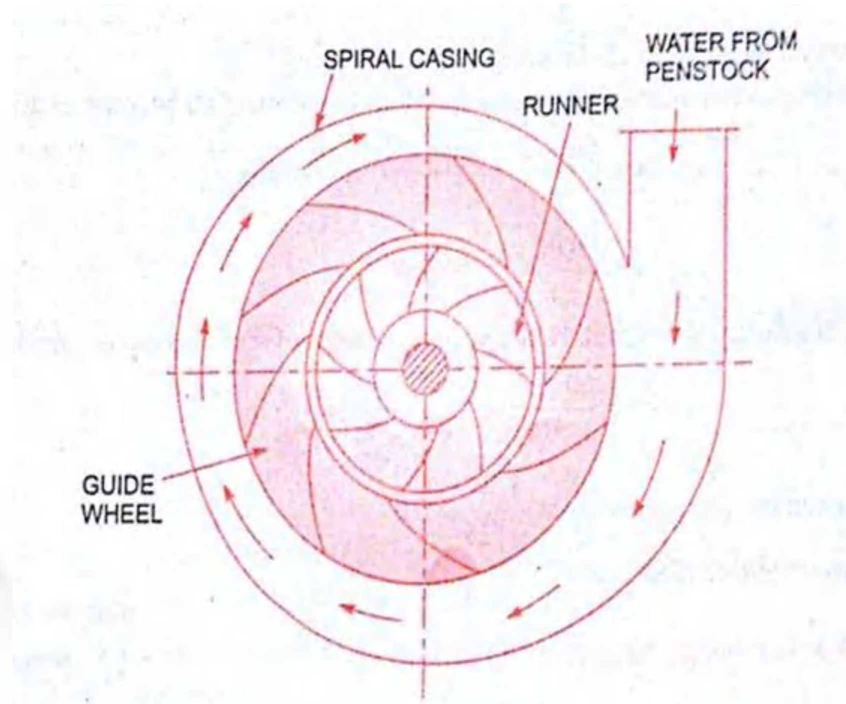


Fig. 18.10 *Main parts of a radial reaction turbines.*

In Inward Radial Flow Turbine, Water enters circumferentially.

So Discharge = Circumferential Area x Flow Velocity

And Tangentially or Peripheral Velocity of Runner at inlet u_1 is not equal to Peripheral velocity at outlet u_2

FRANCIS TURBINE COMPONENTS

- Penstock
- Scroll/Spiral casing
- Stay ring
- Stay vanes
- Guide vanes
- Runner blades
- Draft tube

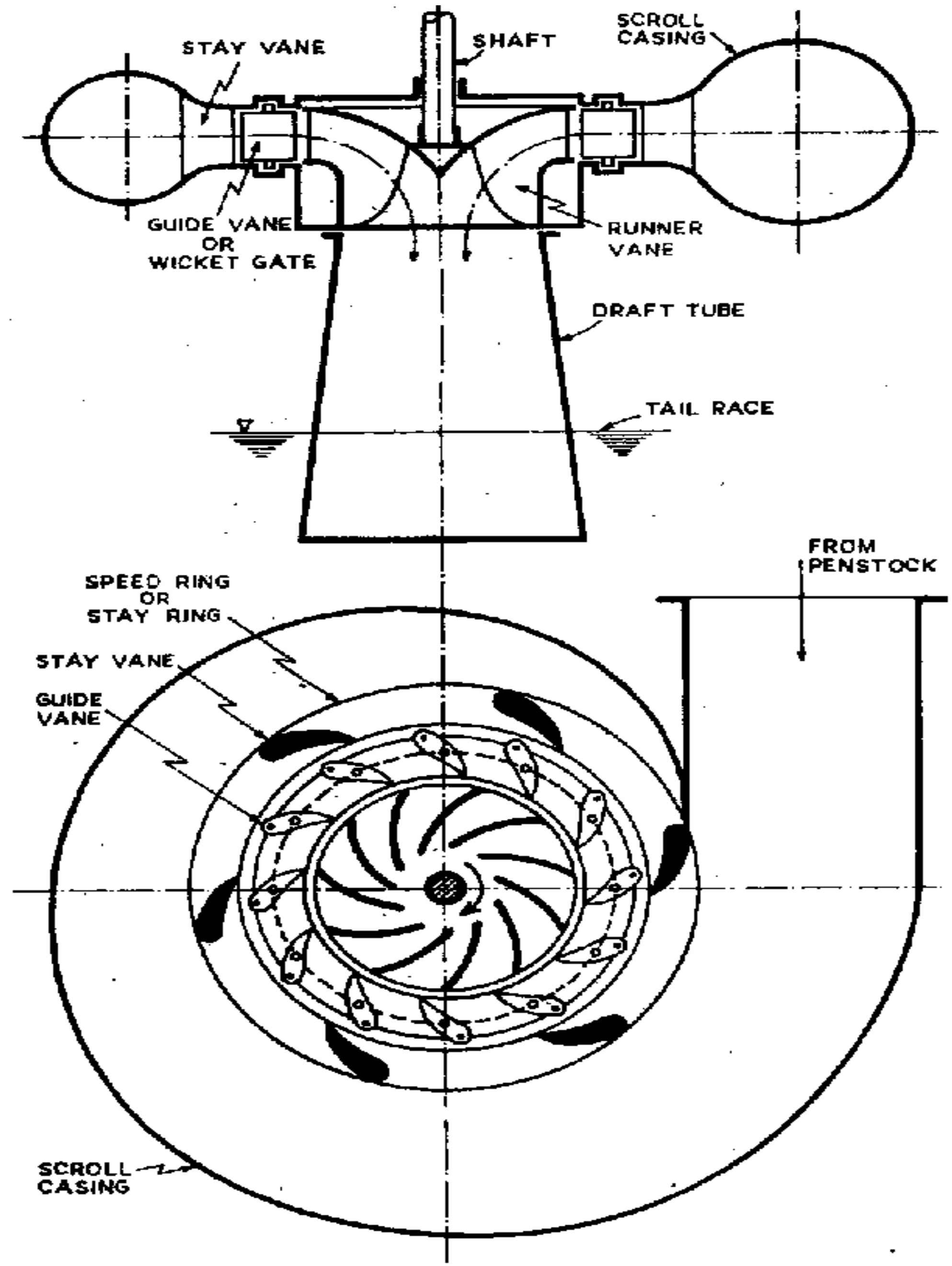
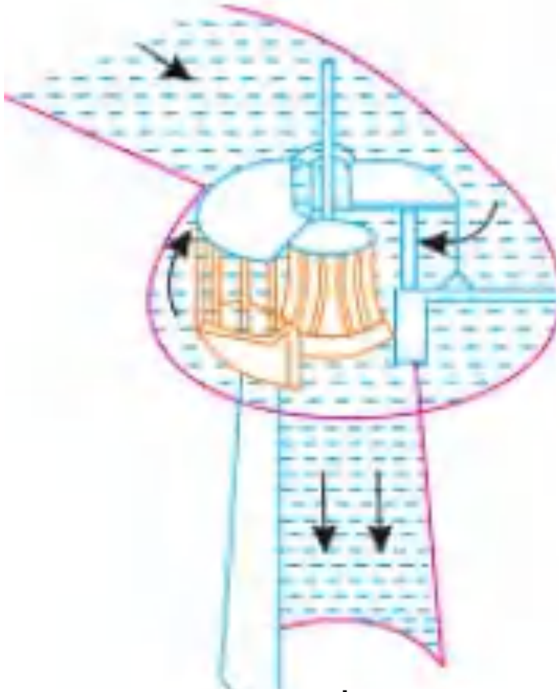
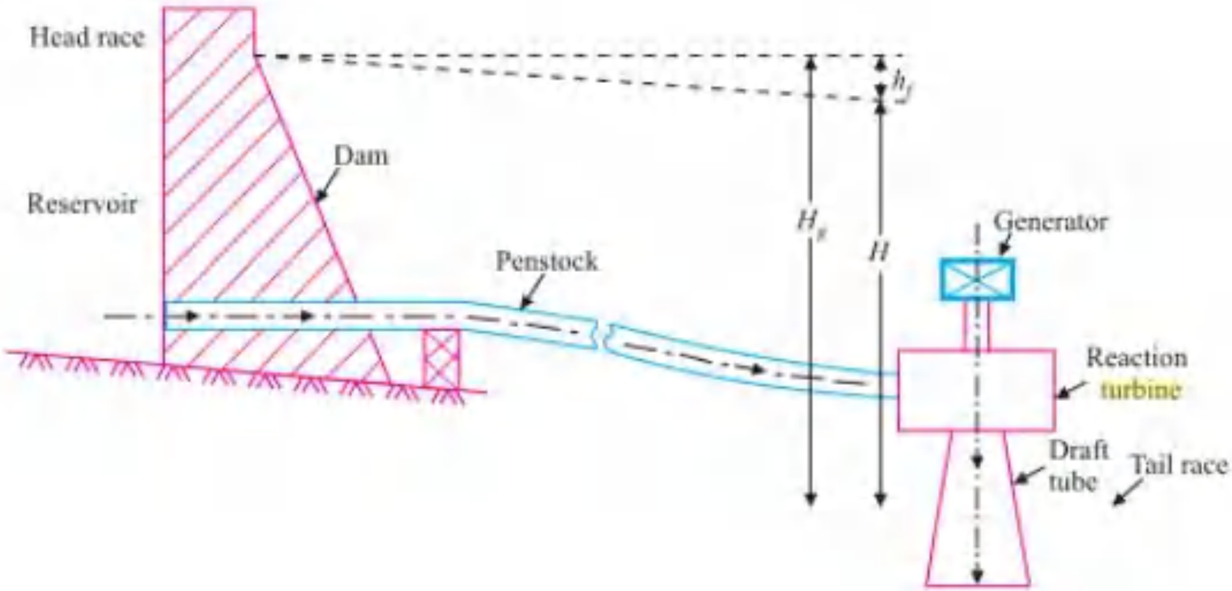
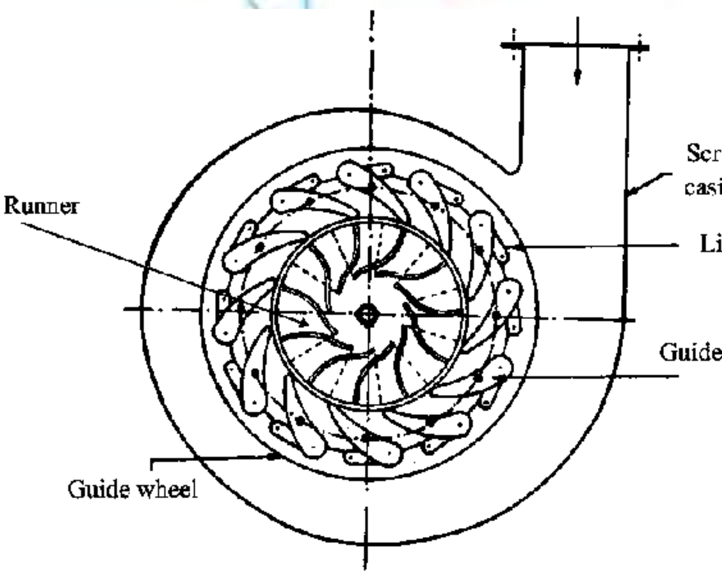


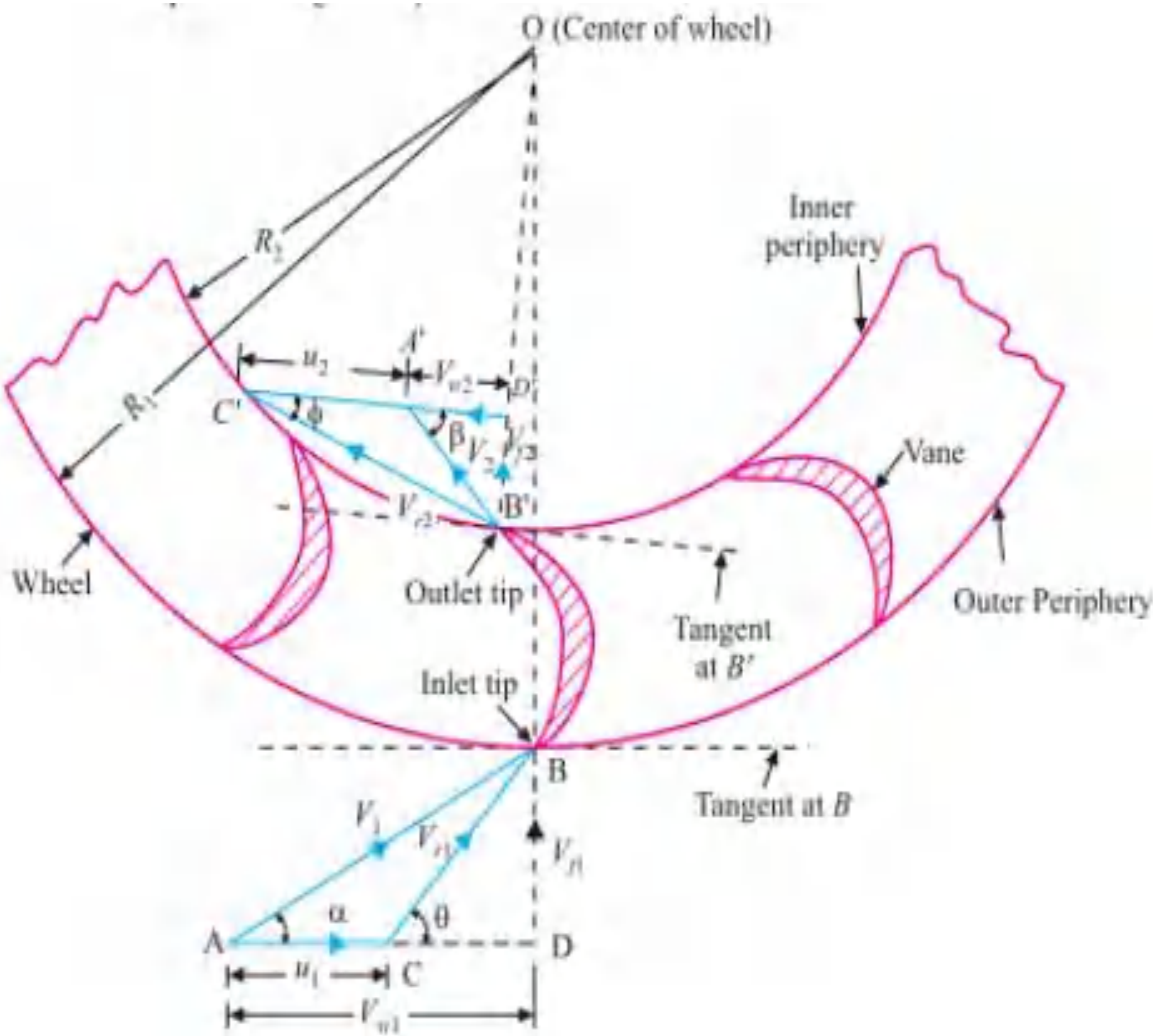
Fig. 21.5 Sectional arrangement of Francis turbine

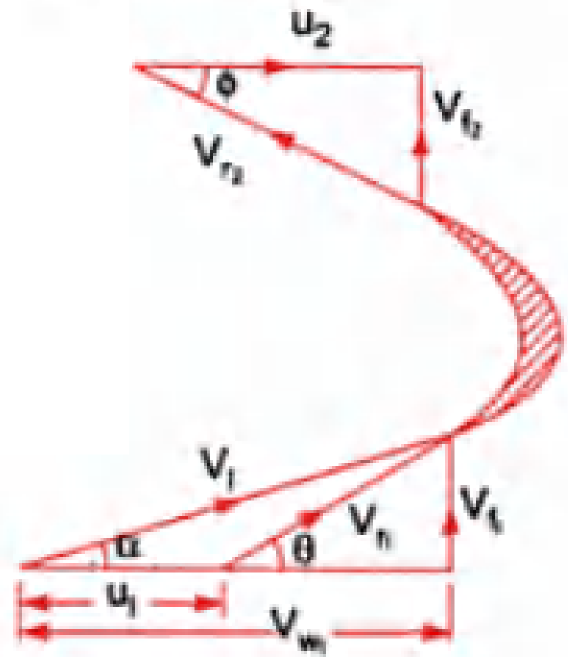






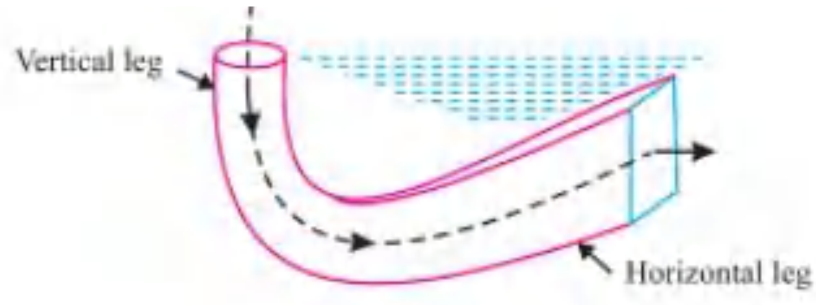
Francis Turbine

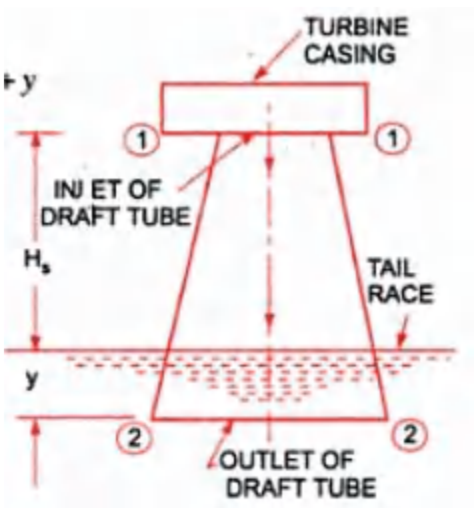

O (Center of wheel)
Inner
periphery
Vane
Wheel
Outlet tip
Tangent
at B'
Outer Periphery
Inlet tip
B
Tangent at B
A'
D'
C'
B'
A
C
D
R2
R1
u2
Vw2
Vf2
V2
Vr2
φ
β
V1
Vr1
Vf1
α
θ
u1
Vw1

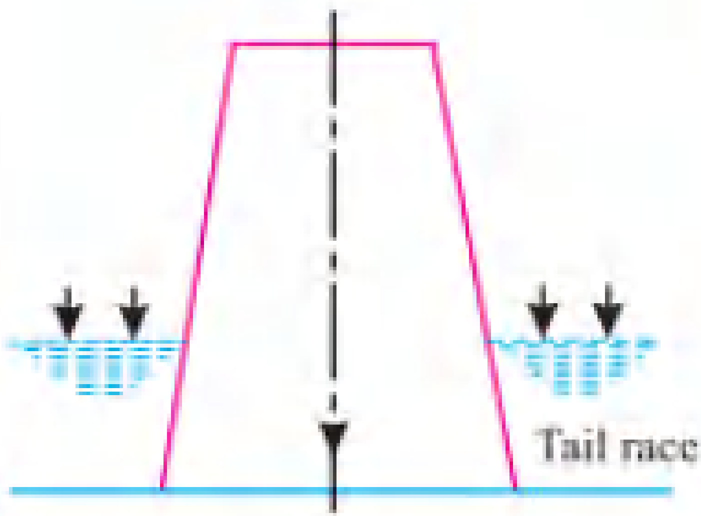

u2
φ
Vf2
Vr2
V1
Vf1
Vr1
α
θ
u1
Vw1

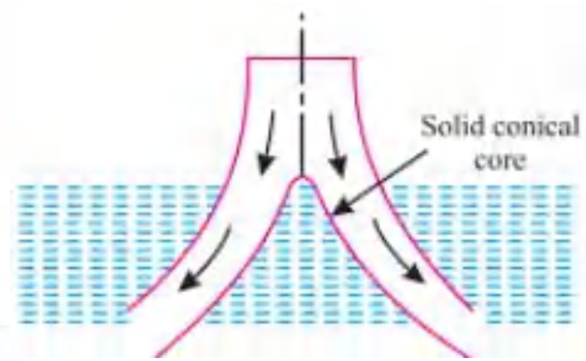
Draft tube

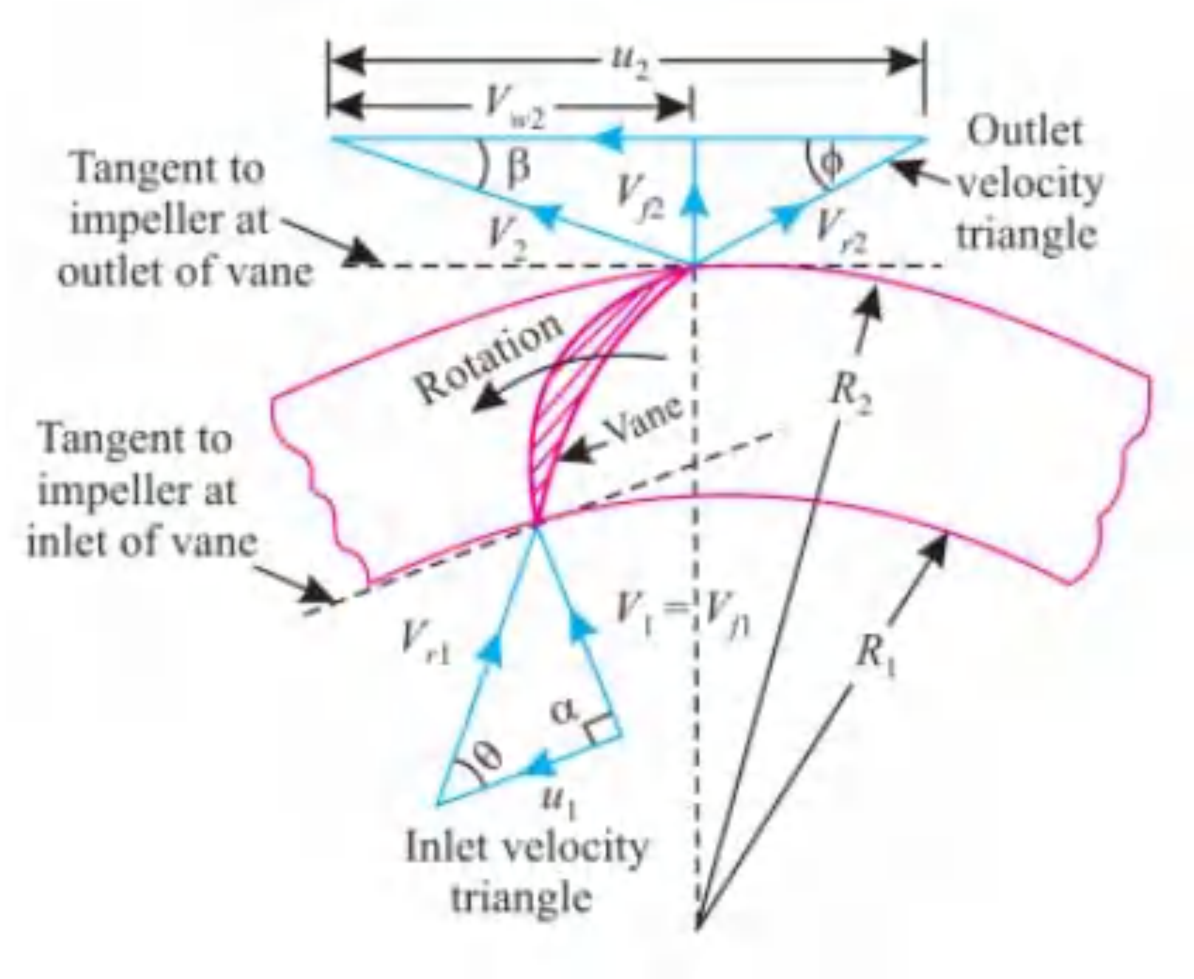
.. **Draft-tube.** The pressure at the exit of the runner of a reaction turbine is generally less than atmospheric pressure. The water at exit cannot be directly discharged to the tail race. A tube or pipe of gradually increasing area is used for discharging water from the exit of the turbine to the tail race. This tube of increasing area is called draft tube.



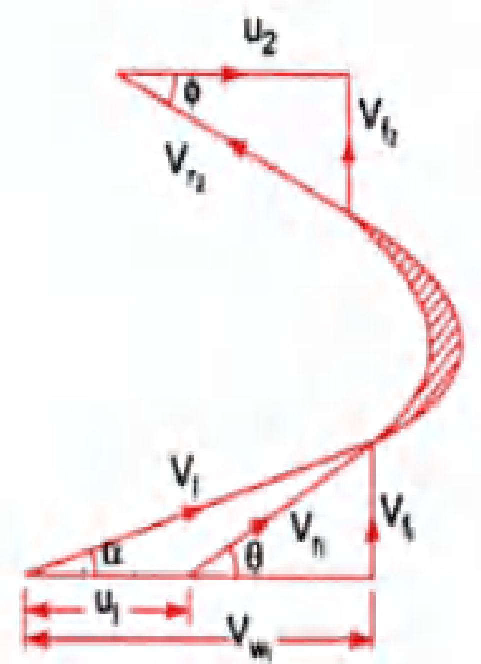








u2
Vw2
β
φ
Outlet velocity triangle
Vf2
V2
Vr2
Tangent to impeller at outlet of vane
Rotation
Vane
R2
Tangent to impeller at inlet of vane
V1 = Vf1
Vr1
R1
α
θ
u1
Inlet velocity triangle



u2
φ
Vr2
V1
α
θ
u1
Vw1

FRANCIS TURBINE-WORKING

- The guide vanes (Stationery vanes)

  -regulate the quantity of water supplied to the runner(to take care of the load variations)

  -It directs water to the runner at an appropriate angle.

- The runner consists of a series of curved vanes (moving vanes) evenly arranged around the circumference.
- At the entrance to the runner only a part of energy of water is converted into kinetic energy and substantial part remains in the form of pressure energy.
- As water flows through the runner the change from pressure to kinetic energy takes place gradually.

FRANCIS TURBINE-WORKING

- The difference in pressure between the inlet and outlet of the runner is called reaction pressure.
- Water enters the runner from the guide vanes towards the centre radially and discharges out axially- Mixed flow turbine. eg-Francis Turbine
- After doing work water is discharged to the tail race through a closed tube of gradually enlarging section called draft tube.

The work done per second on the runner by water is given by equation (17.26) as

$$= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]$$

$$= \rho Q [V_{w_1} u_1 \pm V_{w_2} u_2] \qquad (\because \quad aV_1 = Q) \quad ...(18.18)$$

The equation (18.18) also represents the energy transfer per second to the runner.

where V_{w_1} = Velocity of whirl at inlet,

V_{w_2} = Velocity of whirl at outlet,

u_1 = Tangential velocity of wheel at inlet

$= \frac{\pi D_1 \times N}{60}$, where D_1 = Outer dia. of runner,

u_2 = Tangential velocity of wheel at outlet

$= \frac{\pi D_2 \times N}{60}$, where D_2 = Inner dia. of runner, N = Speed of the turbine in .r.p.m.

The work done per second per unit weight of water per second.

$$= \frac{\text{Work done per second}}{\text{Weight of water striking per second}}$$

$$= \frac{\rho Q \left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{\rho Q \times g} = \frac{1}{g}\left[V_{w_1} u_1 \pm V_{w_2} u_2\right] \quad ...(18.19)$$

This equation is known by **Euler's equation** of hydrodynamics machines. This is also known as fundamental equation of hydrodynamic machines.

In equation (18.19), +ve sign is taken if angle β is an acute angle. If β is an obtuse angle then –ve sign is taken. If $\beta = 90°$, then $V_{w_2} = 0$ and work done per second per unit weight of water striking/s become as

$$= \frac{1}{g} V_{w_1} u_1 \quad \text{...(18.20)}$$

Hydraulic efficiency is obtained from equation (18.2) as

$$\eta_h = \frac{\text{R.P.}}{\text{W.P.}} = \frac{\dfrac{W}{1000g}[V_{w_1} u_1 \pm V_{w_2} u_2]}{\dfrac{W \times H}{1000}} = \frac{(V_{w_1} u_1 \pm V_{w_2} u_2)}{gH} \quad \text{...(18.20}A\text{)}$$

where R.P. = Runner power *i.e.*, power delivered by water to the runner

W.P. = Water power

If the discharge is radial at outlet, then $V_{w_2} = 0$

$$\eta_h = \frac{V_{w_1} u_1}{gH} \quad \text{...(18.20}B\text{)}$$

18.7.3 Degree of Reaction. Degree of reaction is defined as the ratio of pressure energy change inside a runner to the total energy change inside the runner. It is represented by 'R'. Hence mathematically it can be written as

$$R = \frac{\text{Change of pressure energy inside the runner}}{\text{Change of total energy inside the runner}} \quad \text{...(18.20}C\text{)}$$

H_e = Change of total energy per unit weight inside the runner.

$$H_e = \frac{1}{g}[V_{w_1} u_1 \pm V_{w_2} u_2]$$

(*i*) *For a Pelton turbine,*

$$R = 1 - \frac{\left(V_1^2 - V_2^2\right)}{\left(V_1^2 - V_2^2\right)} = 1 - 1 = 0$$

(*ii*) *For an actual reaction turbine,* generally, the angle β is 90° so that the loss of kinetic energy at outlet is minimum (*i.e.*, V_2 is minimum).

$$R = 1 - \frac{V_{f_1}^2 \cot^2 \alpha}{2g \times [\frac{1}{g} V_{f_1}^2 \cot\alpha(\cot\alpha - \cot\theta)]}$$

$$= 1 - \frac{\cot\alpha}{2(\cot\alpha - \cot\theta)}$$

18.7.4 Definitions. The following terms are generally used in case of reaction radial flow turbines which are defined as :

(*i*) **Speed Ratio.** The speed ratio is defind as = $\frac{u_1}{\sqrt{2gH}}$

where u_1 = Tangential velocity of wheel at inlet.

(*ii*) **Flow Ratio.** The ratio of the velocity of flow at inlet (V_{f_1}) to the velocity given $\sqrt{2gH}$ is known as flow ratio or it is given as

$$= \frac{V_{f_1}}{\sqrt{2gH}}, \text{ where } H = \text{Head on turbine}$$

(*iii*) **Discharge of the Turbine.** The discharge through a reaction radial flow turbine is given by

$$Q = \pi D_1 B_1 \times V_{f_1} = \pi D_2 \times B_2 \times V_{f_2} \qquad ...(18.21)$$

where D_1 = Diameter of runner at inlet,

B_1 = Width of runner at inlet,

V_{f_1} = Velocity of flow at inlet, and

D_2, B_2, V_{f_2} = Corresponding values at outiet.

If the thickness of vanes are taken into consideration, then the area through which flow takes place is given by $(\pi D_1 - n \times t)$

where n = Number of vanes on runner and t = Thickness of each vane

The discharge Q, then is given by $Q = (\pi D_1 - n \times t)\, B_1 \times V_{f_1}$ ...(18.22)

(*iv*) The head (H) on the turbine is given by $H = \frac{p_1}{\rho \times g} + \frac{V_1^2}{2g}$ ...(18.23)

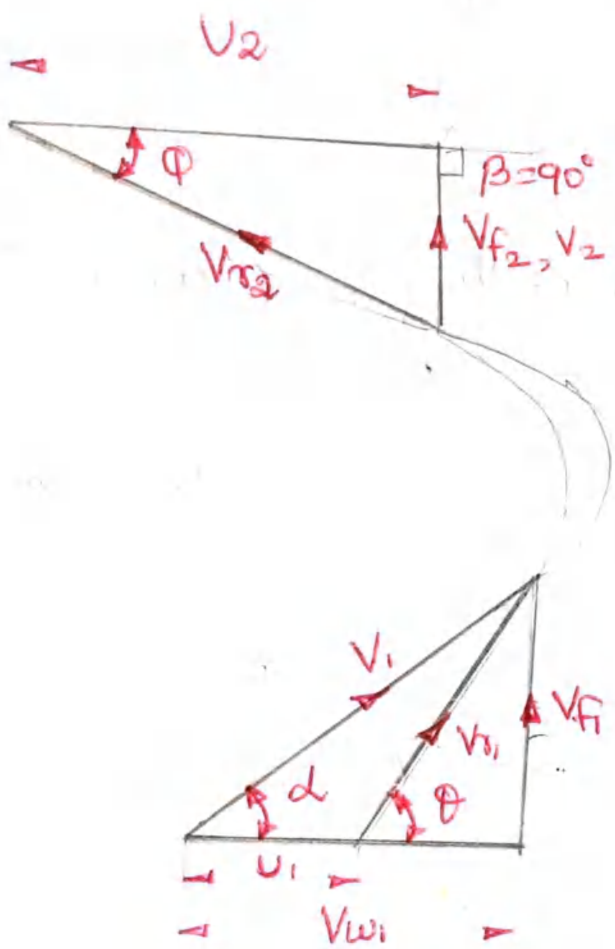
where p_1 = Pressure at inlet.

(*v*) **Radial Discharge.** This means the angle made by absolute velocity with the tangent on the wheel is 90° and the component of the whirl velocity is zero. Radial discharge at outlet means $\beta = 90°$ and $V_{w_2} = 0$, while radial discharge at outlet means $\alpha = 90°$ and $V_{w_1} = 0$.

(*vi*) If there is no loss of energy when water flows through the vanes then we have

$$H - \frac{V_2^2}{2g} = \frac{1}{g}[V_{w_1} u_1 \pm V_{w_2} u_2]. \qquad ...(18.24)$$

INWARD FLOW REACTION TURBINE(FRANCIS TURBINE)



$$V_2 = V_{f2}$$

$$V_{w2} = 0$$

$$V_{r1} = \sqrt{V_{f1}^2 + (V_{w1} - u_1)^2}$$

$$V_{f1} = V_{f2}$$

V_1 – Absolute velocity of jet at inlet

U_1 – Velocity of runner at inlet

U_2 – velocity of runner at outlet

V_{r_1} – relative velocity at inlet

V_{r_2} – relative velocity at outlet.

V_{w_1} – whirl velocity at inlet

V_{f_1} – Vertical velocity Component at inlet [velocity of flow at inlet]

α – Guide blade angle.

θ, ϕ – Runner blade angle. (or) vane angle.

RADIAL INWARD FLOW TURBINE-FORMULA

Tangential or Peripheral Velocity of the wheel at the inlet:

$$u_1 = \frac{\pi D_1 N}{60}$$

Tangential or Peripheral Velocity of the wheel at the outlet:

$$u_2 = \frac{\pi D_2 N}{60}$$

Work done/sec or Power

$$workdone / s = \rho a V_1 \left[V_{W_1} u_1 \right] \; or \; \rho Q \left[V_{W_1} u_1 \right]$$

$$P = \frac{\rho a V_1 \left[V_{W_1} u_1 \right]}{1000} K.W$$

Hydraulic efficiency

$$\eta_h = \frac{V_{W_1} u_1}{gH}$$

Discharge

Discharge = Circumferential Area x flow velocity

$$Q \ at \ inlet = \pi D_1 B_1 V f_1 \qquad Q \ at \ outlet = \pi D_2 B_2 V f_2$$

$$Q = \pi D_1 B_1 V f_1 = \pi D_2 B_2 V f_2$$

Relative velocity at the inlet

$$V_{r1} = \sqrt{{V_{f_1}}^2 + \left(V_{w1} - u_1\right)^2}$$

IMPORTANT RELATIONS FOR FRANCIS TURBINE

Ratio of width of wheel to its diameter

$$n = \frac{B_1}{D_1}$$

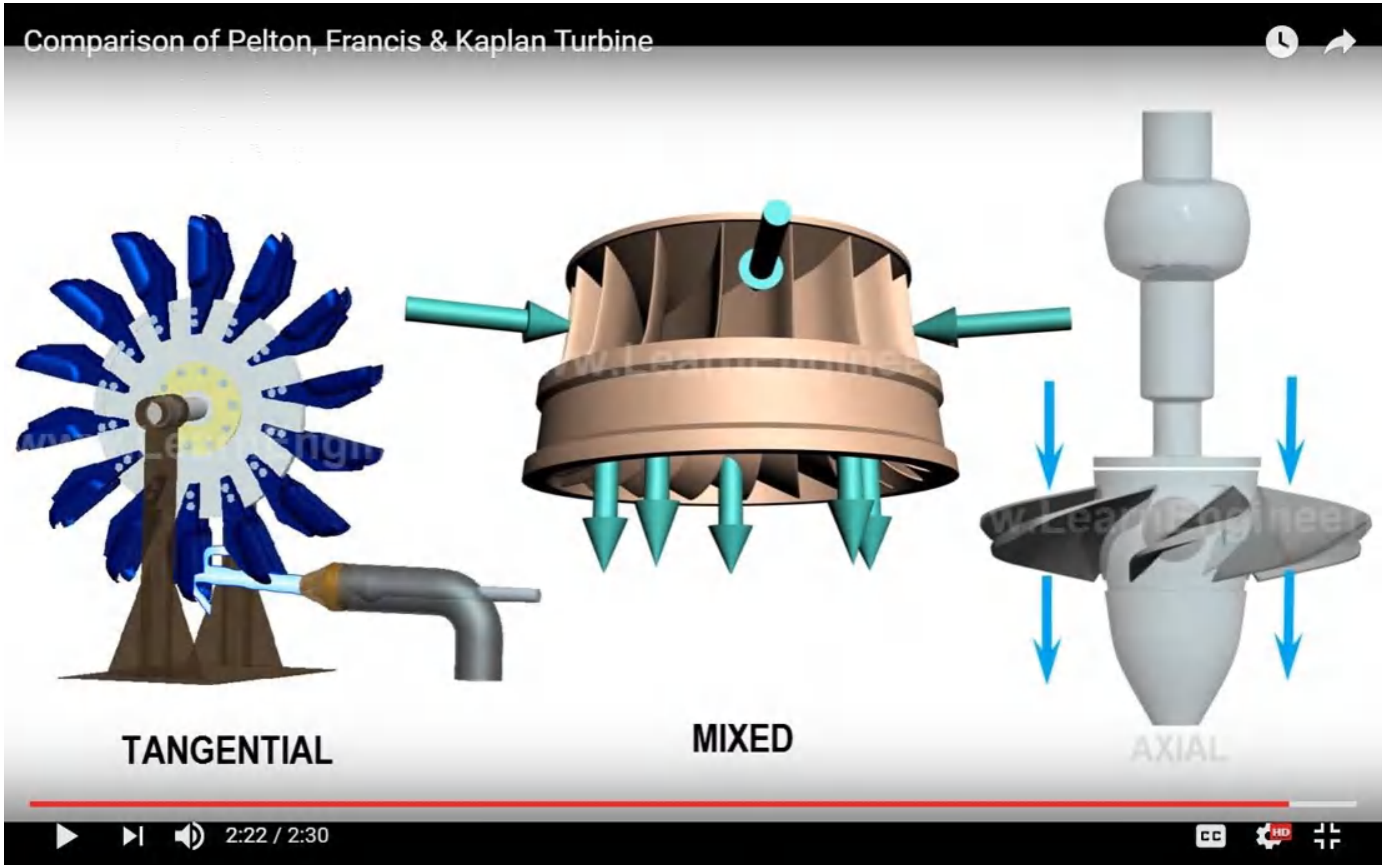
Flow ratio

$$Flow\ ratio = \frac{V_{f_1}}{\sqrt{2gH}}$$

speed ratio

$$speed\ ratio = \frac{u_1}{\sqrt{2gH}}$$

Types of Turbine- Direction of Flow



Problem 18.15 *An inward flow reaction turbine has external and internal diameters as 0.9 m and 0.45 m respectively. The turbine is running at 200 r.p.m. and width of turbine at inlet is 200 mm. The velocity of flow through the runner is constant and is equal to 1.8 mls. The guide blades make an angle of 10° to the tangent of the wheel and the discharge at the outlet of the turbine is radial. Draw the inlet and outlet velocity triangles and determine:*

(i) *The absolute velocity of water at inlet of runner,*

(ii) *The velocity of whirl at inlet, (iii) The relative velocity at inlet,*

(iv) The runner blade angles, (v) Width of the runner at outlet,

(vi) Mass of water flowing through the runner per second,

(vii) Head at the inlet of the turbine,

(viii) Power developed and hydraulic efficiency of the turbine

Solution:

External Dia., $D_1 = 0.9$ m

Internal Dia., $D_2 = 0.45$ m

Speed, $N = 200$ r.p.m.

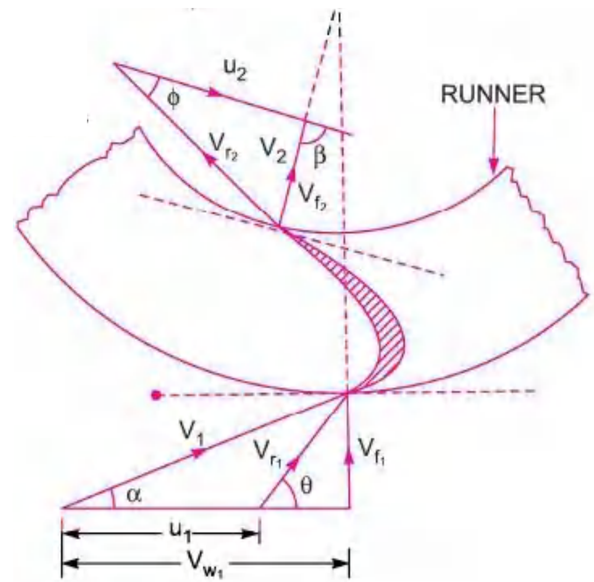
Width at inlet, $B_1 = 200$ mm $= 0.2$ m

Velocity of flow, $V_{f_1} = V_{f_2} = 1.8$ m/s

Guide blade angle, $\alpha = 10°$

Discharge at outlet = Radial

$\therefore$ $\beta = 90°$ and $V_{w_2} = 0$

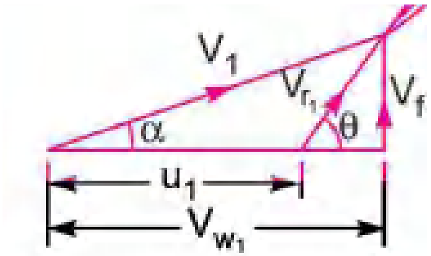


Tangential velocity of wheel at inlet and outlet are:

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times .9 \times 200}{60} = 9.424 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times .45 \times 200}{60} = 4.712 \text{ m/s.}$$

(*i*) *Absolute velocity of water at inlet of the runner i.e.,* V_1



From inlet velocity triangle,

$$V_1 \sin \alpha = V_{f_1}$$

$$\therefore \quad V_1 = \frac{V_{f_1}}{\sin \alpha} = \frac{18}{\sin 10^\circ} = \mathbf{10.365 \text{ m/s. Ans.}}$$

(*ii*) *Velocity of whirl at inlet, i.e.,* V_{w_1}

$$V_{w_1} = V_1 \cos \alpha = 10.365 \times \cos 10^\circ = \mathbf{10.207 \text{ m/s. Ans.}}$$

(*iii*) *Relative velocity at inlet, i.e.,* V_{r_1}

$$V_{r_1} = \sqrt{V_{f_1}{}^3 + (V_{w_1} - u_1)^2} = \sqrt{1.8^2 + (10.207 - 9.424)^2}$$

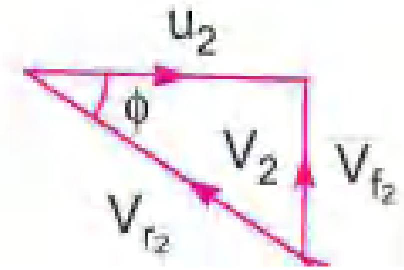
$$= \sqrt{3.24 + .613} = \mathbf{1.963 \text{ m/s. Ans.}}$$

(*iv*) *The runner blade angles* means the angle θ and ϕ

$$\text{Now} \quad \tan \theta = \frac{V_{f_1}}{(V_{w_1} - u_1)} = \frac{1.8}{(10.207 - 9.424)} = 2.298$$

$$\therefore \quad \theta = \tan^{-1} 2.298 = \mathbf{66.48^\circ} \text{ or } \mathbf{66^\circ\ 29'. \text{ Ans.}}$$

From outlet velocity triangle, we have



$$\tan\phi = \frac{V_{f_2}}{u_2} = \frac{1.8}{4.712} = \tan 20.9°$$

$\therefore$ $\phi = \mathbf{20.9°}$ or $\mathbf{20° \ 54.4'}$. **Ans.**

(*v*) *Width of runner at outlet, i.e., B_2*

From equation (18.21), we have

$$\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2} \text{ or } D_1 B_1 = D_2 B_2 \qquad \left(\because \ \pi V_{f_1} = \pi V_{f_2} \text{ as } V_{f_1} = V_{f_2}\right)$$

$\therefore$ $$B_2 = \frac{D_1 B_1}{D_2} = \frac{0.90 \times 0.20}{0.45} = 0.40 \text{ m} = \mathbf{400 \text{ mm}}. \textbf{ Ans.}$$

(*vi*) *Mass of water flowing through the runner per second.*

The discharge, $Q = \pi D_1 B_1 V_{f_1} = \pi \times 0.9 \times 0.20 \times 1.8 = 1.0178 \text{ m}^3/\text{s}$.

$\therefore$ Mass $= \rho \times Q = 1000 \times 1.0178$ kg/s $=$ **1017.8 kg/s. Ans.**

(*vii*) *Head at the inlet of turbine, i.e., H.*

Using equation (18.24), we have

$$H - \frac{V_2^2}{2g} = \frac{1}{g}(V_{w_1} u_1 \pm V_{w_2} u_2) = \frac{1}{g}(V_{w_1} u_1) \qquad (\because \text{ Here } V_{w_2} = 0)$$

$$H = \frac{1}{g} V_{w_1} u_1 + \frac{V_2^2}{2g} = \frac{1}{9.81} \times 10.207 \times 9.424 + \frac{1.8^2}{2 \times 9.81} \quad \left(\because V_2 = V_{f_2}\right)$$

$$= 9.805 + 0.165 = \mathbf{9.97 \text{ m}}. \textbf{ Ans.}$$

(*viii*) *Power developed*, *i.e.*, $P = \dfrac{\text{Work done per second on runner}}{1000}$

$$= \frac{\rho Q \left[V_{w_1} u_1\right]}{1000} \qquad \text{[Using equation (18.18)]}$$

$$= 1000 \times \frac{1.0178 \times 10.207 \times 9.424}{1000} = \mathbf{97.9\ kW.\ Ans.}$$

Hydraulic efficiency is given by equation (18.20*B*) as

$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{10.207 \times 9.424}{9.81 \times 9.97} = 0.9834 = \mathbf{98.34\%.\ Ans.}$$

1. A inward Reaction turbine works at 450 rpm under a head of 120 m. Its diameter at inlet is 1.20 m &the flow area is 0.4 m^2.The angle made by absolute and relative velocity at the inlet are 20^0 am 60^0 respectively with the tangential velocity .find 1.the discharge 2.runner power 3.Hydraulic efficiency.

Give Data:

$N = 450 rpm$

$H = 120m$

$D_1 = 1.2m$

$Flow\ area = \pi D_1 B_1 = 0.4m^2$

$\alpha = 20^0$

$\theta = 60^0$

To find:

$Q, Power, \eta_h$

Solution:

Discharge $Q = \pi D_1 B_1 V f_1$

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} \qquad V_{f_1} = 0.364 V_{w_1}$$

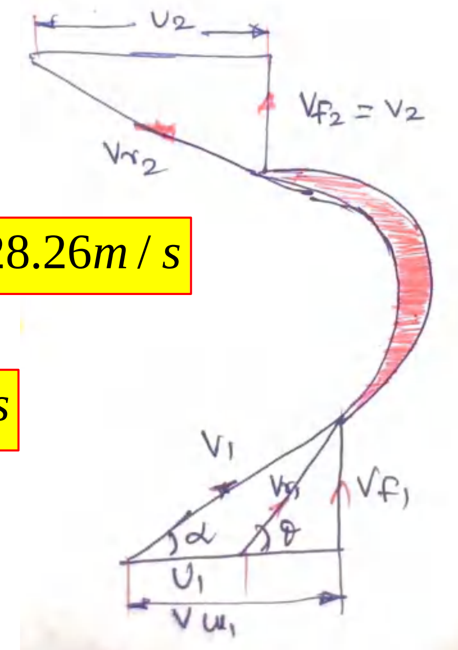
$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} \Rightarrow u_1 = \frac{\pi D_1 N}{60} \Rightarrow u_1 = 28.26 m/s$$

$$\tan 60^0 = \frac{0.364 V_{w_1}}{V_{w_1} - 28.26} \qquad V_{w_1} = 35.789 m/s$$

V_{f1}=13.023 m/s

Runner Power $Q = 5.21 m^3/s$

$$P = \frac{\rho a V_1 \left[V_{W_1} u_1\right]}{1000} K.W \qquad P = 5272.402 K.W$$



Hydraulic efficiency

$\eta_h = v_{w1} u_1/(gH)$ ⇨ η_h = 35.7895 * 28.27 / (9.81 * 120) ⇨ $\eta_h = 0.8595$

⇨ $\eta_h = 85.95\%$

2 A Francis turbine with an overall efficiency of **75%** is required to produce **148.25kW** power. It is Working under a head of 7.62m.the peripheral velocity= **0.26√ 2gH** & the flow velocity at inlet is **= 0.96 √ 2gH**, The wheels runs at **150rpm.** The hydraulic losses in the turbine are **22%** Of the available energy. find 1.The guide blade angle 2.Wheel vane Angle at inlet 3.Diameter &width of wheel at inlet.

Give Data:

$$\eta_0 = 0.75$$

$$H = 7.62m$$

$$N = 150rpm$$

$$u_1 = 0.26\sqrt{2gH}$$

$$V_{f_1} = 0.96\sqrt{2gH}$$

To find:

$$\alpha, \theta, D_1, B_2$$

1.Guide blade angle:

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} \rightarrow \eta_h = \frac{V_{W_1}u_1}{gH} \rightarrow V_{W_1} = \frac{\eta_h gH}{u_1}$$

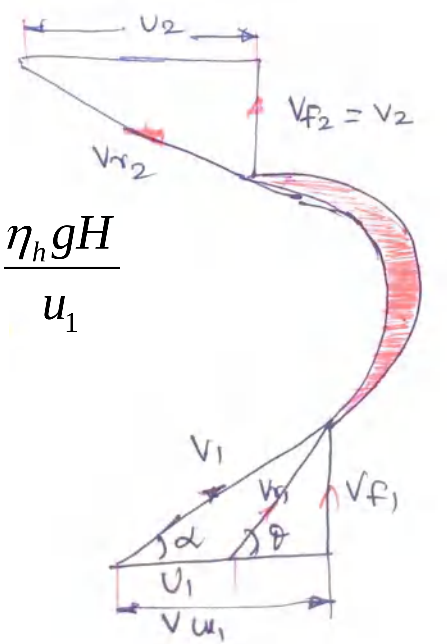
$$\alpha = 32.62^0$$

$$\eta_h = 0.78$$

$$u_1 = 0.26\sqrt{2gH}$$

$$u_1 = 3.17m/s$$

$$V_{W_1} = 18.34m/s$$



2.Vane angle at inlet:

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} \qquad V_{f_1} = 0.96\sqrt{2gH}$$

$$\theta = \tan^{-1}\left[\frac{V_{f_1}}{V_{w_1} - u_1}\right] \qquad \theta = 37.73^0$$

3.Diameter at inlet:

$$u_1 = \frac{\pi D_1 N}{60} \qquad D_1 = \frac{u_1 * 60}{\pi N}$$

$$D_1 = 0.4047m$$

4.Width of the wheel at the inlet

$$Q = \pi D_1 B_1 V f_1$$

$$B_1 = \frac{Q}{\pi D_1 V_{f_1}}$$

$$\eta_0 = \frac{P}{\left[\frac{\rho g Q H}{1000}\right]}$$

$$\eta_0 = \frac{P*1000}{\rho g Q H}$$

$$Q = \frac{P*1000}{\eta_0 \rho g H}$$

$Q = 2.644 m^3 / s$

$B_1 = 0.177 mm$

3. An inward flow reaction turbine has external and internal dia as 0.9m &0.45m respectively. Turbine is running at 200rpm and width of turbine at inlet is 200mm.The velocity of flow through Runner is constant & is equal to 1.8m/s. The guide blades make an angle of 10^0 to the tangent wheel & the discharge is radial at outlet. Find

1.Absolute velocity at inlet of runner
2.Whirl velocity at inlet
3.relative velocity at inlet
4.The runner blade angle
5.Width of the runner at outlet
6.Mass flow through runner per second
7.Head at the inlet of turbine
8.Power
9.Hydraulic efficiency

Give Data:

$D_1 = 0.9m$

$D_2 = 0.45m$

$N = 200rpm$

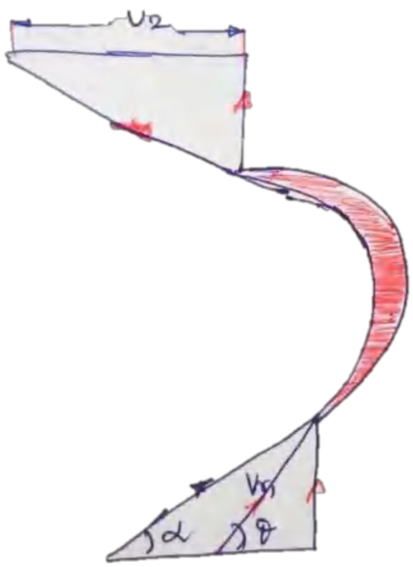
$V_{f_1} = V_{f_2} = 1.8m/s$

$\alpha = 10^0$

$B_1 = 200mm$

To find:

$V_1, V_{w_1}, V_{r_1}, \theta, \phi, B_2, Q, H, P, \eta_h$



Solution:

1.Absolute velocity

$$\sin\alpha = \frac{V_{f_1}}{V_1} \qquad V_1 = \frac{V_{f_1}}{\sin 10}$$

$$V_1 = 10.365 m/s$$

2.Whirl velocity

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}}$$

$$V_{w_1} = 10.208 m/s$$

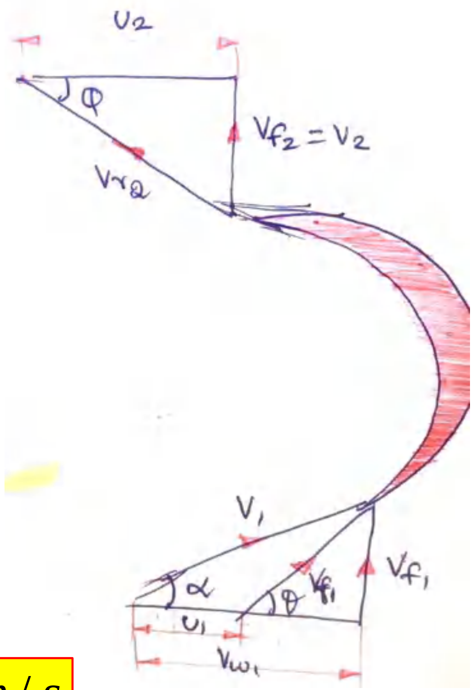
3.Relative velocity at the inlet

$$V_{r1} = \sqrt{V_{f_1}{}^2 + \left(V_{w1} - u_1\right)^2}$$

$$V_{r1} = 1.963 m/s$$

$$u_1 = \frac{\pi D_1 N}{60} \qquad u_1 = 9.42 m/s$$

$$u_2 = \frac{\pi D_2 N}{60} \qquad u_2 = 4.712 m/s$$



4.Runner blade angle

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1}$$

$$\theta = 66.35^0$$

$$\tan\phi = \frac{V_{f_2}}{u_2}$$

$$\phi = \tan^{-1}\left[\frac{V_{f_2}}{u_2}\right]$$

$$\phi = 20.9^0$$

5.Width of the runner blade at outlet

$$Q = \pi D_1 B_1 V f_1 = \pi D_2 B_2 V f_2 \qquad V_{f_1} = V_{f_2}$$

$$B_2 = \frac{D_1 B_1}{D_2}$$

$$B_2 = 400mm$$

6.Mass flow rate through runner

$$Q = \pi D_1 B_1 V_{f_1} \qquad Q = 1.0178 m^3 / s$$

$$m^0 = \rho * Q \qquad m^0 = 1000 * 1.0178 \qquad m^0 = 1017.8 kg / s$$

7.Head at the inlet of the turbine

$$H = \frac{1}{g} V_{w1} u_1 + \frac{V^2}{2g} \qquad V_2 = V_{f_1} = V_{f_2}$$

$$H = 9.97 m$$

8.Hydraulic efficiency:

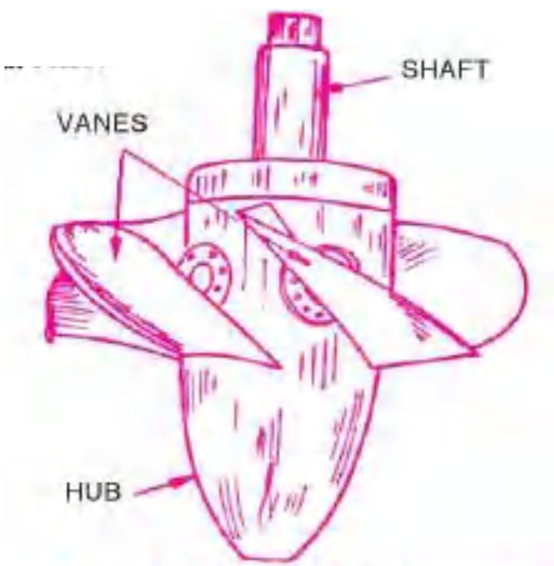
$$\eta_h = \frac{V_{w_1} u_1}{gH} * 100 \qquad \eta_h = 98.34\%$$

KAPLAN TURBINE

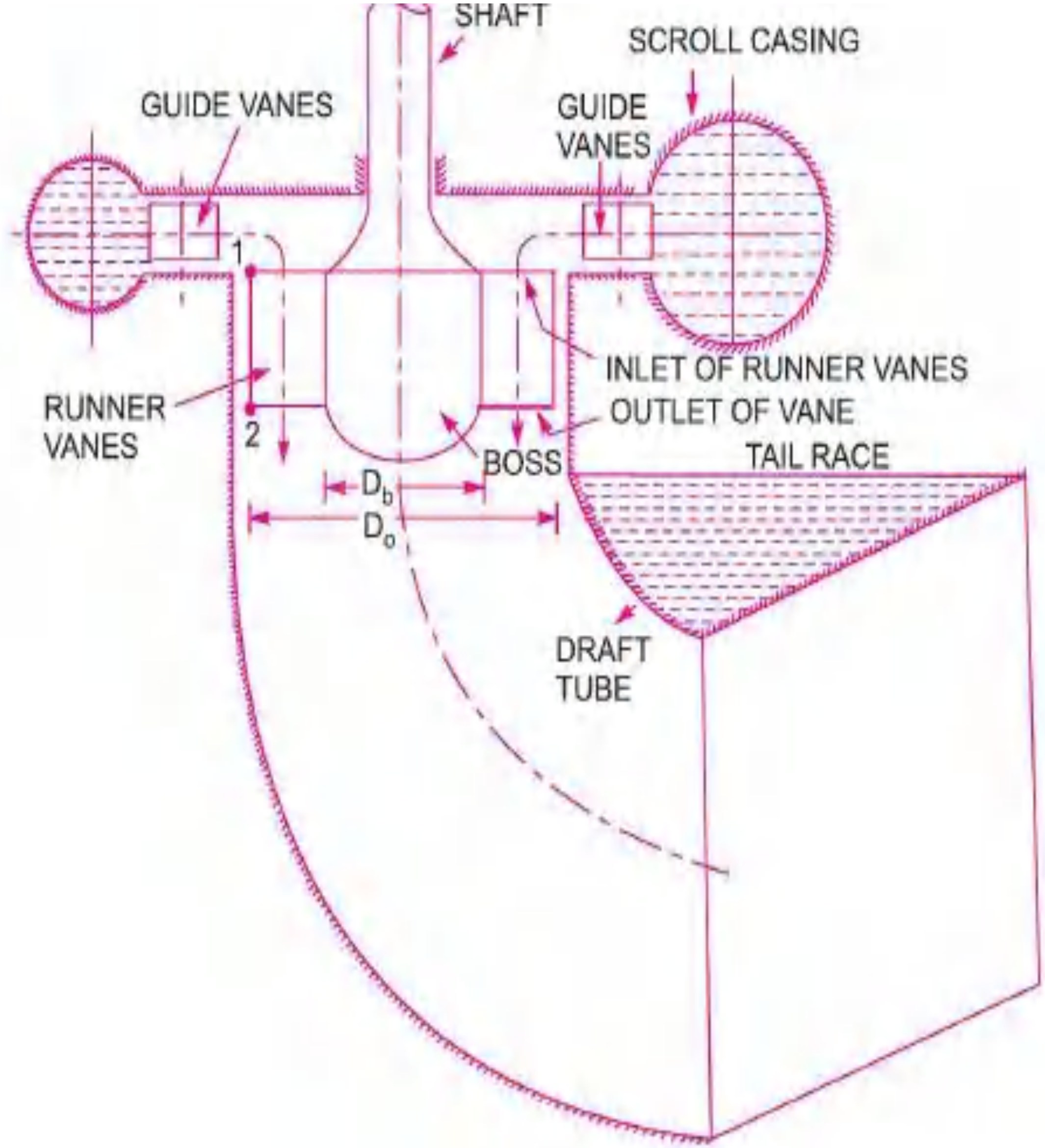
- Developed by Austrian Engineer - Kaplan.
- Kaplan turbine is a reaction turbine – Both Pressure and Kinetic energy is available in the turbine.
- It is an Axially Flow Turbine - water enters and leaves the runner blades axially . So it is called Axial flow turbine
- Low head turbine (Less than 30m)
- So it need high Discharge to create a force on the runner.
- High Specific Speed.(N_s)

Components

- ➢1. Scroll casing
- ➢2. Guide Vanes
- ➢3. Runner Vanes
- ➢4. Hub or Boss
- ➢5. Shaft



. 18.25 *Kaplan turbine runner.*



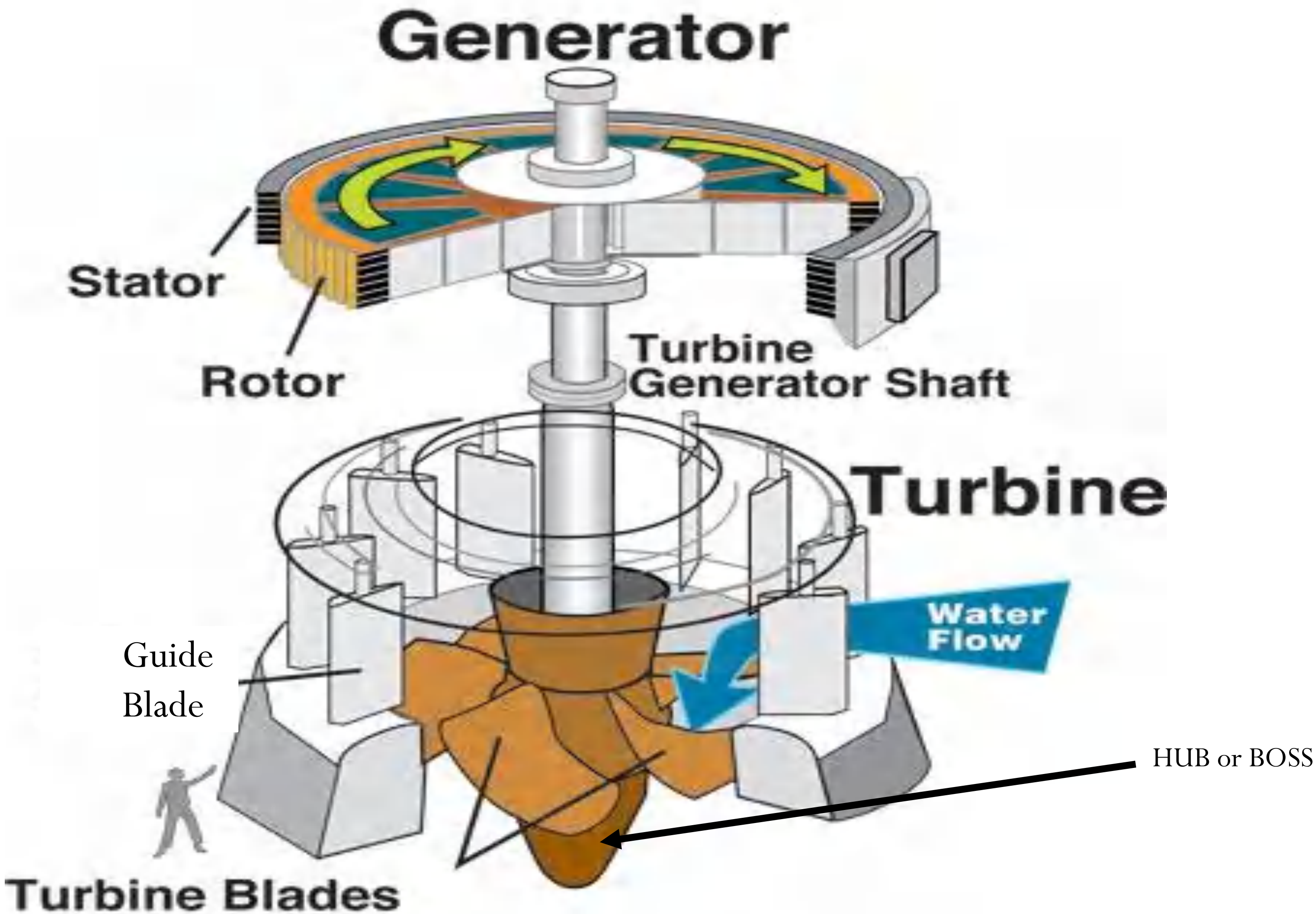
KAPLAN TURBINE- COMPONENTS

- The shaft of an axial flow reaction turbine is vertical.
- The lower end of the shaft is made bigger and is known as hub or boss.
- The runner vanes are fixed on the hub or boss.

PROPELLER TURBINE - if runner blades cannot be adjusted

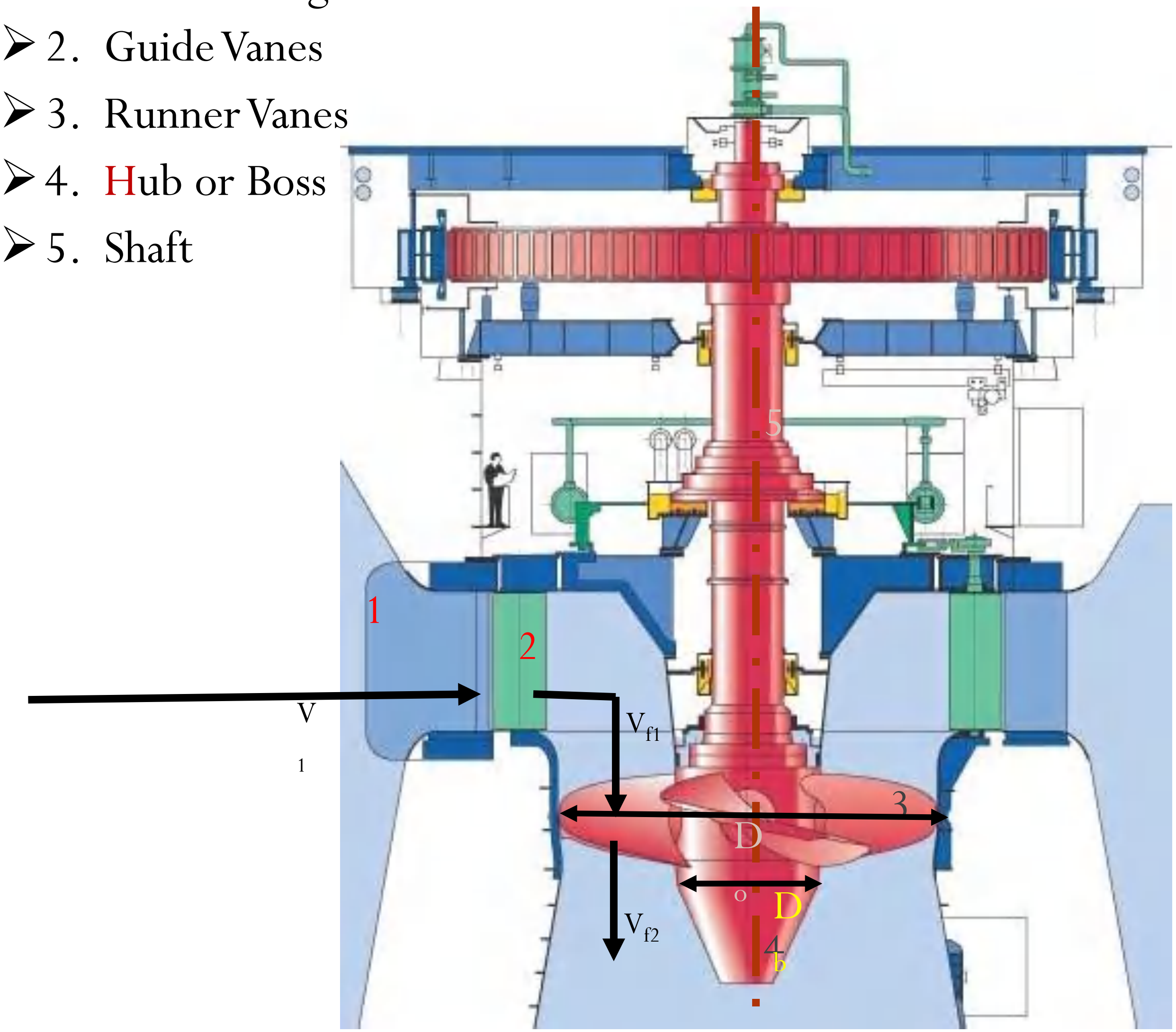
KAPLAN TURBINE - if runner blades can be adjusted

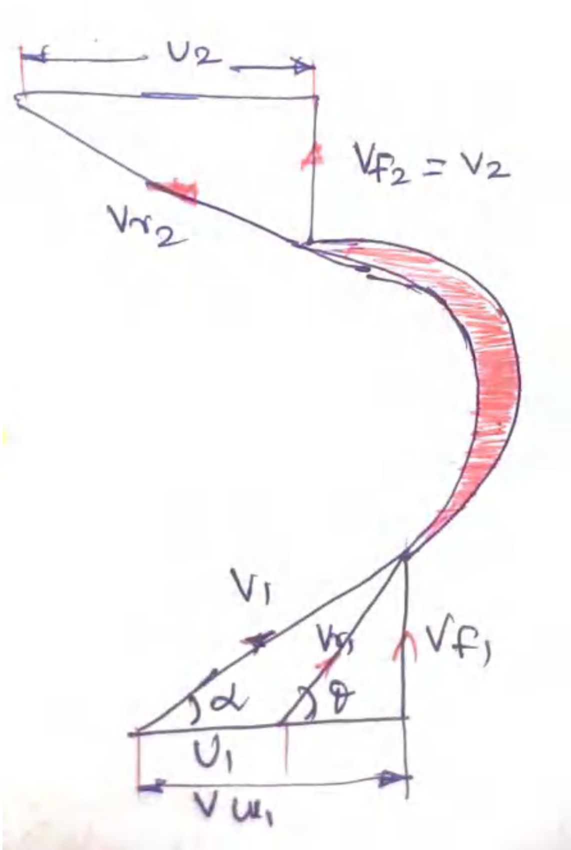
KAPLAN TURBINE ...1


Generator
Stator
Rotor
Turbine
Generator Shaft
Turbine
Water
Flow
Guide
Blade
HUB or BOSS
Turbine Blades

Components

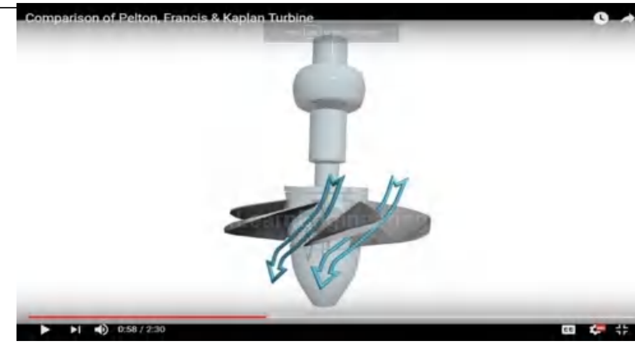
- 1. Scroll casing
- 2. Guide Vanes
- 3. Runner Vanes
- 4. Hub or Boss
- 5. Shaft

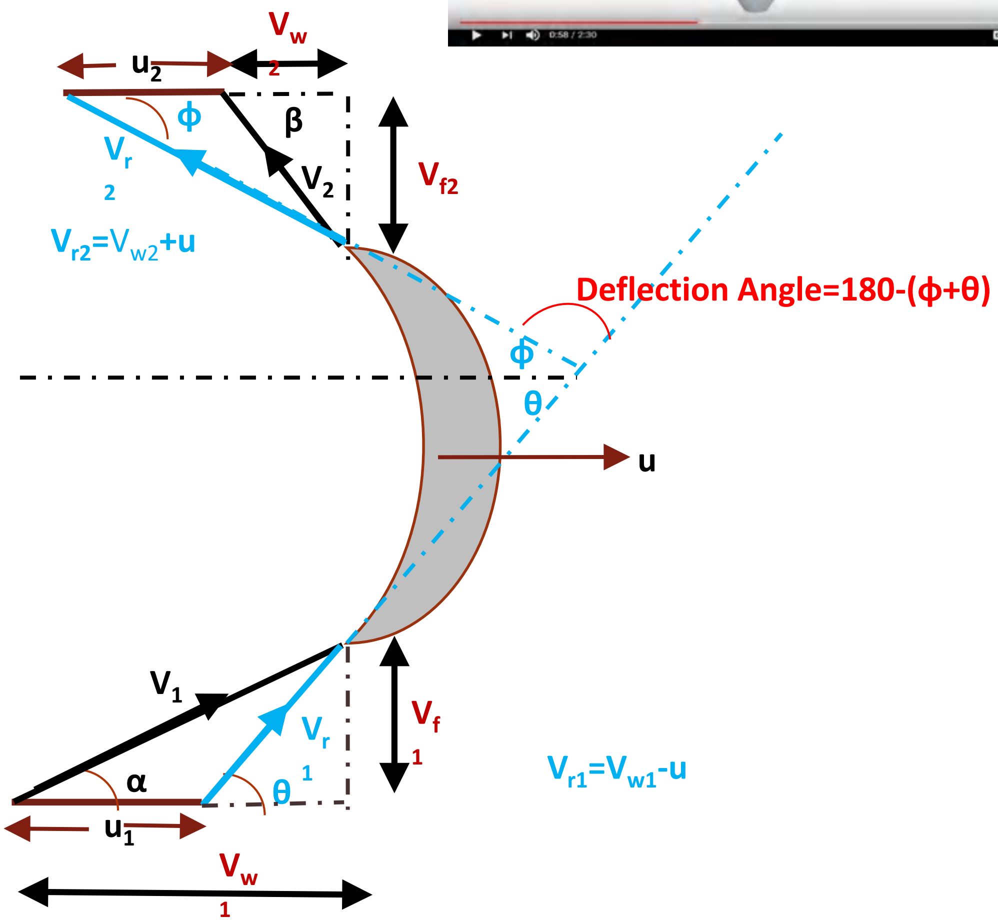




Velocity Triangle for Kaplan

$\beta = 90^o \quad V_{w2}=0 \quad V_{f2}=V_2$





AXIAL FLOW TURBINE-FORMULAS

Velocity of the runner:

$$u_1 = u_2 = \frac{\pi D_0 N}{60}$$

Velocity Triangle for Kaplan

$\beta = 90^o$ $V_{w2}=0$ $V_{f2}=V_2$

Velocity of the flow at the inlet & outlet:

$$V_{f_1} = V_{f_2}$$

Area at the inlet & outlet:

$$A_{inlet} = A_{outlet} = \frac{\pi}{4}\left[D_0^{\ 2} - D_b^{\ 2}\right]$$

D_0 – *Outer dia of runner*

D_b – *diameter of Hub*

Discharge

$$Q = Area * Vf_1$$

TYPE-1 Find Vane Angle – θ & φ, Speed N

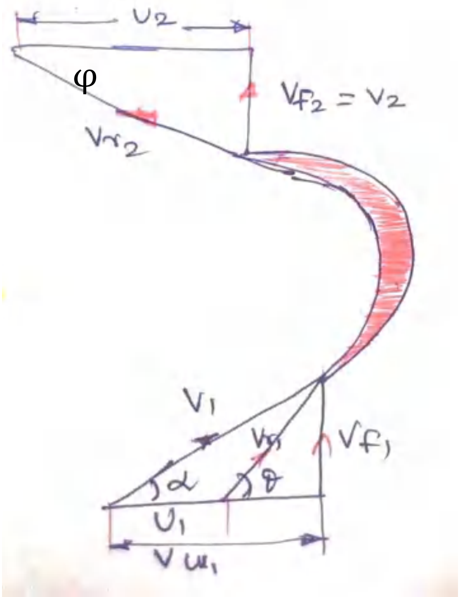
Peripheral Velocity at inlet and outlet are equal,

$$u_1 = u_2 = \frac{\pi * D_0 * N}{60}$$

Flow Velocity @ inlet = outlet

$$V_{f1} = V_{f2}$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1}$$



Step -1 To find V_{f1}

Discharge Q = Area x Velocity

$$Q = \frac{\pi}{4}(D_o^2 - Db^2) * V_{f1}$$

D_o- Outer Dia of Runner

D_b- Dia of Hub

Step- 2 To find V_{w1}

$$\tan\alpha = \frac{V_{f1}}{V_{w1}}$$

Discharge is calculated by using overall efficiency and Shaft Power

---

Step -3 To find u_1

$$\text{Hydraulic Efficiency} = \frac{R.P}{W.P}$$

$$\text{Hydraulic Efficiency} = \frac{\rho * Q * (Vw1 + Vw2) * u}{\rho * g * Q * H} = \frac{V_{w1} * u}{g * H} =$$

TYPE-2 Find diameter of outer runner and hub D_o and D_b and also speed of turbine N. and specific speed - Given Flow ratio , diameter ratio(Do and Db) and speed ratio

Step-1 Flow ratio$=\frac{V_{f1}}{\sqrt{2gHnet}}$ Speed ratio$=\frac{u}{\sqrt{2gHnet}}$

Step-2 Discharge Q = Area x Velocity

$$Q=\frac{\pi}{4}({D_o}^2 - Db^2) * Vf_1$$

D_o- Outer Dia of Runner

D_b- Dia of Hub

Discharge is calculated by using overall efficiency and Shaft Power

In Step -2 , Using Diameter ratio replace D_b in terms of D_o.

So the entire discharge equation has only one unknown D_o

Problem 18.27 *A Kaplan turbine working under a head of 20 m develops 11772 kW shaft power. The outer diameter of the runner is 3.5 m and hub diameter is 1.75 m. The guide blade angle at the extreme edge of the runner is 35°. The hydraulic and overall efficiencies of the turbines are 88% and 84% respectively. If the velocity of whirl is zero at outlet, determine: (i) Runner vane angles at inlet and outlet at the extreme edge of the runner, and*
(ii) Speed of the turbine.

Solution. Given :

Head, $H = 20$ m

Shaft power, S.P. = 11772 kW

Outer dia. of runner, $D_o = 3.5$ m

Hub diameter, $D_b = 1.75$ m

Guide blade angle, $\alpha = 35°$

Hydraulic efficiency, $\eta_h = 88\%$

Overall efficiency, $\eta_o = 84\%$

Velocity of whirl at outlet = 0.

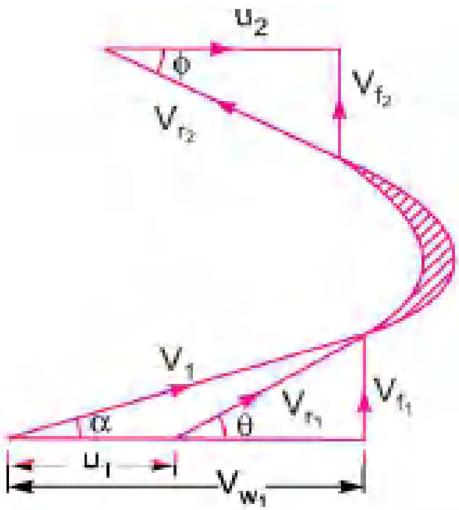


Fig. 18.27

Using the relation, $\eta_o = \dfrac{\text{S.P.}}{\text{W.P.}}$

where $\text{W.P.} = \dfrac{\text{W.P.}}{1000} = \dfrac{\rho \times g \times Q \times H}{1000}$, we get

$$0.84 = \frac{11772}{\dfrac{\rho \times g \times Q \times H}{1000}}$$

$$= \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 20} \qquad (\because \ \rho = 1000)$$

$\therefore$
$$Q = \frac{11772 \times 1000}{0.84 \times 1000 \times 9.81 \times 20} = 71.428 \text{ m}^3/\text{s}.$$

Using equation (18.25),
$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$$

or
$$71.428 = \frac{\pi}{4}\,(3.5^2 - 1.75^2) \times V_{f_1} = \frac{\pi}{4}\,(12.25 - 3.0625)\ V_{f_1}$$
$$= 7.216\ V_{f_1}$$

$\therefore$
$$V_{f_1} = \frac{71.428}{7.216} = 9.9 \text{ m/s}.$$

From inlet velocity triangle, $\tan \alpha = \dfrac{V_{f_2}}{V_{w_1}}$

$\therefore$
$$V_{w_1} = \frac{V_{f_1}}{\tan \alpha} = \frac{9.9}{\tan 35^\circ} = \frac{9.9}{.7} = 14.14 \text{ m/s}$$

Using the relation for hydraulic efficiency,

$$\eta_h = \frac{V_{w_1} u_1}{gH} \qquad (\because \; V_{w_2} = 0)$$

$$0.88 = \frac{14.14 \times u_1}{9.81 \times 20}$$

$$\therefore \quad u_1 = \frac{0.88 \times 9.81 \times 20}{14.14} = 12.21 \text{ m/s.}$$

(*i*) Runner vane angles at inlet and outlet at the extreme edge of the runner are given as :

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{9.9}{(14.14 - 12.21)} = 5.13$$

$$\therefore \quad \theta = \tan^{-1} 5.13 = \mathbf{78.97^\circ} \text{ or } \mathbf{78^\circ\ 58'}. \textbf{ Ans.}$$

For Kaplan turbine, $u_1 = u_2 = 12.21$ m/s and $V_{f_1} = V_{f_2} = 9.9$ m/s

$\therefore$ From outlet velocity triangle, $\tan\phi = \frac{V_{f_2}}{u_2} = \frac{9.9}{12.21} = 0.811$

$$\therefore \quad \phi = \tan^{-1} .811 = \mathbf{39.035^\circ} \text{ or } \mathbf{39^\circ\ 2'}. \textbf{ Ans.}$$

(*ii*) Speed of turbine is given by $u_1 = u_2 = \frac{\pi D_o N}{60}$

$$12.21 = \frac{\pi \times 3.5 \times N}{60}$$

$$\therefore \quad N = \frac{60 \times 12.21}{\pi \times 3.50} = \mathbf{66.63 \text{ r.p.m.}} \textbf{ Ans.}$$

Problem 18.31 *A Kaplan turbine runner is to be designed to develop 7357.5 kW shaft power. The net available head is 5.50 m. Assume that the speed ratio is 2.09 and flow ratio is 0.68, and the overall efficiency is 60%. The diameter of the boss is 1/3rd of the diameter of the runner. Find the diameter of the runner, its speed and its specific speed.*

Solution. Given :

Shaft power, $P = 7357.5 \text{ kW}$

Head, $H = 5.50 \text{ m}$

Speed ratio $= \dfrac{u_1}{\sqrt{2gH}} = 2.09$

$\therefore$ $u_1 = 2.09 \times \sqrt{2 \times 9.81 \times 5.50} = 21.71 \text{ m/s}$

Flow ratio $= \dfrac{V_{f_1}}{\sqrt{2gH}} = 0.68$

$\therefore$ $V_{f_1} = 2.68 \times \sqrt{2 \times 9.81 \times 5.50} = 7.064 \text{ m/s}$

Overall efficiency, $\eta_o = 60\% = 0.60$

Diameter of boss, $D_b = \dfrac{1}{3} \times D_o$

Using relation, $$\eta_o = \frac{\text{Shaft power}}{\text{Water power}} = \frac{7357.5}{\dfrac{\rho \times g \times Q \times H}{1000}}$$

or $$0.60 = \frac{7357.5 \times 1000}{\rho \times g \times Q \times H} = \frac{7357.5 \times 1000}{1000 \times 9.81 \times Q \times 5.5}$$

∴ $$Q = \frac{7357.5 \times 1000}{1000 \times 9.81 \times 5.5 \times 0.60} = 227.27 \text{ m}^3\text{/s.}$$

Using equation (18.25) for discharge,

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$$

or $$227.27 = \frac{\pi}{4}\left[D_o^2 - \left(\frac{D_o}{3}\right)^2\right] \times 7.064 \qquad \left(\because \; D_b = \frac{D_o}{3}\right)$$

$$= \frac{\pi}{4} \times \frac{8}{9} D_o^2 \times 7.064 = 4.9316 \; D_o^2$$

$$\therefore \qquad D_o = \sqrt{\frac{227.27}{4.9316}} = \mathbf{6.788\ m.\ \ Ans.}$$

And
$$D_b = \frac{1}{3} \times 6.788 = \mathbf{2.262\ m.\ \ Ans.}$$

Using the relation,
$$u_1 = \frac{\pi D_o \times N}{60}$$

$$\therefore \qquad N = \frac{60 \times u_1}{\pi D_o} = \frac{60 \times 21.71}{\pi \times 6.788} = \mathbf{61.08\ r.p.m.\ \ Ans.}$$

The specific speed (N_s) is given by,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{61.08 \times \sqrt{7357.5}}{5.50^{5/4}} = \mathbf{622\ r.p.m.\ \ Ans.}$$

1. A Kaplan turbine under a head of 20m develops 11772 kW shaft power. The outer diameter of the runner 3.5m & hub diameter 1.75m.The guide blade angle of the runner is 35^0 .The hydraulic And overall efficiency are 88% & 84% respectively. If the velocity of the whirl is zero at the outlet. Find 1.Runner vane angle at inlet and outlet 2.speed of turbine. 3. Specific Speed

Give Data:

$$H = 20m$$

$$P = 11772KW$$

$$D_0 = 3.5m$$

$$D_h = 1.75m$$

$$\alpha = 35^0$$

$$\eta_h = 0.88$$

$$\eta_0 = 0.84$$

To find:

θ, ϕ, N N_s

Solution:

Runner angle at inlet- Find V_{f1},V_{w1} and u

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1}$$

$$\tan\alpha = \frac{V_{f1}}{V_{w1}}$$

$$Q = Area * Vf_1$$

$$Q = \frac{\pi}{4}\left[D_0^2 - D_b^2\right] * Vf_1$$

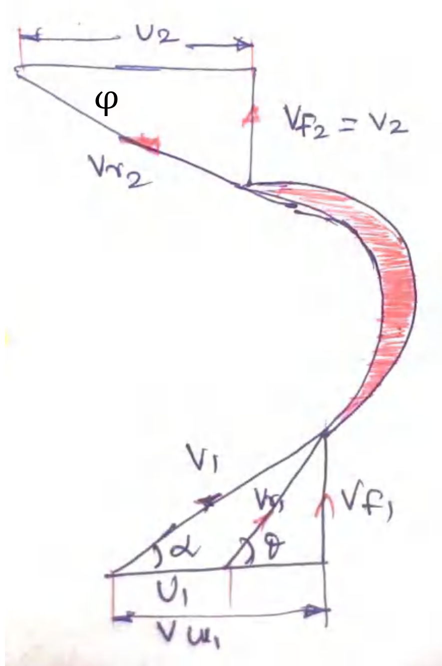
Overall Efficiency = S.P / W.P

W.P = ρ * g * Q * H

To find u_1

Hydraulic Efficiency $= \frac{R.P}{W.P} =$

Hydraulic Efficiency $= \frac{\rho * Q * (Vw1 + Vw2) * u}{\rho * g * Q * H} = \frac{V_{w1} * u}{g * H} =$



$$\eta_0 = \frac{P}{\left[\frac{\rho g Q H}{1000}\right]} \quad Q = \frac{P*1000}{\eta_0 \rho g H}$$

$$Q = 71.428 m^3/s$$

$$Vf_1 = \frac{Q*4}{\pi\left[D_0^{\ 2} - D_b^{\ 2}\right]}$$

$$Vf_1 = 9.9 m/s$$

V_{w_1} :

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}}$$

u_1 :

$$\eta_h = \frac{V_{w_1} u_1}{gH} \qquad u_1 = \frac{\eta_h g H}{V_{w_1}}$$

$$u_1 = u_2 = 12.21 m/s$$

Runner angle at inlet and outlet

$\Theta = 79^o$

$\Phi = 39^0$

Speed :

$$u = \frac{\pi D_0 N}{60}$$

$$N = \frac{u*60}{\pi D_0}$$

$$N = 66.63 rpm$$

Specific Speed :

$$N_s = \frac{N\sqrt{P}}{H^{\frac{5}{4}}}$$

Ns = 114.05

2. A Kaplan Turbine working under a head of 15m develops 7375.5 kW shaft Power. The net available head is 10m. Assume that **speed ratio is 1.8** and **flow ratio is 0.6**. If the overall efficiency is 70% and diameter of Boss (hub) is 0.4 times the diameter of runner. Find the diameter of runner and hub. Find also speed and specific speed of turbine. (tutorial)

Step-1

Flow ratio $= \frac{V_{f1}}{\sqrt{2gH}}$ Speed ratio $= \frac{u}{\sqrt{2gH}}$

Step-2

Discharge Q = Area x Velocity

$$Q = \frac{\pi}{4}(D_o - 0.4D_0)^2 * Vf_1$$

D_o- Outer Dia of Runner

D_b- Dia of Hub =0.4 D_0

Discharge is calculated by using overall efficiency and Shaft Power

$$\eta_0 = \frac{P}{\left[\frac{\rho g Q H}{1000}\right]} \qquad Q = \frac{P*1000}{\eta_0 \rho g H}$$

Answers:

V_1= 14 m/s

V_{f1}=8.4 m/s

u = 25.2 m/s

Q= 107.14 m3/s

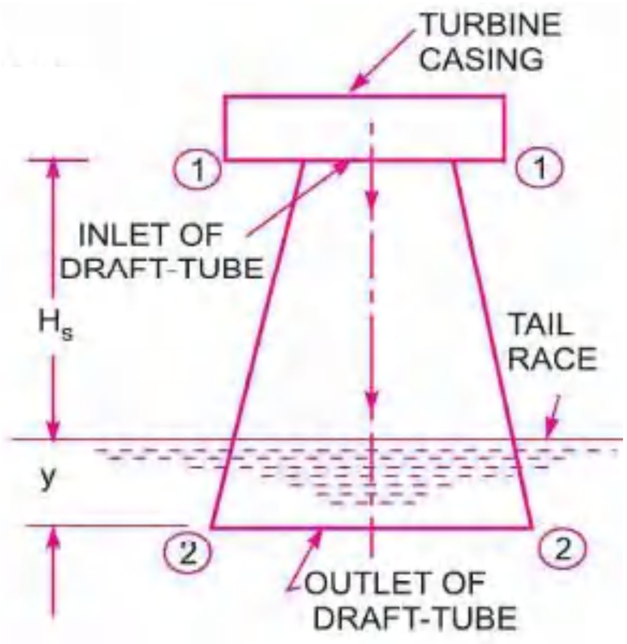
Diameter of Runner= 4.4 m

Speed of turbine N= 109 rpm

Draft-Tube Theory

Hs = Vertical height of draft-tube above the tail race,
y = Distance of bottom of draft-tube from tail race.

Applying Bernoulli's equation to inlet (section 1-1) and outlet (section 2-2) of the draft-tube and taking section 2-2 as the datum line,we get



$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+(H_s+y)=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+0+h_f$$

where *hf* = loss of energy between sections 1-1 and 2-2.

$$\frac{p_2}{\rho g}=\text{Atmospheric pressure head}+y=\frac{p_a}{\rho g}+y$$

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+(H_s+y)=\frac{p_a}{\rho g}+y+\frac{V_2^2}{2g}+h_f$$

$$\frac{p_1}{\rho g}=\frac{p_a}{\rho g}-H_s-\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}-h_f\right)$$

$\frac{p_1}{\rho g}$ is less than atmospheric pressure.

Efficiency of Draft-Tube. The efficiency of a draft-tube is defined as the ratio of actual conversion of kinetic head into pressure head in the draft-tube to the kinetic head at the inlet of the draft-tube. Mathematically, it is written as

$$\eta_d = \frac{\text{Actual conversion of kinetic head into pressure head}}{\text{Kinetic head at the inlet of draft-tube}}$$

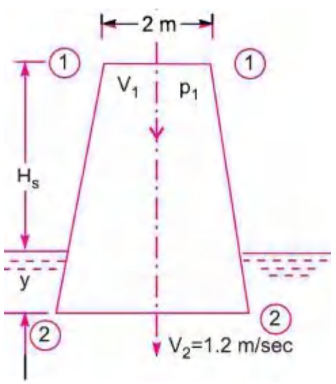
Theoretical conversion of kinetic head into pressure head in draft-tube $= \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$.

Actual conversion of kinetic head into pressure head $= \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - h_f$

V_1 = Velocity of water at inlet of draft-tube,
V_2 = Velocity of water at outlet of draft-tube, and
hf = Loss of head in the draft-tube.

$$\eta_d = \frac{\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - h_f}{\left(\frac{V_1^2}{2g}\right)}$$

Problem 18.34 *A conical draft-tube having diameter at the top as 2.0 m and pressure head at 7 m of water (vacuum), discharges water at the outlet with a velocity of 1.2 m/s at the rate of 25 m3 Is. If atmospheric pressure head is 10.3 m of water and losses between the inlet and outlet of the draft-tubes are negligible, find the length of draft-tube immersed in water. Total length of tube is 5 m.*



Diameter at top, $D_1 = 2.0$ m

Pressure head, $\frac{p_1}{\rho g} = 7$ m (Vacuum)

$= 10.3 - 7.0 = 3.3$ m (abs.)

Velocity at outlet, $V_2 = 1.2$ m/s

Discharge, $Q = 25$ m^3/s

Loss of energy, h_f = Negligible

Let the length of the tube immersed in water = y m.

Total length of the tube = 5 m

The velocity at inlet, $V_1 = \frac{\text{Discharge}}{\text{Area at inlet}}$

$$= \frac{Q}{\frac{\pi}{4} D_1^2} = \frac{25}{\frac{\pi}{4}(2.0)^2} = 7.957 \text{ m/s.}$$

Using equation (18.26), we have

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - H_s - \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f \right)$$

$$3.30 = 10.3 - H_s - \left(\frac{7.957^2}{2 \times 9.81} - \frac{1.2^2}{2 \times 9.81} - 0 \right)$$

$$\left(\because \quad h_f = 0 \text{ and } \frac{p_a}{\rho g} = 10.3 \right)$$

$$= 10.3 - H_s - (3.227 - .0734)$$

$$3.3 = 10.3 - H_s - 3.1536$$

$$\therefore \quad H_s = 10.3 - 3.1536 - 3.3 = 3.8464 \text{ m}$$

$$\therefore \quad y = \text{Total length} - H_s = 5 - 3.8464 = 1.1536 \text{ m. Ans.}$$