Compressible fluid flow which is motion → gas Dynamics

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unit - 5

Fluid dynamics of Compressible flows is generally referred as gas Dynamics

Gas Dynamics deals with unified analysis of dynamics (motion of particles without forces) and thermodynamics (Energy processes by gases & vapours) of high speed flows

Fluid Mechanic analysis of high speed flows of gases and vapours are inadequate without considering compressibility.

Application

high speed aerodynamics, rocket and missile propulsion, steam and gas turbine, high speed turbocompressor, wind tunnels

↳ Compressible fluid dynamics is used to obtain solutions of a number of design problems.

The properties of fluid which are generally considered in compressible flows problems are temp (T), pressure (P), density (ρ), Internal Energy (u), Enthalpy (h), Entropy (s) and viscosity (μ)

→ Recent advances in this area are transonic, supersonic and hypersonic flows, unsteady flows in rotating & reciprocating machines

Fluid dynamics of Compressible flows deals with relations between Force, mass and velocity

Fluid is a substance which is capable of flowing

Following laws are frequently used in dealing with a variety of compressible flow problems

(i) First law of thermodynamics (Energy Equation)
(ii) Second law of thermodynamics (Entropy relations)
(iii) Law of conservation of mass (continuity Equation)
(iv) Newton's second law of motion (momentum Equation)

Definitions and fundamental relations in thermodynamics and fluid Mechanics

Fluid

A fluid is a substance which continuously deforms when shearing forces are applied

liquids, gases and vapours and their mixture are all fluids

real fluid → viscous in nature.

Compressible flow

The relative changes in density in compressible flows are appreciable and cannot be neglected

fluid velocity (c) in such flows are appreciable compared to the local sound velocity of (a)

$M > 0.3$

In compressible flow

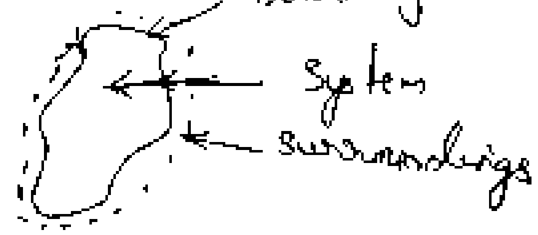
The relative change in density of a fluid in a process is negligibly small

$a >$ than c, $M < 0.3$

System

A system is an arbitrary collection of matter which has a fixed Identity.

Things outside the system are known as Surroundings



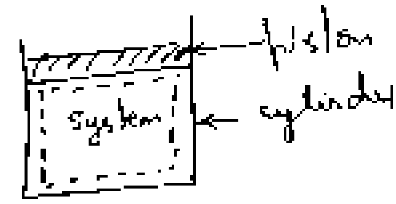
closed system

↳ system as fixed quantity of matter (fluid)

→ no inflow and outflow of fluid from and to the System.

System Interact with its surroundings through heat and work transfer

→ fixed mass of matter.



Example: gas in the cylinder Except → External temp ↑, piston moves

open system (Control Volume)

A System through which there is Continuous flow of matter (fluid)

→ fixed space but mass does not fixed mass

→ air Compressor

mass ≠ C

Isolated system

if there is no flow of mass and Energy to and from a System.

(open system with its surroundings (universe) → Example

Boundary

Boundary is an Imaginary Surface which separates the System from the Surrounding

Control volume

This is a arbitary volume fixed in space through which fluid flows the fluid occupy the control volume changes with time.
∴ control volume open system.

The surface which surrounds the control volume is known as control surface.

State

The state of system is its condition which is defined by its properties.

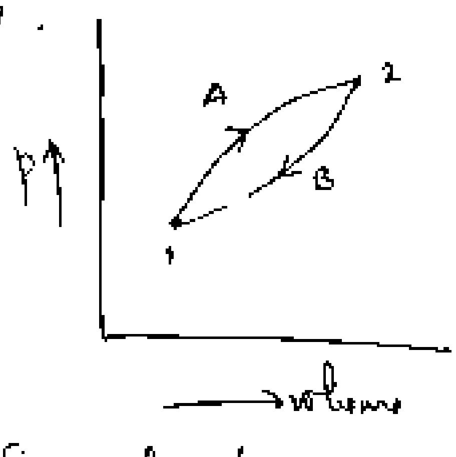
Process

change or a series of change in the state of a system

Cycle

if the Initial and final states of a system experiencing series of processes.

1 & 2 are state
(1A2 & 2B1) process
(1-A-2-B-1) cycle



property

property is an observable characteristic of a system, such as pressure, volume & temperature, P, etc.,
Independent of the process & fixed by the end state of the system.

All the quantities which identify the state of system are called properties.

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Intensive property

A property which is independent of the mass or size of the system.

Ex: Pressure, temperature, viscosity, tension, ρ

Extensive property

A property which depends on the mass or size of the system

Ex: length, area and Volume.

Pure Substance

Pure substance is a system which is homogeneous in composition and chemical aggregation.

Ex: air, steam, A/F mixture etc.,

Pressure (p) bar or [N/m^2 or kgf/cm^2]

$1 \text{ bar} = 1\times10^5 \, N/m^2$

Pressure at a point surrounded by an Elemental area ΔA is the force per unit area

$$p = \lim \frac{\Delta F}{\Delta A}$$

density

$$\rho = \frac{m}{V}$$

$$\rho = \lim_{\Delta V \to \Delta V_c} \left(\frac{\Delta m}{\Delta V}\right) \left(kg/m^3\right)$$

specific volume

$$v = \frac{1}{\rho} \left(m^3/kg\right)$$

Equilibrium

Thermal Equilibrium

if the changes ~~m~~ ~~with~~ in temperature with time is absent

Absence of changes in pressure or force with time → **Mechanical Equilibrium**.

Chemical Equilibrium

~~when a system is in thermal, mechanical and~~

[illegible] All ~~all~~ possible chemical reactions in the system have taken place and there is no change of phase with time

when a system is in thermal, mechanical & chemical Equilibrium it is said to be in thermodynamic Equilibrium

(All chapters assume thermodynamic Equilibrium)

Specific heats of gases

Specific heat of a substance is its heat carrying capacity in a given process.

The amount of heat that is required to raise the temperature of a unit mass of a substance by one degree

Specific heat at constant volume

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v = \left(\frac{dq}{\partial T}\right)_v$$

The amount of heat required to raise the temperature of a unit mass of a gas by one degree at constant volume.

Specific heat at constant pressure

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The amount of heat required to raise the temperature of a unit mass of a gas by one degree at constant pressure.

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p = \left(\frac{\partial q}{\partial T}\right)_p$$

Specific heat varies with temperature.

ratio of specific heat (γ) $= \frac{C_p}{C_v}$

γ_{air} =	1.4
γ_{CO_2} =	1.33
$\gamma_{He, Ar}$ =	1.67

Flow process

A change or series of changes in open system is referred as flow process.

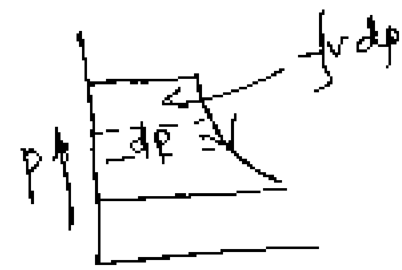
Ex: (i) flow through Nozzles, diffusers and ducts
(ii) flow through a steam, gas or hydraulic turbine

Work done in a steady flow process

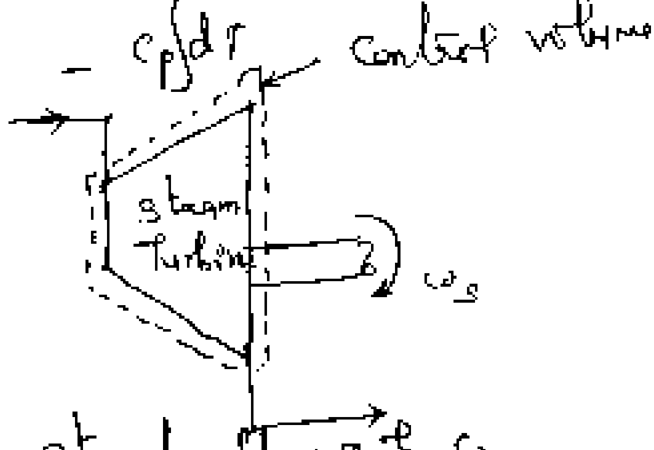
$$W_{fp} = -\int v\,dp$$

$$W_{fp} = -\int \frac{1}{\rho}\,dp = -\int dh$$

for perfect gas $W_{fp} = -C_p \int dT$



work in a flow process (open system)



Steady flow of steam in a turbine (flow process)

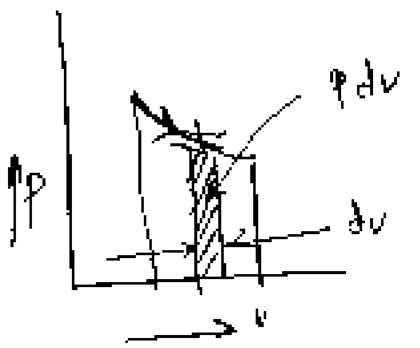
non-flow process

A change or series of change in a closed system is referred to as non-flow process

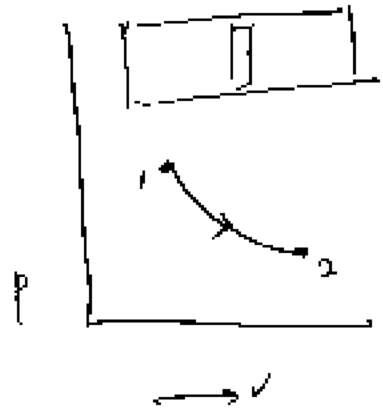
Ex: Constant volume heating & cooling of a gas or a vapour

Expansion or compression in reciprocating Engines & Compressors

$$W_{nfp} = \int p\,dv$$



work in a non-flow process (closed system)



Expansion in an I.C Engine

steady flow

In steady flows the fluid properties (Pressure, temperature, velocity etc.) in a control volume do not change with time.

$$\frac{\partial p}{\partial t} = \frac{\partial \rho}{\partial t} = \frac{\partial T}{\partial t} = \frac{\partial q}{\partial t} = \frac{\partial m}{\partial t} = 0$$

unsteady flow

when one or more fluid properties in the control volume change with time the flow is said to be unsteady flow

$$\frac{\partial p}{\partial t} = \frac{\partial \rho}{\partial t} \neq c$$

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Fluid velocity (c)

Fluid velocity (c) at a given point is the Instantaneous velocity of fluid particles passing through that point.
(vector quantity)

Adiabatic process

In adiabatic process there is no heat transfer between System and Surroundings.

$\text{workdone}_{nfp} = u_1 - u_2$ (for non-flow process)

$w_{fp} = h_1 - h_2$ (flow process)

$w_s = 0$, $h_0 = \text{Constant}$

Isentropic process (reversible adiabatic process)

An isentropic process is one which there is no change in Entropy.

$w_s = 0$

P_0 & T_0 = Constant.

$pv^\gamma = c$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$P_1 v_1^\gamma = P_2 v_2^\gamma$$

$$\frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^\gamma = \left(\frac{\rho_1}{\rho_2}\right)^\gamma$$

First law of thermodynamics

When a system executes a cyclic process, the algebraic sum of the work transfer is proportional to the algebraic sum of the heat transfer.

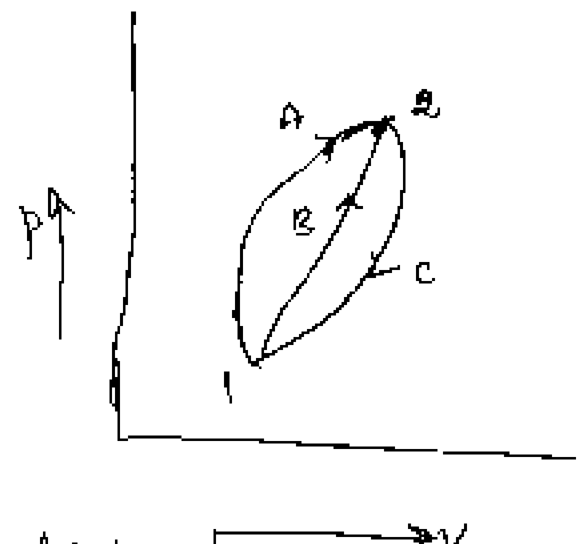
Symbolically

$$\oint dW \propto \oint dQ \qquad J = \text{Constant}, \; \boxed{J = 1}$$

$$\oint dW = J \oint dQ$$

If heat & work are in same units

$$\oint dQ - \oint dW = 0 \longrightarrow (1)$$



different processes b/t two state point.

3-processes, A, B and C occurring b/t two given state points on P-V co-ordinates.
State 2 can be reached by two processes along path A & B.
Return state 2 to 1 is achieved through the process C.

For the cycle 1A2C1

$$\oint (dQ - dW) = \int_{1,A}^{2} (dQ - dW) + \int_{2,C}^{1} (dQ - dW) = 0 \longrightarrow (2)$$

For the cycle 1B2C1

$$\oint (dQ - dW) = \int_{1,B}^{2} (dQ - dW) + \int_{2,C}^{1} (dQ - dW) = 0 \longrightarrow (3)$$

(3) - (2)

$$\int_{A\,1}^{2} (dQ - dW) = \int_{B\,1}^{2} dQ - dW \longrightarrow (4)$$

dQ - dW is not a path function, (i.e.) it does not depend on the path of the process

∴ dQ - dW = change in property called Energy (E)

$$dE = dQ - dW$$

$$E_2 - E_1 = Q - W$$

$$\boxed{Q = W + E_2 - E_1} \longrightarrow (5)$$

This is the well known Energy Equation

Heat transferred = W.D + change in Energy

Internal Energy and Enthalpy

Internal Energy (u) of a gas is the Energy stored in it by virtue of its molecular motion.

$$u = C_v T$$

$$U = m C_v T$$

In compressible flows Internal Energy (u) often appears with the quantity PV. Therefore sum is expressed by one single property called Enthalpy (h)

Thus

$$h = u + PV$$

$$h = u + \frac{P}{\rho}$$

perfect gas

$$\boxed{\frac{P}{\rho} = RT}$$

for a perfect gas

$$h = C_v T + \frac{P}{\rho} = C_v T + RT$$

$$h = (C_v + R) T.$$

$$h = C_p T$$

Ideal gas

A gas which obeys the laws of Boyle and Charles is known as an ideal gas.

Boyle's law @ constant temperature

$$(pv)_T = \text{Constant}$$

Charle's law @ constant pressure

$$\left(\frac{V}{T}\right)_p = \text{constant}$$

@ Constant volume

$$\left(\frac{P}{T}\right)_v = \text{Constant}$$

Perfect gas

A perfect gas is an ideal gas whose specific heats remain constants at all temperatures

$$\frac{d}{dT}(c_v) = 0$$

$$c_v = \text{Constant}$$

$$\frac{d}{dT}(c_p) = 0$$

$$c_p = \text{Constant}$$

Equation of state

The state of a given system is fully defined by a certain minimum number of thermodynamic parameters.

The relationship between these parameters the Equation of state

$$f(P, v, T) = 0$$

(Boyle & Charle's law)

For an Ideal gas

$$PV = mRT$$

R = gas Constant

$R \rightarrow$ gas Constant $= 287 \, J/kg \cdot K$

$$P = \left(\frac{m}{V}\right) RT$$

$$P = \rho RT$$

$$m = n\bar{M}$$

$$PV = n\bar{M}RT$$

$\bar{M}R = \bar{R}$ = universal gas Constant

$$PV = n\bar{R}T$$

$\bar{R} = 8.314 \, kJ/k.mole \, K$

$$\bar{R} = \frac{PV}{nT}$$

$$\bar{R} = \frac{1.0133 \times 10^5 \times 22.4}{273 \times 1000} = 8.314 \, kJ/k.mole \, K$$

The gas Constants are also given by

$$R = C_p - C_v$$

$$R = C_p\left(1 - \frac{C_v}{C_p}\right)$$

$$R = C_p\left(1 - \frac{1}{\gamma}\right)$$

$$R = C_p\left(\frac{\gamma - 1}{\gamma}\right)$$

$$C_p = \frac{\gamma R}{\gamma - 1}, \quad C_v = \frac{1}{\gamma - 1} R$$

Second law of thermodynamics

methods of determining the η of a given pr...

Clausius statement

Heat cannot on its own flow from a body at lower temperature to a body at higher temperature.

~~Coll~~ Corollaries are derived

Entropy

$$S_2 - S_1 = \int_1^2 \frac{dQ_R}{T}$$

Clausius Inequality

$$\oint \frac{dQ}{T} \leq 0$$

for irreversible process

$$S_2 - S_1 > \int_1^2 \frac{dQ}{T}$$

irreversible adiabatic process

$$S_2 - S_1 > 0$$

reversible cycle

$$\oint \frac{dQ_R}{T} = 0$$

isentropic (or) reversible adiabatic process

$$S_2 - S_1 = 0$$

$$\boxed{\Delta S = 0}$$

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reversible flow

A process is reversible if the system and surroundings can be restored to their initial state by reversing the process

Ex: Compression of a spring
frictionless Expansion or compression

Irreversible flow

The system and surroundings cannot be restored to their initial states by reversing the process.

processes involving solid and fluid friction
heat transfer with finite temp. difference and free Expansion.

Bulk modulus of Elasticity

$$K = \frac{\text{Increase in pressure}}{\text{Relative change in volume}}$$

$$K = \rho \cdot \frac{dP}{d\rho}$$

isothermal bulk modulus

$K_T = P$

Adiabatic bulk modulus

$K_S = \gamma P$

Coefficient of Compressibility (β)

$$\beta = \frac{\text{Relative change in volume}}{\text{Change in pressure}}$$

$$\beta = \frac{1}{\rho} \frac{d\rho}{dp}.$$

Reynolds Number (R_e)

$$R_e = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho A c^2}{\mu c l} = \frac{\rho c}{\nu}$$

$$= \frac{\rho c l}{\mu}$$

Mach Number (M)

$$M^2 = \frac{\text{Inertia force}}{\text{Elastic force}} = \frac{\rho A c^2}{KA} = \frac{\rho c^2}{K}$$

$K = \rho a^2$

$$M^2 = \frac{\rho c^2}{\rho a^2} = \left(\frac{c}{a}\right)^2$$

$$M = \frac{c}{a} = \frac{\text{fluid velocity}}{\text{Local sound velocity}}$$

velocity of sound (a) in a medium at temp, T is given by

$$a^2 = \frac{\partial p}{\partial \rho} \qquad a^2 = \left(\frac{\partial p}{\partial \rho}\right)_{\text{isentropic process}} = \gamma R T$$

$$a = \sqrt{\gamma R T}$$

$$M = \frac{c}{a} = \frac{c}{\sqrt{\gamma R T}}$$

Energy Equation

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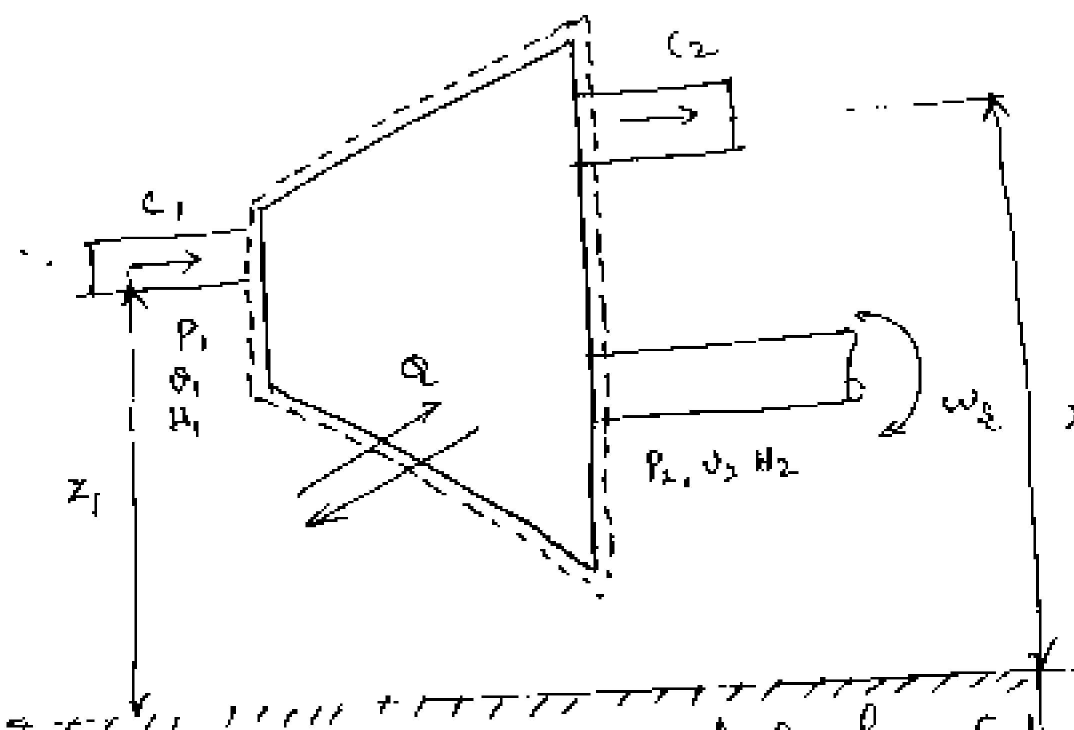
Energy Equation is derived in its various forms and stagnation state & stagnation values are defined



Steady flow through a Control volume (open system)

Figure shows the flow of fluid through the control volume. Various thermodynamic parameters at Entry and Exit Sections are shown with 1 & 2. & w (work) output or Input, Q (heat supplied or rejected)

The Energy Equation $\boxed{Q = W + E_2 - E_1}$ → (A), was derived from first law of thermodynamics.

The Energy terms (E) – Include gravitational, potential Energy, Kinetic Energy, Internal Energy, strain Energy, magnetic Energy, etc

$$E = U + mgz + \frac{1}{2}mc^2 \longrightarrow (1)$$

The differential form of (1) is

$$dE = dU + mg\,dz + m\,d\left(\frac{1}{2}c^2\right) \longrightarrow (2)$$

Integrating Equation (2)

$$\int_1^2 dE = \int_1^2 dU + \int_1^2 mg\,dZ + \int_1^2 m\,d\left(\tfrac{1}{2}c^2\right)$$

$$E_2 - E_1 = U_2 - U_1 + mg(Z_2 - Z_1) + \tfrac{1}{2}m\left(C_2^2 - C_1^2\right) \longrightarrow (3)$$

(3) in (A)

$$Q = W + (U_2 - U_1) + mg(Z_2 - Z_1) + \tfrac{1}{2}m\left(C_2^2 - C_1^2\right) \longrightarrow (3A)$$

In specific terms

$$q = w + u_2 - u_1 + g(Z_2 - Z_1) + \tfrac{1}{2}\left(C_2^2 - C_1^2\right) \longrightarrow (4)$$

Energy Equation for a non-flow process

Processes like the Expansion & Compression of gas in cylinder with a piston are non-flow process in closed system

$P.E = 0, \quad K.E = 0$

$$Q = W_s + U_2 - U_1 \longrightarrow (5)$$

$$dQ = dW_s + dU$$

perfect gas

$$dQ = PdV + mC_v dT \longrightarrow (6)$$

(6) on Integration

$$Q = \int_1^2 P\,dV + mC_v(T_2 - T_1)$$

Energy Equation for a flow process

Expansion of steam & gas in Turbines and Compression of air & gases in turbo compressors are examples of flow process in open system

$$W = W_s + (P_2V_2 - P_1V_1) \longrightarrow (7)$$

W_{s} ext

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(7) in (6) 3A

$$Q = W_s + P_2V_2 - P_1V_1 + U_2 - U_1 + mg(z_2 - z_1) + \frac{m}{2}(C_2^2 - C_1^2)$$

~~$Q = W_s + U_2 + P_2V_2 - (U_1 + P_1V_1)$~~

$H = U + PV$

$$Q = W_s + P_2V_2 + U_2 - P_1V_1 - U_1 + mg(z_2 - z_1) + \frac{1}{2}m(C_2^2 - C_1^2)$$

$$Q = W_s + H_2 - H_1 + mg(z_2 - z_1) + \frac{1}{2}m(C_2^2 - C_1^2)$$

$$Q + H_1 + mgz_1 + \frac{1}{2}mC_1^2 = W_s + H_2 + mgz_2 + \frac{1}{2}mC_2^2$$

In specific terms

$$q + h_1 + gz_1 + \frac{1}{2}C_1^2 = w_s + h_2 + gz_2 + \frac{1}{2}C_2^2 \rightarrow \text{steady flow Energy Equation.}$$

(gz_2 & gz_1 are) are small compared to other quantities

$$h_1 + \frac{1}{2}C_1^2 + q = h_2 + \frac{1}{2}C_2^2 + w_s \rightarrow (8)$$

Adiabatic Energy Equation

$q = 0$, In Some Engg. flows

expansion of gases & vapours in turbines are Examples of such processes

$$h_1 + \frac{1}{2}C_1^2 + 0 = h_2 + \frac{1}{2}C_2^2 + w_s \rightarrow (9)$$

Work in flow process

$$h_1 + \frac{1}{2} c_1^2 = h_2 + \frac{1}{2} c_2^2 + w_s$$

$$\frac{1}{2} c_1^2 = \frac{1}{2} c_2^2 = 0$$

$$w_s = h_1 - h_2 = -c_p \int_1^2 dT$$

$$w_s = c_p (T_1 - T_2)$$

for reversible process

$$w_s = -\int \frac{dp}{\rho} = -\int_1^2 v\, dp$$

Adiabatic Energy Transformation

adiabatic processes that Involve only Energy transformation eg. Expansion of gases in nozzles & their compression in diffusers

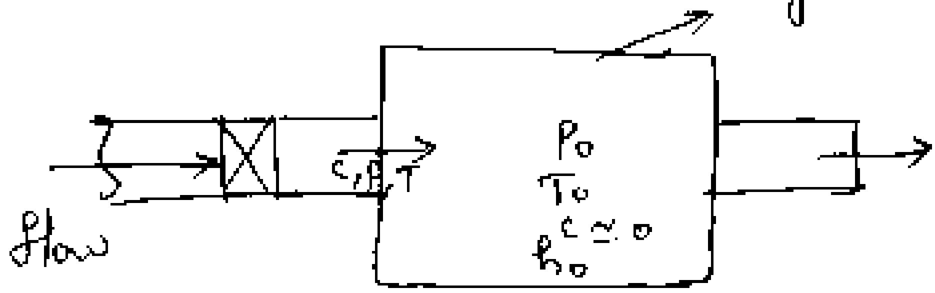
($w_s = 0$) ($z = 0$) ignored.

$$h_1 + g z_1 + \frac{1}{2} c_1^2 = h_2 + g z_2 + \frac{1}{2} c_2^2$$

$$h_1 + \frac{1}{2} c_1^2 = h_2 + \frac{1}{2} c_2^2$$

Stagnation Enthalpy

large settling chamber



deceleration of gas to stagnation state.

deceleration of a gas stream at velocity c, pressure p & temp. T to almost zero velocity (c = 0) in a large settling chamber.

Stagnation enthalpy of a gas or vapour is its enthalpy when it is adiabatically decelerated to zero velocity at zero elevation

Put $h_1 = h$, $z_1 = z$, & $c_1 = c$ for initial state ($h_2 = h_0$, $z_2 = 0$, $c_2 = 0$)

$$h_0 = h + gz + \frac{1}{2} c^2$$

gz compared to other quantities is $\simeq 0$

$$h_0 = h + \frac{1}{2} c^2 \longrightarrow$$

$h_0 = 0$ (adiabatic energy transformula

$$dh_0 = dh + \frac{2cdc}{2}$$

$$0 = dh + 2cdc \longrightarrow$$

Stagnation temperature

Stagnation temperature is the temperature of a gas when it is adiabatically decelerated to zero velocity at zero elevation

We know that
$$h_0 = h + \frac{1}{2} c^2 \longrightarrow (1)$$

$$c_p T_0 = c_p T + \frac{1}{2} c^2$$

$$T_0 = T + \frac{c^2}{2 c_p} \longrightarrow (2)$$

$$T_0 = T + T_c$$

velocity temperature $T_c = \frac{c^2}{2c_p}$

$$\frac{T_0}{T} = 1 + \frac{c^2}{2 c_p T}$$

$$\frac{T_0}{T} = 1 + \frac{c^2}{\frac{2\gamma R}{\gamma-1} T}$$

$$= 1 + \frac{c^2(\gamma-1)}{a^2\, 2}$$

$$\frac{T_0}{T} = 1 + \frac{c^2}{a^2}\frac{\gamma-1}{2}$$

$$\boxed{\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2}$$

Stagnation velocity of sound (a_0)

$$a_0 = \sqrt{\gamma R T_0}$$

$$C_p = \frac{\gamma R}{\gamma-1} \qquad a_0 = \sqrt{\gamma \frac{C_p(\gamma-1)}{\gamma} T_0}$$

$$R = \frac{C_p(\gamma-1)}{\gamma} \qquad a_0 = \sqrt{C_p T_0 (\gamma-1)}$$

$$a_0 = \sqrt{(\gamma-1) h_0}$$

Stagnation pressure (P_0)

Stagnation pressure is the pressure of a fluid which is attained when it is decelerated to zero velocity at zero elevation in a reversible adiabatic process (isentropic)

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\gamma/\gamma-1}$$

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Stagnation density (ρ_0)

Stagnation density (ρ_0) is given by

$$\rho_0 = \frac{P_0}{R T_0}$$

From isentropic relations.

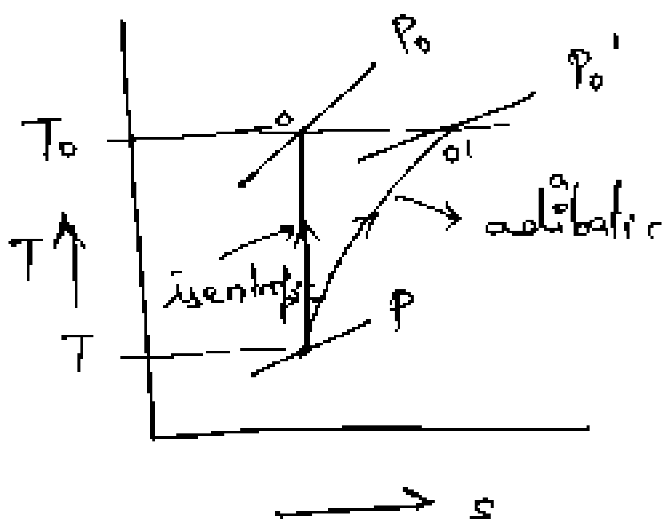
$$\frac{\rho_0}{\rho} = \left(\frac{P_0}{P}\right)^{1/\gamma}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/\gamma-1} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{1}{\gamma-1}}$$

Stagnation state

Fig shows deceleration ~~the~~ of a gas in both the isentropic and adiabatic process.

adiabatic deceleration the gas experiences Increase in decrease in stagnation pressure & Increase in Entropy



~~The stag~~

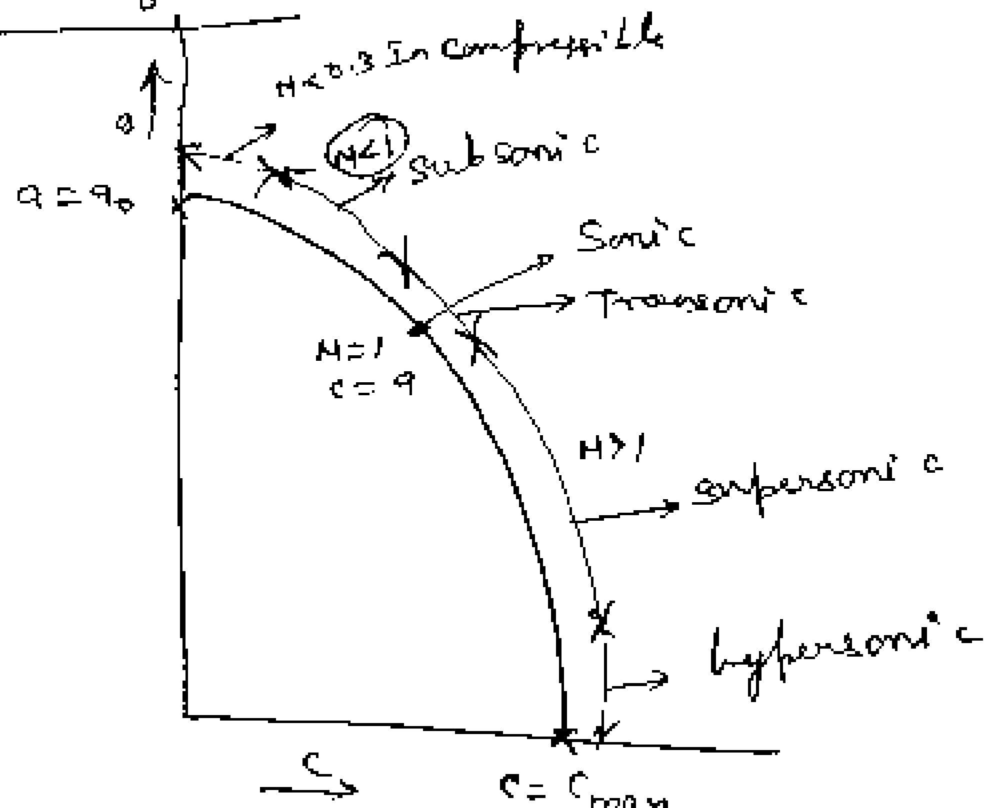
deceleration of gas to stagnation conditions

The state of a fluid attained by isentropically decelerating it to zero velocity at zero elevation is referred as stagnation state. — as 0, (h_0 & T_0 & P_0)

deceleration process isentropic only for (P_0 & 0)

remaining h_0, T_0 & are adiabatic

Various regions of flow



Various regions of flow

The adiabatic Energy Equation for a perfect gas is derived in terms of velocities c & a., this is graphically plotted.

We know that

$$h_0 = h + \frac{1}{2} c^2 \longrightarrow (1)$$

$$h = c_p T, \quad h_0 = c_p T_0 \longrightarrow (2)$$

(2) in (1)

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h_0 ~~$c_p T_0 = c_p T + \frac{1}{2} c^2$~~

~~$T_0 = T + c^2$~~

$h = c_p T$

$= \frac{\gamma R T}{\gamma - 1}$ $\gamma R T = a^2$

$PV = mRT$

$$\boxed{h = \frac{a^2}{\gamma - 1}}$$

$P = \rho R T$

$$\boxed{\frac{P}{\rho} = RT}$$

$h = c_p T$

$= \frac{\gamma R T}{\gamma - 1} = \frac{a^2}{\gamma - 1} = \frac{\gamma}{\gamma - 1} \frac{P}{\rho} \longrightarrow (3)$

(3) in (2)

$h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2} c^2 = \text{Constants} \longrightarrow (4)$

@ $T = 0$, $h = 0$, $c = C_{max}$

$h_0 = \cancel{h} + \frac{1}{2} c^2$

$h_0 = \frac{1}{2} C_{max}^2 \longrightarrow (5)$

$C_{max}^2 = 2 h_0$

$$\boxed{C_{max} = \sqrt{2 c_p T_0}}$$

@ $c = 0$, $a = a_0$ in $h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2} c^2$

$$\boxed{h_0 = \frac{a_0^2}{\gamma - 1}} \longrightarrow (6)$$

Combining (4), (5) & (6)

$$\frac{a^2}{\gamma-1} + \frac{1}{2} c^2 = \frac{1}{2} c_{max}^2 = \frac{a_0^2}{\gamma-1} = h_0 \quad \rightarrow (7)$$

Equation (7) is the another form of Energy Equation, this Expresses the total Energy of a fluid in a numbers of ways

---

Incompressible

region close to a-axis

$c < a$, $M \simeq 0.3$.

Subsonic

↳ right to Incompressible

$M < 1$

Transonic

~~M≤1~~ $M \geq 1$

$M = 1$, Sonic, $M = 1$, $c = a$.

↳ Extending either

Supersonic

$c > a$, $M > 1$

Hypersonic

region close to c-axis

$c > a$ (more higher)

$M > 5$

---

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Reference velocities

In compressible flow analyses it is often convenient to express fluid ~~velocity~~ velocities in non-dimensional form.

This is done by dividing the flow velocity by some reference or standard velocity.

The reference velocities are

(i) local velocity of sound, a.

(ii) stagnation velocity of sound, a_0

(iii) maximum velocity, c_{max}.

(iv) critical velocity of fluid / sound $c^* = a^*$

(1) velocity of sound, Mach number, Ernst Mach.

$$M = \frac{c}{a} = \frac{\text{fluid velocity}}{\text{sound velocity}} = \frac{c}{\sqrt{\gamma R T}}$$

(2) stagnation velocity of sound

$$a_0 = \sqrt{\gamma R T_0}$$

$$a_0 = \sqrt{(\gamma-1) h_0} = \sqrt{(\gamma-1) c_p T_0} = c \text{ for adiabatic flow for stagnation temp.}$$

(3) maximum fluid velocity.

We know that

$$h_0 = h + \frac{1}{2} c^2$$

if $c = 0$, (i.e.) $\frac{1}{2} c^2 = 0$, $h = h_0$

@ if $h = 0$, ($c = C_{max}$) (i.e.) only $\frac{1}{2}c^2$ is present.

$h_0 = \frac{1}{2} C_{max}^2$ $T = 0$, $h = 0$ (Imaginary adiabatic expansion process)

$$C_{max}^2 = 2h_0$$

$$C_{max} = \sqrt{2 c_p T_0}$$

$$C_{max} = \sqrt{\frac{2 \gamma R}{\gamma - 1} T_0}$$

$$C_{max} = \sqrt{\frac{2}{\gamma - 1}}\, a_0 \qquad \gamma = 1.4$$

$$\frac{C_{max}}{a_0} = \sqrt{\frac{2}{\gamma - 1}} = 2.24 \longrightarrow (A)$$

critical velocity of sound

The critical velocity of a fluid is its velocity at a Mach Number of unity.

$$M_{critical} = \frac{c^*}{a^*} = 1$$

$$c^* = a^* = \sqrt{\gamma R T^*}$$

@ $h_0 = h + \frac{1}{2}c^2$, applying this to a critical temp. is

$$h_0 = h^* + \frac{1}{2} c^{*2} \longrightarrow (1)$$

for perfect gas

$$C_p T_0 = c_p T^* + \frac{1}{2} c^{*2}$$

$$T_0 = T^* + \frac{1}{2 C_p} c^{*2} \longrightarrow (2)$$

$$c^{*2} = 2 C_p (T_0 - T^*)$$

$$c^* = \sqrt{2 c_p (T_0 - T^*)} \longrightarrow (3)$$

we know that stagnation temp. ratio @ $M_{critical}$, $c = a =$, $T = T^*$, $M_{critical} = 1$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2, \qquad \frac{T_0}{T^*} = \frac{\gamma + 1}{2} \longrightarrow (4)$$

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(4) in (3)

$$c^{*} = \sqrt{2 c_p T_0 \left(1 - \frac{T^{*}}{T_0}\right)}$$

$$c^{*} = \sqrt{2 c_p T_0 \left(1 - \frac{2}{\gamma+1}\right)}$$

$$c^{*} = \sqrt{\frac{2 \gamma R}{\gamma-1} T_0 \left(\frac{\gamma+1-2}{\gamma+1}\right)}$$

$$c^{*} = \sqrt{\frac{2 \gamma R T_0}{\gamma-1} \left(\frac{\gamma-1}{\gamma+1}\right)}$$

$$c^{*} = \sqrt{\frac{2}{\gamma+1}}\, a_0$$

@ $\gamma = 1.4$

$$\frac{c^{*}}{a_0} = \sqrt{\frac{2}{\gamma+1}} = 0.913 \longrightarrow (5)$$

divide

$$\frac{\frac{c_{max}}{a_0}}{\frac{c^{*}}{a_0}} = \frac{\sqrt{\frac{2}{\gamma-1}}}{\sqrt{\frac{2}{\gamma+1}}} = \sqrt{\frac{\gamma+1}{\gamma-1}}$$

$$\frac{c_{max}}{c^{*}} = \sqrt{\frac{\gamma+1}{\gamma-1}}$$

$c^{*} = a^{*}$

$$\therefore \frac{c_{max}}{a^{*}} = \sqrt{\frac{\gamma+1}{\gamma-1}}$$

$$\frac{c_{max}^2}{a^{*2}} = \frac{\gamma+1}{\gamma-1}$$

$$c_{max}^2 = a^{*2} \frac{\gamma+1}{\gamma-1} \longrightarrow (6)$$

(6) in

$$\frac{a^2}{\gamma-1} + \frac{1}{2} c^2 = \frac{1}{2} c_{max}^2$$

$$\boxed{\frac{a^2}{\gamma-1} + \frac{1}{2} c_{max}^2 = \frac{1}{2} a^{*2} \frac{\gamma+1}{\gamma-1} \longrightarrow (7)}$$

We know that

(a) $$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \longrightarrow (1)$$

$$\frac{T_0}{T^*} = \frac{\gamma + 1}{2} \longrightarrow (2)$$

$\frac{(2)}{(1)}$ $$\frac{\frac{T_0}{T}}{\frac{T_0}{T^*}} = \frac{1 + \frac{\gamma - 1}{2} M^2}{\frac{\gamma + 1}{2}} = \frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2$$

$$\boxed{\frac{T^*}{T} = \frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2}$$

(x) Mach Number, M^*

Second kind of Mach Number M^* can be defined by non-dimensionalizing the fluid velocity by the critical sound velocity
fluid velocity or the

$$M^* = \frac{c}{c^*} = \frac{c}{a^*} \longrightarrow (1)$$

$$M^{*2} = \frac{c^2}{a^{*2}} = \frac{c^2}{a^2} \times \frac{a^2}{a^{*2}} = M^2 \times \frac{a^2}{a^{*2}} \longrightarrow (2)$$

It is often convenient to use M^* instead of M

(a) at high fluid velocity (c) M approaches infinity

(i.e) $c \gg$, $M \simeq \infty$

(a) M is not proportional to the fluid velocity alone

M^* does not mean $M = 1$, This is another type of Mach number

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We know that

$$\frac{a^2}{\gamma-1} + \frac{1}{2}c^2 = \frac{1}{2}a^{*2}\frac{\gamma+1}{\gamma-1}$$

$\times 2$

$$\frac{2a^2}{\gamma-1} + c^2 = a^{*2}\frac{\gamma+1}{\gamma-1} \longrightarrow (3)$$

$$\frac{2}{\gamma-1}\frac{a^2}{a^{*2}} + \frac{c^2}{a^{*2}} = \frac{\gamma+1}{\gamma-1}$$

$$\frac{2}{\gamma-1}\frac{M^{*2}}{M^2} + M^{*2} = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2}\left[\frac{2}{\gamma-1}\frac{1}{M^2} + 1\right] = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2} \times \frac{2}{\gamma-1}\left[\frac{1}{M^2} + \frac{\gamma-1}{2}\right] = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2} = \left[\frac{\frac{\gamma+1}{2}}{\frac{1}{M^2}+\frac{\gamma-1}{2}}\right] = \frac{\frac{\gamma+1}{2}M^2}{1+\frac{\gamma-1}{2}M^2}$$

Take out $\frac{1}{M^2}$ out

$$M^* = \sqrt{\frac{\frac{\gamma+1}{2}M^2}{1+\frac{\gamma-1}{2}M^2}}$$

Crocco Number

$$C_r = \frac{c}{C_{max}} = \frac{\text{fluid velocity}}{\text{maximum fluid velocity.}}$$

Bernoulli Equation

We know

$$h_0 = h + \frac{1}{2}c^2 \longrightarrow (1)$$

$$0 = dh + \frac{2cdc}{2}$$

$$dh + cdc = 0$$

$$\frac{dp}{\rho} + cdc = 0 \longrightarrow (2)$$

$$dc^2 = 2cdc$$

$$\frac{dc^2}{2} = cdc$$

If the flow is assumed is incompressible $\rho \simeq$ Constant.

$$\int \frac{1}{\rho} dp + \frac{1}{2}\int d(c^2) = \text{Constant}.$$

$$\frac{p}{\rho} + \frac{1}{2}c^2 = \text{constant}.$$

isentropically decelerated to zero velocity at zero Elevation $p = p_0$, $\rho = \rho_0$

$$c = 0$$

$$\frac{p_0}{\rho_0} = \text{Constant}$$

Incompressible flow $\rho_0 \simeq \rho = c$

$$\frac{p}{\rho} + \frac{1}{2}c^2 = \frac{p_0}{\rho_0}.$$

$$\frac{p}{\rho} + \frac{1}{2}c^2 = \frac{p_0}{\rho} \longrightarrow (3)$$

$$p_0 = p + \frac{1}{2}\rho c^2 \longrightarrow \text{Bernoulli Equation}$$

Valid only for isentropic & Incompressible flow.

The adiabatic Energy Equation can also be Expressed in terms of pressure

$$h_0 = C_p T_0, \quad = \frac{\gamma}{\gamma - 1} R T_0$$

$$\frac{p_0}{\rho_0} = R T_0$$

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$$h_0 = \frac{\gamma}{\gamma-1} \frac{p_0}{\rho_0}$$

$$h_0 = h + \frac{1}{2} c^2$$

$$\frac{\gamma}{\gamma-1} \frac{p_0}{\rho_0} = \frac{\gamma}{\gamma-1} \frac{p}{\rho} + \frac{1}{2} c^2 \longrightarrow (4)$$

We know that

$$\frac{\rho}{\rho_0} = \left(\frac{p}{p_0}\right)^{1/\gamma}$$

$$\frac{\rho_0}{\rho} = \left(\frac{p_0}{p}\right)^{1/\gamma}$$

$$\frac{1}{\rho} = \frac{1}{\rho_0} \left(\frac{p_0}{p}\right)^{1/\gamma}$$

$$\frac{p}{\rho} = \frac{p}{\rho_0} \times \frac{p_0}{p_0} \times \left(\frac{p_0}{p}\right)^{1/\gamma}$$

$$\frac{p}{\rho} = \frac{p_0}{\rho_0} \times \left(\frac{p}{p_0}\right) \left(\frac{p_0}{p}\right)^{1/\gamma}$$

$$= \frac{p_0}{\rho_0} \times \left(\frac{p_0}{p}\right)^{-1} \left(\frac{p_0}{p}\right)^{1/\gamma}$$

$$\frac{p}{\rho} = \frac{p_0}{\rho_0} \times \left(\frac{p_0}{p}\right)^{-\frac{\gamma-1}{\gamma}} \longrightarrow (5)$$

(5) in (4)

$$\frac{\gamma}{\gamma-1} \frac{p_0}{\rho_0} = \frac{\gamma}{\gamma-1} \times \frac{p_0}{\rho_0} \times \left(\frac{p_0}{p}\right)^{-\frac{\gamma-1}{\gamma}} + \frac{1}{2} c^2 \longrightarrow (6)$$

Effect of Mach Number on Compressibility

If the flow is assumed incompressible, the value of the pressure coefficient (sometimes referred as compressibility factor) obtained by Bernoulli's Equation is unity.

$P_0 = P + \frac{1}{2}\rho c^2$

$$\frac{P_0 - P}{\frac{1}{2}\rho c^2} = 1 \longrightarrow (1)$$

For compressible flow the value of pressure coefficient deviates from unity, the magnitude of deviation increases with the Mach Number of flow.

For isentropic compressible flow, the ratio of stagnation & static pressures is given by

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1}$$

$$\frac{P_0}{P} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\gamma/\gamma-1} \longrightarrow (2)$$

For isentropic compressible flow this can be expanded by the following Taylor's series.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \longrightarrow (3)$$

Here $n = \frac{\gamma}{\gamma-1}$, $x = \frac{\gamma-1}{2}M^2$

$$\frac{P_0}{P} = 1 + \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\gamma-1}{2}M^2\right) + \frac{\frac{\gamma}{\gamma-1}\left(\frac{\gamma}{\gamma-1}-1\right)}{2!}\left(\frac{\gamma-1}{2}M^2\right)^2 + \frac{\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\gamma}{\gamma-1}-1\right)\left(\frac{\gamma}{\gamma-1}-2\right)}{3!}\left(\frac{\gamma-1}{2}M^2\right)^3 + \cdots$$

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$$\frac{P_0}{P} = 1 + \frac{\gamma}{2}M^2 + \frac{\frac{\gamma}{\gamma-1}\left(\frac{\gamma-\gamma+1}{\gamma-1}\right)}{2}\left(\frac{\gamma-1}{2}\right)^2 \frac{M^4}{4} + \cdots$$

$$\frac{\frac{\gamma}{\gamma-1}\left(\frac{\gamma-\gamma+1}{\gamma-1}\right)\left(\frac{\gamma-2\gamma+2}{\gamma-1}\right)}{3\times 2}\left(\frac{(\gamma-1)^3}{8}\right)M^6 + \cdots$$

$$\frac{P_0}{P} = 1 + \frac{\gamma}{2}M^2 + \frac{\gamma}{8}M^4 + \frac{\frac{\gamma}{\gamma-1}\left(\frac{1}{\gamma-1}\right)\left(\frac{-\gamma+2}{\gamma-1}\right)}{3\times 2}\frac{(\gamma-1)^3}{8}M^6$$

$$\frac{P_0}{P} = 1 + \frac{\gamma}{2}M^2 + \frac{\gamma}{8}M^4 + \frac{(2-\gamma)\gamma}{48}M^6 + \cdots$$

$$\frac{P_0}{P} = 1 + \frac{\gamma}{2}M^2 + \frac{\gamma}{8}M^4 + \frac{(2-\gamma)\gamma}{48}M^6 + \cdots$$

$$\frac{P_0}{P} - 1 = \frac{\gamma}{2}M^2 + \frac{\gamma}{8}M^4 + \frac{(2-\gamma)\gamma}{48}M^6 + \cdots$$

$$\frac{P_0 - P}{P} = \frac{\gamma}{2}M^2\left[1 + \frac{M^2}{4} + \frac{2-\gamma}{24}M^4 + \cdots\right]$$

$$\frac{P_0 - P}{P\left(\frac{\gamma}{2}M^2\right)} = 1 + \frac{M^2}{4} + \frac{2-\gamma}{24}M^4 + \cdots$$

but $P = \rho R T$,

$$P\left(\frac{\gamma}{2}M^2\right) = \rho R T\,\frac{\gamma}{2}\left(\frac{c^2}{a^2}\right) = \rho R T\frac{\gamma}{2}\frac{c^2}{\gamma R T} = \frac{1}{2}\rho c^2$$

i.e.) $$\frac{P_0 - P}{\frac{1}{2}\rho c^2} = 1 + \frac{M^2}{4} + \frac{2-\gamma}{24}M^4 + \cdots$$

@ $\gamma = 1.4$

$$\frac{P_0 - P}{\frac{1}{2}\rho c^2} = 1 + \frac{M^2}{4} + \frac{2 - 1.4}{24} M^4 + \cdots$$

$$\frac{P_0 - P}{\frac{1}{2}\rho c^2} = 1 + \frac{M^2}{4} + \frac{M^4}{40} + \cdots$$

@ $M = 0.1$, $\frac{P_0 - P}{\frac{1}{2}\rho c^2} \simeq 0.3$

@ $M = 0.2$ $\frac{P_0 - P}{\frac{1}{2}\rho c^2} = 1$

$M = 0.3$ = $\frac{P_0 - P}{\frac{1}{2}\rho c^2} = 2.3$

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(1)
An air stream at P = 1.0 bar, T = 400 K and c = 400 m/s is brought to rest (i) adiabatically (ii) isentropically. determine stagnation pressure and temperature in the two cases

given data

For air: $\gamma = 1.4$, $R = 287$

For: $P_1 = 1.0$ bar, $T_1 = 400$ k, $C_1 = 400$ m/s

adiabatically

$$T_0 = T_1 + \frac{1}{2c_p} C_1^2$$

$$T_0 = 400 + \frac{1}{2 \times 1005} (400)^2$$

$T_0 = 479.5$ K

$T_0 = 479.5$ K → isentropically

for isentropic

$$\frac{P_0}{P_1} = \left(\frac{T_0}{T_1}\right)^{\gamma/\gamma-1} =$$

$$P_0 = 1 \times 10^5 \left(\frac{479.5}{400}\right)^{\frac{1.4}{1.4-1}}$$

$P_0 = 1.85$ bar

for adiabatic flow P_0 depend on the η of the deceleration process, so, given data is insufficient to find value of (P_0)

(2) The pressure, temperature and ~~Mach~~ Mach Number at the Entry of a flow passage are 2.45 bar, 26.5°C and 1.4 respectively. if the Exit Mach Number is 2.5, determine for adiabatic flow of a perfect gas ($\gamma = 1.3$, $R = 0.469$ KJ/kg.K)
(i) stagnation temp. (ii) temp. & velocity of gas at exit
(iii) the flow rate per square metre of the inlet cross section

given data.

$P_1 = 2.45 \times 10^5 \ N/m^2$

$T_1 = 26.5 + 273 = 299.5$

$M_1 = 1.4$

$M_2 = 2.5$

$\gamma = 1.3$, $R = 469 \ J/kg.k$

for ~~adiabal~~ adiabatic flow

$$T_{o_1} = T_{o_2} = T_o$$

$$\frac{T_{o_1}}{T} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$\frac{T_{o_1}}{299.5} = 1 + \frac{1.3 - 1}{2} (1.4)^2$$

$$T_{o_1} = 388 \ K$$

$$\frac{T_{o_2}}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2$$

$$\frac{T_{o2}}{T_2} = 1 + \frac{1.3 - 1}{2} (2.5)^2$$

$$\frac{T_{o2}}{T_2} = 1.9375$$

$$T_2 = 200.25 \ K$$

$\dot{m} = \rho_1 A_1 C_1$

$$\frac{\dot{m}}{A_1} = \rho_1 C_1$$

$$\rho_1 = \frac{P_1}{R T_1} = \frac{2.45 \times 10^5}{469 \times 299.5}$$

$$\rho_1 = 1.745 \ kg/m^3$$

$$M_1 = \frac{C_1}{a_1}$$

$C_1 = M_1 \times a_1 = 1.4 \times \sqrt{\gamma R T_1}$

$= 1.4 \times \sqrt{1.3 \times 469 \times 299.5}$

$$C_1 = 598 \ m/s$$

$$\frac{\dot{m}}{A} = 1.745 \times 598$$

$$\frac{\dot{m}}{A} = ~~1050~~ \ 1043.95 \ kg/s\,m^2$$

$$M_2 = \frac{C_2}{a_2}$$

$$C_2 = M_2 \times a_2$$
$$= 2.5 \times \sqrt{\gamma R T_2}$$
$$= 2.5 \times \sqrt{1.3 \times 469 \times 201}$$

$$\boxed{C_2 = 875 \, m/s}$$

Ans

$C_2 = 875 \, m/s$, $T_2 = 201 \, K$, $T_{01} = 388 \, K$, $\frac{\dot{m}}{A} = 1043.95 \, kg/sm^2$

---

5) Air ($\gamma = 1.4$, $R = 287.43 \, J/kg.k$) Enters a straight axisymmetric duct at 300 k, 3.45 bar and 150 m/s and leaves it at 277 k, 2.058 bar and 260 m/s. The area of C/s at Entry is 500 cm². Assume adiabatic flow determine
(i) stagnation temp (ii) maximum velocity
(iii) mass flow rate (iv) area of C/s at Exit.

given data

$\gamma = 1.4$, $R = 287 \, J/kg.k$

$T_1 = 300 \, k$, $P_1 = 3.45 \, bar$, $C_1 = 150 \, m/s$

$T_2 = 277 \, k$, $P_2 = 2.058 \, bar$, $C_2 = 260 \, m/s$

$A_1 = 500 \times 10^{-4} \, m^2$

For adiabatic flow $T_{01} = T_{02} = T_0$

To find T_0, C_{max}, $\dot{m}$, A_2

$$C_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4 \times 287.43}{1.4 - 1} = 1006 \, J/kg.k$$

$$T_{01} = T_1 + \frac{C_1^2}{2 C_p} = 300 + \frac{(150)^2}{2 \times 1006}$$

$$\boxed{T_{01} = 311.18 \, k}$$

$$T_{o2} = T_{o1} = 311.18\,K$$ → for adiabatic flow.

(ii) $C_{max} = \sqrt{2 C_p T_o}$

$= \sqrt{2 \times 1006 \times 311.18}$

$$C_{max} = 790\,m/s$$

(iii) $\dot{m} = \rho_1 A_1 C_1$

$\rho_1 = \frac{P_1}{R T_1} = \frac{3.45 \times 10^5}{287.43 \times 300} = 4.0\ kg/m^3$

$\dot{m} = 4.0 \times 500 \times 10^{-4} \times (150)$

$$\dot{m} = 30\ kg/s$$

$\dot{m} = \rho_1 A_1 C_1 = \rho_2 A_2 C_2$

$30 = \rho_2 A_2 C_2$

$30 = 2.59 \times A_2 \times 260$

$$A_2 = 0.0445\ m^2$$

$\rho_2 = \frac{P_2}{R T_2} = \frac{2.058 \times 10^5}{287.43 \times 277}$

$$\rho_2 = 2.59\ kg/m^3$$

---

(4) Air ($C_p = 1.05\ kJ/kg.K$, $\gamma = 1.38$) at $P_1 = 3$ bar and $T_1 = 500K$ flows with a velocity of $200\ m/s$ in a 30cm diameter duct. calculate
(a) mass flow rate, (b) stagnation temperature
(c) Mach Number and (d) stagnation pressure values assuming the flow as Compressible and Incompressible

$R = C_p - C_v =$ $\frac{C_p}{C_V} = \gamma$

$C_V = \frac{1.05}{1.38}$

$R = 1.05 - \frac{1.05}{1.38}$

$$R = 0.289\ kJ/kg.K = 289\ J/kg.K$$

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$$\rho_1 = \frac{P_1}{RT_1} = \frac{3\times10^5}{289\times500}$$

$$\boxed{\rho_1 = 2.076\ kg/m^3}$$

(a) mass flow rate

$$\dot{m} = \rho_1 A_1 C_1$$

$$= 2.076 \times \frac{\pi}{4}(0.3)^2 \times 200$$

$$\boxed{\dot{m} = 29.348\ kg/s}$$

(b) stagnation temp.

$$T_0 = T_1 + \frac{C_1^2}{2C_p} = 500 + \frac{(200)^2}{2\times1050}$$

$$\boxed{T_0 = 519.047\ K}$$

(c) $M_1 = \frac{C_1}{a_1} = \frac{200}{\sqrt{\gamma RT_1}} = \frac{200}{\sqrt{1.38\times289\times500}} = 0.4478.$

(d) stagnation pressure

for compressible flow

$$\frac{P_0}{P_1} = \left(\frac{T_0}{T_1}\right)^{\gamma/\gamma-1}$$

$$P_0 = 3\times10^5\left(\frac{519.047}{500}\right)^{\frac{1.38}{1.38-1}}$$

$$\boxed{P_0 = 3.435\ bar}$$

In compressible flow

$$P_0 = P_1 + \frac{1}{2}\rho_1 C_1^2$$

$$= 3\times10^5 + \frac{1}{2}\times2.076\times(200)^2$$

$$\boxed{P_0 = 3.415\ bar}$$

(5) ~~Air~~ An aircraft flies at 800 km/hr at an altitude 10,000 m. The air is reversibly compressed in an inlet diffuser. If the Mach Number at the exit of the diffuser is 0.26. Determine (a) Entry Mach Number (b) velocity, pressure and temp. of air at the diffuser exit

---

@ $\gamma = 1.4$

$$c_1 = \frac{800 \times 1000}{60 \times 60} = 222.22 \text{ m/s}$$

$Z = 10,000 \text{ m}$,

From gas tables $\hookrightarrow$ $[T_1 = 223.15 \text{ K}, \quad P_1 = 0.264 \text{ bar}]$

$$M_1 = \frac{c_1}{a_1} = \frac{222.22}{\sqrt{\gamma R T_1}} = \frac{222.22}{\sqrt{1.4 \times 287 \times 223.15}}$$

$$\boxed{M_1 = 0.74}$$

For isentropic flow $P_{01} = P_{02} = P_0$, $T_{01} = T_{02} = T_0$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$\frac{T_{01}}{223.15} = 1 + \frac{1.4 - 1}{2} \times (0.74)^2$$

$$\boxed{T_{01} = 247.84 \text{ K}}$$

From isentropic flow table for $\gamma = 1.4$ at

$M_1 = 0.74$, $\frac{P_1}{P_0} = 0.695$, $P_0 = \frac{0.264}{0.695}$

$$\boxed{P_0 = 0.379 \text{ bar}}$$

$M_2 = 0.26$ $\frac{P_2}{P_{02}} = 0.914$, $\frac{T_2}{T_{02}} = 0.975$

$$\boxed{P_{01} = P_{02} = P_{01} = 0.379 \text{ bar}}$$

$$\boxed{T_{01} = 247.84 = T_{02}}$$

$$\frac{P_2}{P_{02}} = 0.914 \qquad \frac{T_2}{T_{02}} = 0.975$$

$$\boxed{P_2 = 0.346 \text{ bar}} \qquad \boxed{T_2 = 241.65 \text{ K}}$$

$$M_2 = \frac{c_2}{a_2} = \frac{c_2}{\sqrt{\gamma R T_2}} = \frac{c_2}{\sqrt{1.4 \times 287 \times 241.65}}$$

$$\boxed{c_2 = 81.17 \text{ m/s}}$$

(22)

(1) An air jet ($\gamma = 1.4$, $R = 287\ J/kg.K$) @ 400 K the sonic velocity. Determine
(i) Velocity of sound at 400 K
(ii) Velocity of sound at stagnation conditions
(iii) maximum velocity of the jet
(iv) stagnation enthalpy
(v) Crocco Number

$M = 1$, $T = T^*$, $p = p^*$

$T^* = 400$

$M = 1$, $c = a = \sqrt{\gamma R T}$

$= \sqrt{1.4 \times 287 \times 400}$

$c = a = 400.89\ m/s$

$= 400\ m/s$.

@ $M = 1$ from isentropic flow table

$\frac{T}{T_0} = 0.834$

$T_0 = 479.61\ K$

$a_0 = \sqrt{\gamma R T_0}$

$= \sqrt{1.4 \times 287 \times 479.61}$

$a_0 = 438.98\ m/s$

$C_{max} = \sqrt{2 C_p T_0} = \sqrt{2 \times 1005 \times 479.61}$

$C_{max} = 981.84\ m/s$

$h_0 = \frac{1}{2} C_{max}^2$

$h_0 = 482.00 \times 10^3\ J/kg$

$C_r = \frac{c}{C_{max}}$

$= \frac{400.89}{981.84}$

$C_r = 0.4083$

---

(2) The conditions of an air stream at entry of a duct are $P_1 = 1\ bar$, $T_1 = 300\ K$, $M_1 = 1.3$, if the Mach number at exit of duct is 0.6, determine the temp. and velocity of air at the adiabatic flow the duct exit

$P_1 = 1\ bar$, $T_1 = 300\ K$, $M_1 = 1.3$,

$M_2 = 0.6$,

$T_{0_1} = T_{0_2} = T_0$ adiabatic flow.

To find T_2, C_2

$\gamma = 1.4$, $R = 287$ J/kg.K for air

from isentropic flow table

$M_1 = 1.3$ $\quad \frac{T_1}{T_{01}} = 0.747$, $\quad \frac{P_1}{P_{01}} = 0.361$

$M_2 = 0.6$ $\quad T_{01} = 401.60K$ $\quad P_{01} = 2.77$ bar

$\frac{T_2}{T_{02}} = 0.933$, $\quad \frac{P_2}{P_{02}} = 0.784$

$P_2 = 2.17$ bar

$T_{01} = T_{02} = T_0$

$P_{01} = P_{02} = P_{02}$

adiabatic flow

$T_2 = 374.69$ K

$M_2 = \frac{C_2}{a_2}$

$0.6 = \frac{C_2}{\sqrt{\gamma R T_2}} = \frac{C_2}{\sqrt{1.4 \times 287 \times 374.69}}$

$C_2 = 232.8$ m/s

Q) An aircraft is flying at an altitude of 12,000m, ($T = 216.65k$, $P_1 = 0.193$ bar) at a Mach Number of 0.82, the cross-sectional area of the inlet diffuser before the L.P. compressor stage is $0.5m^2$. determine

(a) $\dot{m}$, (b) speed of the aircraft, (iii) P_0 & T_0 at the diffuser entry.

$T_1 = 216.65k$, $P_1 = 0.193$ bar

$M_1 = 0.82$ $\quad A_1 = 0.5m^2$

$M_1 = \frac{C_1}{a_1} = \frac{C_1}{\sqrt{\gamma R T_1}}$

$0.82 = \frac{C_1}{\sqrt{1.4 \times 287 \times 216.65}}$

$C_1 = 241.93$ m/s

$\dot{m} = \rho_1 A_1 C_1$

$= \frac{P_1}{R T_1} \times 0.5 \times 241.93$

$= \frac{0.193 \times 10^5}{287 \times 216.65} \times 0.5 \times 241.93$

$\dot{m} = 37.54$ kg/s

$= \frac{241.93 \times 3600}{1000}$

$C_1 = 870.94$ Km/hr

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To $\gamma = 1.4$ (isentropic flow table)

@ $M_1 = 0.82$

$$\frac{P_1}{P_{o_1}} = 0.643 \quad \frac{T_1}{T_{o_1}} = 0.881$$

$P_{o_1} = 0.3001$ bar

$T_{o_1} = 246$ K

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Wave motion in a medium is the movement of a disturbance relative to the medium. The effect of changes at a given point in the medium is communicated to other points through wave motion.

Various types of waves
(a) Infinitesimal pressure waves (Sound waves)
(b) Non-steep pressure waves
(c) steep pressure waves
(d) Expansion waves

<u>Propagation of Infinitesimal waves (Sound waves)</u>

Sound waves is an Infinitesimal pressure wave. The changes across such a wave are small and the speed of the process corresponding to these changes is fast.

~~Q=0~~, heat transfer = 0
changes across an Infinitesimal pressure wave are assumed as reversible adiabatic or isentropic.

velocity of sound a → depend on
(i) bulk modulus of Elasticity (K)
(ii) rate of change of pressure with density $\frac{\partial p}{\partial \rho}$

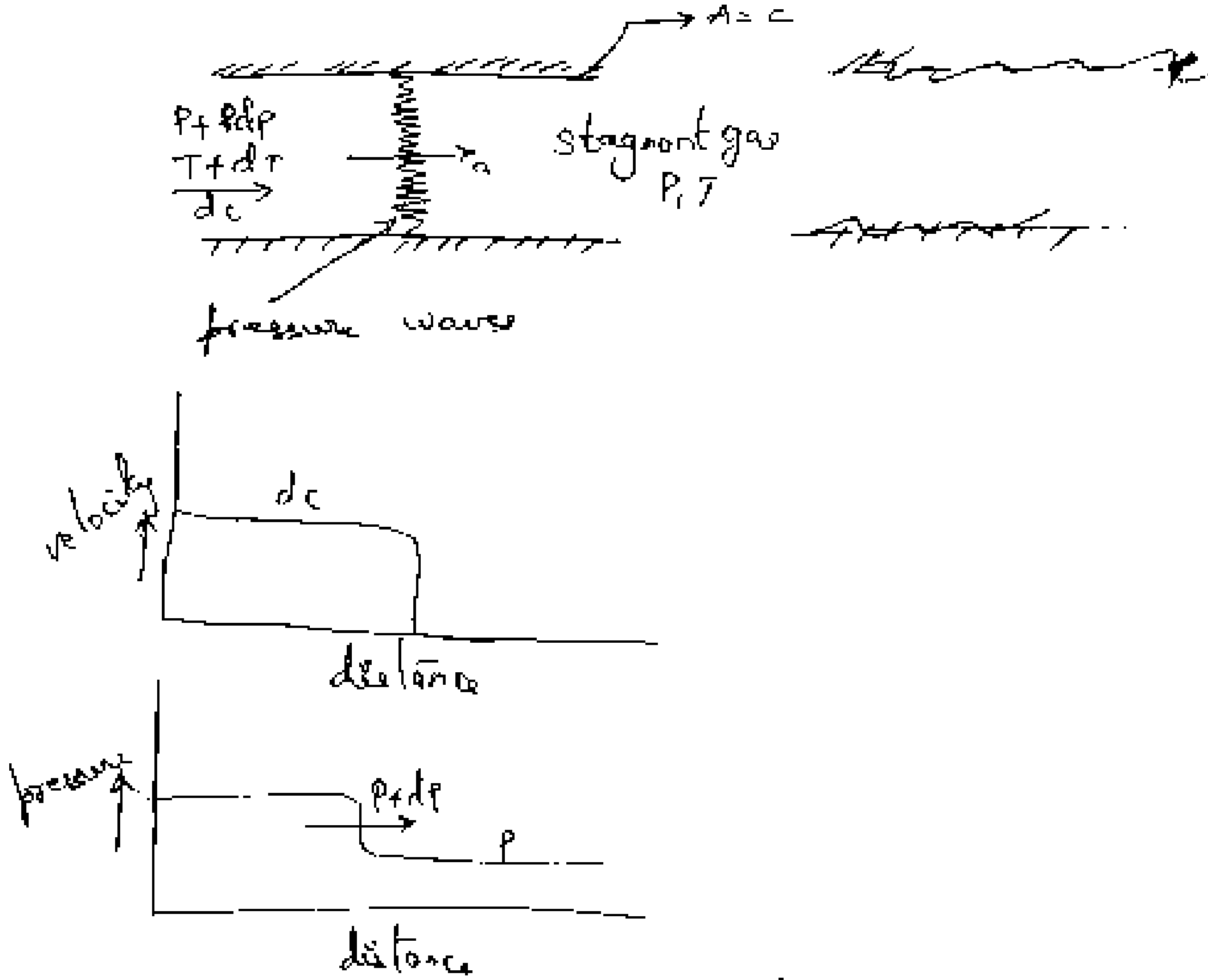


fig. Propagation of Infinitesimal wave in a Constant area duct (observer at rest)

The wave moves with a velocity a towards the right into a stagnant gas i.e. at pressure p and temp. T.

The pressure and temp. of the gas that has been traversed by the wave are raised to p+dp and T+dp respectively, and velocity dc.

The above pattern will be observed when the observer moves with the wave

---

The stagnant gas at pressure p on the right appears to flow towards the left with a velocity, a.

When the flow has passed through the wave to the left its pressure is raised to p+dp and velocity (a-dc)

The wave can be considered through the wave to the left its pressure is raised to $p+dp$ and velocity lowered to $a-dc$. (25)

wave can be considered as a stationary wave contained within a control surface through which flow occurs from right to left

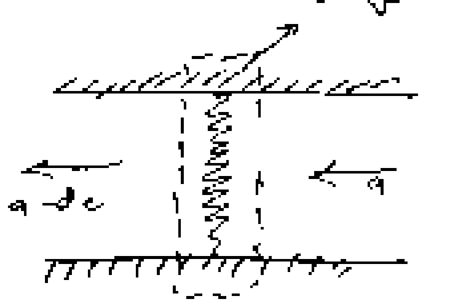
momentum & Continuity Equations are written for the Control Surface

Momentum Equation for this process gives

$\Sigma F_x = \dot{m}(c_x)$

$$A[p-(p+dp)] = \dot{m}[(a-dc)-a]$$

$$A\,dp = \dot{m}\,dc \rightarrow (1)$$



From Continuity Equation

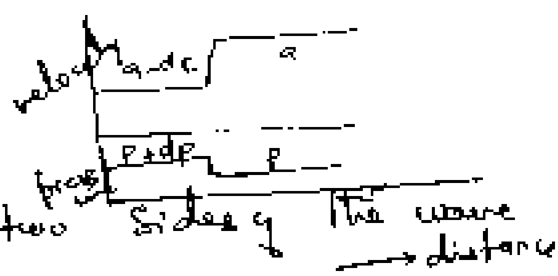
$$\dot{m} = \rho A c$$

$$\dot{m} = \rho A a \rightarrow (2)$$

(2) in (1)

$$A\,dp = (\rho A a)\,dc$$

$$dp = \rho a\,dc \rightarrow (3)$$



From Continuity Equation for the two sides of the wave

$$\dot{m} = \rho A a = (\rho+d\rho)A(a-dc) \rightarrow (4)$$

$$\rho A a = \rho a A - \rho A\,dc + d\rho\,aA - d\rho A\,dc$$

$$\rho A\,dc = d\rho\,a A$$

$$\rho\,dc = a\,d\rho \rightarrow (5)$$

(5) in (3)

$$dp = a^2 d\rho$$

$$\left(\frac{dp}{d\rho}\right)_{isentropic} = a^2 \qquad a = \sqrt{\frac{dp}{d\rho}}$$

$a^2 = \frac{dp}{d\rho}$ → Compressible fluid → there is a large density change for a given pressure change.

$a_{air} = 340\, m/s$ (air) and of compressible <<

$a_{H_2O} = 1400\, m/s$ >> (Incompressible)

$a_{steel} = 5000\, m/s$ >> (").

We know that

$K = \rho \cdot \frac{dp}{d\rho}$ (Bulk modulus)

$K_s = \gamma p$ (adiabatic bulk modulus) $\quad \frac{p}{\rho} = RT$

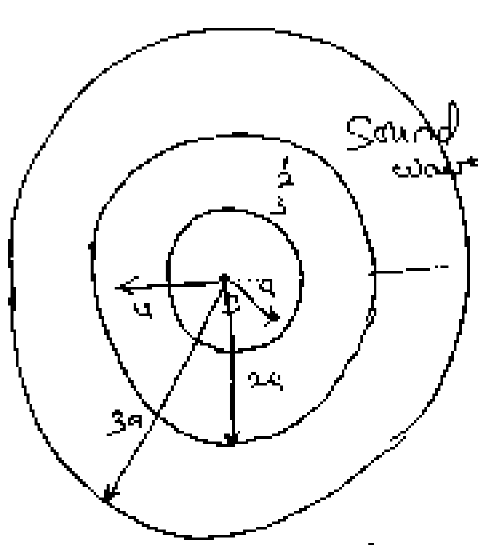
$a = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma R T} = \sqrt{\gamma \cdot \frac{\bar{R}}{W} T}$

@ at a given fluid, a >> at higher temp

@ fluids with higher values of K, have higher velocities of sound

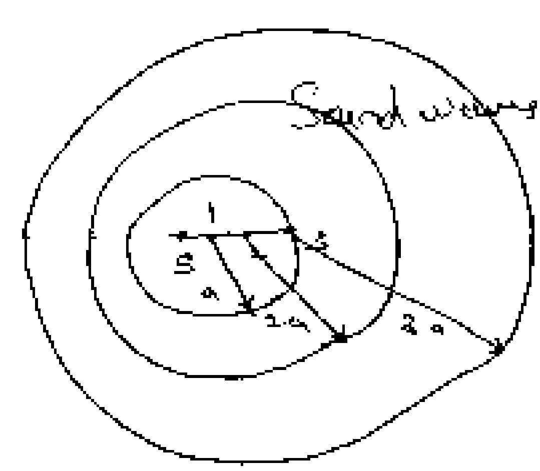
---

Mach angle & Mach Cone



Incompressible flow

$\frac{c}{a} \simeq 0 \quad M \simeq 0$



Subsonic flow

$M = \frac{c}{a}$

$c = 0.5a$

$\frac{1}{2} = \frac{c}{a}$

$0.5 = c$

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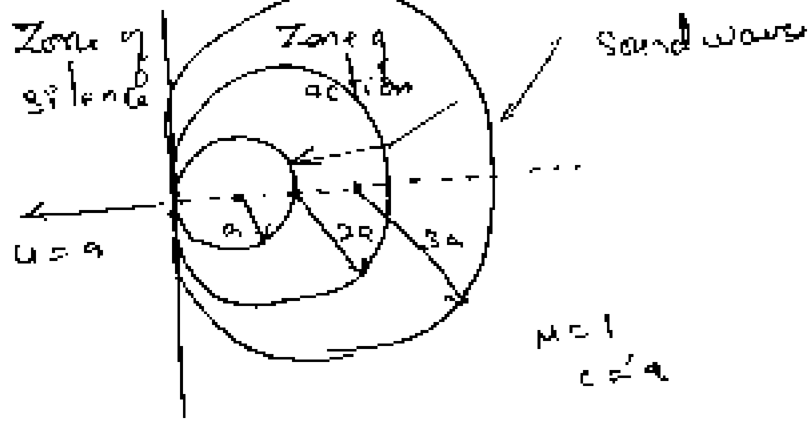
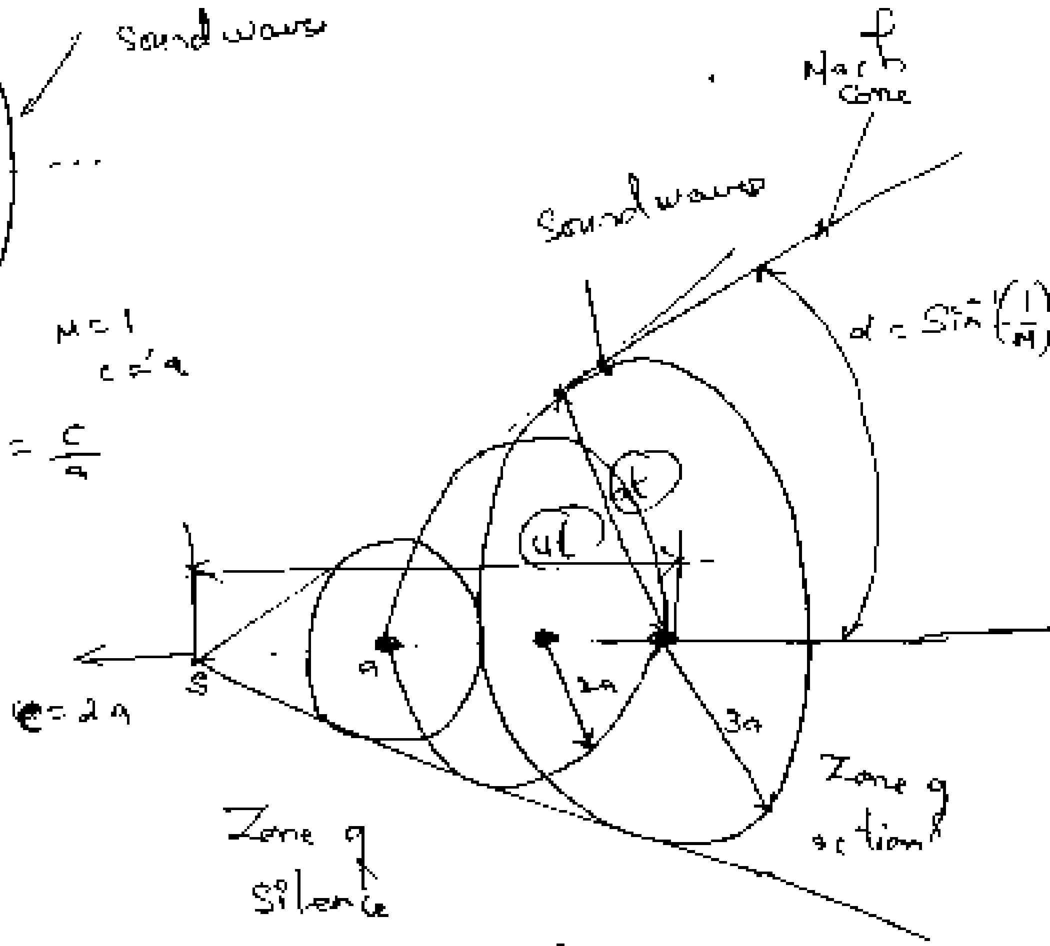


Sonic flow $M = \frac{c}{a}$



Supersonic flow

$C = 2a$

$M = 2 = \frac{c}{a}$

Subsonic flow

S → movement of a source of disturbance (S) at a velocity C in a fluid from right to left, present position

S → 1, 2, and 3 shows its position before (1, 2 and 3 seconds)

disturbance travels are a, 2a and 3a, metres 1, 2, & 3 seconds

Incompressible flow

c (velocity of source << small (negligibly small) compared to a.

Infinitesimal spherical waves (sound waves) are generated to a velocity a in all direction

displacement (s) is very small in 3 second

Subsonic flow

$M = \frac{c}{a} = 0.5$,

→ S (travel) half the velocity of the wave
source of disturbance

→ Spherical sound waves generated t = 3, 2, 1 seconds before the present position.

→ It is observed that wave front move ahead the point source and intensity is not symmetrical

Sonic flow

M = 1, point source travel with the same velocity as that of wave

c = a. Exist at present position of the wave front always

point and cannot move ahead on it.
(region down streaming point source) (i.e.) the wave front is a the zone lying on the left of because the waves do not reach this zone.

Zone of silence

The zone on the right of the wave front is traversed by the waves and is therefore a Zone of action.

Super Sonic flow

$c = 2a$

$M = 2 = \frac{c}{a}$,

The waves generated at positions 3, 2 and 1 and S are shown always ahead of the wave fronts.

The point source are

Tangents drawn from the point S on the sphere defines a conical surface referred as Mach cone

All the waves are confined to the region within the Mach cone, ∴ → zone of action

The waves do not reach the region outside the Mach cone, this zone is zone of silence (27)

Semi angle of cone ⟶ Mach angle

$$\sin\alpha = \frac{\partial T}{\partial S}$$

$$\alpha = \sin^{-1}\left(\frac{\partial T}{\partial S}\right) = \sin^{-1}\left(\frac{at}{ct}\right)$$

$$= \sin^{-1}\left(\frac{1}{c/a}\right)$$

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right)$$

$$\boxed{\sin\alpha = 1/M}$$

Isentropic flow in variable area Unit-I

(1) (28)

→ A steady one-dimensional isentropic flow in variable area passages is considered.

→ The flow parameters do not vary with time it is referred as steady flow

→ Along with its steady flow the flow parameters do not vary in directions normal to the flow directions it is said to one-dimensional flow

Assumptions
heat transfer Q = 0
no irreversibilities due to fluid friction.

one-dimensional flow occur in stream tube where the boundary layer effects and heat conduction are absent.

Comparision of isentropic and adiabatic process

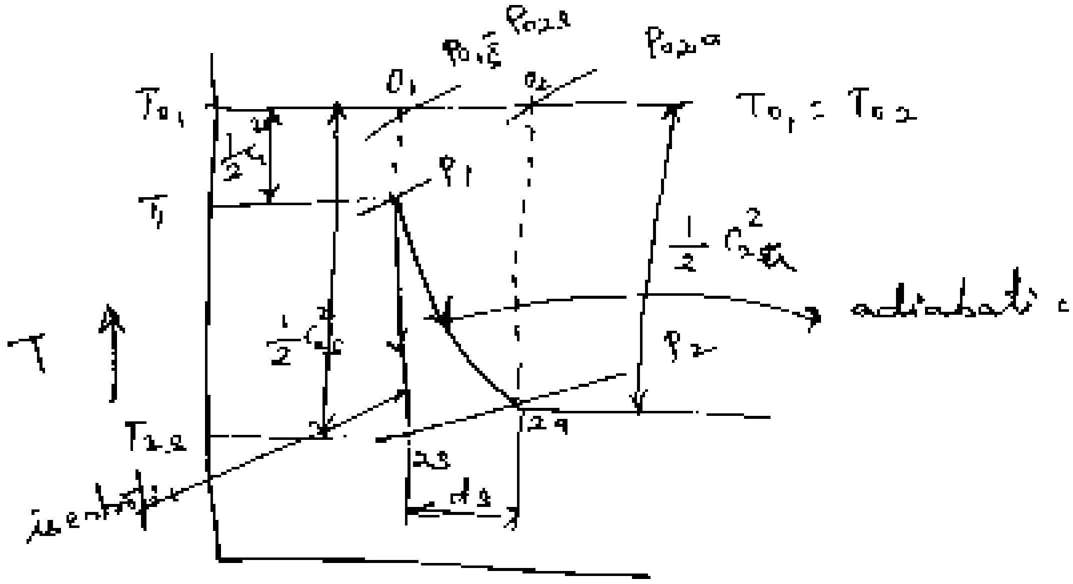
Fig(1) → shows isentropic and adiabatic expansion of a perfect gas b/t the two state 1 & 2

P_{01} — stagnation

$\frac{1}{2}c_1^2$ = kinetic Energy

T_{01} & T → Stagnation temp. & static temps

T_{2s} & $\frac{1}{2}c_{2s}^2$ → final temp. & K.E, isentropic process



T-s diagram for Expansion process

There is an increase in Entropy (ds)

$P_{01s} = P_{02} = P_0$ isentropic process

$P_{02a} < P_{01s}$

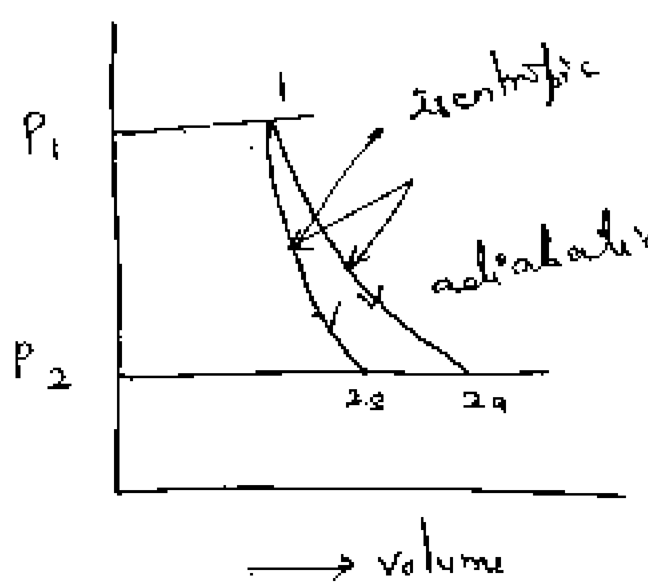
$\frac{1}{2}C_{2a}^2 < \frac{1}{2}C_{2s}^2$

$T_{01} = T_{02} = T_0$ (for both isentropic and adiabatic process for a perfect gas)

from Equation of state

$$v_{2s} = \frac{R\,T_{2s}}{P_2}$$

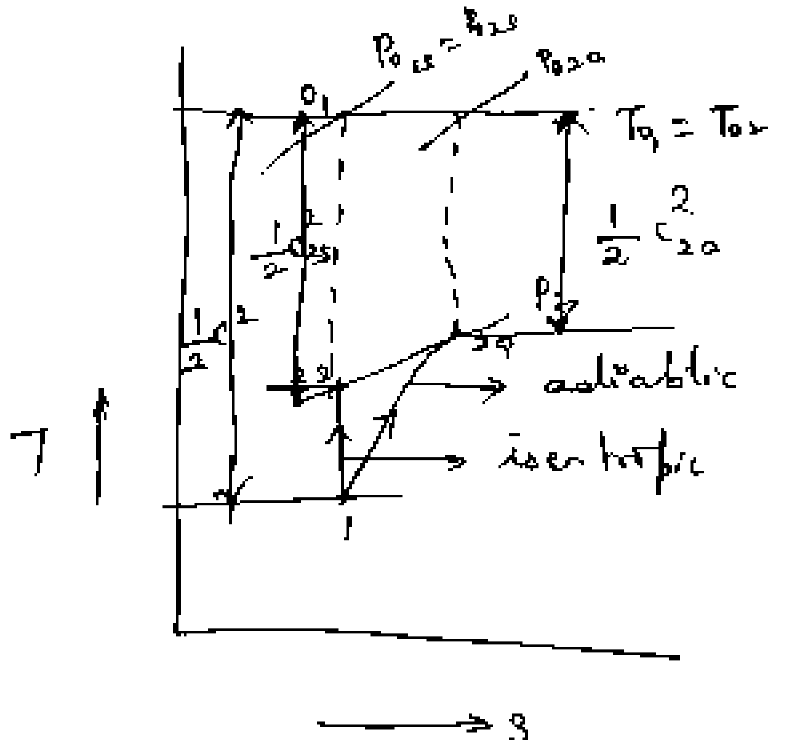
$$v_{2a} = \frac{R\,T_{2a}}{P_2}$$

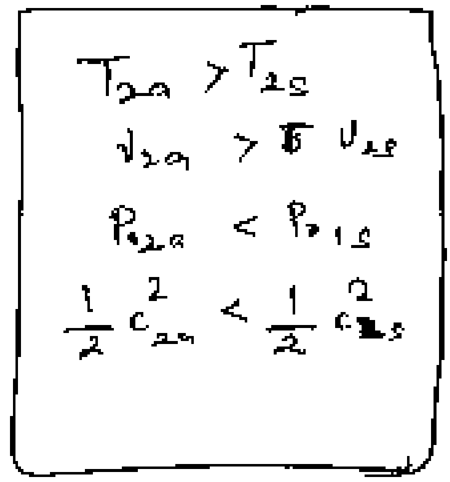


but $T_{2a} > T_{2s}$

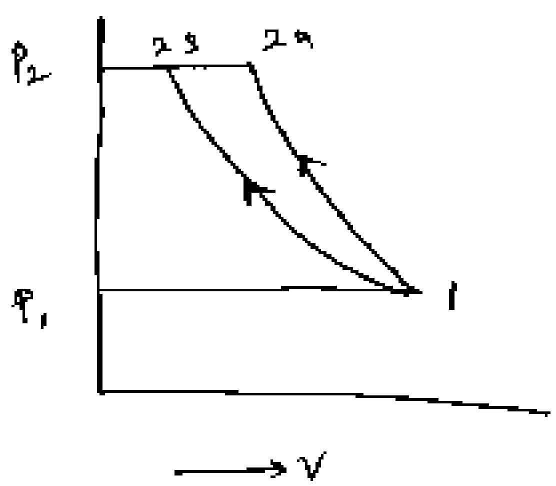
$\therefore \quad v_{2a} > v_{2s}$

The isentropic and adiabatic Compression (29) processes between P_1 and P_2 on T-S and P-V diagram are shown in fig.





T-S diagram for Compression processes



P-V diagram for Compression process

Mach Number variation

(30)

(2)

~~from~~ ~~Conti~~

From Continuity Equation mass flow rate

$$\dot{m} = \rho A c = \text{Constant} \longrightarrow (1)$$

From ~~isentropic~~ adiabatic Energy Equation

$$h_0 = h + \frac{1}{2}c^2$$

$$dh_0 = dh + \frac{2c\,dc}{2}$$

$$0 = dh + c\,dc$$

$$\frac{dp}{\rho} = -c\,dc$$

$$dp = -\rho c\,dc \longrightarrow (2)$$

Taking logs and diff in (1)

$$\ln\rho + \ln A + \ln c = \text{Constant}$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dc}{c} = 0$$

$$\frac{dc}{c} = -\left(\frac{d\rho}{\rho} + \frac{dA}{A}\right)$$

$$dc = -c\left(\frac{d\rho}{\rho} + \frac{dA}{A}\right) \longrightarrow (3)$$

(2) in (3)

$$\frac{dp}{-\rho c} = -c\left(\frac{d\rho}{\rho} + \frac{dA}{A}\right)$$

$$dp = \rho c^2\left(\frac{d\rho}{\rho} + \frac{dA}{A}\right)$$

$$\frac{dp}{\rho c^2} = \frac{d\rho}{\rho} + \frac{dA}{A}$$

$$\frac{dA}{A} = \frac{dp}{\rho c^2} - \frac{d\rho}{\rho}$$

Taking out $\frac{dp}{\rho c^2}$

$$\frac{dA}{A} = \frac{dp}{\rho c^2}\left[1 - c^2 \frac{\partial \rho}{dp}\right]$$

For an isentropic process

$$\frac{dp}{d\rho} = \left(\frac{\partial p}{\partial \rho}\right)_s = a^2$$

$$\frac{dA}{A} = \frac{dp}{\rho c^2}\left[1 - \frac{c^2}{a^2}\right]$$

$$\frac{dA}{A} = \frac{dp}{\rho c^2}\left[1 - M^2\right] \longrightarrow (4)$$

Equation (4) can now be considered for both accelerating & decelerating passages for various values of Mach Number.

Expansion in Nozzles

Gases and vapours are Expanded in Nozzles by providing a pressure ratio them.

The shape of the passage depends on local Mach number, M, is shown in Equation $\frac{dA}{A} = \frac{dp}{\rho c^2}[1-M^2)$

The purpose of nozzle is to accelerate the flow by providing a pressure drop, (dp)

So the pressure drop (dp) in Equation is Negative.

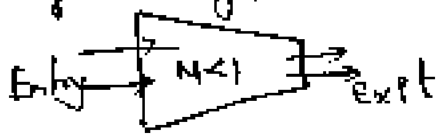
3- possible conditions are considered

$dp(-)$ $M \rightarrow 0$ to 1

(i.e.)

(a) for $M < 1$, $\frac{dA}{A}$ = Negative (<)

This is Convergent type Nozzle. & flow is subsonic



(b) for $M = 1$, Sonic velocity

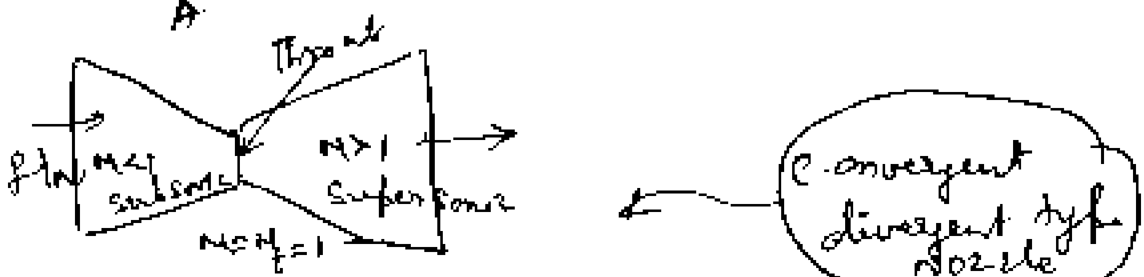
$\frac{dA}{A} = 0$ [no change in area] (c/s)

Throat of the passage

(c) ~~for M>1~~ ~~Supers~~

(c) for $M > 1$, $\frac{dA}{A}$ = positive

$\frac{dA}{A} >>$, divergent passage → divergent Nozzle



Isentropic flow of gas in a Nozzle ($\Delta p <<$)

Compression in diffusers

Diffusers are employed to obtain pressure rise in flowing gases for a given value of the initial kinetic energy.

$\Delta p >>$, (static pressure rise) at the cost of deceleration of flow in a diffuser.

$\Delta p >>$, decrease in velocity

dp in the Equation (4) is always positive

(i.e.) $\frac{dA}{A} = \frac{dp}{\rho c^2}\left[1 - M^2\right]$

→ positive

3 - cases are considered

(a) for $M < 1$, $\frac{dA}{A}$ = positive as (Subsonic flow)

$dp = (+)$ divergent passage

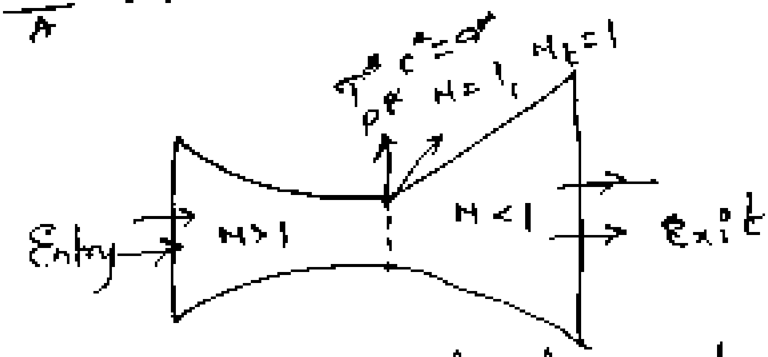
$A >> \; << M$

(b) for $M = 1$, $\frac{dA}{A} = 0$.

There is no change in area.

(c) for $M > 1$, $\frac{dA}{A}$ = Negative.

$\frac{dA}{A} <<$ with Mach Number Convergent type



Convergent - divergent diffuser

~~Variation in flow parameters in isentropic flow~~

Variation in flow parameters in isentropic flow

Mach Number	Nozzles A	Nozzles P	Nozzles C	Diffusers A	Diffusers P	Diffusers C
M<1 Sub sonic	<	<	>	>	>	<
M=1 Sonic	A^*	p^*	$c^* = a^*$	A^*	p^*	$c^* = a^*$
M>1 Super sonic	>	<	>	<	>	<

Stagnation and critical states

The Equation $\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2 \rightarrow (1)$ was derived for adiabatic flow, this holds for isentropic flow also. The ~~statistical~~ stagnation state was defined for isentropic process.

The pressure and density ratio are

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\gamma/\gamma-1} \rightarrow (2)$$

$$\frac{\rho_0}{\rho} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{1/\gamma-1} \rightarrow (3)$$

From the fig (1) stagnation temp. (T_0) & stagnation pressure (P_0) fully define stagnation state.

The isentropic expansion of gas to its C_{max}, through the critical velocity $c^* = a^*$, the critical point ($M = 1$).

In adiabatic flow, the critical point ($M=1$) isentropic and adiabatic flow are different (fig 2)

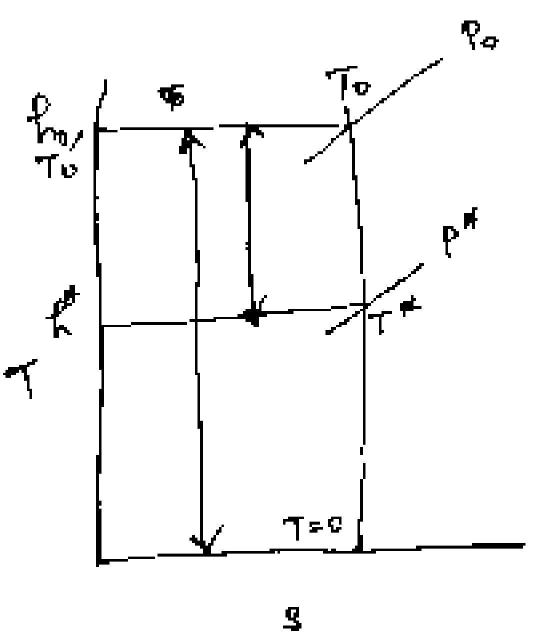


fig (1) critical & maximum velocities

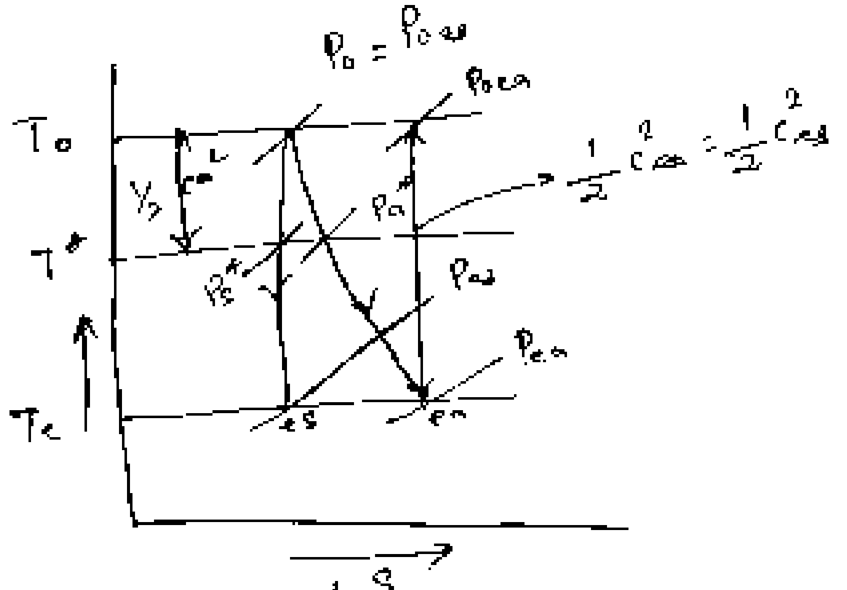


fig (2) Comparision of adiabatic & isentropic Expansion processes

The critical velocities are same

@ critical point $c_s^* = c_a^* = a_s^* = a_a^* = \sqrt{\gamma R T^*}$

$P_s^* > P_a^*$

$P_{0s}^* > P_{0a}^*$

@ exit point

$P_{es} > P_{ea}$

$P_{0es} > P_{0ea}$

for isentropic flow

$P_0 = P_{0s}^* = P_{0es}$

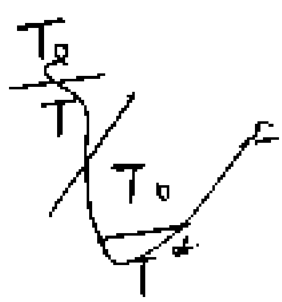
@ $M = 1$ in ①, ② + ③ $(\gamma = 1.4)$

$$\frac{T_0}{T^*} = 1 + \frac{\gamma - 1}{2}(1) = \frac{\gamma + 1}{2} = 0.833 \rightarrow ④$$

$$\frac{P^*}{P_0} = \left(\frac{2}{\gamma + 1}\right)^{\gamma/\gamma - 1} = 0.528 \rightarrow ⑤$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma + 1}\right)^{1/\gamma - 1} = 0.634 \rightarrow ⑥$$

④, ⑤, ⑥ in



$$\frac{T^*}{T_0} = \frac{2}{\gamma + 1}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{T^*}{T_0} \times \frac{T_0}{T} = \frac{2}{\gamma + 1} \times \left(1 + \frac{\gamma - 1}{2} M^2\right)$$

$$\frac{T^*}{T} = \frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2 \rightarrow ⑦$$

$$\frac{P^*}{P} = \left(\frac{T^*}{T}\right)^{\gamma/\gamma - 1} = \left[\frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2\right]^{\gamma/\gamma - 1} \rightarrow ⑧$$

$$\frac{\rho^*}{\rho} = \left[\frac{2}{\gamma+1} \frac{-1}{\gamma+1} M^2\right]^{1/\gamma-1} \longrightarrow (9)$$

Area ratio has a function of Mach Number

The ratio of area (A) at the given Mach Number (M) and the reference ratio A^* at sonic conditions ($M = M_t = 1$) is expressed as a function of the local Mach number (M)

(i.e) $$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

From Continuity equation $\dot{m} = \rho A C \longrightarrow (1)$

@ critical condition $\dot{m} = \rho^* A^* c^* \longrightarrow (2)$

$$\rho^* A^* c^* = \rho A C$$

$$\frac{\rho^*}{\rho} \times \frac{c^*}{c} = A/A^* \longrightarrow (3)$$

We know that M^* (Characteristic Mach Number)

$$M^{*2} = \frac{\frac{\gamma+1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2}$$

$M^* = \frac{c}{a^*} = \frac{c}{c^*}$

$c^* = $

$$M^* = \left[\frac{\frac{\gamma+1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2}\right]^{1/2}$$

$$\frac{c}{a^*} = \left[\frac{\frac{\gamma+1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2}\right]^{1/2}$$

$$\frac{c}{c^*} = \left[\frac{\frac{\gamma+1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2}\right]$$

$$\frac{c^*}{c} = \frac{1}{\left[\dfrac{\frac{\gamma+1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2}\right]^{1/2}} = \left[\frac{\frac{\gamma-1}{2} M^2}{\frac{\gamma+1}{2} M^2}\right]^{1/2}$$

Multiplying 2 on RHS

$$\frac{c^*}{c} = \left[\frac{2 + \gamma - 1\ M^2}{\gamma+1\ M^2}\right]^{1/2} = \left[\frac{2}{(\gamma+1)M^2} + \frac{\gamma-1}{\gamma+1}\right]^{1/2}$$

Taking out $1/M^2$ out side

$$\frac{c^*}{c} = \frac{1}{M}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M^2\right]^{1/2} \longrightarrow (4)$$

Also we know that

$$\frac{\rho^*}{\rho} = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M^2\right]^{1/\gamma-1} \longrightarrow (5)$$

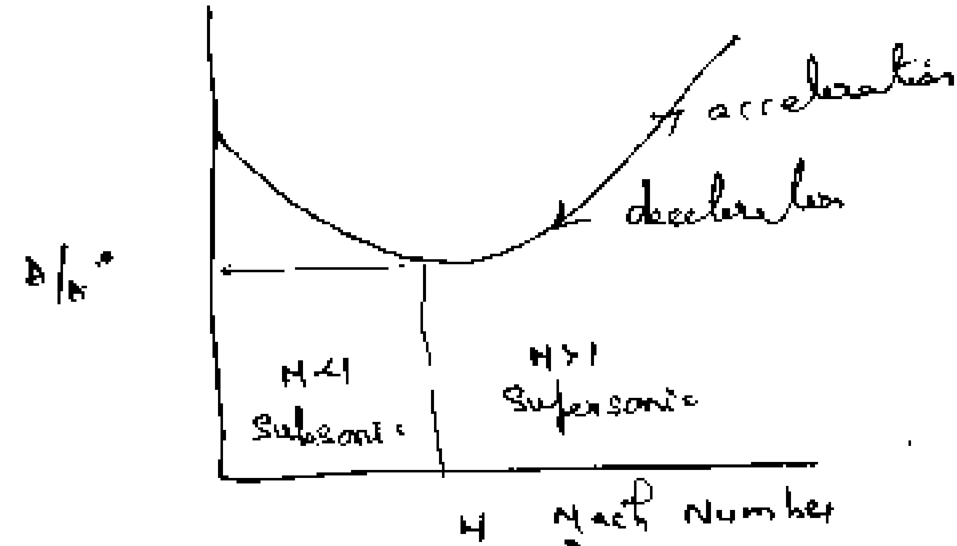
(4) and (5) in 3

$$A/A^* = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M^2\right]^{1/\gamma-1} \times \frac{1}{M}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M^2\right]^{1/2}$$

$$/_* = \frac{1}{M}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}} \longrightarrow (6)$$

$$\left(\frac{1}{\gamma-1} + \frac{1}{2} \quad \frac{2+(\gamma-1)}{2(\gamma-1)}\right)$$

The variation of area ratio for subsonic and supersonic isentropic acceleration & deceleration (34)



Variation of area ratio as a function of Mach Number.

Mass flow rate

From continuity equation the mass flow rate

$$\dot{m} = \rho A c \longrightarrow (1)$$

stagnation pressure & stagnation density relation

$$\frac{\rho_0}{\rho} = \left(\frac{P_0}{P}\right)^{1/\gamma} \longrightarrow (2)$$

$$\rho = \frac{\rho_0}{\left(\frac{P_0}{P}\right)^{1/\gamma}} = \rho_0 \times \left(\frac{P_0}{P}\right)^{-1/\gamma}.$$

$$\rho = \rho_0 \times \left(\frac{P}{P_0}\right)^{1/\gamma} \longrightarrow (3)$$

Stagnation temp.

$$T_0 = T + \frac{c^2}{2c_p}$$

$$c^2 = 2c_p \left(T_0 - T\right)$$

$$c^2 = 2c_p T_0 \left[1 - \frac{T}{T_0}\right]$$

$$c^2 = \frac{2\gamma R}{\gamma-1} T_0 \left[1 - \frac{T}{T_0}\right]$$

$$c = \sqrt{\frac{2\gamma R}{\gamma-1} T_0 \left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad \frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$

→ (4)

mass flow rate in terms of area ratio

$\dot{m} = \rho A c$ → (6) → (A)

sub (3) & (4) in (A)

$$\dot{m} = \rho_0 \times \left(\frac{p}{p_0}\right)^{1/\gamma} \times A \sqrt{\frac{2\gamma R}{\gamma-1} T_0 \left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\rho_0 = \frac{p_0}{R T_0}$$

$$\dot{m} = \frac{p_0}{R T_0} \times A \times \left(\frac{p}{p_0}\right)^{1/\gamma} \sqrt{\frac{2\gamma R}{\gamma-1} T_0 \left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \frac{p_0}{R T_0} \times A \times \left(\frac{p}{p_0}\right)^{1/\gamma} \sqrt{R T_0} \sqrt{\frac{2\gamma}{\gamma-1} \left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \frac{A p_0}{\sqrt{R T_0} \times \sqrt{R T_0}} \times \left(\frac{p}{p_0}\right)^{1/\gamma} \sqrt{R T_0} \sqrt{\frac{2\gamma}{\gamma-1} \left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\dot{m} = \frac{A p_0}{\sqrt{R T_0}} \times \left(\frac{p}{p_0}\right)^{1/\gamma} \sqrt{\frac{2\gamma}{\gamma-1} \left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\frac{\dot{m}\sqrt{R T_0}}{A p_0} = \sqrt{\gamma} \times \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{p}{p_0}\right)^{2/\gamma} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

$$\left(\frac{\gamma-1}{\gamma} + \frac{2}{\gamma} = \frac{\gamma+1}{\gamma}\right)$$

~~$$\frac{\dot{m}\sqrt{T_0}}{A p_0} \times \sqrt{\frac{R}{\gamma}} = \frac{2}{\gamma-1} \left[\left(\frac{p}{p_0}\right)^{2/\gamma} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right]$$~~

$$\frac{\dot{m}\sqrt{T_0}}{A P_0} \times \sqrt{\frac{R}{\gamma}} = \sqrt{\frac{2}{\gamma-1}\left\{\left(\frac{P}{P_0}\right)^{2/\gamma} - \left(\frac{P}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\right\}} \quad \text{(5)} \quad \text{(35)}$$

Let $q = P/P_0$

$$\frac{d}{dp}\left[\left(\frac{P}{P_0}\right)^{2/\gamma} - \left(\frac{P}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\right] = 0$$

$$\frac{d}{dp}\left[(q)^{2/\gamma} - (q)^{\frac{\gamma+1}{\gamma}}\right] = 0$$

$$\frac{2}{\gamma} q^{2/\gamma - 1} - \frac{\gamma+1}{\gamma} q^{\frac{\gamma+1}{\gamma} - 1} = 0$$

$$\frac{2}{\gamma} q^{\frac{2-\gamma}{\gamma}} - \frac{\gamma+1}{\gamma} q^{\frac{\gamma+1-\gamma}{\gamma}} = 0$$

$$\frac{2}{\gamma} q^{\frac{2-\gamma}{\gamma}} = \gamma+1 \; q^{1/\gamma}$$

$$\frac{2}{\gamma+1} = \frac{(q)^{1/\gamma}}{(q)^{\frac{2-\gamma}{\gamma}}} = q^{1/\gamma + \frac{2-\gamma}{\gamma}}$$

$$= q^{\frac{1-2+\gamma}{\gamma}} = q^{\frac{-1+\gamma}{\gamma}}$$

$$\frac{2}{\gamma+1} = q^{\frac{\gamma-1}{\gamma}}$$

$$\left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1} = q = P/P_0 \longrightarrow \text{(6)}$$

For maximum flow rate condition, $\dot{m} = \dot{m}_{max}$, $A = A^*$

$$\left(\frac{P}{P_0}\right)_{max} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1} \quad \text{(i.e.)} \quad \frac{P^*}{P_0} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}$$

(7) in (5)

$$\frac{\dot{m}_{max}\sqrt{T_0}}{A P_0} \times \sqrt{\frac{R}{\gamma}} = \sqrt{\frac{2}{\gamma-1}\left\{\left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1 \times \frac{2}{\gamma}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1} \times \frac{\gamma+1}{\gamma}}\right\}}$$

$$\frac{\dot m_{max}\sqrt{T_0}}{A\times P_0}\times\sqrt{\frac{R}{\gamma}} = \sqrt{\frac{2}{\gamma-1}\left\{\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right\}}$$

$$\frac{\dot m_{max}\sqrt{T_0}}{A\, P_0}\times\sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\sqrt{\frac{2}{\gamma-1}\left\{\left(\frac{2}{\gamma+1}\right)\right.}$$

$$=\sqrt{\frac{2}{\gamma-1}\left\{\left(\frac{2}{\gamma+1}\right)^{2/\gamma-1}\times\left(\frac{2}{\gamma+1}\right)^{1}\times\frac{\gamma+1}{2} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right\}}$$

$$=\sqrt{\frac{2}{\gamma-1}\left\{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{\gamma+1}{2}\right) - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right\}}$$

divide & Mult $\frac{2}{\gamma+1}$

$\frac{2}{\gamma-1}+1$

$\frac{2+\gamma-1}{\gamma-1}$

$\frac{\gamma+1}{\gamma-1}$

$$=\sqrt{\frac{2}{\gamma-1}\times\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[\frac{2}{\gamma+1}-1\right]}$$

$$=\sqrt{\frac{2}{\gamma-1}\times\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[\frac{2-\gamma-1}{\gamma+1}\right]}$$

$$=\sqrt{\left(\frac{2}{\gamma-1}\right)\times\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[\frac{-\gamma+1}{\gamma+1}\right]}$$

$$=\sqrt{\frac{2}{\gamma-1}\left\{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\times\frac{\gamma+1}{2} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right\}}$$

$$=\sqrt{\frac{2}{\gamma-1}\times\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[\frac{\gamma+1}{2}-1\right]}$$

$$=\sqrt{\frac{2}{\gamma-1}\times\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[\frac{\gamma+1-2}{2}\right]}$$

$$=\sqrt{\frac{2}{\gamma-1}\times\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\frac{\gamma-1}{2}}$$

$$\frac{\dot m_{max}\sqrt{T_0}}{A\times P_0}\times\sqrt{\frac{R}{\gamma}} = \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

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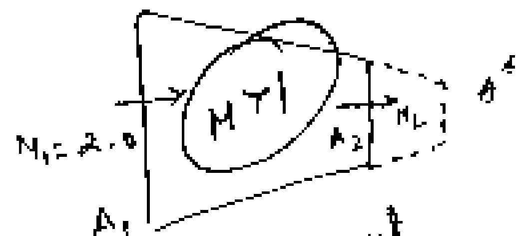
A supersonic diffuser has a following data $P_1 = 1.0$ bar, $T_1 = 300$ k, $M_1 = 2.0$. The area ratio of the diffuser is 1.5945. Determine the exit Mach Number, pressure and temperature of air for isentropic flow

Taking $\gamma = 1.4$, $R = 287$ J/kg.k

$P_1 = 1.0$ bar, $T_1 = 300$ k, $M_1 = 2.0$



@ $M_1 = 2.0$ (isentropic flow Table)

$\frac{A_1}{A^*} = 1.687$, $\frac{P_1}{P_{01}} = 0.128$, $\frac{T_1}{T_{01}} = 0.55$

$\frac{A_1}{A_2} = 1.5945$

$\frac{P_1}{P_{01}} = \frac{1}{P_{01}} = 0.128$ $\quad$ $P_{01} = 7.8125$ bar

$\frac{300}{T_{01}} = 0.55$ $\quad$ $T_{01} = 540.54$ K

@ $\frac{A_2}{A^*} = \frac{A_2}{A_1} \times \frac{A_1}{A^*} = \frac{1}{1.5945} \times 1.687$

$\frac{A_2}{A^*} = 1.058$

(i.e.) $\frac{A_1}{A_2} = \frac{A_1}{A^*} \times \frac{A^*}{A_2}$ ✓

From isentropic flow Table $M_2 > 1 = 1.28$

From isentropic flow table

$M_2 = 1.28$, $\frac{P_2}{P_{02}} = 0.371$, $\frac{T_2}{T_{02}} = 0.753$

For isentropic flow

$P_{01} = P_{02} = P_0 = 7.8125$ bar

$T_{01} = T_{02} = T_0 = 540.54$ K

$$\frac{p_2}{p_{02}} = 0.371$$

$$p_2 = 0.371 \times 7.8125 = \boxed{2.898 \text{ bar} = p_2}$$

$$\frac{T_2}{T_{02}} = 0.753$$

$$T_2 = 0.753 \times 540.54, \quad \boxed{T_2 = 407.027 \text{ K}}$$

(2) Air is discharged from a reservoir at $P_0 = 6.91$ bar, $T_0 = 325°C$, through a Nozzle to an Exit pressure of 0.98 bar. if the flow rate is 3600 kg/hr. determine for isentropic flow

(i) throat area, pressure & velocity

(ii) Exit area, Mach Number, Temp. exit.

(iii). maximum velocity.

Taking $\gamma = 1.4$, $R = 287$ J/kg.K

given data

$P_0 = 6.91$ bar, $T_0 = 325 + 273$, $p_2 = 0.98$ bar

$T_0 = 598$ K

for isentropic flow $\boxed{T_{01} = T_{02} = T_0}$, $\boxed{P_{01} = P_{02} = P_0}$

@ M = 1 throat condition, isentropic flow table

$c^* = a^*$, $\gamma = 1.4$

T^*, P^*: $\frac{T^*}{T_0} = 0.834$, $\frac{p^*}{p_0} = 0.528$, $\frac{\rho^*}{\rho_0} = 0.634$

$$\rho_0 = \frac{P_0}{R T_0} = \frac{6.91 \times 10^5}{287 \times 598}$$

$$\boxed{\rho_0 = 4.04 \text{ kg/m}^3}$$

$p^* = 0.528 \times 6.91$

$\boxed{p^* = 3.65 \text{ bar}}$

$T^* = 0.834 \times 598$

$\boxed{T^* = 498.73 \text{ K}}$

$$c^* = a^* = \sqrt{\gamma R T^*}$$

$$= \sqrt{1.4 \times 287 \times 498.73}$$

$$\boxed{c^* = a^* = 448.2 \text{ m/s}}$$

$\dot{m} = 3600 \ kg/hr$

$\dot{m} = 1 \ kg/sec$

$\frac{\rho^*}{\rho_0} = 0.634$

$\dot{m} = \rho^* A^* c^*$

$\rho^* = 0.634 \times 4.04$

$1 = 2.56 \times A^* \times 448.2$

$\rho^* = 2.56 \ kg/m^3$

$A^* = 8.68 \times 10^{-4} \ m^2$

$\gamma = 1.4$

$\frac{P_2}{P_{02}} = \frac{0.98}{6.91} = 0.14$ from isentropic flow Table (check)

$= 0.14$

$M_2 = 1.93$. $\frac{A_2}{A^*} = 1.599$. $\frac{T_2}{T_{02}} = 0.572$ ✓

$A_2 = 13.85 \times 10^{-4} \ m^2$

$T_2 = 342.65 K$

$C_{max} = \sqrt{2 C_p T_0} = \sqrt{\frac{2 \gamma R}{\gamma - 1} T_0} = \sqrt{\frac{2 \times 1.4 \times 287 \times 598}{1.4 - 1}}$

$= 1096 \ m/s$

---

(3) A gas is isentropically expanded from $P = 10$ bar Temp = 525°C in a Nozzle to a pressure of 7.6 bar. If the rate of flow of a gas is 1.5 kg/s, determine

(i) Pressure, temp and velocity at throat & Exit.

(ii) C_{max}. $\gamma = 1.2$, $R = 464 \ J/kg.K$.

(iii) The type of the Nozzle and its throat area

given data

$P_0 = 10$ bar $\quad P_2 = 7.6$ bar

$T_0 = 525 + 273 = 798 K \quad \dot{m} = 1.5 \ kg/s$

$T_0 = 798 K$

@ $M=1$ T^*, c^*, a^* $\gamma=1.2$

$$\frac{T^*}{T_0} = 0.870 \qquad \frac{P^*}{P_0} = 0.546$$

$T^* = 694.26\,k$ $P^* = 5.46$

$c^* = a^* = \sqrt{\gamma R T^*}$

$= \sqrt{1.3 \times 464 \times 694.26}$

From isentropic flow table

@ $\frac{P_2}{P_{02}} = \frac{7.6}{10} = 0.76$

$\gamma = 1.3$

$M_2 = 0.66$

$$\frac{T_2}{T_{02}} = 0.938$$

$T_2 = 748.5\,k$

for isentropic flow

$T_{01} = T_{02} = T_0$

$P_{01} = P_{02} = P_0$

$$M_2 = \frac{c_2}{a_2} = \frac{c_2}{\sqrt{\gamma R T_2}}$$

$c_2 = 443.48\ m/s$

$$0.66 = \frac{c_2}{\sqrt{1.3 \times 464 \times 748.5}}$$

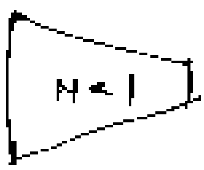
Since $M_2 < 1$, the type of Nozzle is Convergent

So, $P_2 = P_2^* = 7.6\,bar$

$T_2 = T_2^* = 748.5\,k$

$c_2 = c^* = 443.48\ m/s$

exit & Throat are same



$C_{max} = \sqrt{2 c_p T_0}$

$= \sqrt{\frac{2 \gamma R}{\gamma - 1} T_0}$

$$C_{max} = \sqrt{\frac{2 \times 1.3 \times 464}{1.3 - 1} \times 798}$$

$C_{max} = 1771\ m/s$

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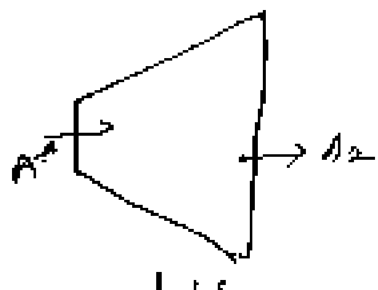
$\dot{m} = \rho_2 A_2 C_2$

$1.5 = 2.188 \times A_2 \times 442.48$ $\boxed{A_2 = 1.54 \times 10^{-3}\ m^2}$

$\rho_2 = \frac{P_2}{R T_2} = \frac{7.6 \times 10^5}{464 \times 748.5} = 2.188\ kg/m^3$

A diffuser has exit to throat area ratio of 1.5 to 1. The inlet Mach Number is 0.8. The initial pressure & temp. are 1 bar & 15°C. Assuming the flow to be isentropic, calculate the following for exit air, Pressure, temp. & Mach Number

<u>given data</u>



diffuser

$\frac{A_2}{A^*} = \frac{1.5}{1}$

$M_1 = 0.8$

$P_1 = 1 \times 10^5\ N/m^2$, $T_1 = 15 + 273 = 288 K$

For isentropic flow $P_{01} = P_{02}$, $T_{01} = T_{02}$

From isentropic flow Table $\underline{\gamma = 1.4}$

$M_1 = 0.8$

$\frac{T_1}{T_{01}} = 0.886$ $\frac{P_1}{P_{01}} = 0.656$ $\frac{A_1}{A^*} = 1.038$

$T_{01} = 325.05 k$, $\frac{1 \times 10^5}{P_{01}} = 0.656$

$\boxed{T_{01} = T_{02}}$ $\boxed{P_{01} = 1.524 bar}$

$\frac{A_2}{A^*} = 1.5 \quad \gamma = 1.4$

$M_2 = 1.86$ ~~$M_2 = 1.86$~~ $M_2 = 0.43$

$M_2 < 1$ → 0.43

divergent type of diffuser

$\frac{T_2}{T_{02}} = 0.964$

$\frac{P_2}{P_{02}} = 0.881$

$P_2 = 0.881 \times 1.524$

$T_2 = 0.964 \times 325.05$

$T_2 = 313.35\,K$

$P_2 = 1.343$ bar

mass flow rate interms of area ratio

(39)

From Continuity Equation

$$\dot{m} = \rho A c = \rho^* A^* c^* \rightarrow (1)$$

$$\frac{\dot{m}}{A} = \rho c = \rho^* \times \frac{A^*}{A} \times c^*$$

$$\rho^* = \frac{p^*}{R T^*}, \quad c^* = a^* = \sqrt{\gamma R T^*}$$

$$\frac{\dot{m}}{A} = \frac{p^*}{R T^*} \times \sqrt{\gamma R T^*} \times \frac{A^*}{A} = \frac{p^*}{R\sqrt{T^*} \times \sqrt{T^*}} \sqrt{\gamma R} \times \sqrt{T^*} \times \frac{A^*}{A}$$

@ M = 1, we know that

$$\frac{T^*}{T_0} = \frac{2}{\gamma+1}, \quad \frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}$$

~~$$\frac{\dot{m}}{A} = \frac{p_0 \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}}{T_0 \left(\frac{2}{\gamma+1}\right) R} \times \sqrt{\gamma R T^*}$$~~

~~$$\frac{\dot{m}}{A} = \frac{p_0 \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}}{T_0 \left(\frac{2}{\gamma+1}\right) \sqrt{R} \times \sqrt{R}} \sqrt{\gamma R T^*}$$~~

~~$$\frac{\dot{m}\sqrt{T_0}}{A\, p_0} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{A^*}{A}$$~~

$$\frac{\dot{m}}{A} = \frac{p^*}{R\sqrt{T^*}} \sqrt{\gamma R} = \frac{p^*}{\sqrt{R} \times \sqrt{R T^*}} \sqrt{\gamma}\sqrt{R} \times \frac{A^*}{A}$$

$$\frac{\dot{m}}{A} \times \sqrt{\frac{R}{\gamma}} = \frac{p_0 \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}}{\sqrt{T_0 \left(2/\gamma+1\right)}} \times \frac{A^*}{A}$$

$$\frac{\dot{m}\sqrt{T_0}}{A\, p_0} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1 - 1/2} \times \frac{A^*}{A}$$

$$\frac{\dot{m}\sqrt{T_0}}{A P_0}\sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times \frac{A^*}{A}$$

for maximum mass flow rate conditions

$\dot{m} = \dot{m}_{max}$ & $A = A^*$

$$\frac{\dot{m}_{max}\sqrt{T_0}}{A^* P_0} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \longrightarrow (4)$$

Numerical value of the non-dimensional maximum mass flow parameter

The value of the maximum mass flow parameter is fixed by the properties of the gas (i.e.) R and γ. Therefore for the same gas various combinations of P_0 and T_0 can be determined to give maximum mass flow density from the following expression

$$\frac{\dot{m}_{max}\sqrt{T_0}}{A^* P_0} = \sqrt{\frac{\gamma}{R}} \times \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

for S.I. units, $R = 287\ J/kg \cdot K$, $\gamma = 1.4$

$$\frac{\dot{m}_{max}\sqrt{T_0}}{A^* P_0} = 0.0404 \longrightarrow (1)$$

$\dot{m}_{max}$ kg/s
T_0 is in K
$A^* = m^2$
$P = N/m^2$

for m.k.s units $R = 29.3$, kgf-m/kg.K $\gamma = 1.4$

$$\frac{\dot{m}_{max}\sqrt{T_0}}{A^* P_0} = 0.397 \longrightarrow (2)$$

Fliegner's formula

$\dot{m}_{max}$ kg/s
T_0 is in K
A^* is in m^2
P_0 is in kgf/m^2

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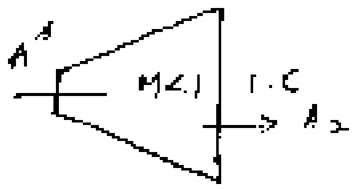
Refacturel

A diffuser has exit to throat area ratio of 1.5 to 1. The inlet Mach Number is 0.8. The Initial pressure and temperature are 1 bar and 15°C. Assuming the flow is isentropic. Calculate the following for exit air, P_2, T_2 and C_2. M_2

$$\frac{A_2}{A^*} = 1.5 \qquad P_1 = 1\ \text{bar}$$

$M_1 = 0.8$ $\qquad T_1 = 15 + 273°C$

$T_1 = 288\ K$



Isentropic flow, $T_{01} = T_{02} = T_0$, $P_{01} = P_{02} = P_0$

Taking $\gamma = 1.4$, $R = 287\ J/kg \cdot K$.

From isentropic flow table ($\gamma = 1.4$)

$M_1 = 0.8$

$$\frac{T_1}{T_{01}} = 0.886 \qquad \frac{P_1}{P_{01}} = 0.656 \qquad \frac{A_1}{A^*} = 1.038.$$

$$T_{01} = \frac{288}{0.886} \qquad \frac{1}{0.656} = P_{01}$$

$T_{01} = 325.05K$ $\qquad P_{01} = 1.524\ bar$

$P_{01} = P_{02} = 1.524\ bar.$

$T_{01} = T_{02} = 325.05\ K.$

$\frac{A_2}{A^*} = 1.5$ $\quad M_2 < 1$ → divergent type nozzle

From isentropic flow Table

$M_2 = 0.43$ $\quad \frac{T_2}{T_{02}} = 0.964, \qquad \frac{P_2}{P_{02}} = 0.881.$

$$\frac{T_2}{325.05} = 0.964$$

$T_2 = 313.35\,K$

$M_2 = 0.43$

$$\frac{P_2}{P_{02}} = 0.881$$

$$\frac{P_2}{\text{}}$$

$P_2 = 0.881 \times 1.524$

$P_2 = 1.343\,bar$

A Supersonic Nozzle expands air from a stagnation pressure of 25 bar and stagnation temperature of 1050 K to an exit pressure of 4.35 bar. If the exit area of the Nozzle is 100 cm². determine the (i) throat area (ii) Pressure and temperature at throat. (iii) Temp. at exit (iv) exit velocity as fraction of the maximum attainable velocity (v) mass flow rate.

given data

Taking $\gamma = 1.4$, $R = 287\ J/kg.k$

$P_0 = 25\,bar$ $T_0 = 1050\,k$

$P_2 = 4.35\,bar$ $A_2 = 100\,cm^2$.

Assume isentropic flow

$T_{01} = T_{02} = 1050\,k$

$P_{01} = P_{02} = 25\,bar$

A^*, P^*, T^*, T_2, $\frac{C_2}{C_{max}}$, $\dot{m}$

$$\frac{P_2}{P_0} = \frac{4.35}{25} = 0.174$$

isentropic flow Table ($\gamma = 1.4$)

$M_2 = 1.80$ $\frac{T_2}{T_{02}} = 0.607$ $\frac{A_2}{A^*} = 1.439$

$T_2 = 1050 \times 0.607$

$T_2 = 637.4\,K$

$$A^* = \frac{100 \times 10^{-4}}{1.439}$$

$A^* = 69.49\,cm^2$

$$M_2 = \frac{c_2}{a_2} = \frac{c_2}{\sqrt{\gamma R T_2}}$$

$$1.80 = \frac{c_2}{\sqrt{1.4 \times 287 \times 637.4}}$$

$c_2 = 910\,m/s$

(41)

@ N=1,

T^*, p^*

$$\frac{T^*}{T_0} = 0.834, \quad \frac{p^*}{p_0} = 0.528$$

$T^* = 0.834 \times 1050$ $\quad p^* = 0.528 \times 25$

$$\boxed{T^* = 875.7\,K} \quad \boxed{p^* = 13.2\,bar}$$

$$C_{max} = \sqrt{2 C_p T_0} = \sqrt{2 \times \frac{\gamma R}{\gamma - 1} T_0} = \sqrt{2 \times \frac{1.4 \times 287}{1.4 - 1} \times 1050}$$

$$\boxed{C_{max} = 1452.37\ m/s}$$

$$\frac{C_{max}}{C_2} = \frac{1452.39}{910} = 0.62 \checkmark$$

$$\dot{m} = \rho_2 A_2 C_2 \ (\text{or})\ \rho^* A^* c^*$$

$$\rho_2 = \frac{P_2}{R T_2} = \frac{4.35 \times 10^5}{287 \times 637.4}$$

$$= \frac{4.35 \times 10^5 \times}{287 \times 637.4} \ 100 \times 10^{-4} \times$$

$$\boxed{\dot{m} = 21.65\ kg/sec}$$

A Nozzle in a wind tunnel gives a test Section Mach Number of 2.0. Air Enters the Nozzle from a large reservoir at 0.69 bar and 310 K. The cross-section area of the throat of $1000\,cm^2$. Determine the following quantities for the tunnel for 1-dimensional isentropic flow.

(i) pressures, temps and velocities at the throat and test sections

(ii) area of c/s of the test section

(iii) mass flow rate (iv) power required to drive the Compressor.

given data

$M_{test} = 2.0$.

$P_0 = 0.69$ bar , $T_0 = 310$ K.

$A^* = 1000$ cm^2.

$P_{01} = P_{02}$, $T_{01} = T_{02}$ — isentropic flow

To find P^*, T^* & C^*, P_{test}, T_{test} & C_{test}

A_{test}.

$\dot{m}$, Power.

$$\rho_0 = \frac{P_0}{RT_0} = \frac{0.69 \times 10^5}{287 \times 310} = 0.775 \text{ kg/m}^3$$

$$a_0 = \sqrt{\gamma R T_0}$$
$$= \sqrt{1.4 \times 287 \times 310}$$

$$a_0 = 353 \text{ m/s}$$

From gas table
M = 1 (throat section)
isentropic flow gas table

$$\frac{P^*}{P_0} = 0.528 \qquad \frac{T^*}{T_0} = 0.834 , \qquad \frac{\rho^*}{\rho_0} = 0.632$$

$$P^* = 0.528 \times 0.69 \qquad T^* = 0.834 \times 310$$

$$P^* = 0.365 \text{ bar} \qquad T^* = 258.54 \text{ K}$$

$$a^* = \sqrt{\gamma R T^*} = \sqrt{1.4 \times 287 \times 258.54}$$

$$a^* = 323 \text{ m/s} = c^*$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{0.365 \times 10^5}{287 \times 258.54} = 0.49 \text{ kg/m}^3$$

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$N_{test} = 2$ (Test section)

$\frac{P_{test}}{P_0} = 0.128$, $\frac{T_{est}}{T_0} = 0.555$, $\frac{A_{test}}{A^*} = 1.687$

$P_{test} = 0.128 \times 0.69$

$\boxed{P_{test} = 0.0885 \text{ bar}}$

$T_{est} = 0.555 \times 310$

$\boxed{T_{est} = 172 K}$

$A_{test} = 1.687 \times 1000 \times 10^{-4}$

$\boxed{A_{test} = 1687 \text{ cm}^2}$

$N_{test} = \frac{c_t}{a_t} = \frac{c_t}{\sqrt{\gamma R T_t}} = \frac{c_{test}}{\sqrt{1.4 \times 287 \times 172}}$

$2 = \frac{c_t}{\sqrt{1.4 \times 287 \times 172}}$

$\boxed{c_{test} = 528 \text{ m/s}}$

$\dot{m} = \rho^* A^* c^*$

$= 0.49 \times 1000 \times 10^{-4} \times 323$

$\boxed{\dot{m} = 15.9 \text{ kg/s}}$

Power required

$w_c = \dot{m} c_p \Delta T$ $\qquad T_2 = T_{est}$

$= \dot{m} c_p (T_{0_1} - T_2)$

$= 15.9 \times 1005 (310 - 172)$

$\boxed{w_c = 2189.19 \text{ kW}}$

A reservoir of Infinite size maintains air with a density of 1.12 kg/m^3, the velocity of Sound in the reservoirs measured at 500m/s. determine

(a) the Value of maximum mass flow rate through a Nozzle of 1 cm throat diameter

(b) Pressure and temperature at the throat for this flow rate.

Taking $\gamma = 1.4$
$R = 287$ J/kg.K

$\rho_0 = 1.12$ kg/m^3 $\quad a_0 = 500$ m/s

$$a_0 = \sqrt{\gamma R T_0}$$

$$500^2 = 1.4 \times 287 \times T_0$$

$$\boxed{T_0 = 622 K}$$

$$\rho_0 = \frac{P_0}{R T_0}$$

$$1.12 = \frac{P_0}{287 \times 622}$$

$$\boxed{P_0 = 2 bar}$$

$$A^* = \frac{\pi}{4} D^{*2}$$

$$= \frac{\pi}{4}(1 \times 10^{-2})^2$$

$$\boxed{A^* = 7.85 \times 10^{-5} m^2}$$

From isentropic flow Table ($\gamma = 1.4$)

@ M = 1, $\frac{P^*}{P_0} = 0.528$ $\quad \frac{T^*}{T_0} = 0.834$

$$P^* = 0.528 \times 2$$

$$\boxed{P^* = 1.056 bar}$$

$$T^* = 0.834 \times 622$$

$$\boxed{T^* = 518 K}$$

maximum mass flow rate

$$\frac{\dot{m}_{max} \sqrt{T_0}}{A^* P_0} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

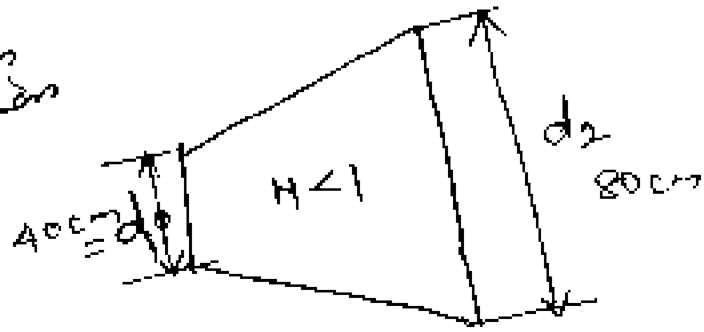
$$\frac{\dot{m}_{max} \times \sqrt{622}}{7.85 \times 10^{-5} \times 2 \times 10^5} \times \sqrt{\frac{287}{1.4}} = \left(\frac{2}{1.4+1}\right)^{\frac{1.4+1}{2(1.4-1)}}$$

$$\boxed{\dot{m}_{max} = 0.0254 \text{ kg/s}}$$

(43)

A Conical air diffuser has an inlet diameter of 40cm and an Exit diameter of 80cm. Air Enters the diffuser with a static pressure of 200kPa, static temperature of 37°C and velocity of 265m/s

Calculate (i) mass flow rate
(ii) properties at Exit.
(iii) Impulse function



given data

$d_1 = 40cm = 0.4m$

$d_2 = 80cm = 0.8m$

$P_1 = 200 \times 10^3$

$T_1 = 37 + 273$

$T_1 = 310 K$

Taking

$\gamma = 1.4$

$R = 287 J/kg.K$

$d_2 > d_1$

$A_2 > A_1$ ∴ C.D. diffuser

[For divergent type diffuser M < 1]

$A_1 = \frac{\pi}{4} d_1^2$

$= \frac{\pi}{4} (0.40)^2$

$\boxed{A_1 = 0.126 m^2}$

$\dot{m} = \rho_1 A_1 C_1 = 2.24 \times 0.126 \times 265 = \boxed{75.05 kg/s = \dot{m}}$

$\rho_1 = \frac{P_1}{RT_1} = \frac{200 \times 10^3}{287 \times 310} = 2.24 kg/m^3$

$M_1 = \frac{C_1}{a_1} = \frac{C_1}{\sqrt{\gamma R T_1}} = \frac{265}{\sqrt{1.4 \times 287 \times 310}}$

$\boxed{M_1 = 0.75}$

From isentropic flow Table ($\gamma = 1.4$)

$M_1 = 0.75$

$\frac{T_1}{T_{01}} = 0.899$ $\quad \frac{P_1}{P_{01}} = 0.688$ $\quad \frac{A_1}{A^*} = 1.062$ $\quad \frac{F_1}{F_1^*} = 1.037$

$\frac{310}{T_{01}} = 0.899$ $\quad \frac{2\times10^5}{P_{01}} = 0.688$

$T_{01} = 344.83\,K$ $\quad P_{01} = 2.905\times10^5\,N/m^2$

$T_{01} = T_{02} = 344.83\,K$ $\quad P_{01} = P_{02} = 2.905\,bar$ isentropic flow.

$\frac{A_1}{A_1^*} = 1.062$

$A_1^* = 0.1186\,m^2 = A_2^*$

$A_2 = \frac{\pi}{4}(0.8)^2 = 0.5026$

$\frac{A_2}{A^*} = 4.238 \approx 4.180$

$M_2 = 0.14$ $\quad 2.99$

$d_2 > d_1$, $A_2 > d_1$ → divergent type diffuser, $M < 1$

choosing

$M_2 = 0.14$

$\frac{P_2}{P_{02}} = 0.986$ $\quad \frac{T_2}{T_{02}} = 0.996$ $\quad \frac{F_2}{F_1^*} = 3.343$

$P_2 = 0.986 \times 2.9$

$P_2 = 2.86\,bar$

$T_2 = 0.996 \times 344.83$

$T_2 = 343.54\,K$

$M_2 = \frac{c_2}{a_2} = \frac{c_2}{\sqrt{\gamma R T_2}}$

$c_2 = 0.14 \times \sqrt{1.4 \times 287 \times 343.54}$

$c_2 = 52\,m/s$

$\rho_2 = \frac{P_2}{R T_2} = \frac{2.86\times10^5}{287 \times 343.54}$

$\rho_2 = 2.90\,kg/m^3$

@ $M = 1$, $P = P^*$ From isentropic flow Table ($\gamma = 1.4$)

$P^*/P_{0_1}^* = 0.528$

(44)

$P^* = 0.528 \times 2.905$

$\boxed{P_1^* = 1.533 \text{ bar}}$

$F_1^* = P_1^* A_1^* (1+\gamma)$

$= 1.533 \times 10^5 \times 0.1186 (1 + 1.4)$

$\boxed{F_1^* = 43.635 \text{ kN}} = F_2^*$

$T = F_2 - F_1$

$= 3.343 F_2^* - 1.031 F_1^*$

$= 2.312 \times F_1^* =$

$\boxed{T = 100 \text{ kN}}$

1

A supersonic wind tunnel settling chamber expands freon-21 through a nozzle from a pressure of 10 bar to 4 bar in the test section. Calculate the stagnation temp. to be maintained in the settling chamber to obtain a velocity of 500 m/s in the test section for

Freon-21, $C_p = 785$ J/kg.K, $C_v = 675$

$c = 500$ m/s

$P_0 = 10$ bar, $P = 4$ bar

$\gamma = \frac{C_p}{C_v} = 1.163$

$$T_0 = T + \frac{c^2}{2C_p}$$

for isentropic flow

$$\frac{T}{T_0} = \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T = T_0 \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_0 = T_0 \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}} + \frac{c^2}{2C_p}$$

$$T_0 = T_0 \left(\frac{4}{10}\right)^{\frac{1.163-1}{1.163}} + \frac{(500)^2}{2 \times 785}$$

$$\boxed{T_0 = 1322.50\,K}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

$$\frac{1322.50}{T} = 1 + \frac{1.163-1}{2} M^2 = \left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{M = 1.296}$$

(46)

The Mach Numbers and pressure at the Entry of a Subsonic diffuser are 0.9 and 4.165 bar. Determine the area ratio required if the pressure rise of the Mach Numbers at Exit of diffuser is 0.20. Assume isentropic diffusion of air.

$M_1 = 0.9$, $P_1 = 4.165$, $\frac{A_2}{A_1}$, $M_2 = 0.20$

From isentropic flow Table, $\gamma = 1.4$, $R = 287\ J/kg \cdot K$

pressure rise $= P_2 - P_1$

$M_1 = 0.9$

$\frac{P_1}{P_0} = 0.591$ $\quad \frac{T_1}{T_{01}} =$ $\quad \frac{A_1}{A^*} = 1.009$

$P_{0_1} = 7.047\ bar$

$P_{0_1} = P_{02} = 7.047\ bar$

$M_2 = 0.20$ $\quad \frac{A_2}{A^*} = 22.964$

$\frac{P_2}{P_{02}} = 0.973$

$A_1^* = A_2^* = A^*$

$P_2 = 0.973 \times 7.047$

$P_2 = 6.857\ bar$

$$\frac{A_2}{A_1} = \frac{A_2/A_1^*}{A_1/A_1^*} = \frac{22.964}{1.009} = 2.937$$

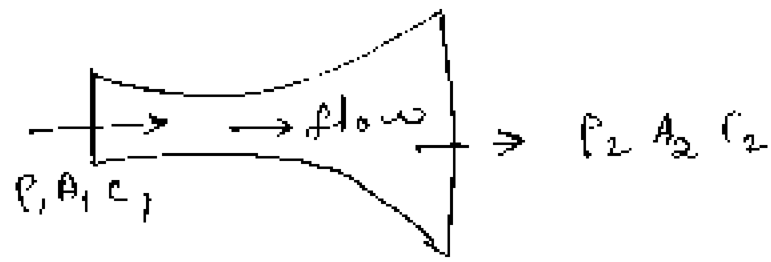
Pressure rise $= P_2 - P_1 = 6.857 - 4.165$
$= 2.692\ bar$

Impulse function

The quantities PA and ρAC^2 occur frequently in some compressible flow problems since the units of both these quantities are the units of force they are conveniently expressed together as an important gas dynamic parameter.

this is referred as Impulse function (F) or wall force function

$$F = PA + \rho AC^2$$



Direction of flow & Thrust in a duct

one-dimensional flow through a control volume (a - symmetrical straight duct) is shown.

The Thrust or wall force experienced by the duct in the direction shown is a result of change in pressure and Mach Number between the cross-section 1 & 2

Momentum Equation

$$\tau_2 = P_2 A_2 + \rho_2 A_2 C_2^2$$

$$\tau_1 = P_1 A_1 + \rho_1 A_1 C_1^2$$

$$\tau = \tau_2 - \tau_1 = \left(P_2 A_2 + \rho_2 A_2 C_2^2\right) - \left(P_1 A_1 + \rho_1 A_1 C_1^2\right)$$

For a perfect gas

$$\rho C^2 = \frac{P}{RT} C^2 = \frac{\gamma P}{\gamma RT} C^2 = \frac{\gamma P C^2}{a^2} = \gamma P M^2$$

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$F = PA + \gamma PAM^2$

$F = PA(1+\gamma M^2) \rightarrow (1)$

$T = F_2 - F_1$

$T = P_2 A_2 (1+\gamma M_2^2) - P_1 A_1 (1+\gamma M_1^2) \rightarrow (2)$

Equation (2) is very convenient in obtaining the thrust exerted by the flowing fluid.

The thrust exerted by the fluid due to its flow b/t two sections of a duct can be obtained by the change of the Impulse function b/t these section.

1-dimensional Impulse function & Mach number the flow is assumed isentropic

@ $M=1$, $F = F^*$, $P = P^*$, $T = T^*$

$F^* = P^* A^* (1+\gamma) \rightarrow (3)$

$\frac{(1)}{(3)} = \frac{F}{F^*} = \frac{P}{P^*} \times \frac{A}{A^*} \frac{1+\gamma M^2}{1+\gamma} \rightarrow (3a)$

We know that

$P/P^* = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M^2\right]^{-\gamma/\gamma-1} \rightarrow (4)$

$A/A^* = \frac{1}{M}\left(\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}} \rightarrow (5)$

(4) & (5) in (3a)

$$\frac{F}{F^*} = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{-\gamma/\gamma-1} \times \frac{1}{M}\left(\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times \frac{1+\gamma M^2}{1+\gamma}$$

$$\frac{F}{F^*} = \frac{1}{M}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{-\frac{\gamma}{\gamma-1} \times \frac{\gamma+1}{2(\gamma-1)}} \times \left(\frac{1+\gamma M^2}{1+\gamma}\right)$$

$$F/F^* = \frac{1}{M}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{-1/2} \times \left(\frac{1+\gamma M^2}{1+\gamma}\right)$$

$$\frac{-\gamma}{\gamma-1} + \frac{\gamma+1}{2(\gamma-1)} = \frac{-2\gamma+\gamma+1}{2(\gamma-1)} = \frac{-\gamma+1}{2(\gamma-1)} = \left(-\tfrac{1}{2}\right)$$

~~$\frac{F}{F^*} = \frac{1+\gamma M^2}{}$~~

This is simplification

Taking out $2/\gamma+1$ outside & re-arrangement

$$F/F^* = \frac{1+\gamma M^2}{\frac{1}{M}\sqrt{2(1+\gamma)\left(1+\frac{\gamma-1}{2}M^2\right)}}$$

~~$\frac{1}{M}\sqrt{\frac{2}{\gamma+1}}\left[1+\frac{\gamma-1}{2}M^2\right]$ $\sqrt{1+\gamma}$~~

Another non-dimensional expression for the Impulse function

$$\frac{F}{p_0 A^*} = \frac{pA(1+\gamma M^2)}{p_0 A^*}$$

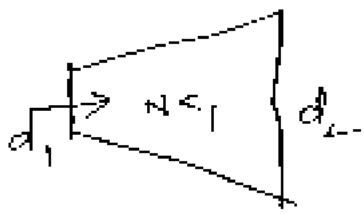
1 A Conical diffuser has Entry and Exit diameters of 15 cm and 30 cm respectively. (18)
The pressure and temp. & velocity of air at Entry are 0.69 bar, 340 K and 180 m/s. determine
the Exit pressure
Exit velocity
force Exerted on the diffuser walls
($\gamma = 1.4$, c_p = ~~1.005 kJ/kg.K~~ = 1000 J/kg.K)

$d_1 = 15\,cm$, $d_2 = 30\,cm$

$d_1 = 0.15\,m$, $d_2 = 0.30\,m$

$P_1 = 0.69\,bar$, $T_1 = 340\,K$

$C_1 = 180\,m/s$



$M_1 = \frac{C_1}{a_1} = \frac{180}{\sqrt{\gamma R T_1}} = \frac{180}{\sqrt{1.4 \times 287 \times 340}} = \frac{180}{369.6}$

~~$M_1 = 0.48$ (isentropic~~

$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(0.15)^2 = 176.8\,cm^2$

$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(0.30)^2 = 706\,cm^2$

isentropic flow

$T_{01} = T_{02}$
$P_{01} = P_{02}$

$T_{01} = T_1 + \frac{C_1^2}{2c_p}$

$T_{01} = 340 + \frac{(180)^2}{2 \times 1000}$

$T_{01} = 356.2\,K$

($\gamma = 1.4$)

$\frac{T_1}{T_{01}} = \frac{340}{356.2} = 0.92$ — $M_1 = 0.50$ From isentropic flow table

$M_1 = 0.50$, $\frac{P_1}{P_{01}} = 0.843$, $\frac{A_1}{A_1^*} = 1.34$ $\frac{F_1}{F_1^*} = 1.203$

$\frac{P_1}{P_{01}} = 0.843$, $P_{01} = \frac{0.69}{0.843}$

$P_{01} = 0.818\,bar$

$A_1^* = A_2^*$

$$\frac{A_1}{A_1^*} = 1.34$$

$$A_1^* = \frac{176.8 \times 10^{-4}}{1.34} = 0.0131\, m^2 = A_1^* = A_2^*$$

@

$M = 1$

$$\frac{P^*}{P_0} = 0.528 \qquad \frac{T^*}{T_0} =$$

$P_1^* = 0.528 \times 0.818$

$P_1^* = 0.432$ bar $\qquad P_1^* = P_2^*$

$$F_1^* = F_2^* = P_1^* A_1^* (1+\gamma)$$

$$= 0.432 \times 10^5 \times (0.0131)(1+1.4)$$

$$F_1^* = F_2^* = 1370\, N$$

$$\frac{A_2}{A_2^*} = \frac{706 \times 10^{-4}}{0.0131} = 5.37 \quad \begin{cases} M_2 = 0.107 \; (\checkmark) \\ M_2 = 3.25 \; (\times) \end{cases}$$

choosing $M_2 < 1$ (since it is a divergent type of Nozzle)

($\gamma = 1.4$) isentropic flow Table

$M_2 = 0.107 \qquad \frac{P_2}{P_{0_2}} = 0.992 \qquad \frac{F_2}{F_2^*} = 4.30$

$P_2 = 0.992 \times 0.818 = $ ~~0.432 bar~~ 0.81145 bar

~~M_2~~ $\dot{m} = \rho_1 A_1 C_1 = \rho_2 A_2 C_2 \rightarrow C_2 = 45\, m/s$

$\dot{m}$

$$\tau = F_2 - F_1$$

$$= (4.30\, F_2^*) - (1.203\, F_1^*)$$

$$= (4.30 - 1.203)\, 1370$$

$$\tau = 4243\, N$$

P. Raghu

(1)

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unit - II

Flow through constant area ducts with friction (Fanno flow)

Flow in a constant area duct with friction and without work transfer and heat transfer.

Fanno flow occurs in a number of practical situations - flow processes occuring in gas ducts of aircraft Engines. (aircraft propulsion)

- Engg. Industrial plants.
- air conditioning systems
- transport of fluids in chemical process plants & various types of flow machinery, etc.;

Steam pipe lines in power plants and other Industrial applications are well insulated to minimise heat losses. the flow through them can be assumed adiabatic (Q=0)

The effect of wall friction on the fluid parameters is considerably significant in long pipes and ducts.

Fanno line describes an adiabatic flow process in a constant area duct with friction.

h_0 = constant
$\dot{m}/A$ = constant

on account of friction, the process is irreversible

Assumptions

The process is governed by the Equations of Continuity, Energy and state.

(1) perfect gas
(2) Constant area duct
(3) one-dimensional steady frictional flow.
(4) Absence of heat transfer, work transfer & body forces.

Flow in a constant area duct with friction and without heat transfer is described by a curve is known as Fanno line or Fanno curve

From continuity equation

mass flow rate = G = mass flow density

$$\dot{m} = \rho A c \longrightarrow (1)$$

$$\frac{\dot{m}}{A} = \rho c = G.$$

$$c = \frac{G}{\rho} \longrightarrow (2)$$

The Energy Equation

$$h_0 = h + \frac{1}{2} c^2 \longrightarrow (3)$$

$$h_0 = h + \frac{1}{2} \frac{G^2}{\rho^2}$$

$$h = h_0 - \frac{1}{2}\left(\frac{G}{\rho}\right)^2 \longrightarrow (4)$$

Equation of state can be written

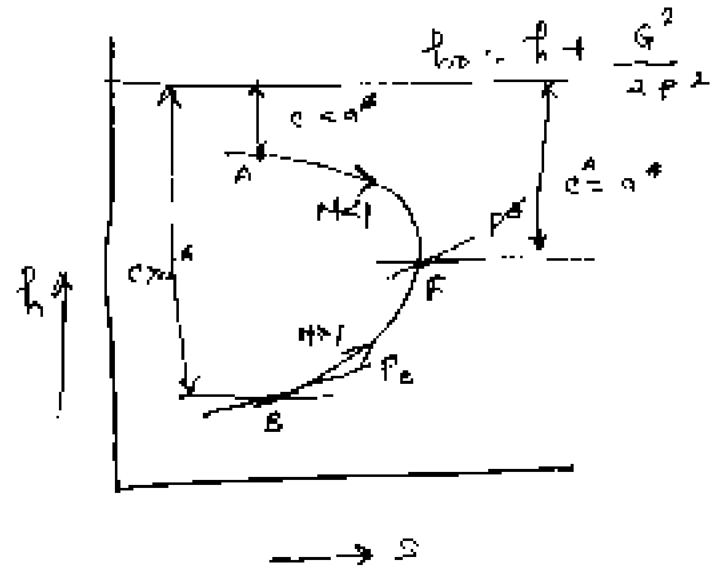
$$\rho = f(s, h) \longrightarrow (5)$$

$$h = h_0 - \frac{1}{2} \frac{G^2}{[f(s,h)]^2} \longrightarrow (6)$$

The above Equation is known as Fanno line equation

with the help of Equations (4) & (6) for the given values of h_0 and G a curve in the $(h-s)$ plane as shown in fig (1)

(2)



Fanno line or curve

The curve consist of two branches AF and BF above and below the sonic point F ($M = 1$, $c = c^* = a^*$)

AF

$c < a^*$

process occurs in direction AF,

FA ≠ possible, since they lead to decrease in Entropy which violates the IInd law of thermodynamics

subsonic region ($M < 1$) cause irreversible acceleration of flow with pressure drop

max. velocity obtainable is Sonic at F.

FB ≠ not possible. Impossibility of < in Entropy

BF

$c > a^*$

process occur in direction BF

FB ≠ possible,
since Δs << violates the 2nd law of thermodynamics

$M > 1$ (supersonic region) wall friction causes irreversible deceleration of flow >> in Δp

minimum velocity obtainable is sonic at point F.

From the above fig seen that upper & lower branches of the Fanno line represents the subsonic & supersonic flows respectively and for given initial conditions, the final state always lies towards the sonic state

1. The non-dimensional friction factor (Fanning coefficient of skin friction) (4)

$$f = \frac{\text{wall shear stress}}{\text{Dynamic head}} = \frac{\tau_w}{\frac{1}{2}\rho c^2}$$

$$\tau_w = f \times \frac{1}{2}\rho c^2$$

The area of the duct within the control volume

dA_w = perimeter × length

$dA_w = P \cdot dx$

The hydraulic mean diameter

$$D = \frac{4A}{P}$$

$$P = \frac{4A}{D}$$

$$dA_w = \frac{4A}{D}\, dx$$

momentum Equation for the control surface

$$\dot{m}\left[(c+dc) - c\right] = pA - (p+dp)A - \tau_w\, dA_w$$

$$\dot{m}\, dc = -A\, dp - \tau_w\, dA_w$$

$$\frac{\dot{m}\, dc}{A} = -\left[dp + f \times \frac{1}{2}\rho c^2 \times \frac{4A}{D}dx\right]$$

$\rho c^2 = \gamma p M^2$

$$\frac{\dot{m}}{A}dc = \frac{\rho A c}{c}\, dc$$

$$= \rho c\, dc$$

$$= \rho c^2 \frac{dc}{c}$$

$$= \gamma p M^2 \cdot \frac{dc^2}{2c^2}$$

$$\gamma p M^2 \cdot \frac{dc^2}{2c^2} = -\left[dp + \frac{f}{2}\gamma p M^2 \times \frac{4\, dx}{D}\right]$$

$\div p$

$$\frac{\gamma M^2}{2} \cdot \frac{dc^2}{c^2} + \frac{dp}{p} + \frac{\gamma M^2}{2}\left(\frac{4f}{D}dx\right) = 0$$

Variation of flow properties

Flow properties at $M = M^* = 1$ are used as reference values for non-dimensionalizing various ~~prop~~ properties at any section of the duct.

The flow properties are pressure, density, temp & velocity.

Temperature

The ratio of stagnation temp & static temp of state is given by

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2 \longrightarrow (1)$$

@ critical state $M = 1$, $T_0 = T_0^*$, $T = T^*$

$$\frac{T_0^*}{T^*} = 1 + \frac{\gamma-1}{2} = \frac{\gamma+1}{2} \longrightarrow (2)$$

for ~~Fanno~~ flow
$T_0^* = T_0 = \text{constant}$

$$\frac{T}{T^*} = \frac{\frac{T_0^*}{T^*}}{\frac{T_0}{T}} = \frac{\frac{\gamma+1}{2}}{1 + \frac{\gamma-1}{2} M^2} = \frac{\gamma+1}{2\left[1 + \frac{\gamma-1}{2} M^2\right]} \longrightarrow (3)$$

Applying the Equation (1) for section (1) & (2)

$T_{01} = T_{02} = C$

$$\frac{T_2}{T_1} = \frac{T_{01}/T_1}{T_{02}/T_2}$$

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \longrightarrow (4)$$

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velocity

$$M = \frac{c}{a}$$

$$c = M \times a = M \times \sqrt{\gamma R T}.$$

@ critical state

$$M = 1, \quad c = c^* \quad T = T^*$$

$$c^* = \sqrt{\gamma R T^*}$$

$$\frac{c}{c^*} = \frac{M\sqrt{\gamma R T}}{\sqrt{\gamma R T^*}} = M \times \sqrt{\frac{T}{T^*}} = M \times \left[\frac{\gamma+1}{2\left[1+\frac{\gamma-1}{2}M^2\right]}\right]^{1/2}$$

apply the above Equation ① & ②

$$M_2 = \frac{c_2}{a_2}, \quad M_1 = \frac{c_1}{a_1}$$

$$\frac{c_2}{c_1} = \frac{M_2 a_2}{M_1 a_1} = \frac{M_2}{M_1} \times \sqrt{\frac{\gamma R T_2}{\gamma R T_1}}$$

$$= \frac{M_2}{M_1} \times \sqrt{\frac{T_2}{T_1}} = \frac{M_2}{M} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{1/2}$$

density

$$\dot{m} = \rho c A = \rho^* A^* c^* \qquad A = \text{constant}$$

$$\frac{\rho}{\rho^*} = \frac{c^*}{c}$$

$$\frac{\rho}{\rho^*} = \frac{1}{c/c^*} = \frac{1}{M\left[\frac{\gamma+1}{2\left[1+\frac{\gamma-1}{2}M^2\right]}\right]^{1/2}}$$

applying ① & ②

$$\frac{\rho_2}{\rho_1} = \frac{c_1}{c_2} = \frac{1}{\frac{c_2}{c_1}} = \frac{1}{\frac{M_2}{M_1}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{1/2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{1/2}$$

Pressure from Equation of state

$$P = \rho R T$$

@ $M=1$, $P=P^*$, $\rho=\rho^*$, $T=T^*$

$$P^* = \rho^* R T^*$$

$$P/P^* = \rho/\rho^* \times \frac{T}{T^*}$$

$$= \frac{\gamma+1}{2\left[1+\frac{\gamma-1}{2}M^2\right]} \times \left[\frac{1}{M\left[\frac{\gamma+1}{2\left[1+\frac{\gamma-1}{2}M^2\right]}\right]}\right]^{1/2}$$

$$\frac{P}{P^*} = \frac{1}{M}\left[\frac{\gamma+1}{2\left[1+\frac{\gamma-1}{2}M^2\right]}\right]^{1/2}$$

$$\frac{P_2}{P_1} = \frac{\rho_2 R T_2}{\rho_1 R T_1} = \frac{M_1}{M_2} \times \left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{1/2} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]$$

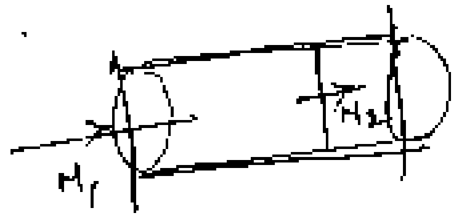
$$= \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{1/2} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{1/2} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]$$

$$\frac{P_2}{P_1} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{1/2}$$

(b) P. Kagru

(1) A circular duct passes 9 kg/s of air at an Exit Mach Number of 0.5. The Entry pressure and temperature are 3.5 bar and 313 K respectively, and the coefficient of friction of 0.005. if the Mach Number at the Entry is 0.15, determine.

(a) the diameter of the duct
(b). Length of the duct
(c) pressure and temp. at the Exit.
(d). stagnation pressure loss.



given data

$\dot{m} = 9\ kg/s$ $\qquad f = 0.005$

$M_2 = 0.5$, $P_1 = 3.5\ bar$, $T_1 = 313K$

$M_1 = 0.15$ $\qquad$ Take $\gamma = 1.4$, $R = 287\ J/$

To find

(i) D (ii) L (iii) P_2, T_2 & $P_{01} - P_{02}$

$\dot{m} = \rho_1 A_1 C_1$

$\rho_1 = \frac{P_1}{RT_1} = \frac{3.5 \times 10^5}{287 \times 313} = 3.89\ kg/m^3$, $\quad M_1 = \frac{C_1}{a_1}$

$C_1 = 0.15 \times \sqrt{\gamma R T}$

$= 0.15 \times \sqrt{1.4 \times 287 \ldots}$

$\boxed{C_1 = 53.19\ m/s}$

$\dot{m} = \rho_1 A_1 C_1$

$9 = 3.89 \times A_1 \times 53.19$

$A_1 = 0.0434$

$\frac{\pi}{4} D_1^2 = 0.0434$

$\boxed{D_1 = 0.2353\ m}$

From Isentropic flow table ($\gamma = 1.4$)

$\frac{P_1}{P_{01}} = 0.984$ $\qquad \frac{T_1}{T_{01}} =$

$P_{01} = \frac{3.5}{0.984} = 3.556\ bar$ $\qquad T_{01} =$

To find the length of duct

$$\frac{4fL}{D} = \left(\frac{4fL_{max}}{D}\right)_1 - \left(\frac{4fL_{max}}{D}\right)_2$$

from Fanno flow Table $\gamma = 1.4$

M	P/P^*	T/T^*	P_0/P_0^*	C/C^*	$\frac{4fL_{max}}{D}$
$M_1 = 0.15$	7.319	1.194	3.928	0.164	28.354
$M_2 = 0.50$	2.138	1.143	1.340	0.534	1.069

$$\frac{4fL}{D} = 28.354 - 1.069$$

$$L = \frac{(28.354 - 1.069) \times D}{4 \times f} = \frac{27.285 \times 0.2352}{4 \times 0.005}$$

$L = 319.91\,m$

$C_2^* = C_1^* = C^*$
$P_2^* = P_1^* = P^*$
$T_2^* = T_1^* = T^*$

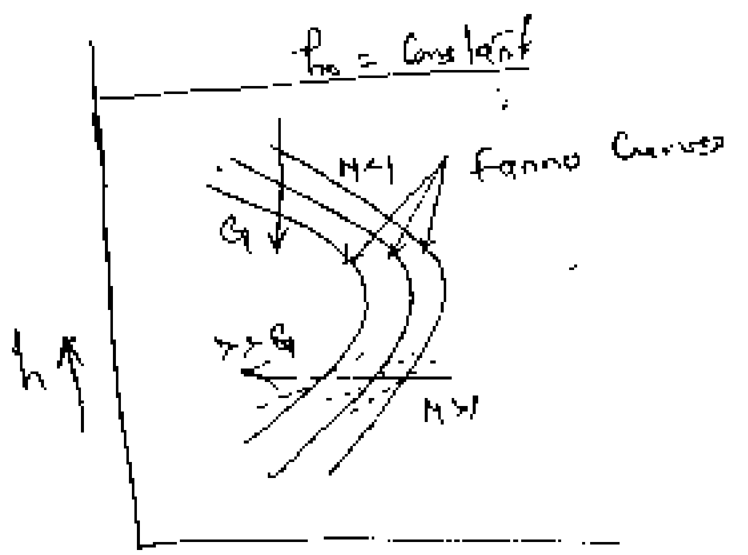
exit pressure & temp

$$\frac{P_2}{P_1} = \frac{(P/P^*)_2}{(P/P^*)_1}$$

$$\frac{P_2}{P_1} = \frac{2.138}{7.319}$$

$P_2 = 0.29211 \times 3.5$

$P_2 = 1.022\,bar$



Fanno Curves for various values of G

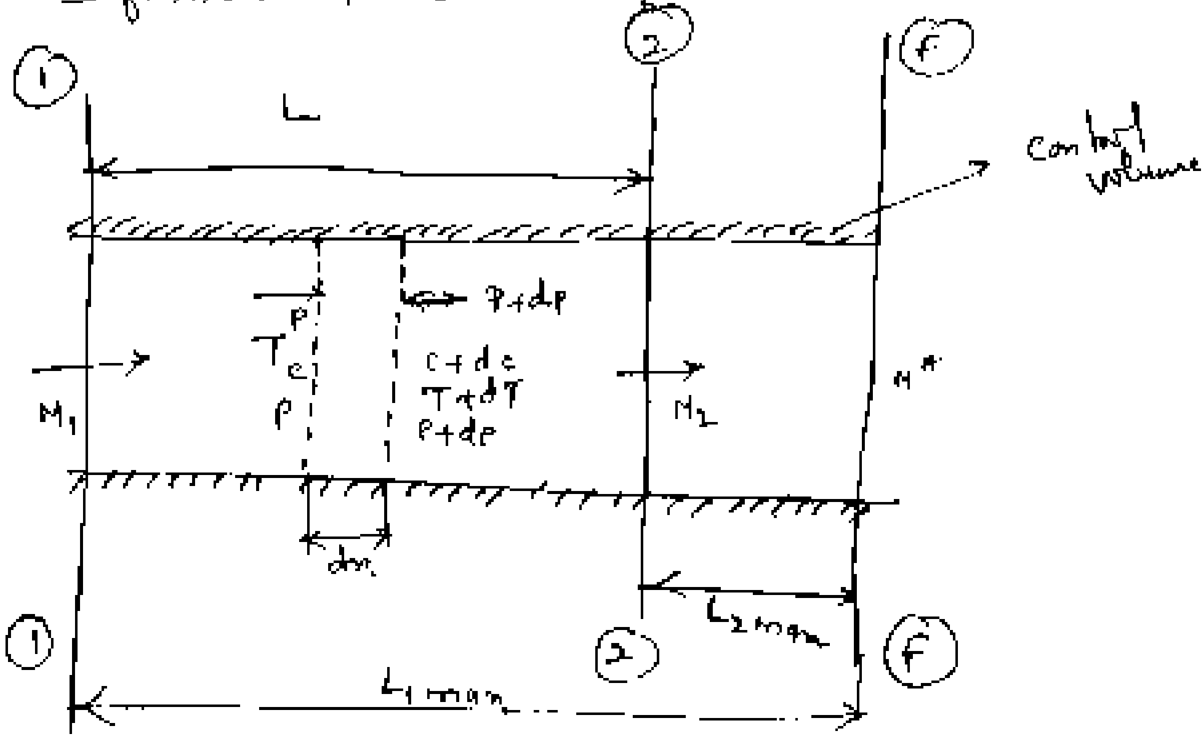
For different values of mass flow density (G), the fanno curves are drawn

M < 1, (G >>, c >> and p <<)

M > 1 (G >> p >> & c <<)

Fanno flow Equation

The below fig the variation of flow properties in fanno flow processes along the Insulated duct. An Infinitesimal element of flow is considered



Variation of flow properties in fanno flow process

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at a distance x from state 1. The changes between states 1 & 2, 1 & f, 2 & f are finite, but across the control volume the changes are infinitesimal

We know that, mass flow density

$$G = \rho c \longrightarrow (1)$$

To the log. differentiation

$G =$ Constant

$$\ln G = \ln(\rho c)$$

$$\ln G = \ln \rho + \ln c$$

$$0 = \frac{d\rho}{\rho} + \frac{dc}{c}$$

$$\frac{d\rho}{\rho} = -\frac{dc}{c} = -\frac{d[c^2]}{2c^2} \longrightarrow (2)$$

from Equation of state

$$p = \rho R T$$

taking log. diff

$$\ln p = \ln \rho + \ln R + \ln T$$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + 0 + \frac{dT}{T} \longrightarrow (3)$$

$$M = \frac{c}{a} = \frac{c}{\sqrt{\gamma R T}}, \quad M^2 = \frac{c^2}{\gamma R T}$$

Taking log. diff

$$\ln M^2 = \ln c^2 - \ln(\gamma R T)$$

$$\ln M^2 = \ln(c^2) - \ln T$$

$$\frac{dM^2}{M^2} = \frac{dc^2}{c^2} - \frac{dT}{T} = 0$$

Stagnation Enthalpy

$$h_0 = h + \frac{c^2}{2} = \text{Const}$$

diff

$$dh_0 = dh + \frac{2c\,dc}{2} = 0$$

$$0 = dh + c\,dc$$

$$c_p\,dT + d\left[\frac{c^2}{2}\right] = 0$$

$$\frac{\gamma R}{\gamma - 1} \times \frac{T\,dT}{T} + \frac{dc^2}{2} = 0 \quad \text{divide \& multiply by } T$$

$$\frac{\gamma R T}{(\gamma-1)} \frac{dT}{T} + \frac{dc^2}{2} = 0$$

$$\frac{a^2}{\gamma-1} \frac{dT}{T} + \frac{dc^2}{2} = 0 \qquad \times\ \gamma - 1$$

$$a^2 \cdot \frac{dT}{T} + (\gamma-1)\frac{dc^2}{2} = 0 \qquad M^2 = \frac{c^2}{a^2}$$

$$a^2 = \frac{c^2}{M^2}$$

$$\frac{c^2}{M^2} \cdot \frac{dT}{T} + (\gamma-1)\frac{dc^2}{2} = 0$$

$$\boxed{\frac{dT}{T} + \frac{\gamma-1}{2} \times M^2 \times \frac{dc^2}{c^2} = 0}$$

1. The non-dimensional friction factor (fanning coefficient of skin friction) (4)

$$f = \frac{\text{wall shear stress}}{\text{Dynamic head}} = \frac{\tau_w}{\frac{1}{2}\rho c^2}$$

$$\tau_w = f \times \frac{1}{2}\rho c^2$$

The area of the duct within the control volume

dA_w = perimeter × length

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The hydraulic mean diameter

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$$\dot m\left[(c+dc) - c\right] = pA - (p+dp)A - \tau_w\,dA_w$$

$$\dot m\,dc = -A\,dp - \tau_w\,dA_w$$

$$\frac{\dot m\,dc}{A} = -\left[dp + f \times \frac{1}{2}\rho c^2 \times \frac{4A\,dx}{D}\right]$$

$$\rho c^2 = \gamma p M^2$$

$$\frac{\dot m}{A}\,dc = \frac{\rho A c}{c}\,dc$$

$$= \rho c\,dc$$

$$= \rho c^2\,\frac{dc}{c}$$

$$= \gamma p M^2 \cdot \frac{dc^2}{2c^2}$$

$$\gamma p M^2 \cdot \frac{dc^2}{2c^2} = -\left[dp + \frac{f}{2}\gamma p M^2 \times \frac{4\,dx}{D}\right]$$

$\div\, p$

$$\frac{\gamma M^2}{2} \cdot \frac{dc^2}{c^2} + \frac{dp}{p} + \frac{\gamma M^2}{2}\left(\frac{4f\,dx}{D}\right) = 0$$

Variation of flow properties

Flow properties at $M = M^* = 1$ are used as reference values for non-dimensionalizing various flow properties at any section of the duct. The flow properties are pressure, density, temp & velocity.

Temperature

The ratio of stagnation temp & static temp of state is given by

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \longrightarrow (1)$$

@ critical state $M = 1$, $T_0 = T_0^*$, $T = T^*$

$$\frac{T_0^*}{T^*} = 1 + \frac{\gamma - 1}{2} = \frac{\gamma + 1}{2} \longrightarrow (2)$$

for ~~fanno~~ flow: $T_0^* = T_0 = \text{constant}$

$$\frac{T}{T^*} = \frac{\frac{T_0^*}{T^*}}{\frac{T_0}{T}} = \frac{\frac{\gamma+1}{2}}{1 + \frac{\gamma-1}{2} M^2} = \frac{\gamma + 1}{2\left[1 + \frac{\gamma-1}{2} M^2\right]} \longrightarrow (3)$$

Applying the Equation (1) for section (1) & (2)

$T_{01} = T_{02} = C$

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$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \longrightarrow (4)$$

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<u>velocity</u>

$$M = \frac{c}{a}$$

$$c = M \times a = M \times \sqrt{\gamma R T}.$$

@ critical state

$$M = 1, \quad c = c^* \quad T = T^*$$

$$c^* = \sqrt{\gamma R T^*}$$

$$\frac{c}{c^*} = \frac{M\sqrt{\gamma R T}}{\sqrt{\gamma R T^*}} = M \times \sqrt{\frac{T}{T^*}} = M \times \left[\frac{\gamma+1}{2\left[1+\frac{\gamma-1}{2}M^2\right]}\right]^{1/2}$$

apply the above Equation ① & ②

$$M_2 = \frac{c_2}{a_2}, \quad M_1 = \frac{c_1}{a_1}$$

$$\frac{c_2}{c_1} = \frac{M_2 a_2}{M_1 a_1} = \frac{M_2}{M_1} \times \sqrt{\frac{\gamma R T_2}{\gamma R T_1}}$$

$$= \frac{M_2}{M_1} \times \sqrt{\frac{T_2}{T_1}} = \frac{M_2}{M_1} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{1/2}$$

<u>density</u>

$$\dot{m} = \rho c A = \rho^* A^* c^* \qquad A = \text{constant}$$

$$\frac{\rho}{\rho^*} = \frac{c^*}{c}$$

$$\frac{\rho}{\rho^*} = \frac{1}{c/c^*} = \frac{1}{M\left[\frac{\gamma+1}{2\left[1+\frac{\gamma-1}{2}M^2\right]}\right]^{1/2}}$$

applying ① & ②

$$\frac{\rho_2}{\rho_1} = \frac{c_1}{c_2} = \frac{1}{\frac{c_2}{c_1}} = \frac{1}{\frac{M_2}{M_1}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{1/2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{1/2}$$

Pressure from Equation of state

$$P = \rho R T$$

@ $M=1$, $P=P^*$, $\rho=\rho^*$, $T=T^*$

$$P^* = \rho^* R T^*$$

$$P/P^* = \rho/\rho^* \times \frac{T}{T^*}$$

$1/2 \times 1/2$

$$= \frac{\gamma+1}{2\left[1+\frac{\gamma-1}{2}M^2\right]} \times \left[\frac{1}{M\left[\frac{\gamma+1}{2\left[1+\frac{\gamma-1}{2}M^2\right]}\right]}\right]^{1/2}$$

$$\frac{P}{P^*} = \frac{1}{M}\left[\frac{\gamma+1}{2\left[1+\frac{\gamma-1}{2}M^2\right]}\right]^{1/2}$$

$$\frac{P_2}{P_1} = \frac{\rho_2 R T_2}{\rho_1 R T_1} = \frac{M_1}{M_2}\times\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{1/2} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]$$

$$= \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{1/2} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{1/2} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]$$

$$\frac{P_2}{P_1} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{1/2}$$

(6) P. Kagnu

(1) A circular duct passes 9 kg/s of air at an Exit Mach Number of 0.5. The Entry pressure and temperature are 3.5 bar and 313 K respectively, and the Coefficient of friction of 0.005. If the Mach Number at the Entry is 0.15, determine.

(a) the diameter of the duct
(b) Length of the duct
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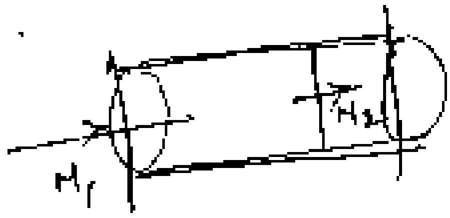


given data

$\dot{m} = 9$ kg/s

$M_2 = 0.5$, $P_1 = 3.5$ bar, $T_1 = 313$ K, $f = 0.005$

$M_1 = 0.15$

Take $\gamma = 1.4$

$R = 287$ J/[illegible]

To find

(i) D (ii) L (iii) P_2, T_2 & $P_{01} - P_{02}$

$\dot{m} = \rho_1 A_1 C_1$

$\rho_1 = \frac{P_1}{RT_1} = \frac{3.5 \times 10^5}{287 \times 313} = 3.89$ kg/m³, $M_1 = \frac{C_1}{a_1}$

$C_1 = 0.15 \times \sqrt{\gamma R T}$[illegible]

$= 0.15 \times \sqrt{1.4 \times 287 \ldots}$

$C_1 = 53.19$ m/s

$\dot{m} = \rho_1 A_1 C_1$

$9 = 3.89 \times A_1 \times 53.19$

$A_1 = 0.0434$

$\frac{\pi}{4} D_1^2 = 0.0434$

$D_1 = 0.2353$ m

From Isentropic flow table ($\gamma = 1.4$)

$\frac{P_1}{P_{01}} = 0.984$ $\frac{T_1}{T_{01}} =$

$P_{01} = \frac{3.5}{0.984} = 3.556$ bar $T_{01} =$

To find the length of duct

$$\frac{4fL}{D} = \left(\frac{4fL_{max}}{D}\right)_1 - \left(\frac{4fL_{max}}{D}\right)_2$$

from fanno flow Table $\gamma = 1.4$

M	P/P^*	T/T^*	P_0/P_0^*	C/C^*	$\frac{4fL_{max}}{D}$
$M_1 = 0.15$	7.319	1.194	3.928	0.164	28.354
$M_2 = 0.50$	2.138	1.143	1.340	0.534	1.069

$$\frac{4fL}{D} = 28.354 - 1.069$$

$$L = \frac{(28.354 - 1.069) \times D}{4 \times f} = \frac{27.285 \times 0.2352}{4 \times 0.005}$$

$$\boxed{L = 319.91\,m}$$

$c_2^* = c_1^* = c^*$

$P_2^* = P_1^* = P^*$

$T_2^* = T_1^* = T^*$

exit pressure & temp

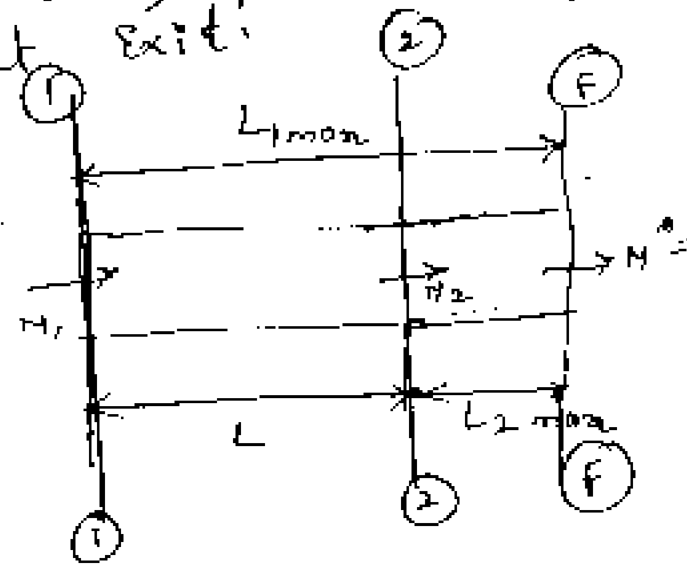
$$\frac{P_2}{P_1} = \frac{(P/P^*)_2}{(P/P^*)_1}$$

$$\frac{P_2}{P_1} = \frac{2.138}{7.319}$$

$$P_2 = 0.29211 \times 3.5$$

$$\boxed{P_2 = 1.022\,bar}$$

The friction factor for an 25mm diameter 11.5m long pipe is 0.004. The conditions of air at Entry are $P_1 = 2.0$ bar, $T_1 = 301$ K, $M_1 = 0.25$. Determine the mass flow rate, pressure, temp. & velocity, Mach Number at Exit.



given data

$D = 25m$, $L = 11.5m$

$f = 0.004$.

$P_1 = 2.0$ bar, $T_1 = 301$ K, $M_1 = 0.25$

$\dot{m}$, P_2, T_2, C_2, M_2

from isentropic flow Table ($\gamma = 1.4$)

$M_1 = 0.26$

$$\frac{T_1}{T_{01}} = 0.986, \quad \frac{P_1}{P_{01}} = 0.954$$

$$T_{01} = \frac{301}{0.986} \qquad P_{01} = \frac{2.0}{0.954}$$

$T_{01} = 305.27$ K $P_{01} = 2.096$ bar

from fanno flow Table ($\gamma = 1.4$)

M	P/P^*	T/T^*	$\frac{P_0}{P_0^*}$	C/C^*	$\frac{4fL_{max}}{D}$
$M_1 = 0.26$	4.185	1.184	2.3#	0.272	7.687
$M_2 = 0.64$	1.645	1.109	1.145		0.372

We know that
$$\frac{4fL}{D} = \left(\frac{4fL_{max}}{D}\right)_1 - \left(\frac{4fL_{max}}{D}\right)_2$$

$$\frac{4 \times 0.004 \times 11.5}{25 \times 10^{-3}} = 7.689 - \left(\frac{4fL_{max}}{D}\right)_2$$

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$$\frac{T_2}{T_1} = \frac{(T/T^*)_2}{(T/T^*)_1}$$

$$\frac{T_2}{T_1} = \frac{1.143}{1.194}$$

$$T_2 = \frac{1.143}{1.194} \times 313$$

$$\boxed{T_2 = 299.63\,K}$$

$$\frac{c_2}{c_1} = \frac{(c/c^*)_2}{(c/c^*)_1}$$

$$c_2 = \frac{0.534}{0.164} \times 53.19$$

$$\boxed{c_2 = 173.19\ m/s}$$

$$\frac{P_{02}}{P_{01}} = \frac{(P_0/P_0^*)_2}{(P_0/P_0^*)_1} = \frac{1.34}{3.928}$$

$$P_{02} = \frac{1.34}{3.928} \times 3.556$$

$$\boxed{P_{02} = 1.213\ bar}$$

Stagnation pressure loss $= P_{01} - P_{02} = 3.556 - 1.213$

$$\boxed{\Delta P_0 = 2.342\ bar}$$

$$7.36 \times 10 = 7.687 - \left(\frac{4f L_{max}}{D}\right)_2$$ (8)

$$\left(\frac{4f L_{max}}{D}\right)_2 = 0.327$$ → See the Fanno flow Table ($\gamma = 1.4$)

$M_2 = 0.64$ ($M_2 < 1$) for Insulated duct or line

Exit properties.

$$\frac{P_2}{P_1} = \frac{(P/P^*)_2}{(P/P^*)_1} = \frac{1.645}{4.185}$$

$P_1^* = P_2^* = P^*$
$T_1^* = T_2^* = T^*$
$P_{01}^* = P_{02}^* = P_0^*$

$P_2 = 0.393 \times 2$

$P_2 = 0.786$ bar

P. Raghu

$$\frac{T_2}{T_1} = \frac{(T/T^*)_2}{(T/T^*)_1} = \frac{1.109}{1.184} = 0.93$$

$T_2 = 0.93 \times 301$

$T_2 = 281.93$ K

$$\frac{C_2}{C_1} = \frac{(C/C^*)_2}{(C/C^*)_1}$$

$C_2 =$

$\dot{m} = \rho_2 A_2 C_2$

$= 0.9314 \times 4.9 \times 10^{-4} \times 215$

$\dot{m} = 0.098$ kg/s

$A_1 = \frac{\pi}{4} D_1^2 = 4.9 \times 10^{-4}$ m^2

$$\rho_2 = \frac{P_2}{RT_2} = \frac{0.786 \times 10^5}{287 \times 281.93}$$

$\rho_2 = 0.9314$ kg/m^3

$$M_2 = \frac{C_2}{a_2} = \frac{C_2}{\sqrt{\gamma R T_2}}$$

(corrected)

$C_2 = 215$ m/s

A long pipe of 25mm diameter has a mean coefficient of friction 0.003. Air Enters the pipe at a Mach Number of 2.5, stagnation temperature 310K and static pressure 0.507 bar. Determine the section at which mach Number reaches 1.2. (a) static pressure & temperature at Exit (b) stagnation pressure and temp. (c) Velocity of air (d) distance of this section from the inlet and (e) mass flow rate of air.

given data. [$\gamma = 1.4$, $R = 287$ J/kg.K]

$D = 25mm = 25 \times 10^{-3}$ m

$f = 0.003$

$M_1 = 2.5$, $T_{o1} = 310$ k

$P_1 = 0.507$

$M_2 = 1.2$

find

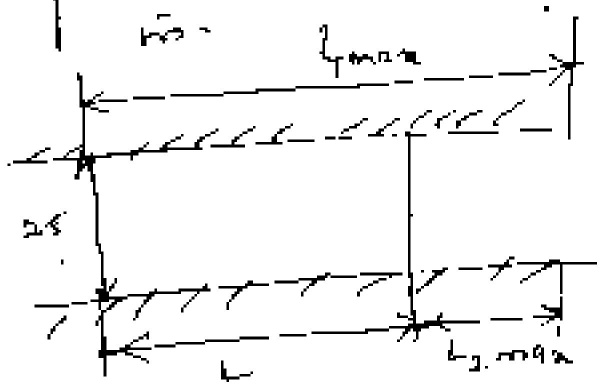
P_{o1} & T_{o1}

P_2, T_2

$C_2 =$

$L =$

$\dot{m} =$



From isentropic flow Table $\gamma = 1.4$

$M_1 = 2.5$

$\frac{T_1}{T_{o1}} = 0.425$, $\frac{P_1}{P_{o1}} = 0.0501$

$P_{o1} = \frac{0.507}{0.0501}$

$T_1 = 0.425 \times 310$

$T_1 = 131.75$ k

$P_{o1} = 10.12$ bar

From fanno flow Table $\gamma = 1.4$

M.	P/P^*	$\frac{T}{T^*}$	$\frac{P_o}{P_o^*}$	$\frac{4f L_{max}}{D}$
$M_1 = 2.5$	0.275	0.510	2.896	0.453
$M_2 = 1.2$	0.804	0.932	1.030	0.034

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$$\frac{4fL}{D} = \left(\frac{4f L_{max}}{D}\right)_1 - \left(\frac{4f L_{max}}{D}\right)_2$$

$$\frac{4 \times 0.003 \times L}{25 \times 10^{-3}} = (0.453 - 0.034)$$

$$\boxed{L = 0.8729 \, m}$$

Exit properties

$$\frac{P_2}{P_1} = \frac{\left(\frac{p}{p^*}\right)_2}{\left(p/p^*\right)_1}$$

$$\frac{P_2}{P_1} = \frac{0.804}{0.275}$$

$$P_2 = 2.92 \times 0.507$$

$$\boxed{P_2 = 1.482 \, bar}$$

$$M_2 = \frac{C_2}{a_2} = \frac{C_2}{\sqrt{\gamma R T_2}}$$

$$1.2 = \frac{C_2}{\sqrt{1.4 \times 287 \times 240.76}}$$

$$\boxed{C_2 = 373.23 \, m/s}$$

$$\frac{T_2}{T_1} = \frac{\left(\frac{T}{T^*}\right)_2}{\left(\frac{T}{T^*}\right)_1} = \frac{0.932}{0.510}$$

$$T_2 = 131.75 \times 1.827$$

$$\boxed{T_2 = 240.76 \, K}$$

$$\dot{m} = \rho_1 A_1 C_1$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{0.507 \times 10^5}{287 \times 131.75} = 1.34 \, kg/m^3$$

$$A_1 = \frac{\pi}{4} d_1^2 = 4.9 \times 10^{-4}$$

$$M_1 = \frac{C_1}{a_1} = \frac{C_1}{\sqrt{\gamma R T_1}}$$

$$2.5 \times \sqrt{1.4 \times 287 \times 131.75} = C_1$$

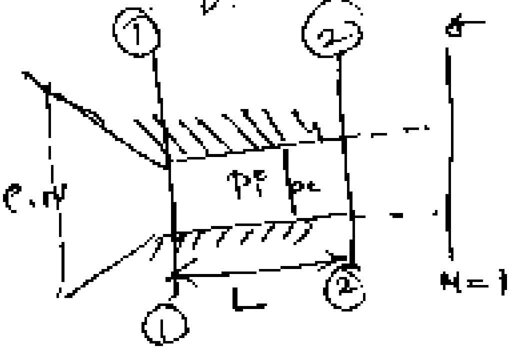
$$\boxed{C_1 = 575.20 \, m/s}$$

$$= 1.34 \times 4.9 \times 10^{-4} \times 575.20$$

$$\boxed{\dot{m} = 0.3934 \, kg/s}$$

Air is Expanded from stagnation properties of 200 kPa $M_1 = 0.2$ and 30°C through a Convergent Nozzle for which the Exit Mach Number is 0.8. Assume the flow through the Nozzle is isentropic and flow through pipe is adiabatic. find the length of the pipe and % decrease in stagnation pressure. Also find the change of Entropy for friction factor $4f = 0.005$.

$D = 25mm$



$M_2 = 0.8$

$P_{o_1} = 2$ bar

$M_1 = 0.2$

$T_1 = 30 + 273 = 303$ K

Taking $\gamma = 1.4$

$R = 287$ J/kg·K

L = ?

@ $M = 0.2$ from isentropic flow Table $\gamma = 1.4$

$\frac{T_1}{T_{o_1}} = 0.992$ $\frac{P_1}{P_{o_1}} = 0.973$

$T_1 = 0.992 \times 303 = 300.5K$

$P_1 = 0.973 \times 2$

$P_1 = 1.94$ bar

from Fanno flow Table $\gamma = 1.4$

M	P/P^*	C/C^*	T/T^*	$\frac{P_o}{P_o^*}$	$\frac{4f L_{max}}{D}$
$M_1 = 0.2$	5.455	0.218	1.190	2.964	14.533
$M_2 = 0.8$	1.289	0.825	1.064	1.038	0.072

$$\frac{P_{02}}{P_{01}} = \frac{(P_0/P_0^*)_2}{(P_0/P_0^*)_1} = \frac{1.038}{2.914} = 0.35$$

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$P_{02} = 0.35 \times P_{01}$

$$\boxed{P_{02} = 0.70 \text{ bar}}$$

$$\frac{4fL}{D} = \left(\frac{4fL_{max}}{D}\right)_1 - \left(\frac{4fL_{max}}{D}\right)_2$$

$$= 14.533 - 0.073$$

$$\frac{4fL}{D} = 14.46$$

$$\frac{0.005 \times L}{25 \times 10^{-3}} = 14.46$$

$$\boxed{L = 72.3 \text{ m}}$$

% decrease in stagnation pressure

$$= \frac{P_{01} - P_{02}}{P_{01}} = \frac{2 - 0.70}{2}$$

$$= 65\%$$

Change in Entropy

$$\frac{\Delta s}{R} = \ln\left(\frac{P_{01}}{P_{02}}\right) = \ln\left(\frac{2}{0.7}\right)$$

$$\Delta s = R \times \ln\left(\frac{2}{0.7}\right)$$

$$\boxed{\Delta s = 301.29 \text{ J/kg.K}}$$

Air is decelerated from $M=3$ to sonic speed in a 4 cm inner diameter pipe having a friction factor of 0.002. Find the length of the pipe to achieve this deceleration.

$M_1 = 3$, $M_2 = 1$ Taking $\gamma = 1.4$

$d = 4\,cm$, $f = 0.002$

$L = ?$

From Fanno flow Table $\gamma = 1.4$

@ $M_1 = 3$

$$\frac{4f L_{max}}{D} = 0.522$$

$\gamma = 1.4$

@ $M_2 = 1$,

$$\frac{4f L_{max}}{D} = 0$$

We know that

$$\frac{4fL}{D} = \left(\frac{4f L_{max}}{D}\right)_1 - \left(\frac{4f L_{max}}{D}\right)_2$$

$$= 0.522 - 0$$

$$\frac{4 \times f L}{D} = 0.522$$

$$\frac{4 \times 0.002 \times L}{4 \times 10^{-2}} = 0.522$$

$$\boxed{L = 2.61\,m}$$

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Stagnation pressure

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1}$$

$$P_0 = P\left[\frac{T_0}{T}\right]^{\gamma/\gamma-1} \longrightarrow (1)$$

@ critical state

$M = 1, \quad P = P^*, \quad P_0 = P_0^*, \quad T_0 = T_0^*$

$$P_0^* = P^*\left[\frac{T_0^*}{T^*}\right]^{\gamma/\gamma-1} \longrightarrow (2)$$

$$\frac{P_0}{P_0^*} = \frac{P\left[\frac{T_0}{T}\right]^{\gamma/\gamma-1}}{P^*\left[\frac{T_0^*}{T^*}\right]^{\gamma/\gamma-1}} = \frac{P}{P^*} \times \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1} \times \left(\frac{T^*}{T_0^*}\right)^{\gamma/\gamma-1}$$

for fanno flow

$T_0 = T_0^* = C$

$$\frac{P_0}{P_0^*} = P/P^* \times \left(\frac{T^*}{T}\right)^{\gamma/\gamma-1}$$

$$\frac{P_0}{P_0^*} = \frac{1}{M}\left[\frac{\gamma+1}{2\left[1+\frac{\gamma-1}{2}M^2\right]}\right]^{1/2} \times \left[\frac{\gamma+1}{2\left[1+\frac{\gamma-1}{2}M^2\right]}\right]^{\gamma/\gamma-1}$$

$$= \frac{1}{M}\left[\frac{2\left[1+\frac{\gamma-1}{2}M^2\right]}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{P_{01}}{P_1} = \left(\frac{T_{01}}{T_1}\right)^{\gamma/\gamma-1}$$

$$\frac{P_{02}}{P_2} = \left(\frac{T_{02}}{T_2}\right)^{\gamma/\gamma-1}$$

$$\frac{\frac{P_{01}}{P_1}}{\frac{P_{02}}{P_2}} = \frac{\left(\frac{T_{01}}{T_1}\right)^{\gamma/\gamma-1}}{\left(\frac{T_{02}}{T_2}\right)^{\gamma/\gamma-1}}$$

$$\frac{P_{01}}{P_{02}} = \frac{P_2}{P_1} \times \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1}$$

$$= \ln \frac{M_2}{M_1} \left[\frac{1+\frac{\gamma-1}{2} M_1^2}{1+\frac{\gamma-1}{2} M_2^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

<u>Entropy</u>

$$\frac{\Delta s}{R} = -\ln \frac{P_0}{P_0^*}$$

$$\frac{S-S^*}{R} = -\ln \frac{1}{M} \left[\frac{2\left[1+\frac{\gamma-1}{2} M^2\right]}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Similarly

$$\frac{S_2-S_1}{R} = \ln\left(\frac{P_{01}}{P_{02}}\right) = \ln \frac{M_2}{M_1}\left[\frac{1+\frac{\gamma-1}{2} M_1^2}{1+\frac{\gamma-1}{2} M_2^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}.$$

$= \ln[$

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Air Enters an Insulated tube of 0.03m diameter through a convergent divergent Nozzle with throat diameter 0.015m, the stagnation properties of air at Entry to Nozzle are 750 kPa and 450 K. the flow through the Nozzle is isentropic the friction factor for the duct 4f = 0.02. Calculate man. length of tube & Condition of air at duct if the length of pipe is 25cm

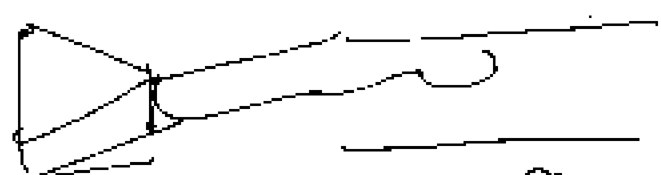
$L = 25cm = 0.25m$

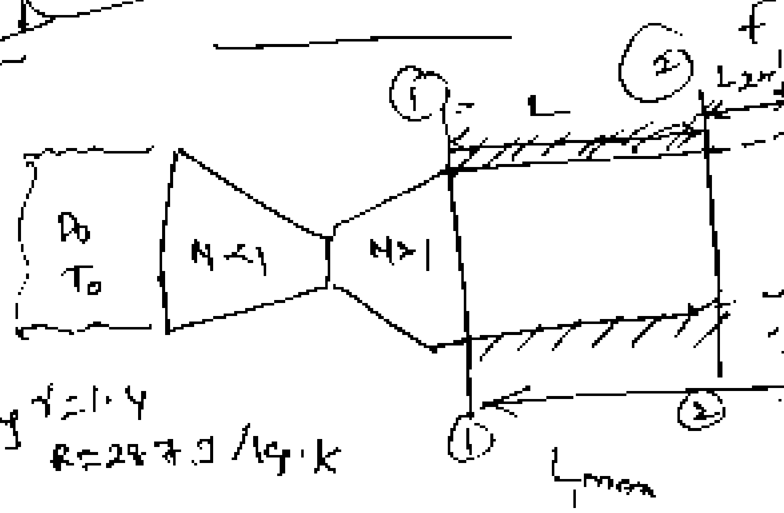
$d_1 = 0.03m$

$d^* = 0.015$

$P_{0_1} = 7.5 bar, \quad T_{0_1} = 450K$

$4f = 0.02$





Taking $\gamma = 1.4$, $R = 287 J/kg \cdot K$

(i) L_{max},

(ii) P_2, T_2, C_2, M_2, t_2

Area at Entry

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(0.03)^2 = 7.06 \times 10^{-4}$$

$$A_1^* = \frac{\pi}{4} d^{*2} = \frac{\pi}{4}(0.015)^2 = 1.767 \times 10^{-4} m^2$$

From isentropic flow Table $\gamma = 1.4$

From

$$\frac{A_1}{A_1^*} = \frac{7.06 \times 10^{-4}}{1.767 \times 10^{-4}} = 3.995 \simeq 4 \quad \begin{cases} M_1 = 2.94 \\ M_1 = 0.14 \end{cases}$$

$\boxed{M_1 > 1}$ since it is ~~Co Nozzle~~ Convergent, divergent Nozzle isentropic flow Table $\gamma = 1.4$ [$M_1 = 2.94$]

$$\frac{T_1}{T_{01}} = 0.367 \qquad \frac{p_1}{p_{01}} = 0.0298$$

$T_1 = 0.367 \times 450$, $P_1 = 0.0298 \times$ ~~P~~ 0.0298

[$T_1 = 165.15 K$] [$P_1 = 0.223$ bar]

$M_1 = \frac{c_1}{a_1} = \frac{c_1}{\sqrt{\gamma R T_1}}$

[$c_1 = 756.9 m/s$]

From fanno flow Table ($\gamma = 1.4$)

M	P/P^*	c/c^*	T/T^*	$\frac{P_0}{P_0^*}$	$\frac{4 f L_{max}}{D}$
$M_1 = 2.94$	0.226	1.949	0.439	4.00	0.512
$M_2 = 0.64$	1.645	0.674	1.109	1.145	0.353

~~4f Lmax~~ $\frac{4fL}{D} = \left(\frac{4 f L_{max}}{D}\right)_1 - \left(\frac{4 f L_{max}}{D}\right)_2$

[Pg. No: 82]

$$\frac{~~4\times~~ 0.02 \times 0.25}{0.03} = 0.512 - \left(\frac{4 f L_{max}}{D}\right)_2$$

$$\left(\frac{4 f L_{max}}{D}\right)_2 = 0.346 \simeq 0.353$$

[$M_2 < 1$] for Insulated duct or tube

[$M_2 = 0.64$]

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$$\frac{P_2}{P_1} = \frac{(P/P^*)_2}{(P/P^*)_1}$$

$$= \frac{1.645}{0.226}$$

$$= 7.2 \times P_1$$

$$\boxed{P_2 = 1.62 \text{ bar}}$$

$$\frac{T_2}{T_1} = \frac{(T/T^*)_2}{(T/T^*)_1}$$

$$T_2 = \frac{1.109}{0.439} \times T_1$$

$$\boxed{T_2 = 416.82}$$

$P_1^* = P_2^*$
$T_1^* = T_2^*$
$c_1^* = c_2^*$

$$\frac{c_2}{c_1} = \frac{(c/c^*)_2}{(c/c^*)_1}$$

$$= \frac{0.674}{1.949} \times 756$$

$$\boxed{c_2 = 261.78 \text{ m/s}}$$

Air is flowing in Insulated duct. The inlet Mach Number is 0.25. The friction factor $4f = 0.01$. The diameter of duct is 15cm. What length of pipe would give a 10% loss in stagnation pressure, what is the Mach Number at this section? what is the % of stagnation pressure loss from inlet to a section at which the Mach Number is 0.8? what is the maximum length to reach Choking condition.

$M_1 = 0.25$, $4f = 0.01$

$D = 15\text{cm}$

① $\frac{P_{01} - P_{02}}{P_{01}} = 10\%$ @ $M = ?$

② $\% \frac{P_{01} - P_{02}}{P_{01}}$ @ $M_2 = 0.8$

L_{max}, $M_2 = 1$

Case (i)

$$\frac{P_{0_1} - P_{0_2}}{P_{0_1}} = 1 - \frac{P_{0_2}}{P_{0_1}} = 0.1$$

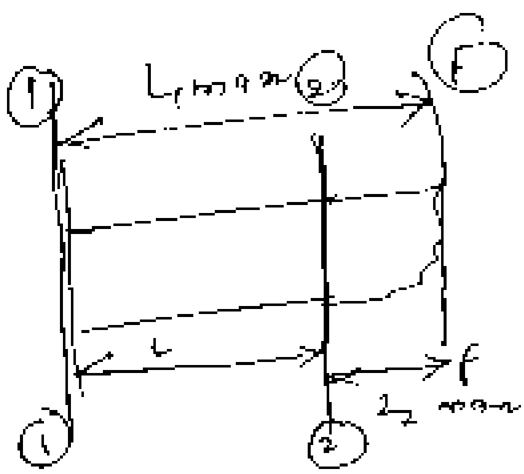
$$\frac{P_{0_2}}{P_{0_1}} = 0.9$$

From Fanno flow Table $\gamma = 1.4$, $R = 287 \ J/kg \cdot K$

~~$M_1 = 0.25$~~

$\frac{P_{0_1}}{P_{0_1}^*} = 2.4065$

	$\frac{P_{0_1}}{P_{0_1}^*}$	$\frac{4 f L_{max}}{D}$
$M_1 = 0.25$		
$M_1 = 0.25$	2.4065	8.537
$M_2 = 0.28$	2.165	6.357



$P_{0_1}^* = P_{0_2}^*$

We know

$$\frac{P_{0_2}}{P_{0_2}^*} = \frac{P_{0_2}}{P_{0_1}} \times \frac{P_{0_1}}{P_{0_2}^*}$$

$$= \frac{P_{0_2}}{P_{0_1}} \times \frac{P_{0_1}}{P_{0_1}^*} = 0.9 \times 2.4065$$

$$\boxed{\frac{P_{0_2}}{P_{0_2}^*} = 2.165}$$

$M_2 = 0.28$ ✓

$M_2 = 2.28$ ✗

$[M_2 < 1]$ for Insulated duct

$$\frac{4fL}{D} = \left(\frac{4 f L_{max}}{D}\right)_1 - \left(\frac{4 f L_{max}}{D}\right)_2$$

$$= 8.537 - 6.357$$

$$\frac{0.01 \times L}{0.15} = 2.18$$

$$\boxed{L = 32.7 \ m}$$

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Case 2

@ $M_2 = 0.8$, $\frac{P_{o_1} - P_{o_2}}{P_{o_1}} = ?$

M	$\frac{P_{o_k}}{P_o^*}$	$\frac{4 f L_{max}}{D}$
$M_1 = 0.25$	2.4065	8.537
$M_2 = 0.8$	1.038	0.073

$$\frac{P_{o_2}}{P_{o_1}} = \frac{P_{o_2}}{P_{o_2}^*} \times \frac{P_{o_2}^*}{P_{o_1}} \qquad P_{o_2}^* = P_{o_1}^*$$

$$= 1.038 \times \frac{P_{o_1}^*}{P_{o_1}}$$

$$= 1.038 \times \frac{1}{2.4065} = 0.4313$$

stagnation pressure loss

$$\Delta P_o = \frac{P_{o_1} - P_{o_2}}{P_{o_1}} = 1 - 0.4313$$

$$= 56.8\%$$

Case : 3

$M_2 = 1$, $L = L_{max}$

$$\frac{4 f L_{max}}{D} = 0$$

$$\frac{4 f L}{D} = \left(\frac{4 f L_{max}}{D}\right)_1 - \left(\frac{4 f L_{max}}{D}\right)_2^{0}$$

$$\frac{0.01 \times L}{0.15} = 8.537$$

$L = L_{max}$

$L_{max} = 128.1\,m$

Air is flowing in a Insulated duct with the Mach Number of 0.25. At the Section the downstream Entropy is greater by amount of 0.124 kJ/kg.K, as a result of friction. What is the mach Numbers at these Section. The static properties at inlet are 700 kPa, 60°C. find the velocity, temp. & Mach Number at exit? Also find the properties at critical section

$\gamma = 1.4$

$M_1 = 0.25$, $\Delta S = 0.124$ kJ/kg.K

$P_1 = 7$ bar, $T_1 = 60 + 273$

$T_1 = 333$ K

@ $\gamma = 1.4$ isentropic flow table

$\frac{P_1}{P_{01}} = 0.957$ $\frac{T_1}{T_{01}} = 0.987$

$P_{01} = \frac{7}{0.957} = 7.3$ bar $= P_{01}$

$\frac{333}{0.987} = T_{01} = 337.3$ K

From Fanno flow table $\gamma = 1.4$

M	P/P^*	C/C^*	T/T^*	$\frac{P_0}{P_0^*}$	$\frac{4 f L_{max}}{D}$
$M_1 = 0.25$	4.36	0.272	1.185	2.4065	8.54
$M_2 = 0.4$	2.695	0.431	1.163	1.59	2.308

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$M_1 = \frac{c_1}{a_1} = \frac{c_1}{\sqrt{\gamma R T_1}} = \frac{c_1}{\sqrt{1.4 \times 287 \times 333}}$

$0.25 = \frac{c_1}{\sqrt{1.4 \times 287 \times 333}}$

$c_1 = 446 \, m/s$

$\frac{p_1}{p_1^*} = 4.36,$ $p_1^* = 1.60 \, bar$

$\frac{T_1}{T_1^*} = 1.185$ $T_1^* = 281.012 \, K$

$\frac{p_{01}}{p_{01}^*} = 2.406$ $p_{01}^* = 3.033 \, bar$

$p_1^* = p_2^* = p^*$
$T_1^* = T_2^* = T^*$
$p_{01}^* = p_{02}^* = p_0^*$

$\frac{\Delta s}{R} = \ln\left(\frac{p_{01}}{p_{02}}\right)$

$e^{\frac{124}{287}} = e^{\ln \frac{p_{01}}{p_{02}}}$

$\frac{p_{01}}{p_{02}} =$

$p_{02} = 4.74 \, bar$

$\frac{p_{02}}{p_{02}^*} = \frac{4.74}{3.031} = 1.564$

$[M_2 < 1]$ for Rayleigh duct

$\frac{p_2}{p_1} = \frac{(p/p^*)_2}{(p/p^*)_1} = \frac{2.695}{4.36}$

$p_2 = 4.32 \, bar$

$\frac{T_2}{T_1} = \frac{(T/T^*)_2}{(T/T^*)_1} = \frac{1.1632}{1.185}$

$T_2 = 326 \, K$

$\frac{c_2}{c_1} = \frac{(c/c^*)_2}{(c/c^*)_1} = \frac{0.431}{0.272}$

$c_2 = 144.9 \, m/s$

A circular duct is to deliver 225 m³/min of air at 20°C and 1.25 bar. If it is necessary to hold the length of pipe of to 30 m with diameter 15 cm, determine the required inlet condition. Take $4f = 0.02$.

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Take $\gamma = 1.4$, $R = 287$ J/kg.K

$Q = \frac{225}{60} = 3.75\ m^3/sec$, $T_2 = 20+273 = 293\ K$

$P_2 = 1.25$ bar

$L = 30\ m$,

To find

P_1, T_1, c_1, M_1

$Q = A_2 \times c_2$

$\frac{Q}{A_2} = c_2$ $\quad$ $c_2 = 212.207\ m/s$

$\frac{3.75}{\frac{\pi}{4}(0.15)^2} = c_2$ $\quad$ $c_2 = 212.207\ m/s$

$M_2 = \frac{c_2}{a_2} = \frac{c_2}{\sqrt{\gamma R T_2}} = \frac{212}{\sqrt{1.4 \times 287 \times 293}}$

$M_2 = 0.6184 \simeq 0.62$

M	P/P_0^*	$\frac{c}{c^*}$	$\frac{T}{T^*}$	$\frac{4f L_{max}}{D}$
$M_2 = 0.62$	1.703	0.654	1.1154	0.417
$M_1 = 0.32$	3.388	0.347	1.176	4.417

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$$\frac{4fL}{D} = \left(\frac{4f L_{max,1}}{D}\right)_1 - \left(\frac{4f L_{max}}{D}\right)_2$$

$$\frac{0.02 \times 30}{0.15} = \qquad - 0.417$$

$$\left(\frac{4f L_{max}}{D}\right)_1 = 4.417$$

$M_1 < 1$ Corresponding to Fanno flow

$M_1 = 0.32$

$$\frac{P_2}{P_1} = \frac{\left(P/P^*\right)_2}{\left(P/P^*\right)_1} = \frac{1.703}{3.388}$$

$P_1 = 2.48 \text{ bar}$

$$\frac{T_2}{T_1} = \frac{\left(\frac{T}{T^*}\right)_2}{\left(T/T^*\right)_1}$$

$$\frac{T_2}{T_1} = \frac{1.114}{1.176}$$

$$\frac{T_2}{0.947} = T_1$$

$T_1 = 309.30 \text{ K}$

$$\frac{c_2}{c_1} = \frac{\left(\frac{c}{c^*}\right)_2}{\left(c/c^*\right)_1}$$

$$\frac{c_2}{c_1} = \frac{0.654}{0.347}$$

$c_1 = 112.57 \text{ m/s}$

Variation of Mach Number with duct length

L_{max} = maximum length

The mean coefficient of friction

$$\bar{f} = \frac{1}{L_{max}} \int_0^{L_{max}} f \, dx \longrightarrow (1)$$

We know that

$$4f \frac{dx}{D} = \frac{1-M^2}{\gamma M^2 \left[1 + \frac{\gamma-1}{2} M^2\right]} \cdot \frac{dM^2}{M^2} \longrightarrow (2)$$

@ the critical state, $M=1$, $L=0$

@ the actual state $M=M$, $L=L_{max}$

∴ by Integrating the above equation

$x=0$, $x=L_{max}$

$M=0$, to $M=1$

$$\int_0^{L_{max}} 4f \frac{dx}{D} = \int_M^1 \frac{1-M^2}{\gamma \left[1 + \frac{\gamma-1}{2} M^2\right]} \times \frac{dM^2}{M^4}$$

These are integration

$$\frac{4f L_{max}}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma} \ln \left[\frac{(\gamma+1) M^2}{2\left(1 + \frac{\gamma-1}{2} M^2\right)} \right] \longrightarrow (3)$$

$$\frac{4f L_{max}}{D} = 0 \quad \text{for} \quad M = 1$$

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The distance b/t the two sections of the duct where the Mach Number are M_1 & M_2 is given by

$$\frac{4\bar{f}L}{D} = \left(\frac{4\bar{f}L_{max}}{D}\right)_1 - \left(\frac{4\bar{f}L_{max}}{D}\right)_2 \longrightarrow (4)$$

(4) in 3 for M_1 & M_2

$$\frac{4\bar{f}L}{D} = \frac{M_2^2 - M_1^2}{\gamma M_1^2 M_2^2} + \frac{\gamma+1}{2\gamma} \ln \frac{M_1^2}{M_2^2} \frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}$$

Explain choking in fanno flow

In a fanno flow, $M<1$, the effect of friction will increase the velocity & Mach Number & to decrease the Enthalpy & pressure of the gas

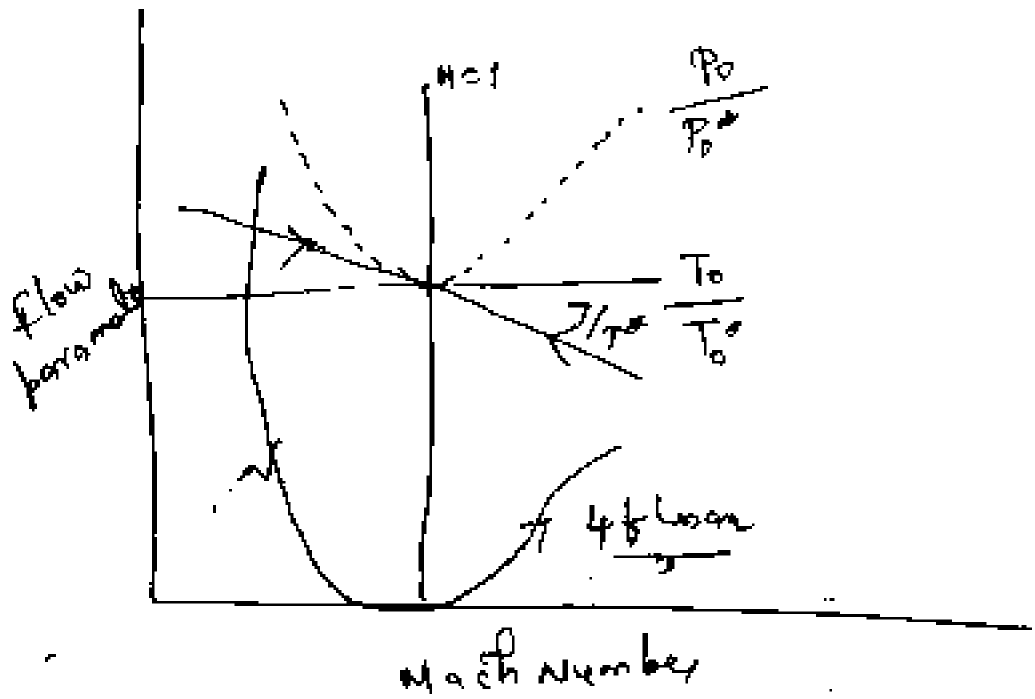
In $M>1$, (supersonic flow) the effect of friction & to will decrease the velocity & Mach Number of gas & Increase the Enthalpy & pressure of gas

In both cases Entropy >> upto limiting state where the Mach Number is one ($M=1$) so the mass flow rate is maximum @ $M=1$, & it is constant afterwards. At this point the flow is a choked flow

4/2/013 Absent C.D., and Prav

12, 14, 11, 26
28, 32, 8 39, 40, 47,
57, 58,

Variation of flow parameter in fanno flow



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unit - II
chapter - II

Flow in a constant area ducts with heat transfer
(Rayleigh flow)

A flow in a constant area duct with heat transfer and without friction is known as Rayleigh flow.

Ex:- Flow in many thermal systems can be closely approximated this process

Heat transfer processes in heat exchangers and combustion chambers

the changes in flow parameters on account of wall friction are negligible compared to those due to heat transfer.

Rayleigh line

The frictionless flow process with heat transfer in a constant area duct is described by a curve known as Rayleigh line.

The locus of all state points during the Rayleigh process satisfies the equation of state, continuity & momentum.

Assumptions

(1) perfect gas (2) constant area duct
(3) 1-dimensional steady frictionless flow.
(4) absence of body forces

We know that mass flow rate

$$\dot{m} = \rho A c$$

mass flow density → $G = \frac{\dot{m}}{A} = \rho c$

$$c = \frac{G}{\rho} \longrightarrow (1)$$

The momentum equation

$$p + \rho c^2 = \text{Constant}$$

$$p + \rho \times \frac{G^2}{\rho^2} = \text{Constant}$$

$$p + \frac{G^2}{\rho} = \text{Constant} \longrightarrow (2) \qquad v = \frac{1}{\rho}$$

$$p + G^2 v = \text{Constant} \longrightarrow (3)$$

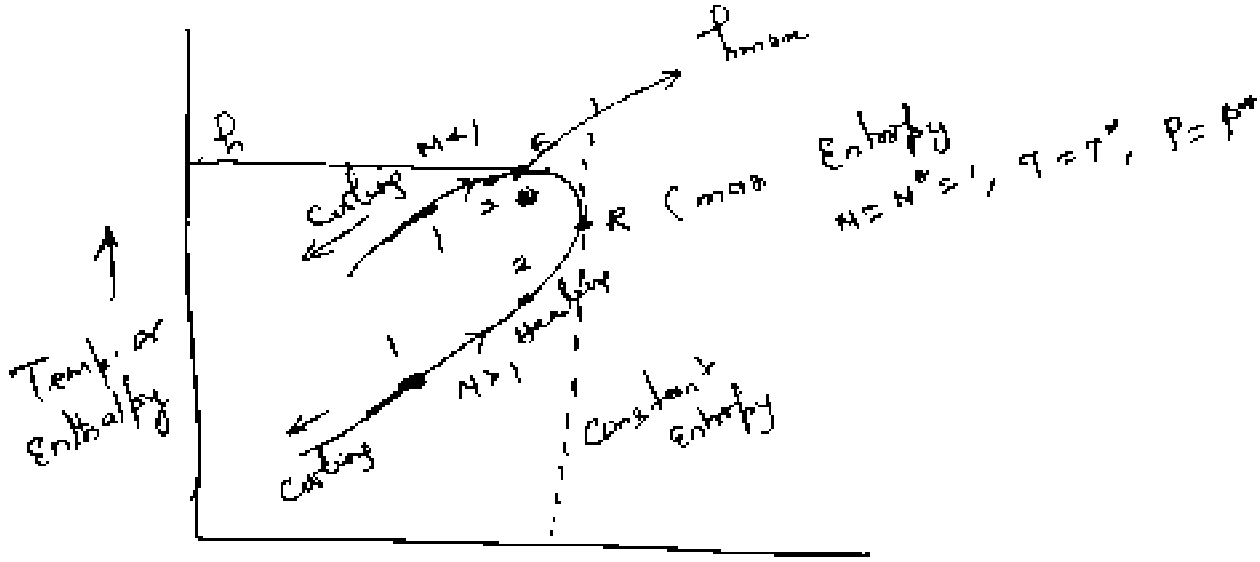
Equation of state

$$p = p(h, s), \quad \rho = \rho(h, s) \longrightarrow (4)$$

(4) in (2)

$$p(h, s) + \frac{G^2}{\rho(h, s)} = \text{Constant} \longrightarrow (5)$$

Equation (5) may be used for representing Rayleigh line on h-s diagram



~~A Rayleigh~~ A Rayleigh line or curve

* heat transfer causes heating or cooling of the gas changes its stagnation temperature.

In heating Entropy >, i.e. the state of gas moves towards the right on the Rayleigh line app. max. Entropy point R.

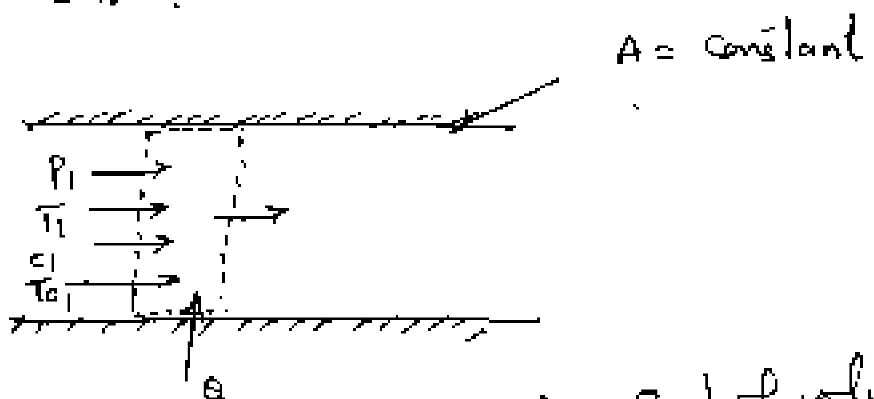
* heating beyond R not possible since Entropy <

∴ ~~during heating processes,~~ the limit of the heating processes on both the subsonic and supersonic branches of the Rayleigh line is the max. Entropy point R.

The direction of cooling processes is away from the limiting point R leading to decrease in Entropy, which confirms the 2nd law of thermodynamics

Fundamentals Equations

Fig shows a Rayleigh flow through a control volume. The flow parameters in a finite process at entry and exit of the control volume are also shown.



Rayleigh flow through control volume.

The changes in the flow parameters occur due to the heat transfer Q.

The initial & final state points during the process are shown in fig.

Continuity Equations

mass flow rate

$$\dot{m} = \rho_1 A_1 c_1 = \rho_2 A_2 c_2 \qquad A_1 = A_2 = c$$

$$\dot{m} = \rho_1 c_1 = \rho_2 c_2$$

$$\frac{c_1}{c_2} = \frac{\rho_2}{\rho_1} \rightarrow (1) \quad , \quad \frac{\rho_1}{\rho_2} = \frac{c_2}{c_1}$$

Equation of state

$$PV = mRT$$

$$P_1 = \rho_1 R T_1 \quad , \quad P_2 = \rho_2 R T_2$$

$$\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2} \times \frac{T_1}{T_2} = \frac{c_2}{c_1} \times \frac{T_1}{T_2} \rightarrow (2)$$

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Momentum Equations

momentum equation between state 1 & 2 is given by

$$P_1 A + \dot{m} c_1 = P_2 A + \dot{m} c_2$$

$$P_1 A - P_2 A = \dot{m}(c_2 - c_1)$$

$\dot{m} = \rho A c$

$A_2 = A_1 = c$

$M = \frac{c}{a}$

$$(P_1 - P_2) A = \dot{m}(c_2 - c_1)$$

$$= \rho_2 A_2 c_2^2 - \rho_1 A_1 c_1^2$$

$$(P_1 - P_2) A = (\rho_2 c_2^2 - \rho_1 c_1^2) A$$

$$P_1 - P_2 = \frac{P_2}{R T_2} c_2^2 - \frac{P_1}{R T_1} c_1^2$$

$$= \frac{P_2}{R T_2} \times M_2^2 \times a_2^2 - \frac{P_1}{R T_1} M_1^2 \times a_1^2$$

$$= \frac{P_2}{R T_2} M_2^2 \times \gamma R T_2 - \frac{P_1}{R T_1} M_1^2 \times \gamma R T_1$$

$$P_1 - P_2 = \gamma P_2 M_2^2 - \gamma P_1 M_1^2$$

$$P_1 + \gamma P_1 M_1^2 = P_2 + \gamma P_2 M_2^2$$

$$P_1 [1 + \gamma M_1^2] = P_2 [1 + \gamma M_2^2]$$

$$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \longrightarrow (3)$$

Mach Number

$M_1 = \frac{c_1}{a_1}$, $M_2 = \frac{c_2}{a_2}$

$$\frac{M_2}{M_1} = \frac{\frac{c_2}{a_2}}{\frac{c_1}{a_1}} = \frac{c_2}{c_1} \times \frac{a_1}{a_2}$$

$$= \frac{c_2}{c_1} \times \frac{\sqrt{\gamma R T_1}}{\sqrt{\gamma R T_2}}$$

$$\frac{M_2}{M_1} = \frac{c_2}{c_1} \times \left(\frac{T_1}{T_2}\right)^{1/2}$$

Energy

The heat transfer during the process

$$Q = m\, c_p\, (T_{02} - T_{01})$$

m = mass flow rate kg/s.

c_p = specific heat J/kg.K

T_{02} & T_{01} = stagnation temp in K.

Rayleigh flow relation

The flow parameters (p, T, p_0, T_0, e, f) are Expressed in terms of machnumbers & γ. All the above parameters are Expressed in non-dimensional form

$F = [1 + \gamma M_1^2]\, p$

$F_1 = [1 + \gamma M_1^2]\, p_1$

$F_2 = [1 + \gamma M_2^2]\, p_2$

$$\frac{F_2}{F_1} = \frac{1 + \gamma M_2^2}{1 + \gamma M_1^2} \times \frac{p_2}{p_1}$$

$$\frac{F_2}{F_1} = \frac{p_1}{p_2} \times \frac{p_2}{p_1}$$

$$\boxed{\frac{F_2}{F_1} = 1}$$

Stagnation pressure

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\gamma/\gamma-1}$$

for state 1 & 2

$$\frac{P_{01}}{P_1} = \left[1 + \frac{\gamma-1}{2} M_1^2\right]^{\gamma/\gamma-1}$$

$$\frac{P_{02}}{P_2} = \left[1 + \frac{\gamma-1}{2} M_2^2\right]^{\gamma/\gamma-1}$$

$$\frac{\frac{P_{02}}{P_2}}{\frac{P_{01}}{P_1}} = \frac{\left[1 + \frac{\gamma-1}{2} M_2^2\right]^{\gamma/\gamma-1}}{\left[1 + \frac{\gamma-1}{2} M_1^2\right]^{\gamma/\gamma-1}}$$

$$\frac{P_{02}}{P_{01}} = \frac{P_2}{P_1} \times \left[\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2}\right]^{\gamma/\gamma-1}$$

$$\frac{P_{02}}{P_{01}} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \left[\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2}\right]^{\gamma/\gamma-1}$$

Temperature

$$\frac{M_2}{M_1} = \frac{C_2}{C_1} \times \left(\frac{T_1}{T_2}\right)^{1/2}$$

$$\frac{T_1}{T_2} = \left[\frac{M_2}{M_1} \times \frac{C_1}{C_2}\right]^2 \longrightarrow (1)$$

We know that

$P_1 = \rho_1 R T_1$, $\quad P_2 = \rho_2 R T_2$

$$\frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \times \frac{T_1}{T_2}$$

$$\frac{C_1}{C_2} = \frac{\rho_2}{\rho_1}$$

$$\frac{C_1}{C_2} = \frac{P_2}{P_1} \times \frac{T_1}{T_2}$$

$$\frac{T_1}{T_2} = \left[\frac{P_2}{P_1} \times \frac{T_1}{T_2} \times \frac{M_2}{M_1}\right]^2$$

$$\frac{T_1}{T_2} = \left[\frac{P_2 \times M_2}{P_1 \times M_1}\right]^2 \times \frac{T_1^2}{T_2^2}$$

$$\frac{T_1}{T_2} \times \frac{T_2^2}{T_1^2} = \left(\frac{M_2}{M_1}\right)^2 \times \left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^2$$

$$\frac{T_2}{T_1} = \frac{M_2^2}{M_1^2} \times \left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^2$$

stagnation temp

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

for state (1) & (2)

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2 \rightarrow (1), \qquad \frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2 \rightarrow (2)$$

$$\frac{(2)}{(1)} = \frac{\frac{T_{02}}{T_2}}{\frac{T_{01}}{T_1}} = \frac{1+\frac{\gamma-1}{2} M_2^2}{1+\frac{\gamma-1}{2} M_1^2}$$

$$= \frac{T_{02}}{T_{01}} = \frac{T_2}{T_1} \times \frac{1+\frac{\gamma-1}{2} M_2^2}{1+\frac{\gamma-1}{2} M_1^2}$$

$$= \frac{M_2^2}{M_1^2} \times \left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^2 \times \left[\frac{1+\frac{\gamma-1}{2} M_2^2}{1+\frac{\gamma-1}{2} M_1^2}\right]$$

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Change in Entropy

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - C_p \ln \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= C_p \ln \frac{\frac{T_2}{T_1}}{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$= C_p \ln \left[\frac{M_2^2}{M_1^2} \times \frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^{1} \Big/ \left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^{\frac{\gamma-1}{\gamma}}$$

$$= \frac{\gamma R}{\gamma-1} \ln \frac{\frac{M_2^2}{M_1^2} \times \left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^{1}}{\left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^{\frac{\gamma-1}{\gamma}}}$$

$$S_2 - S_1 = \ln \frac{M_2}{M_1}\left[\frac{M_2}{M_1}\left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^{\frac{\gamma+1}{\gamma-1}}\right.$$

Heat transfer

$$Q = m C_p (T_{o_2} - T_{o_1})$$

$$\frac{Q}{C_p T_1} = \frac{T_{o_1}}{T_1}\left[\frac{T_{o_2}}{T_{o_1}} - 1\right] = \left[1 + \frac{\gamma-1}{2} M_1^2\right]\left[\frac{T_{o_2}}{T_{o_1}} - 1\right]$$

$$= \left[1+\frac{\gamma-1}{2}M_1^2\right]\left\{\frac{M_2^2}{M_1^2} \times \left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^2 \times \left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right] - 1\right\}$$

$$\frac{q}{c_p T_1} = \frac{M_2^2}{M_1^2}\left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^2\left[1+\frac{\gamma-1}{2}M_2^2\right] - \left[1+\frac{\gamma-1}{2}M_1^2\right]$$

$$= \frac{M_2^2}{M_1^2}\left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^2\left[\frac{2+(\gamma-1)M_2^2}{2} - \left(\frac{2+(\gamma-1)M_1^2}{2}\right)\right]$$

simplify the above Equation

$$= \frac{M_2^2 - M_1^2\left[2(1-\gamma M_1^2 M_2^2)+(\gamma-1)(M_2^2+M_1^2)\right]}{M_1^2(1+\gamma M_2^2)\,2}$$

Variations of flow parameters

The variation of various flow parameters with the Mach Number i.e flow properties values for $M = M^* = 1$, are used as reference for non-dimensionalizing various properties at any section of the duct

We know that $\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}$

$P_2 = P^*$, $P_1 = P$, $M_2 = 1$, $M_1 = M$,

$$\frac{P^*}{P} = \frac{1+\gamma M^2}{1+\gamma}$$

Similarly $P_{0_2} = P_0^*$, $P_{0_1} = P_0$

$$\frac{P_0}{P_0^*} = \frac{1+\gamma}{1+\gamma M^2}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\gamma/\gamma-1}$$

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The condition of gas in a chamber at Entry are $P_1 = 0.343$ bar, $T_1 = 310$ K, $c_1 = 60$ m/s. Determine the Mach Number, Pressure, temp and velocity at Exit of the gas if the Increase in stagnation Enthalpy between Entry & Exit are 1172.5 kJ/kg. ($C_p = 1005$, $\gamma = 1.4$)

$P_1 = 0.343$ bar | $c_1 = 60$ m/s | M_2, P_2, T_2, C_2

$T_1 = 310$ K | $\Delta H = 1172.5$ kJ/kg.

$a_1 = \sqrt{\gamma R T_1}$

$a_1 = \sqrt{1.4 \times 287 \times 310}$

$\boxed{a_1 = 353 \text{ m/s}}$

$M_1 = \frac{c_1}{a_1} = \frac{60}{353}$

$M_1 = 0.17$

from Isentropic flow Table $\gamma = 1.4$

$\frac{T_1}{T_{01}} = 0.9943$, $\frac{P_1}{P_{01}} = 0.980$

$\boxed{T_{01} = 311.78 \text{ K}}$

Rayleigh flow Table $\gamma = 1.4$

M	P/P^*	P_0/P_0^*	T/T^*	T_0/T_0^*	C/C^*
$M_1 = 0.18$ ~~2.295~~	2.295 ~~1.237~~	1.241 ~~0.227~~	0.171 ~~0.143~~	0.142	0.079
$M_2 = 0.45$	1.87	1.155	0.7075	0.61	0.378

$$\Delta h_0 = c_p\,(T_{o2} - T_{o1})$$

$$10^3 \times 1175.8 = 1.005 \times 10^5\,(T_{o2} - 311.78)$$

$$\boxed{T_{o2} = 1478.45\,k}$$

$$\frac{T_{o2}}{T_{o2}^*} = \frac{1478.45}{2416.89} = 0.612$$

Rayleigh flow

$\gamma = 1.4$

$$\boxed{M_2 = 0.45} \qquad \boxed{M_2 = 2.6}$$

$\boxed{M_2 < 1}$ for heating process

$$\frac{T_{o1}}{T_{o1}^*} = 0.143 \quad (\text{~~0.129~~})$$

$$\boxed{T_{o1}^* = 2416.87\,k} \qquad \boxed{T_{o1}^* = T_{o2}^*}$$

$$\frac{P_2}{P_1} = \frac{(P/P^*)_2}{(P/P^*)_1}$$

$$\boxed{P_2 = 0.278\,\text{bar}}$$

$$P_2 = \frac{1.87}{2.316} \times 0.343$$

$$\frac{T_2}{T_1} = \frac{\left(\frac{T}{T^*}\right)_2}{\left(T/T^*\right)_1} = \frac{0.7075}{0.154}$$

$$T_2 = \frac{0.7075}{0.154} \times T_1 = \frac{0.7075}{0.154} \times 310$$

$$\boxed{T_2 = 1424.18\,K}$$

$$\boxed{C_2 = 341.05\,m/s}$$

$$\frac{C_2}{C_1} = \frac{\left(\frac{C}{C^*}\right)_2}{\left(\frac{C}{C^*}\right)_1} = \frac{0.378}{0.0665}$$

$$C_2 = \frac{0.378}{0.0665} \times 60$$

(24)

A Combustion chamber in a gas Turbine receives air at 350 K, 0.55 bar and 75 m/s. The air-fuel ratio is 29 and the calorific value of the fuel is 41.87 MJ/kg. Taking $\gamma = 1.4$, $R = 287$ J/kg.K for the gas determine

(i) the initial & final Mach Number

(ii) final pressure, temp. & velocity of the gas

(iii) percentage stagnation pressure loss in the Combustion chamber

(iv) the maximum attainable temp condition

$\gamma = 1.4$

$R = 287$ J/kg.K

$T_1 = 350$ K, $P_1 = 0.55$ bar $C_1 = 75$ m/s

$A/F = 29$, $C.V = 41.87 \times 10^6$ J/kg.

(i) M_1, M_2

(ii) P_2, T_2, C_2

(iii) $\% \frac{P_{01} - P_{02}}{P_{01}}$

(iv) $T_{01}^* = T_{02}^* = T_0^*$

$$M_1 = \frac{C_1}{a_1} = \frac{C_1}{\sqrt{\gamma R T_1}} = \frac{75}{\sqrt{1.4 \times 287 \times 350}}$$

$M_1 = 0.20$

From isentropic flow Table $\gamma = 1.4$

$\frac{T_1}{T_{01}} = 0.992$ $\frac{P_1}{P_{01}} = 0.55$

$T_{01} = 352.82$ K

$P_{01} = 0.565$ bar

From Rayleigh Table $\gamma = 1.4$

M	P/P^*	$\frac{P_0}{P_0^*}$	T/T^*	$\frac{T_0}{T_0^*}$	$\frac{c}{c^*}$
$M_1 = 0.20$	2.273	1.235	0.207	0.174	0.091
$M_2 = 0.64$	1.525	1.061	0.953	0.859	0.625

$$\Delta h_0 = \frac{C \cdot V}{A/F + 1} = \frac{41.87 \times 10^6}{29 + 1} = 1395.67 \text{ KJ/kg}$$

$$\Delta h_0 = C_p \left(T_{0_2} - T_{0_1} \right)$$

$$1395.67 \times 10^3 = 1005 \left(T_{0_2} - 352.82 \right)$$

$$\boxed{T_{0_2} = 1741.55 \text{ K}}$$

$$\frac{T_{02}}{T_{0_2}^*} = \frac{T_{02}}{T_{0_2}^*} \times \frac{T_{0_1}}{T_{0_1}}$$

$$\boxed{T_{0_1}^* = T_{0_2}^* = T_0^*}$$

$$= \frac{T_{02}}{T_{0_1}} \times \frac{T_{0_1}}{T_{0_1}^*}$$

$$= \frac{1741.55}{352.82} \times 0.174$$

$$\frac{T_{02}}{T_{0_2}^*} = 0.859$$ — Rayleigh flow Table $\gamma = 1.4$

$$\boxed{M_2 = 0.64}$$

(24)

A Combustion chamber in a gas [sic, 24 written over gas] Turbine receives air at 350 K, 0.55 bar and 75 m/s. The air-fuel ratio is 29 and the calorific value of the fuel is 41.87 MJ/kg. Taking $\gamma = 1.4$, R = 287 J/kg for the gas determine

(i) the initial & final Mach Numbers

(ii) final pressure, temp. & velocity of the gas

(iii) percentage stagnation pressure loss in the Combustion chamber

(iv) the maximum attainable temp condition

$T_1 = 350\,K$, $P_1 = 0.55\,bar$, $C_1 = 75\,m/s$, $\gamma = 1.4$, $R = 287\,J/kg\,K$

$A/F = 29$, $C.V = 41.87 \times 10^6\,J/kg$.

(i) M_1, M_2

(ii) P_2, T_2, C_2

(iii) $\% \frac{P_{01} - P_{02}}{P_{01}}$

(iv) $T_{01}^* = T_{02}^* = T_0^*$

$$M_1 = \frac{C_1}{a_1} = \frac{C_1}{\sqrt{\gamma R T_1}} = \frac{75}{\sqrt{1.4 \times 287 \times 350}}$$

$M_1 = 0.20$

From isentropic flow Table $\gamma = 1.4$

$$\frac{T_1}{T_{01}} = 0.992 \qquad \frac{P_1}{P_{01}} = 0.55\ \text{[sic]}$$

$$\boxed{T_{01} = 352.82\,K} \qquad \boxed{P_{01} = 0.565\,bar}$$

From Rayleigh Table $\gamma = 1.4$

M	P/P^*	$\frac{P_0}{P_0^*}$	T/T^*	$\frac{T_0}{T_0^*}$	$\frac{c}{c^*}$
$M_1 = 0.20$	2.273	1.235	0.207	0.174	0.091
$M_2 = 0.64$	1.525	1.061	0.953	0.859	0.625

$$\Delta h_0 = \frac{C.V}{A/F + 1} = \frac{41.87 \times 10^6}{29 + 1} = 1395.67 \text{ KJ/kg}$$

$$\Delta h_0 = C_p (T_{02} - T_{01})$$

$$1395.67 \times 10^3 = 1005 (T_{02} - 352.82)$$

$$\boxed{T_{02} = 1741.55 \text{ K}}$$

$$\frac{T_{02}}{T_{02}^*} = \frac{T_{02}}{T_{02}^*} \times \frac{T_{01}}{T_{01}}$$

$$\boxed{T_{01}^* = T_{02}^* = T_0^*}$$

$$= \frac{T_{02}}{T_{01}} \times \frac{T_{01}}{T_{01}^*}$$

$$= \frac{1741.55}{352.82} \times 0.174$$

$$\frac{T_{02}}{T_2^*} = 0.859 \quad \text{— Rayleigh flow Table } \gamma = 1.4$$

$$\boxed{M_2 = 0.64}$$

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$$P_1^* = P_2^* = P^*$$

$$\frac{P_2}{P_1} = \frac{\left(\frac{P}{P^*}\right)_2}{\left(P/P^*\right)_1}$$

$$P_2 = \frac{1.525}{2.273} \times P_1 = \frac{1.525}{2.275} \times 0.55$$

$$P_2 = 0.369 \text{ bar}$$

$$\frac{T_2}{T_1} = \frac{\left(T/T^*\right)_2}{\left(T/T^*\right)_1} = \frac{0.953}{0.207}$$

$$T_2 = \frac{0.952}{0.207} \times 350$$

$$T_2 = 1611.35 K$$

$$\frac{C_2}{C_1} = \frac{\left(\frac{C}{C^*}\right)_2}{\left(C/C^*\right)_1} = \frac{0.625}{0.091}$$

$$C_2 = \frac{0.625}{0.091} \times C_1 = \frac{0.625}{0.091} \times 75 = 515.1 \text{ m/s}$$

$$\frac{P_{02}}{P_{01}} = \frac{\left(\frac{P_0}{P_0^*}\right)_2}{\left(P_0/P_0^*\right)_1} = \frac{1.061}{1.235}$$

$$P_{02} = \frac{1.061}{1.235} \times P_{01} = \frac{1.061}{1.235} \times 0.565 = 0.485 \text{ bar} = P_{02}$$

Stagnation pressure loss =

$$\Delta P_0 = P_{0_1} - P_{0_2}$$

$$= 0.565 - 0.485$$

$$\Delta P_0 = 0.080 \text{ bar}$$

% stagnation $\frac{\Delta P_0}{P_{0_1}} = 14.16\,\%$

$$\frac{T_{02}}{T_{0_2}^*} = 0.859$$

$$\boxed{T_{0_2}^* = 2027.4} = T_{0_1}^* = \boxed{T_{0\,max}}$$

The Mach Number at the Exit of the Combustion Chamber is 0.9. The ratio of stagnation temp. at Exit & Entry is 3.74. If the pressure & temp of the gas at Exit are 2.5 bar, 1000°C & determine (a) Mach Number, pressure & temp. of the gas at Entry (b) heat supplied per kg of gas (c) maximum heat that can be

Take $\gamma = 1.3$, $c_p = 1218 \text{ J/kg.K}$

$M_2 = 0.9$, $\frac{T_{02}}{T_{01}} = 3.74$

$P_2 = 2.5 \text{ bar}$, $T_2 = 1000 + 273 = 1273 \text{ K}$

$M_1, P_1, T_1, Q_{supp}, Q_{max} = ?$

$\gamma = 1.3$, $M_2 = 0.9$

From isentropic flow Table

(2b)

$\frac{T_2}{T_{02}} = 0.892$ $\quad \frac{P_2}{P_{02}} = 0.608$

$T_{02} = \frac{1273}{0.892} = 1427.13\,K = T_{02}$

$\frac{T_{02}}{T_{01}} = 3.74$, $\quad T_{01} = 381.58\,K$

$\gamma = 1.3$ Rayleigh flow Table

M	T/T^*	$\frac{T_0}{T_0^*}$	P/P^*
$M_1 = 0.26$	0.302	0.265	2.114
$M_2 = 0.9$	1.017	0.991	1.12

$\frac{T_{02}}{T_{02}^*} = 0.991$, $\quad T_{02}^* = 1440.09\,K$ $\quad T_{01}^* = T_{02}^* = T_0^*$

$\frac{T_{01}}{T_{01}^*} = \frac{381.58}{1440.09} = 0.265 \Rightarrow$ Rayleigh flow Table $\gamma = 1.3$

$M_1 = 0.26$

$\frac{T_2}{T_1} = \frac{\left(\frac{T}{T^*}\right)_2}{\left(\frac{T}{T^*}\right)_1} = \frac{1.017}{0.302}$

$\frac{T_2}{T_1} = 3.36$

$T_1 = 378\,K$

$\frac{P_2}{P_1} = \frac{\left(\frac{P}{P^*}\right)_2}{\left(P/P^*\right)_1} = \frac{1.12}{2.114}$

$\frac{P_2}{P_1} = 0.529$, $\quad \frac{2.5}{0.529} = P_1$

$P_1 = 4.7\,bar$

$Q = \dot{m} c_p (T_{02} - T_{01})$ $\quad m = 1$

$= 1218 (1427.12 - 381.58)$

$Q = 1273.48 \text{ kJ/kg}$

$$q_{max} = \frac{c_p T_1 (1 - M_1^2)^2}{2(1+\gamma) M_1^2} = \frac{1218 \times 378 (1 - (0.26)^2)^2}{2(1+1.3)(0.26)^2}$$

$q_{max} = 1287.18 \text{ kJ/kg}$

(or)

Alternatively $\quad T_{02} = (T_{02}^*)$

$q_{max} = c_p (T_0^* - T_{01})$

$= 1218 (1440.09 - 381.58)$

$q_{max} = 1293.60 \text{ kJ/kg}$

The data of for a gas ($\gamma = 1.3$, $R = 466$ J/kg.K) at the Entry of the Constant area duct are $P_1 = 0.345$ bar, $T_1 = 312$ K, $C_1 = 65.5$ m/s. If 4592 kJ/kg of heat is added to the gas in the duct between Entry & Exit sections. Determine at the Exit, (P_2, T_2, C_2 & M_2). How much of heat is required to accelerate the gas from initial to sonic conditions, what are the pressure & temp. of gas at sonic conditions

$\gamma = 1.3$, $R = 466$ J/kg.K $\quad P_1 = 0.345$ bar, $T_1 = 312$ K, $C_1 = 65.5$ m/s

$q = 4592$ kJ/kg.

P_2, T_2, C_2, M_2, $\quad q_{max} = c_p (T_{02}^* - T_{01})$, $\quad P^*, T^*$

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$\gamma = 1.3$

(a) $M_1 = \frac{c_1}{a_1} = \frac{65.5}{\sqrt{\gamma R T_1}} = \frac{65.5}{\sqrt{1.3 \times 466 \times 312}} = 0.15$

isentropic flow

$\frac{T_1}{T_{01}} = 0.9965$ $\frac{P_1}{P_{01}} = 0.985$

~~$T_{01} = 0.346\,K$~~ $P_{01} = 0.35\,bar$

$T_{01} = 313.09\,K$

from Rayleigh flow table

M	P/P^*	$\frac{P_0}{P_0^*}$	T/T^*	$\frac{T_0}{T_0^*}$	$\frac{c}{c^*}$
$M_1 = 0.15$	2.2515	1.237	0.1125	0.098	0.0505
$M_2 = 0.60$	1.567	1.074	0.884	0.810	0.564

$\frac{T_{01}}{T_{01}^*} = 0.098$ $T_{01}^* = 3194.79\,K$ $T_{02}^* = T_0^*$

$c_p = \frac{\gamma R}{\gamma - 1} = \frac{1.3 \times 466}{1.3 - 1}$

$c_p = 2019.33$

$Q = c_p (T_{02} - T_{01})$

$4592 \times 10^3 = 2019.33 (T_{02} - 313.09)$

$2274.0 = T_{02} - 313.09$

$T_{02} = 2587.10\,K$

$$\frac{T_{02}}{T_{02}^*} = \frac{2567.10}{3194.79} = 0.809$$

$M_2 = 0.60$

$$\frac{P_2}{P_1} = \frac{\left(\frac{P}{P^*}\right)_2}{\left(P/P^*\right)_1} = \frac{1.074}{1.237} = 0.868$$

$P_2 = 0.868 \times P_1$

$P_2 = 0.299$ bar

$$\frac{T_2}{T_1} = \frac{\left(\frac{T}{T^*}\right)_2}{\left(T/T^*\right)_1} = \frac{0.884}{0.1125} = 7.85 \times T_1$$

$T_2 = 7.85 \times T_1$

$T_2 = 24.511$ bar

$$\frac{c_2}{c_1} = \frac{\left(\frac{c}{c^*}\right)_2}{\left(c/c^*\right)_1} = \frac{0.564}{0.0505} = 11.16$$

$c_2 = 11.16 \times 65.5$

$c_2 = 731.2$ m/s

$q_{max} = c_p\left(T_{02}^* - T_{01}\right)$

$= 2019.33\,(3194.79 - 313.09)$

$q_{max} = 5819.103$ kJ/kg

@ $M_1 = 0.75$ Rayleigh Ha

$\frac{P_1}{P_1^*} = 2.2515$ $\qquad \frac{T_1}{T^*} = 0.1125$

$P^* = 0.1532$ bar $\qquad T^* = 2773.3$ K

(28)

The data for a combustion chamber employing a hydrocarbon fuel is given below

Entry

Gas velocity = 152 m/s
pressure = 4 bar
Temperature = 400 K

Exit

Mach Number = 0.8

Take $\gamma = 1.3$, $c_p = 2.144$ kJ/kg.K for the products of combustion. Calorific value of the fuel burnt = 44×10^6 J/kg. determine
(a) Entry Mach number, (b) pressure, temp & velocity of gas at Exit
(c) stagnation pressure loss
(d) Air-fuel ratio required.

$C_1 = 152$ m/s
$P_1 = 4$ bar
$T_1 = 400$ K

$M_2 = 0.8$
$\gamma = 1.3$, $c_p = 2144$ J/kg.K
C.V = 44×10^6 J/kg.

M_1, P_2, T_2 & C_2, $P_{01} - P_{02}$, A/F

$$c_p = \frac{\gamma R}{\gamma - 1}$$

$$2144 = \frac{1.3 \times R}{1.3 - 1}$$

$R =$

(i) $$M_1 = \frac{c_1}{a_1} = \frac{152}{\sqrt{\gamma R T_1}}$$

$$= \frac{152}{\sqrt{1.3 \times R \times 400}}$$

$M_1 = 0.299$ ≃ 0.3

From isentropic flow table $\gamma = 1.3$

$$\frac{T_1}{T_{01}} = 0.987 \qquad \frac{P_1}{P_{01}} = 0.944$$

$$T_{01} = \frac{400}{0.987} \qquad P_{01} = \frac{4}{0.944}$$

$T_{01} = 405.27$ K

$P_{01} = 4.23$ bar

From Rayleigh flow table

M	p/p^*	T/T^*	P_0/P_0^*	T_0/T_0^*	c/c^*
$M_1=0.30$	2.059	0.882	1.191	0.34	0.185
$M_2=0.89$	1.255	1.009	1.019	0.961	0.804

$$\frac{P_2}{P_1} = \frac{\left(\frac{P}{P^*}\right)_2}{\left(P/P^*\right)_1} = \frac{1.255}{2.059} = 0.6095$$

$$P_2 = 0.6095 \times P_1 = 0.6095 \times 4$$

$$\boxed{P_2 = 2.438\ bar}$$

$$\frac{T_2}{T_1} = \frac{\left(\frac{T}{T^*}\right)_2}{\left(\frac{T}{T^*}\right)_1} = \frac{1.009}{0.882}$$

$$T_2 = \frac{1.009}{0.882} \times T_1 = \frac{1.009}{0.882} \times 400, \quad \boxed{T_2 = 1056.59\ K}$$

$$\frac{c_2}{c_1} = \frac{\left(\frac{c}{c^*}\right)_2}{\left(\frac{c}{c^*}\right)_1} = \frac{0.804}{0.185}$$

$$c_2 = \frac{0.804}{0.185} \times c_1 = \frac{0.804}{0.185} \times 152$$

$$\boxed{c_2 = 660.58\ m/s}$$

$$\frac{P_{02}}{P_{01}} = \frac{\left(\frac{P_0}{P_0^*}\right)_2}{\left(\frac{P_0}{P_0^*}\right)_1} = \frac{1.019}{1.191}$$

$$P_{02} = \frac{1.019}{1.191} \times P_{01} = \frac{1.019}{1.191} \times 4.23$$

$$\boxed{P_{02} = 3.619\ bar}$$

$$\frac{T_{02}}{T_{01}} = \frac{\left(\frac{T_0}{T_0^*}\right)_2}{\left(\frac{T_0}{T_0^*}\right)_1} = \frac{0.961}{0.336}$$

$$T_{02} = \frac{0.961}{0.336} \times T_{01} = \frac{0.961}{0.336} \times 405.27$$

$$\boxed{T_{02} = 1159.12\ K}$$

Stagnation pressure loss = $P_{01} - P_{02}$

$= 4.23 - 3.619$ (29)

$\Delta P_0 = 0.611$ bar

Stagnation Enthalpy rise

$$\Delta h_0 = \frac{C.V}{A/F + 1} = \frac{44\times10^6}{A/F+1}$$

$$\Delta h_0 \text{ or } Q = C_p (T_{02} - T_{01})$$

$$\Delta h_0 \text{ or } Q = 2144 (1159.12 - 405.27)$$

$=$

$$2144 (1159.12 - 405.27) = \frac{44\times10^6}{A/F+1}$$

$$\boxed{A/F = 26.22 : 1}$$

Air at Pressure = 3 bar, temp = 288 K, $M_1 = 1.6$ is brought to sonic velocity in a frictionless constant area duct through which heat transfer occurs. The stagnation temp of air, in a combustion chamber is increased by 1.04 times its initial value. Calculate Mach Number, pressure & temp. of air
(i) what will be the Mach number if this heat is extracted from the air?

Taking $\gamma = 1.4$
$R = 287$ J/kg.K
$C_p = 1005$ J/kg.K

$P_1 = 3$ bar, $T_1 = 288$ K, $M_1 = 1.6$

$T_{02} = 1.04\, T_{01}$

M_2, P_2, T_2, Q

@ $M_1 = 1.6$ from isentropic flow table ($\gamma = 1.4$)

$$\frac{T_1}{T_{01}} = 0.661, \quad \frac{P_1}{P_{01}} = 0.235$$

$$T_{01} = \frac{288}{0.661} \qquad P_{01} = \frac{3}{0.235}$$

$$\boxed{T_{01} = 435.70\,K} \qquad P_{01} =$$

From Rayleigh flow Table ($\gamma = 1.4$)

M	P/P^*	T/T^*	P_0/P_0^*	T_0/T_0^*
1.6	0.524	0.702	1.176	0.884
$M_2 = 1.46$	0.602	1.103		0.919

$$\frac{T_{01}}{T_0^*} = 0.884$$

$$T_{01}^* = 492.87\,k$$

given that

$$T_{02} = 1.04\,T_{01}$$

$$T_{02} = 453.128\,k$$

$$\frac{T_{02}}{T_{02}^*} = \frac{453.128}{492.87}$$

$$\frac{T_{02}}{T_{02}^*} = 0.919$$

$T_{01}^* = T_{02}^* = T_0^*$

$P_{01}^* = P_{02}^* = P_0^*$

$P_1^* = P_2^* = P^*$

$T_1^* = T_2^* = T^*$

Rayleigh flow Table (0.92) → 1.46

$M_2 > 1$ (since for cooling process)

$M_2 = 1.46$

$$\frac{P_2}{P_1} = \frac{(P/P^*)_2}{(P/P^*)_1} = \frac{0.602}{0.524}$$

$$P_2 = \frac{0.602}{0.524} \times 3 = 3.45\,bar = P_2$$

$$\frac{T_2}{T_1} = \frac{(T/T^*)_2}{(T/T^*)_1}$$

$$\frac{T_2}{T_1} = \frac{1.102}{0.702}$$

$$T_2 = \frac{1.102}{0.702} \times T_1$$

$$= \frac{1.102}{0.702} \times 288$$

$$T_2 = 317.12\,k$$

$$Q = C_p(T_{02} - T_{01})$$

$$= 1005(453.128 - 435.70)$$

$$Q = 17.575\,kJ/kg$$

Unit - III

Shock wave is a special kind of wave referred as steep finite pressure wave.

The propagation of sound in fluid flow was restricted to infinitesimal changes in fluid properties across the wave front.

If the changes are finite and abrupt, the wave is shock wave.

A shock is a discontinuity in supersonic flow and is accompanied with rise in static pressure and fall in velocity. (From supersonic to subsonic).

As the process is irreversible and adiabatic there is increase in entropy across the shock.

$Q = 0,$

$T = 0$

Types of shock

1) Normal shock
2) Oblique shock

Normal shock

If the plane of shock is normal to the direction of the flow of gas, it is called a normal shock.

In normal shock there is no change in the direction of fluid flow before & after the shock.

Oblique shock

If the plane shock is inclined to the direction of flow of gas, the shock is oblique.

i.e the fluid passing through a shock changes its direction.

Strength of shock depends on Mach Number before shock.

M = 1 → strength of shock is zero.

M > 1 → more strength.

Examples

wind tunnels.

diverging portion of a converging-diverging Nozzle supersonic wind tunnels.

Infront of intakes of turbojets & ramjets

shock waves supersonic compressor / Exit of Turbine blade

Governing Equations

(1) Continuity Equation

Continuity Equation for the shock waves gives the flow rate

$A_x = A_y = A = c$

$\dot{m} = \rho_x A_x C_x = \rho_y A_y C_y$

$\frac{\dot{m}}{A} = \rho_x C_x = \rho_y C_y \longrightarrow (1)$

Energy Equation

$Q = 0$, In the absence of shaft work,

$h_{ox} = h_{oy} = h_o = \text{Constant}$

$T_{ox} = T_{oy} = T_o = \text{Constant}.$

$h_{ox} = h_x + \frac{c_x^2}{2}, \quad h_{oy} = h_y + \frac{c_y^2}{2} \longrightarrow (2)$

Equation of a perfect gas

$PV = mRT$

$\frac{p_x}{\rho_x} = RT_x, \quad \frac{p_y}{\rho_y} = RT_y$

$\frac{p_x}{\rho T_x} = \frac{p_y}{RT_y} \longrightarrow (3)$

momentum Equation

Applying momentum Equation

$(p_x - p_y) A = \dot{m} (c_y - c_x)$

$p_x - p_y = \frac{\dot{m}}{A} (c_y - c_x)$

$p_x - p_y = \rho_x c_x (c_y - c_x)$

$p_x - p_y = -\rho_x c_x^2 + \rho_y c_y^2$

$p_x + \rho_x c_x^2 = p_y + \rho_y c_y^2$ $= \text{Constant} \longrightarrow (4)$

$F_x = F_y$

The Equation of state gives

$h = f(s, \rho)$

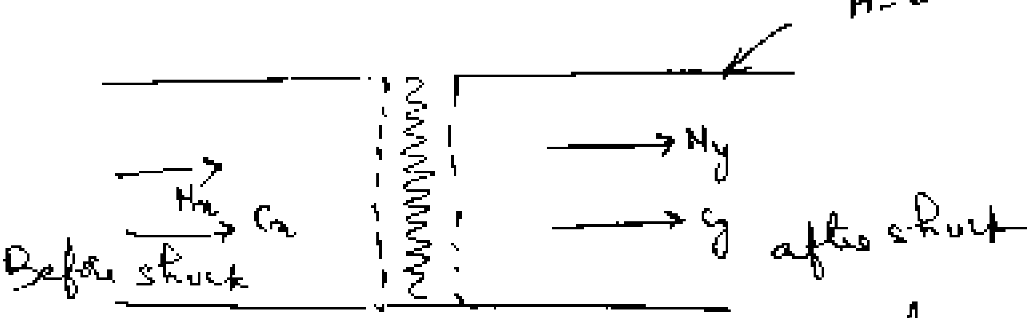
$h = f(P, \rho)$ } to

are to $d(x)$ Curves known as Fanno & Rayleigh lines

Prandtl- Meyer relation

It is a fundamental equations which give the relationship between the gas velocities before and after the Normal shock. and critical velocity of sound.

Prandtl- Meyer Equation the basis for other Equations for shock wave



Adiabatic Energy Equation on stagnation Enthalpy Equation

$$h_0 = \frac{a^2}{\gamma-1} + \frac{c^2}{2} = \frac{1}{2}\frac{\gamma+1}{\gamma-1} a^{*2} \longrightarrow (1)$$

Applying the above Equation the before shock & after shock

before shock wave

$$\frac{a_x^2}{\gamma-1} + \frac{c_x^2}{2} = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)a^{*2}$$

$$\frac{a_x^2}{\gamma-1} = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)a^{*2} - \frac{c_x^2}{2}$$

multiply $\gamma-1$

$$a_x^2 = \frac{\gamma-1}{2}\frac{(\gamma+1)}{\gamma-1} a^{*2} - \frac{\gamma-1}{2} c_x^2$$

$$a_x^2 = \frac{\gamma+1}{2} a^{*2} - \frac{\gamma-1}{2} c_x^2$$

divide by c_x

$$\frac{a_x^2}{c_x} = \frac{\gamma+1}{2}\frac{a^{*2}}{c_x} - \frac{\gamma-1}{2} c_x \longrightarrow (2)$$

after shock wave

Similarly,

$$\frac{a_y^2}{c_y} = \frac{\gamma+1}{2}\frac{a^{*2}}{c_y} - \frac{\gamma-1}{2} c_y^2 \longrightarrow (3)$$

from momentum equation

$$P_x - P_y = \frac{\dot{m}}{A}(c_y - c_x)$$

We know that

$$c_y - c_x = \frac{P_x - P_y}{\dot{m}/A} = \frac{P_x}{\dot{m}/A} - \frac{P_y}{\dot{m}/A} \longrightarrow (4)$$

$$\frac{\dot{m}}{A} = \rho c = \rho_x c_x = \rho_y c_y \longrightarrow (5)$$

$$c_y - c_x = \frac{P_x}{\rho_x c_x} - \frac{P_y}{\rho_y c_y}$$

multiply γ

$$\gamma(c_y - c_x) = \frac{\gamma P_x}{\rho_x c_x} - \frac{\gamma P_y}{\rho_y c_y} \longrightarrow (5)$$

Perfect gas equation

$$\frac{P}{\rho} = RT \qquad \times \gamma$$

$$\frac{P}{\rho} = \frac{\gamma R T}{\gamma} = \frac{a^2}{\gamma}$$

$$\frac{\gamma P}{\rho} = a^2$$

applying before & after shock

$$a_x^2 = \frac{\gamma P_x}{\rho_x}, \quad a_y^2 = \frac{\gamma P_y}{\rho_y} \longrightarrow (6)$$

(6) in (5)

$$\gamma(C_y - C_x) = \frac{a_x^2}{C_x} - \frac{a_y^2}{C_y} \longrightarrow (7)$$

(2) & (3) in (7)

$$\frac{a_x^2}{C_x} - \frac{a_y^2}{C_y} = \gamma(C_y - C_x)$$

$$\left[\left(\frac{\gamma+1}{2}\right)\frac{a^{*2}}{C_x} - \left(\frac{\gamma-1}{2}\right)C_x\right] - \left[\left(\frac{\gamma+1}{2}\right)\frac{a^{*2}}{C_y} - \frac{(\gamma-1)}{2}C_y\right] = \gamma(C_y - C_x)$$

$$\frac{\gamma+1}{2}\frac{a^{*2}}{C_x} - \frac{\gamma+1}{2}\frac{a^{*2}}{C_y} + \frac{\gamma-1}{2}C_y - \frac{\gamma-1}{2}C_x = \gamma(C_y - C_x)$$

$$\frac{\gamma+1}{2}a^{*2}\left[\frac{1}{C_x} - \frac{1}{C_y}\right] + \frac{\gamma-1}{2}(C_y - C_x) = \gamma(C_y - C_x)$$

$$\frac{\gamma+1}{2}a^{*2}\left[\frac{C_y - C_x}{C_x C_y}\right] + \frac{\gamma-1}{2}(C_y - C_x) = \gamma(C_y - C_x)$$

multiply by $\frac{C_x C_y}{C_y - C_x}$

$$\left(\frac{\gamma+1}{2}\right)a^{*2} + \left(\frac{\gamma-1}{2}\right)C_x \cdot C_y = \gamma C_x C_y$$

multiply by 2

$$(\gamma+1)a^{*2} + (\gamma-1)\,C_x \cdot C_y = 2\gamma C_x C_y$$

$$(\gamma+1)a^{*2} + \gamma C_x C_y - C_x C_y = 2\gamma C_x C_y$$

$$(\gamma+1)a^{*2} = 2\gamma C_x C_y - \gamma C_x C_y + C_x \cdot C_y$$

$$(\gamma+1)a^{*2} = \gamma C_x C_y + C_x \cdot C_y$$

$$(\gamma+1)a^{*2} = (\gamma+1)C_x \cdot C_y$$

$$a^{*2} = C_x \cdot C_y \longrightarrow \text{prandtl - Meyer relation}$$

The above equation is known as Prandtl-Meyer relation (4)

$$a^* \times a^* = c_x \cdot c_y$$

$$1 = \frac{c_x}{a^*} \times \frac{c_y}{a^*}$$

$$\boxed{1 = M_x^* \times M_y^*}$$

Mach Number downstream of the Normal shock wave

M_x = upstream Mach Number (or) Mach Number before the Normal shock

M_y = downstream mach Number (or) Mach Number after the shock

adiabatic flow of a perfect gas gives

$$a^{*2} = \frac{2\gamma}{\gamma+1} R T_0 \longrightarrow (1)$$

We know that

$$c_x \times c_y = a^{*2} \longrightarrow (2)$$

$$M_x = \frac{c_x}{a_x}, \quad M_y = \frac{c_y}{a_y}$$

$$\left.\begin{aligned} c_x &= M_x a_x = M_x\sqrt{\gamma R T_x} \\ c_y &= M_y a_y = M_y\sqrt{\gamma R T_y} \end{aligned}\right\} (3)$$

in

$$c_x \times c_y = \frac{2\gamma}{\gamma+1} R T_0$$

$$M_x\sqrt{\gamma R T_x} \times M_y\sqrt{\gamma R T_y} = \frac{2\gamma}{\gamma+1} R T_0$$

Squaring on both sides

$$M_x^2 (\gamma R T_x) M_y^2 (\gamma R T_y) = \left(\frac{2}{\gamma+1}\right)^2 \gamma^2 R^2 T_0^2.$$

$$M_x^2 \times M_y^2 \times T_x \times T_y = \left(\frac{2}{\gamma+1}\right)^2 \times T_0^2$$

$$M_x^2 \times M_y^2 = \frac{\left(\frac{2}{\gamma+1}\right)^2 \times T_0^2}{T_x \times T_y} = \left(\frac{2}{\gamma+1}\right)^2 \times \frac{T_0}{T_x} \times \frac{T_0}{T_y} \longrightarrow (5)$$

We know that

applying before & after shock

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

$$\frac{T_{0x}}{T_x} = 1 + \frac{\gamma-1}{2} M_x^2 \quad / \quad \frac{T_{0y}}{T_y} = 1 + \frac{\gamma-1}{2} M_y^2 \longrightarrow (6)$$

$T_{0x} = T_{0y}$

$$M_x^2 \times M_y^2 = \left(\frac{2}{\gamma+1}\right)^2 \times \left(1 + \frac{\gamma-1}{2} M_x^2\right)\left(1 + \frac{\gamma-1}{2} M_y^2\right)$$

Taking M_y^2 outside

$$M_x^2 \times M_y^2 = \frac{4}{(\gamma+1)^2}\left[1 + \frac{\gamma-1}{2} M_x^2\right] \times M_y^2 \left[\frac{1}{M_y^2} + \frac{\gamma-1}{2}\right]$$

separating x & y term

$$\frac{M_x^2}{\frac{4}{(\gamma+1)^2}\left[1 + \frac{\gamma-1}{2} M_x^2\right]} = \frac{M_y^2\left[\frac{1}{M_y^2} + \frac{\gamma-1}{2}\right]}{M_y^2}$$

$$\frac{M_x^2}{\frac{4}{(\gamma+1)^2}\left[1 + \frac{\gamma-1}{2} M_x^2\right]} - \frac{\gamma-1}{2} = \frac{1}{M_y^2}$$

Taking L.C.M

$$\frac{1}{M_y^2} = \frac{M_x^2(\gamma+1)^2}{4\left[1 + \frac{\gamma-1}{2} M_x^2\right]} - \frac{\gamma-1}{2}$$

$$\frac{1}{M_y^2} = \frac{2 M_x^2 (\gamma+1)^2 - 4(\gamma-1)\left(1+\frac{\gamma-1}{2} M_x^2\right)}{8\left[1+\frac{\gamma-1}{2} M_x^2\right]} \quad (3)$$

$$M_y^2 = \frac{8\left[1+\frac{\gamma-1}{2} M_x^2\right]}{2 M_x^2 (\gamma+1)^2 - 4(\gamma-1)\left(1+\frac{\gamma-1}{2} M_x^2\right)}$$

$$= \frac{8\left[1+\frac{\gamma-1}{2} M_x^2\right]}{2 M_x^2 [\gamma+1]^2 - 4(\gamma-1) - 2(\gamma-1)^2 M_x^2}$$

$$= \frac{8\left[\frac{2+(\gamma-1) M_x^2}{2}\right]}{2 M_x^2 (\gamma+1)^2 - 4(\gamma-1) - 2(\gamma-1)^2 M_x^2} \quad \div 2$$

$$\frac{8 + 4(\gamma-1) M_x^2}{2 M_x^2 (\gamma+1)^2 - 4(\gamma-1) - 2(\gamma-1)^2 M_x^2} \quad \div 2$$

$$M_y^2 = \frac{4 + 2(\gamma-1) M_x^2}{(\gamma+1)^2 M_x^2 - 2(\gamma-1) - (\gamma-1)^2 M_x^2}$$

Divide both nr & dr by $2(\gamma-1)$

$$M_y^2 = \frac{\frac{2}{\gamma-1} + M_x^2}{\frac{(\gamma+1)^2}{2(\gamma-1)} M_x^2 - 1 - \frac{(\gamma-1)}{2} M_x^2}$$

∴ This is simplification & re-arrangement we get

$$\boxed{M_y^2 = \frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}}$$

static pressure ratio across the shock

Force, $F = P + \rho c^2$

force acting before the shock $F_x = P_x + \rho_x C_x^2$

" " after the shock, $F_y = P_y + \rho_y C_y^2$

$F_x = F_y$

$P_x + \rho_x C_x^2 = P_y + \rho_y C_y^2$

$$P_x + \frac{\gamma P_x}{\gamma R T_x} C_x^2 = P_y + \frac{\gamma P_y}{\gamma R T_y} C_y^2$$

$$P_x + \frac{\gamma P_x C_x^2}{a_x^2} = P_y + \frac{\gamma P_y}{a_y^2} C_y^2$$

$P_x + \gamma P_x M_x^2 = P_y + \gamma P_y M_y^2$

$P_x [1 + \gamma M_x^2] = P_y [1 + \gamma M_y^2]$

$$\frac{P_y}{P_x} = \frac{1+\gamma M_x^2}{1+\gamma M_y^2} \longrightarrow (1)$$

We know that $M_y^2 = \dfrac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1} \longrightarrow (2)$

(2) in (1)

$$\frac{P_y}{P_x} = \frac{1+\gamma M_x^2}{1+\gamma\left[\dfrac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}\right]} = \frac{(1+\gamma M_x^2)}{\dfrac{\frac{2\gamma}{\gamma-1} M_x^2 - 1 + \gamma\left[\frac{2}{\gamma-1} + M_x^2\right]}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}}$$

$$\frac{P_y}{P_x} = \frac{(1+\gamma M_x^2)\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right)}{\frac{2\gamma}{\gamma-1} M_x^2 - 1 + \frac{2\gamma}{\gamma-1} + \gamma M_x^2} = \frac{(1+\gamma M_x^2)\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right)}{\left[\frac{2\gamma}{\gamma-1} M_x^2 + \gamma M_x^2 - 1 + \frac{2\gamma}{\gamma-1}\right]}$$

$$\frac{P_y}{P_x} = \frac{\left(1+\gamma M_x^2\right)\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right)}{\gamma M_x^2\left[\frac{2}{\gamma-1}+1\right] + \left(\frac{2\gamma}{\gamma-1}-1\right)}$$ (4)

$$= \frac{\left(1+\gamma M_x^2\right)\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right)}{\gamma M_x^2\left[\frac{2+\gamma-1}{\gamma-1}\right] + \left(\frac{2\gamma-\gamma+1}{\gamma-1}\right)}$$

$$= \frac{\left(1+\gamma M_x^2\right)\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right)}{\gamma M_x^2\left[\frac{1+\gamma}{\gamma-1}\right] + \frac{\gamma+1}{\gamma-1}}$$

$$\frac{P_y}{P_x} = \frac{\left(1+\gamma M_x^2\right)\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right)}{\frac{\gamma+1}{\gamma-1}\left[\gamma M_x^2 + 1\right]} = \frac{\frac{2\gamma}{\gamma-1} M_x^2 - 1}{\frac{\gamma+1}{\gamma-1}}$$

Multiply $\frac{\gamma-1}{\gamma+1}$ both Nr & Dr

$$= \frac{\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right) \times \frac{\gamma-1}{\gamma+1}}{\frac{\gamma+1}{\gamma-1} \times \frac{\gamma-1}{\gamma+1}}$$

So

$$\boxed{\frac{P_y}{P_x} = \frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1}} \rightarrow (3)$$

Equation (3) is applicable for $M_x > 1$. there will be pressure rise ratio P_y/P_x.

$\frac{P_y}{P_x}$ to $M_x < 1$

but $M_x = 1$, $\frac{P_y}{P_x} = 1$, for $M_x = \infty$, $\frac{P_y}{P_x} = \infty$

(7)

The following data refers to a supersonic wind tunnel.

Nozzle throat area = 200 cm^2

$\gamma = 1.4$

Test Section c/s = 337.5 cm^2 R = 287 J/kg.K

determine the test section Mach Number and diffuser throat area if a Normal shock is located in the test section

$A^* = 200$ cm^2

$A_t = 337.5$ cm^2

Isentropic flow Table $\gamma = 1.4$

$\frac{A_t}{A^*} =$

From isentropic flow table $\gamma = 1.4$

$\frac{A_t}{A^*} = \frac{337.5}{200} = 1.619$ → $M_y = 0.32$, $M_x = 1.95$

$M_x > 1$ Always greater than 1

Normal shock wave Table $\gamma = 1.4$

$M_x = 1.95$, $M_y = 0.586$

@ $M_y = 0.586$ isentropic flow Table

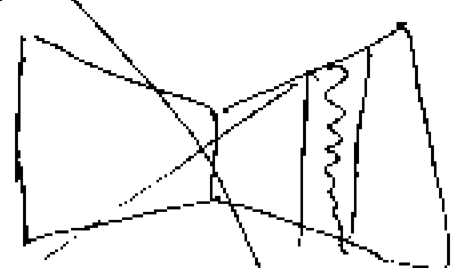
$\frac{A_y}{A_y^*} = 1.213$

$A_y = 337.5$ cm

$A_y^* = 280.29$ cm^2 diffuser throat area

Q(11) A Normal shock occurs in the diverging section of a convergent-divergent air nozzle. The throat area 1/3 exit area & the static pressure at Exit is 0.4 times the stagnation pressure at the Entry, the flow is isentropic except through the shock.
determine (i) M_x & M_y
(ii) static pressure
(iii) area of C/S of the nozzle at the section of the nozzle where the normal shock occurs

$A^* = \frac{1}{3} A_2$ $\frac{A_2}{A^*} = 3$



$P_2 = 0.4\, P_{01}$

isentropic
$M_x = 2.64$ $\frac{T_x}{T_{0x}} = 0.417$, $\frac{P_x}{P_{0x}} = 0.0464$

Normal
$M_y = 0.50$, $\frac{P_y}{P_x} = 7.963$, $\frac{T_y}{T_x} = 2.279$, $\frac{P_{0y}}{P_{0x}} = 0.43$

isentropic
$M_y = 0.50$
$\frac{A_y}{A_y^*} = 1.340$

Ans (i)
isentropic flow table $\gamma = 1.4$

$\frac{A_2}{A_2^*} \times \frac{P_2}{P_{02}} = 3 \times 0.4 = 1.2$

$M_2 = 0.45$, $\frac{P_2}{P_{02}} = 0.870$

$P_{02} = \frac{P_2}{0.870} = \frac{0.4\, P_{01}}{0.870} = 0.459\, P_{01}$

(8)

$$\frac{P_{oy}}{P_{ox}} = \frac{P_{o2}}{P_{o1}} = 0.459 \longleftarrow \text{Normal shock wave}$$

$$M_x = 2.58, \quad M_y = 0.506$$

$$\frac{P_{oy}}{P_x} = 9.115$$

From Isentropic flow Table $M = M_x = 2.58$

$$\frac{A_x}{A^*} = 2.842$$

The shock occurs in the divergent portion

$$\boxed{A_x = 2.842\,A^*} \text{ ans}$$

$$P_x = \frac{P_{oy}}{9.115} = \frac{P_{o2}}{9.115} = \frac{0.466\,P_{o1}}{9.115}$$

$$\boxed{P_x = 0.051\,P_{o1}} \longrightarrow \text{answer}$$

The state of a gas ($\gamma = 1.2$, $R = 469\ J/kg.K$) upstream of a normal shock wave is given by following data.

$M_x = 2.5$, $P_x = 2$ bar, $T_x = 275k$.

Calculate the Mach Number, Pressure, temp. & velocity of gas downstream of shock, Check the Values those given in gas Table

---

$\gamma = 1.3$, $R = 469\ J/kg.k$

$M_x = 2.5$, $P_x = 2$ bar, $T_x = 275\ k$

M_y, P_y, T_y & C_y

$$M_y^2 = \frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1} = \frac{\frac{2}{\gamma-1} + (2.5)^2}{\frac{2\times1.2}{1.3-1} (2.5)^2 - 1} = \boxed{0.242 = M_y}$$

$$\frac{P_y}{P_x} = \frac{2\gamma}{\gamma-1} M_x^2 - \frac{\gamma-1}{\gamma+1} = \frac{2\times1.3}{1.3-1} (2.5)^2 - \frac{1.3-1}{1.3+1} = 6.935$$

$$\boxed{P_y = 13.870\ \text{bar}}$$

$$\frac{T_y}{T_x} = \frac{1 + \frac{\gamma-1}{2} M_x^2}{1 + \frac{\gamma-1}{2} M_y^2} = \boxed{T_y = 513.975k}$$

$$M_y = \frac{C_y}{a_y} = \frac{C_y}{\sqrt{\gamma R T_y}} \qquad \boxed{C_y = 275.375}$$

(9)

The stagnation pressure and temperature of air at the Entry of a Nozzle are 5 bar, and 500K respectively. The Exit Mach Number is 2, where a Normal shock occurs. Calculate the following quantities before and after shock, static & stagnation pressures & temperatures, air velocities & Mach Number, what are the values of Entropy across stagnation pressure loss & increase in the shock.

To find: T_x, T_y, P_x, P_y, $T_{0x}, T_{0y}, P_{0x}, P_{0y}$, Δp_0, ΔS

$P_{0x} = P_{01} = 5\text{ bar}$, $T_0 = 500\text{ K} = T_{0x}$

$M_x = 2 = M_2$ (Normal shock occurs)

From isentropic flow table $M_x = 2$ and $\gamma = 1.4$

$\frac{T_x}{T_{0x}} = 0.555$ | $\frac{P_x}{P_{0x}} = 0.128$

$T_x = 0.555 \times 500$

$T_x = 277.5\text{K}$

$P_x = 0.128 \times 5$

$P_x = 0.64\text{ bar}$

From Normal shock wave table $\gamma = 1.4$, $M_x = 2$

$M_y = 0.577$, $\frac{P_y}{P_x} = 4.500$ $\frac{T_y}{T_x} = 1.687$ $\frac{P_{0y}}{P_{0x}} = 0.721$

$P_y = 4.500 \times 0.64$, $T_y = 1.687 \times 277.5$, $P_{0y} = 0.721 \times 5$

$P_y = 2.88\text{ bar}$

$T_y = 468.14\text{ K}$

$P_{0y} = 3.605\text{ bar}$

Stagnation Temperature remains Constant $T_{0x} = T_{0y} = 500\text{ K}$

Stagnation pressure loss $\Delta P_0 = P_{0x} - P_{0y} = 5 - 3.605 = 1.395\text{ bar}$

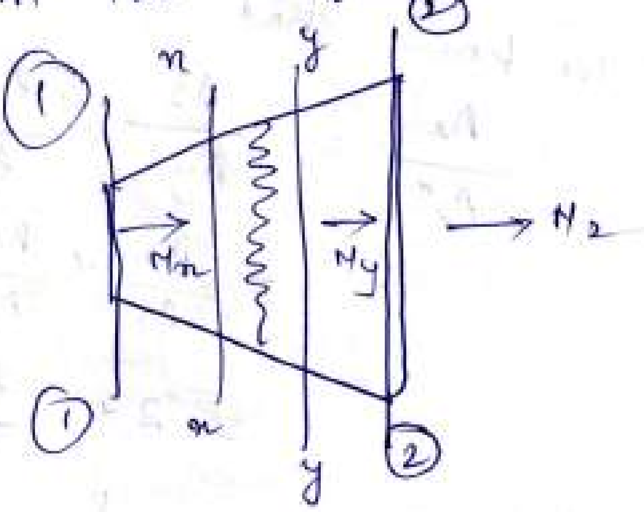
Increase in Entropy

$\Delta S = R \ln\left(\frac{P_{0x}}{P_{0y}}\right) = 287 \ln \frac{5}{3.605}$

$\Delta S = 93.88\text{ J/kg.K}$

(10)

A Mach 2 aircraft Employes a subsonic inlet diffuser of area ratio 2. A Normal shock is formed just upstream of the diffuser inlet. The freestream condition upstream of diffuser $P = 0.10$ bar, $T = 300$ K, determine (a) Mach Number, pressure & temp. at the diffuser Exit (b) diffuser efficiency Including the shock.
Assume isentropic flow in the diffuser downstream of the shock



From isentropic

$M_x = 2$, $\frac{A_2}{A_1} = 2$

$P_{0x} = 0.10$ bar, $T_{0x} = 300$ K

M_2, P_2, T_2

Diffuser

From isentropic flow Table $\gamma = 1.4$

$M_x = 2$

$\frac{P_x}{P_{0x}} = 0.128$ $\frac{T_x}{T_{0x}} = 0.555$

$T_x = 0.555 \times 300 = 165$ K

(540k)

$P_x = 0.128 \times 0.10$

$P_x = 0.0128$ bar

From Normal shock wave

$M_x = 2$, $\gamma = 1.4$ $M_y = 0.577$

$\frac{P_y}{P_x} = 4.5$	$\frac{T_y}{T_x} = 1.687$,	~~$\frac{T_y}{T_x} = 1.687$~~

$P_y = 4.5 \times 0.0128$

$P_y = 0.0576$ bar

$T_y = 278.355$ K

@ $M_y = 0.577$ $\gamma = 1.4$

isentropic flow Table

$\frac{P_y}{P_{0y}} = 0.796$ | $\frac{A_y}{A_y^*} = 1.213$

$P_{0y} = 0.0723$ bar $= P_{02}$

$\frac{A_2}{A_1} = \frac{A_2}{A_n}$

$A_n = A_y$

$A_2^* = A_y^*$

We know that

$\frac{A_2}{A_2^*} = \frac{A_2}{A_y} \times \frac{A_y}{A_2^*}$

$= \frac{A_2}{A_1} \times \frac{A_y}{A_2^*}$

$= 2 \times \frac{A_y}{A_y^*} = 2 \times 1.213$

$\frac{A_2}{A_2^*} = 2.639$ $\gamma = 1.4$

$M < 1$ (Since subsonic diffuser

Isentropic flow Table

$M_2 = 0.16$,

$\frac{T_2}{T_{02}} = 0.9949$

$T_2 = 0.9949 \times 300 = 298.47\,K$

(Sol)

$T_y = 537.2$

$\frac{P_2}{P_{02}} = 0.982$

$P_2 = 0.07$ bar $= 0.554$

$M_2 = \frac{C_2}{a_2} = \frac{C_2}{\sqrt{\gamma R T_2}}$

$C_2 = 0.16\sqrt{1.4 \times 287 \times 298.47}$

$C_2 = 55.408$ m/s

Diffuser Efficiency

$$\eta_D = \frac{\frac{T_{01}}{T_1}\left\{\left[\frac{P_{0y}}{P_{0x}}\right]^{\frac{\gamma-1}{\gamma}} - 1\right\}}{\frac{\gamma-1}{2} M_1^2}$$

check the form ? (11)

$= 80\%$

A Supersonic Nozzle is provided with constant diameter circular duct at its Exit. The duct diameter is same as Nozzle exit diameter. Nozzle exit c/s is 3 times that of throat. The entry conditions are $\gamma = 1.4$, $R = 287$ J/kg.K, are $P_0 = 10$ bar, $T_0 = 600$ K. calculate the static pressure, Mach Number, Temp & velocity at Exit

(i) when a normal shock occurs at a section in the diverging part where the area ratio $A/A^* = 2$

(ii) when a Normal shock occurs at Exit

(iii) when the Nozzle operat at its design condition

$P_0 = 10$ bar, $T_0 = 600$ K

$\frac{A_2}{A^*} = 3$

Case (i)

When a Normal shock occurs at a section in diverging part ($A/A^* = A_x/A_x^* = 2$)

$$\frac{A_x}{A_x^*} = 2.$$

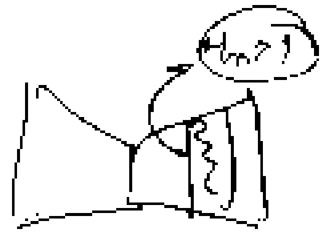
From isentropic flow Table $\frac{A_x}{A_x^*} = 2$, $\gamma = 1.4$

$M_x < 1$ / ($M_x > 1$)

$M_x = 2.2$

$\frac{P_x}{P_{0x}} = 0.0935$, $\frac{T_x}{T_{0x}} = 0.508$

$M_x = 2.2$



$P_x = 0.0935 \times 10$

$P_x = 0.935$ bar

$T_x = 0.508 \times 600$

$T_x = 304.8K$

From normal shock wave Table $\gamma = 1.4$

$M_x = 2.2$

$M_y = 0.547$

$M_y = 0.547$ (isentropic flow Table)

$$\frac{A_y}{A_y^*} = 1.2625.$$

$A_2^* = A_y^*$

$$\frac{A_2}{A_2^*} = \frac{A_2}{A_x^*} \times \frac{A_y}{A_y^*} \times \frac{A_x^*}{A_x}$$

$A_x = A_y$

$$= 3 \times 1.2625 \times \frac{1}{2}$$

$$\frac{A_2}{A_2^*} = 1.89$$

From Isentropic flow Table $\gamma = 1.4$

$M_2 = 0.325$, $\frac{P_2}{P_{0y}} = 0.9295$, $\frac{T_2}{T_{0y}} = 0.9795$

$M_2 < 1$ for supersonic to subsonic

$P_2 = P_{oy} \times 0.9295 = 5.82$ bar

(12)

$T_2 = T_{oy} \times 0.9785$

$= 600 \times 0.9785$

$T_2 = 587.1\ K$ ✓

$M_2 = \dfrac{c_2}{a_2}$,

$c_2 = 0.325 \times \sqrt{\gamma R T_2}$

$= 0.325 \times \sqrt{1.4 \times 287 \times 587.1}$

$c_2 = 157.85\ m/s$

The velocity of a Normal shock moving into stagnant air ($P = 1.0$ bar, $T = 17°C$) is 500 m/s. If the area of the cross-Section of duct is Constant determine (a) pressure (b) Temp (c) velocity (d) Mach Number of the wave front Imparted upstream of the

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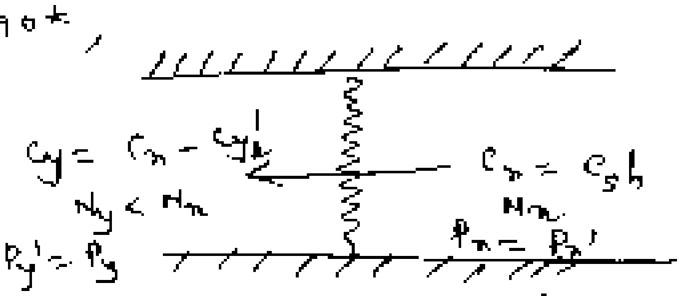
$P_x = 1$ bar, $T_x = 17 + 273 = 290\ K$,

$c_x = 500\ m/s$



$M_x = \dfrac{c_x}{a_x} = \dfrac{c_x}{\sqrt{\gamma R T_x}} = \dfrac{500}{\sqrt{1.4 \times 287 \times 290}}$

$M_x = 1.465$

The shock is first Considered in the moving co-ordinate system shock is stationary & stagnant air on right of the wave appears to be flowing towards the left

from Normal shock wave table

$M_x = 1.465$,

$M_y = 0.715$, $\dfrac{P_y}{P_x} = 2.335$, $\dfrac{T_y}{T_x} = 1.297$

$P_y = 2.335 \times P_x = 2.335 \times 1 = 2.335$ bar

$T_y = 1.297 \times T_x = 1.297 \times 290$

$T_y = T_y' = 376.13\ K$

$C_y = M_y\, a_y = 0.715 \times 388.75$

$\boxed{C_y = 277.95\ m/s}$

$C_y = C_m - C_y'$

$C_y' = C_x - C_y = 500 - 277.95$

$\boxed{C_y' = 222.05\ m/s}$

Now the shock is considered to be moving

$M_y' = \dfrac{C_y'}{a_y} = 0.571$

From isentropic flow table
$M_y' = 0.571$

$\dfrac{T_y'}{T_{oy}'} = 0.939$

$\boxed{T_{oy}' = 400.56\,K}$

Temperature ratio across the shock (17)

We know that $\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2 \longrightarrow (1)$

Applying (1) for both before & after shock

$$\left.\begin{aligned} \frac{T_{0x}}{T_x} &= 1 + \frac{\gamma-1}{2} M_x^2 \\ \frac{T_{0y}}{T_y} &= 1 + \frac{\gamma-1}{2} M_y^2 \end{aligned}\right\} (2)$$

$T_{0x} = T_{0y} = T_0$

$$\frac{T_y}{T_x} = \frac{T_{0x}}{T_x} \times \frac{T_y}{T_{0y}} = \frac{1 + \frac{\gamma-1}{2} M_x^2}{1 + \frac{\gamma-1}{2} M_y^2}$$

(i.e.) $M_y^2 = \dfrac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}$

$$\frac{T_y}{T_x} = \frac{1 + \frac{\gamma-1}{2} M_x^2}{1 + \frac{\gamma-1}{2}\left[\dfrac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}\right]}$$

$$\frac{T_y}{T_x} = \frac{1 + \frac{\gamma-1}{2} M_x^2}{\dfrac{\left[\frac{2\gamma}{\gamma-1} M_x^2 - 1\right] + \left[1 + \frac{\gamma-1}{2} M_x^2\right]}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}} = \frac{\left[1 + \frac{\gamma-1}{2} M_x^2\right]\left[\frac{2\gamma}{\gamma-1} M_x^2 - 1\right]}{\frac{2\gamma}{\gamma-1} M_x^2 - 1 + 1 + \frac{\gamma-1}{2} M_x^2}$$

$$= \frac{\left[1 + \frac{\gamma-1}{2} M_x^2\right]\left[\frac{2\gamma}{\gamma-1} M_x^2 - 1\right]}{\frac{2\gamma}{\gamma-1} M_x^2 + \frac{\gamma-1}{2} M_x^2}$$

$$= \frac{\left(1+\frac{\gamma-1}{2}M_x^2\right)\left(\frac{2\gamma}{\gamma-1}M_x^2-1\right)}{\frac{4\gamma M_x^2+(\gamma-1)^2 M_x^2}{2(\gamma-1)}}$$

$$= \frac{\left[1+\frac{\gamma-1}{2}M_x^2\right]\left[\frac{2\gamma}{\gamma-1}M_x^2-1\right]}{\frac{4\gamma M_x^2+\gamma^2 M_x^2-2\gamma M_x^2+M_x^2}{2(\gamma-1)}}$$

$$\frac{T_y}{T_x} = \frac{\left[1+\frac{\gamma-1}{2}M_x^2\right]\left[\frac{2\gamma}{\gamma-1}M_x^2-1\right]}{\frac{M_x^2(\gamma+1)^2}{2(\gamma-1)}} \longrightarrow (3)$$

Velocity of sound across the shock

We know that velocity of sound $a = \sqrt{\gamma R T}$

for upstream Normal shock $a_x = \sqrt{\gamma R T_x}$

" downstream " " $a_y = \sqrt{\gamma R T_y}$

$$\frac{a_y}{a_x} = \frac{\sqrt{\gamma R T_y}}{\sqrt{\gamma R T_y}} = \sqrt{\frac{T_y}{T_x}} = \left[\frac{\left[1+\frac{\gamma-1}{2}M_x^2\right]\left[\frac{2\gamma}{\gamma-1}M_x^2-1\right]}{\frac{M_x^2(\gamma+1)^2}{2(\gamma-1)}}\right]^{1/2}$$

$\longrightarrow (4)$

Density ratio across the shock (Rankine – Hugoniot Equations)

We know that density $= \rho = P/RT$ (14)

For upstream Normal shock wave

$$\rho_x = \frac{P_x}{RT_x}$$

For downstream Normal shock wave

$$\rho_y = P_y / RT_y$$

$$\frac{\rho_y}{\rho_x} = \frac{P_y}{RT_y} \Big/ \frac{P_x}{RT_x} = \frac{P_y}{P_x} \times \frac{T_x}{T_y} \longrightarrow (1)$$

We know that

$$P_y/P_x = \frac{2}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1}$$

$$T_y/T_x = \frac{\left[\frac{2\gamma}{\gamma-1} M_x^2 - 1\right]\left[1 + \frac{\gamma-1}{2} M_x^2\right]}{\frac{(\gamma+1)^2}{2(\gamma-1)} M_x^2}$$

(2)a & (2b)

(2a) & (2b) in (1)

$$\rho_y/\rho_x = \left[\frac{2}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1}\right] \times \frac{\frac{1}{2}\frac{(\gamma+1)^2}{2(\gamma-1)} M_x^2}{\left[\frac{2\gamma}{\gamma-1} M_x^2 - 1\right]\left[1 + \frac{\gamma-1}{2} M_x^2\right]}$$

Change in entropy across the shock

The change in entropy across the shock is given by (6)

$$\Delta S = C_p \ln \frac{T_y}{T_x} - R \ln \frac{P_y}{P_x}$$

$$= C_p \ln \frac{T_y}{T_x} - C_p\left(\frac{\gamma-1}{\gamma}\right) \ln \frac{P_y}{P_x}$$

$$= C_p \left[\ln \frac{T_y}{T_x} - \ln \left(\frac{P_y}{P_x}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

that $$\Delta S = C_p \ln \frac{T_y/T_x}{\left(P_y/P_x\right)^{\frac{\gamma-1}{\gamma}}} \rightarrow (1)$$

$$\frac{T_y}{T_x} = \frac{1+\frac{\gamma-1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_y^2} = \frac{T_{oy}}{T_{ox}} \rightarrow (2)$$

$$\frac{P_y}{P_x} = \left(\frac{1+\frac{\gamma-1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_y^2}\right)^{\gamma/\gamma-1} \frac{P_{oy}}{P_{ox}} \rightarrow (3)$$

$$\left(\frac{P_y}{P_x}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1+\frac{\gamma-1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_y^2} \left(\frac{P_{oy}}{P_{ox}}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow (4)$$

$\frac{(2)}{(4)}$

$$\frac{\frac{T_y}{T_x}}{\left(P_y/P_x\right)^{\frac{\gamma-1}{\gamma}}} = \frac{\frac{T_{oy}}{T_{ox}}}{\left(\frac{P_{oy}}{P_{ox}}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$\frac{\frac{T_y}{T_x}}{\left(\frac{P_y}{P_x}\right)^{\frac{\gamma-1}{\gamma}}} = \left(\frac{P_{oy}}{P_{ox}}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)} \rightarrow (5)$$

$$\frac{P_{oy}}{P_y} = \left[\frac{\frac{M_n^2}{2(\gamma-1)}(\gamma+1)^2}{\frac{2\gamma}{\gamma-1}M_n^2 - 1}\right]^{\gamma/\gamma-1}$$

That is simplification

$$\frac{P_{oy}}{P_y} = \left[\frac{\frac{\gamma+1}{2}M_n^2}{\frac{2\gamma}{\gamma+1}M_n^2 - \frac{\gamma-1}{\gamma+1}}\right]^{\gamma/\gamma-1}$$

$$\frac{P_n}{P_{on}} = \left(1+\frac{\gamma-1}{2}M_n^2\right)^{-\gamma/\gamma-1}$$

$$\frac{P_{oy}}{P_{on}} = \left[\frac{\frac{\gamma+1}{2}M_n^2}{\frac{2\gamma}{\gamma+1}M_n^2 - \frac{\gamma-1}{\gamma+1}}\right]^{\gamma/\gamma-1} \times \left(\frac{2\gamma}{\gamma+1}M_n^2 - \frac{\gamma-1}{\gamma+1}\right)^{1} \times \left(1+\frac{\gamma-1}{2}M_n^2\right)^{-\gamma/\gamma-1}$$

$$\frac{P_{oy}}{P_{on}} = \left[\frac{\frac{\gamma+1}{2}M_n^2}{1+\frac{\gamma-1}{2}M_n^2}\right]^{\gamma/\gamma-1} \times \left(\frac{2\gamma}{\gamma+1}M_n^2 - \frac{\gamma-1}{\gamma+1}\right)^{-1/\gamma-1}$$

Stagnation pressure ratio across the shock

Shock wave is an irreversibility across which there is a stagnation pressure loss and increase in Entropy.

The stagnation pressure ratio is a function of the upstream Mach Number M_x

$$\frac{P_{oy}}{P_{ox}} = \frac{P_{oy}}{P_{ox}} \times \frac{P_y}{P_y} \times \frac{P_x}{P_x}$$

$$= \frac{P_{oy}}{P_y} \times \frac{P_y}{P_x} \times \frac{P_x}{P_{ox}}$$

$$\frac{P_{oy}}{P_y} = \left(1 + \frac{\gamma-1}{2} M_y^2\right)^{\gamma/\gamma-1}$$

$$= \left\{1 + \frac{\gamma-1}{2}\left[\frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}\right]\right\}^{\gamma/\gamma-1}$$

$$= \left\{1 + \frac{1 + \frac{\gamma-1}{2} M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}\right\}^{\gamma/\gamma-1}$$

$$= \left[\frac{\frac{2\gamma}{\gamma-1} M_x^2 - 1 + 1 + \frac{\gamma-1}{2} M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}\right]^{\gamma/\gamma-1} = \left[\frac{\frac{4\gamma + (\gamma-1)^2 M_x^2}{2(\gamma-1)}}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}\right]^{\gamma}$$

$$\frac{p_y}{p_x} = \frac{\frac{\rho_y}{\rho_x}\left[\frac{\gamma+1}{\gamma-1}\right] - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}} \rightarrow \text{Rankine Hugoniot Equation}$$

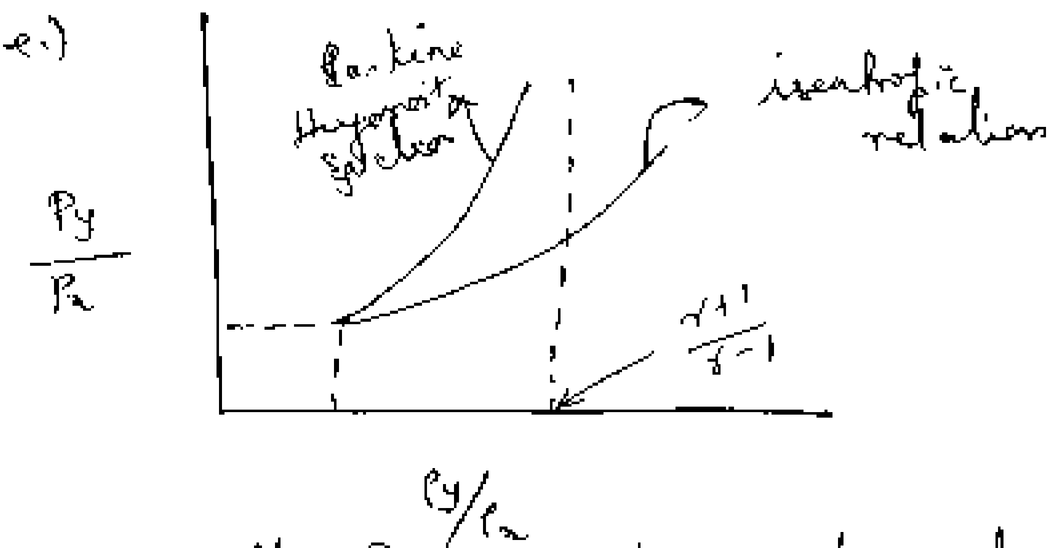
The Pressure & density relation isentropic

$$\frac{p_y}{p_x} = \left(\frac{\rho_y}{\rho_x}\right)^{\gamma}$$

density change across the shock is greater than the isentropic value

but N_x << low change ρ_y/ρ_x are negligible

(i.e.)



Comparison of the Rankine Hugoniot and isentropic relations

$$\frac{\rho_y}{\rho_x} \times \frac{T_x}{T_y} = \frac{\frac{p_y}{p_x} + \frac{\gamma-1}{\gamma+1}}{1 + \frac{\gamma-1}{\gamma+1} \cdot p_y/p_x} \qquad (11)$$

$$\frac{\rho_y}{\rho_x} = \frac{\frac{p_y}{p_x} + \frac{\gamma-1}{\gamma+1}}{1 + \frac{\gamma-1}{\gamma+1}\frac{p_y}{p_x}} = \frac{\frac{\gamma-1}{\gamma+1}\left[\frac{\gamma+1}{\gamma-1}\frac{p_y}{p_x} + 1\right]}{\frac{\gamma-1}{\gamma+1}\left[\frac{\gamma+1}{\gamma-1} + p_y/p_x\right]}$$

$$\frac{\rho_y}{\rho_x} = \frac{1 + \frac{\gamma+1}{\gamma-1}\frac{p_y}{p_x}}{\frac{p_y}{p_x} + \frac{\gamma+1}{\gamma-1}} \longrightarrow \text{Rankine Hugoniot Equation}$$

Also,

$$\rho_y/\rho_x \left[\frac{p_y}{p_x} + \frac{\gamma+1}{\gamma-1}\right] = \left[1 + \frac{\gamma+1}{\gamma-1}\frac{p_y}{p_x}\right]$$

$$\frac{\rho_y}{\rho_x} \times \frac{p_y}{p_x} + \frac{\gamma+1}{\gamma-1}\frac{\rho_y}{\rho_x} = 1 + \frac{\gamma+1}{\gamma-1}\frac{p_y}{p_x}$$

$$\frac{\rho_y}{\rho_x}\left[\frac{\gamma+1}{\gamma-1}\right] = 1 + \frac{\gamma+1}{\gamma-1}\frac{p_y}{p_x} - \frac{\rho_y}{\rho_x} \times \frac{p_y}{p_x}$$

$$\frac{\rho_y}{\rho_x}\left[\frac{\gamma+1}{\gamma-1}\right] = 1 + \frac{p_y}{p_x}\left[\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}\right]$$

$$\frac{\rho_y}{\rho_x}\left[\frac{\gamma+1}{\gamma-1}\right] - 1 = \frac{p_y}{p_x}\left[\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}\right]$$

$$= \frac{\left[\frac{\gamma+1}{\gamma-1}\frac{P_y}{P_x}\right]\left[1+\frac{(\gamma-1)(\gamma+1)}{4\gamma}\frac{P_y}{P_x}+\frac{(\gamma-1)^2}{4\gamma}\right]}{\frac{(\gamma+1)^3}{4\gamma(\gamma-1)}\left[\frac{P_y}{P_x}+\frac{\gamma-1}{\gamma+1}\right]}$$

Simplifying only Nr.

$$\frac{\gamma+1}{\gamma-1}\frac{P_y}{P_x}+\frac{(\gamma+1)^2}{4\gamma}\left(\frac{P_y}{P_x}\right)^2+\frac{(\gamma+1)(\gamma-1)}{4\gamma}\left(\frac{P_y}{P_x}\right)$$

$$\frac{(\gamma+1)^3}{4\gamma(\gamma-1)}\frac{P_y}{P_x}\left[1+\frac{\gamma-1}{\gamma+1}\frac{P_y}{P_x}\right]$$

$$= \frac{\frac{(\gamma+1)^3}{4\gamma(\gamma-1)}\frac{P_y}{P_x}\left[1+\frac{\gamma-1}{\gamma+1}\frac{P_y}{P_x}\right]}{\frac{(\gamma+1)^3}{4\gamma(\gamma-1)}\left[\frac{P_y}{P_x}+\frac{\gamma-1}{\gamma+1}\right]}$$

$$\frac{T_y}{T_x} = \frac{\frac{P_y}{P_x}\left[1+\frac{\gamma-1}{\gamma+1}\frac{P_y}{P_x}\right]}{\frac{P_y}{P_x}+\frac{\gamma-1}{\gamma+1}}$$

We know that

(15)

$$\frac{P_y}{P_x} = \frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1}$$

$$\frac{P_y}{P_x} + \frac{\gamma-1}{\gamma+1} = \frac{2\gamma}{\gamma+1} M_x^2$$

$$M_x^2 = \frac{\gamma+1}{2\gamma}\frac{P_y}{P_x} + \frac{\gamma-1}{2\gamma} \longrightarrow (5)$$

We know that

$$\frac{T_y}{T_x} = \frac{\left[\frac{2\gamma}{\gamma-1} M_x^2 - 1\right]\left[1 + \frac{\gamma-1}{2} M_x^2\right]}{\frac{(\gamma+1)^2}{2(\gamma-1)} M_x^2} \longrightarrow (6)$$

(5) in (6)

$$\frac{T_y}{T_x} = \frac{\left[\frac{2\gamma}{\gamma-1}\left(\frac{\gamma+1}{2\gamma}\frac{P_y}{P_x} + \frac{\gamma-1}{2\gamma}\right) - 1\right]\left[1 + \frac{\gamma-1}{2}\left(\frac{\gamma+1}{2\gamma}\frac{P_y}{P_x} + \frac{\gamma-1}{2\gamma}\right)\right]}{\frac{(\gamma+1)^2}{2(\gamma-1)}\left(\frac{\gamma+1}{2\gamma}\cdot\frac{P_y}{P_x} + \frac{\gamma-1}{2\gamma}\right)}$$

$$\frac{T_y}{T_x} = \frac{\left[\frac{\gamma+1}{\gamma-1}\frac{P_y}{P_x} + 1 - 1\right]\left[1 + \frac{(\gamma-1)(\gamma+1)}{4\gamma}\frac{P_y}{P_x} + \frac{(\gamma-1)^2}{4\gamma}\right]}{\frac{(\gamma+1)^3}{4\gamma(\gamma-1)}\left(\frac{P_y}{P_x}\right) + \frac{(\gamma+1)^2}{4\gamma}}$$

$$= \frac{\frac{2\gamma}{\gamma+1} M_x^2 \times \frac{1}{2}\frac{(\gamma+1)^2}{\gamma-1} M_x^2 - \frac{\gamma-1}{\gamma+1} \times \frac{1}{2}\frac{(\gamma+1)^2}{\gamma-1} M_x^2}{\left[1+\frac{\gamma-1}{2}M_x^2\right]\left[\frac{2\gamma}{\gamma-1}M_x^2-1\right]}$$

$$= \frac{2\gamma M_x^2 \times \frac{1}{2}\frac{\gamma+1}{\gamma-1} M_x^2 - \frac{\gamma+1}{2} M_x^2}{\left[1+\frac{\gamma-1}{2}M_x^2\right]\left[\frac{2\gamma}{\gamma-1}M_x^2-1\right]}$$

$$= \frac{\frac{\gamma+1}{2} M_x^2 \left[\frac{2\gamma}{\gamma-1}M_x^2-1\right]}{\left[1+\frac{\gamma-1}{2}M_x^2\right]\left[\frac{2\gamma}{\gamma-1}M_x^2-1\right]}$$

$$\rho_y/\rho_x = \frac{\frac{\gamma+1}{2} M_x^2}{1+\frac{\gamma-1}{2}M_x^2} \longrightarrow (3)$$

$A_x = A_y = A$

. The Equation of Continuity for constant flow rate through the shock

$$\dot{m} = \rho A C = \rho_x A_x C_x = \rho_y A_y C_y$$

$$\frac{\dot{m}}{A} = \rho_x C_x = \rho_y C_y = C$$

$$\frac{\rho_x}{\rho_y} = \frac{C_y}{C_x}$$

$$\frac{C_y}{C_x} = \frac{1+\frac{\gamma-1}{2}M_x^2}{\frac{\gamma+1}{2}M_x^2} \longrightarrow (4)$$

(5) in (2)

$$\Delta S = C_p \ln\left(\frac{P_{oy}}{P_{ox}}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)}$$

$$= -C_p \ln \frac{P_{oy}}{P_{ox}}$$

$$= -C_p\left(\frac{\gamma-1}{\gamma}\right) \ln \frac{P_{oy}}{P_{ox}} = R$$

$$-\frac{\Delta S}{R} = \ln \frac{P_{oy}}{P_{ox}}$$

$$\boxed{\frac{P_{oy}}{P_{ox}} = e^{-\Delta S / R}}$$

$$\frac{\Delta S}{R} = -\ln\left(\frac{\frac{\gamma+1}{2} M_n^2}{1+\frac{\gamma-1}{2} M_n^2}\right)^{\gamma/\gamma-1} \times \left(\frac{2\gamma}{\gamma+1} M_n^2 - \frac{\gamma-1}{\gamma+1}\right)^{-1/\gamma-1} \rightarrow (6)$$

Impossibility of shock in Subsonic flow

Equation (6) gives values of γ is plotted as shown in fig

Perfect gas applications have γ (1.0 to 1.67)

for the $\frac{\Delta S}{R}$ values Positive for $M_n >> 1$

for $M_n = 1$

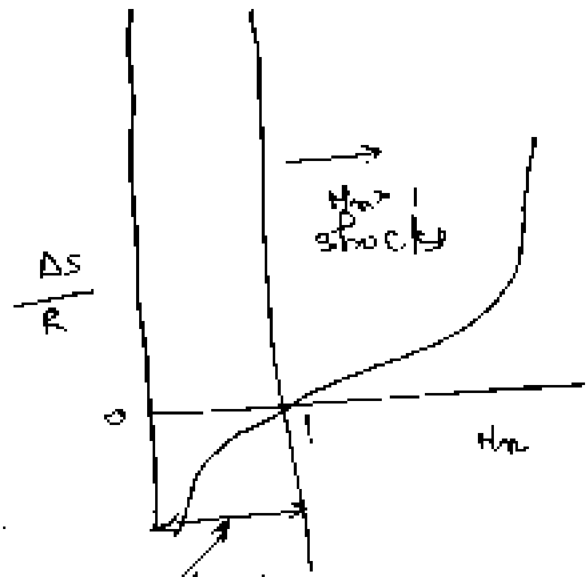
$\frac{\Delta S}{R} = 0$

if $M_n < 1$, Entropy decrease, by the second law of Thermodynamics is Impossible (violates)

∴ Shock wave cannot develop in a Subsonic flow

(i.e) $\frac{\Delta S}{R} << 1$ $P_r > 1$ (Increase in stagnation pressure)

(19)



Change of Entropy across a Normal shock wave

Strength of a shock wave (ξ)

The parameter which define the strength of a shock wave is often used in shock wave analysis.
This is given by

$$\xi = \frac{P_y - P_x}{P_x} = \frac{P_y}{P_x} - 1$$

It is defined as the ratio of difference of pressure in downstream and upstream shock $P_y - P_x$ to upstream shock pressure P_x

for weak shock $\xi \propto \left(\frac{P_y}{P_x} - 1\right)$

$$\xi = \frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1} - \frac{1}{1}$$

$$= \frac{1}{\gamma+1}\left[2\gamma M_x^2 - (\gamma-1) - (\gamma+1)\right]$$

$$= \frac{1}{\gamma+1}\left[2\gamma M_x^2 - \gamma + 1 - \gamma - 1\right]$$

$$= \frac{1}{\gamma+1}\left[2\gamma M_x^2 - 2\gamma\right]$$

$$\xi = \frac{2\gamma}{\gamma+1}\left[M_x^2 - 1\right]$$

for shocks for any strength

$$\boxed{\xi \propto M_x^2 - 1}$$

Shocks of vanishing strength

Shock waves for which $T_y = 0$ are referred to as shocks of vanishing strength

$$M_x \simeq 1$$

$$\rho_y/\rho_x \simeq 1 \;,\quad \frac{P_y}{P_x} \simeq 1,\quad T_y/T_x \simeq 1 \quad \frac{P_{oy}}{P_{ox}} = 1$$

$$\frac{\Delta s}{R} = 0$$

Strong shocks

Strong shocks are a result of very high values of the upstream Mach Number.

(i.e.) $\lim_{M_x \to \infty} M_y = \sqrt{\frac{\gamma-1}{2\gamma}}$

$$\lim_{M_x \to \infty} P_y/P_x = \infty$$

$$\lim_{M_x \to \infty} T_y/T_x \simeq \infty$$

$$\lim_{M_x \to \infty} \rho_y/\rho_x = \frac{\gamma+1}{\gamma-1}$$

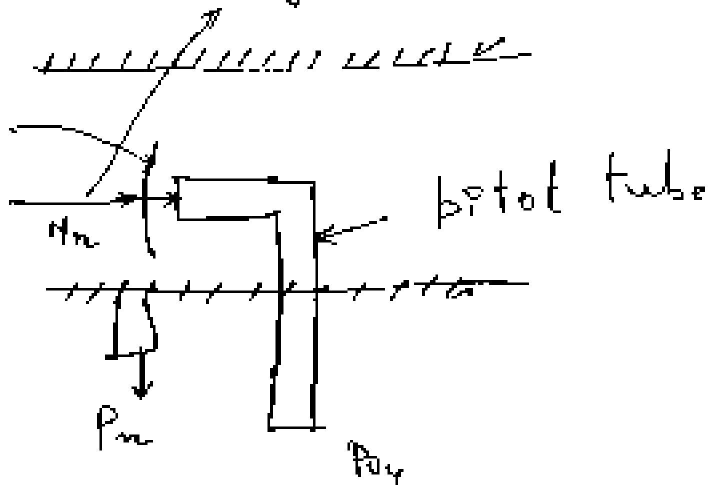
$$\lim_{M_x \to \infty} \frac{P_{oy}}{P_{ox}} = 0$$

Determination of Mach Number of Supersonic flows

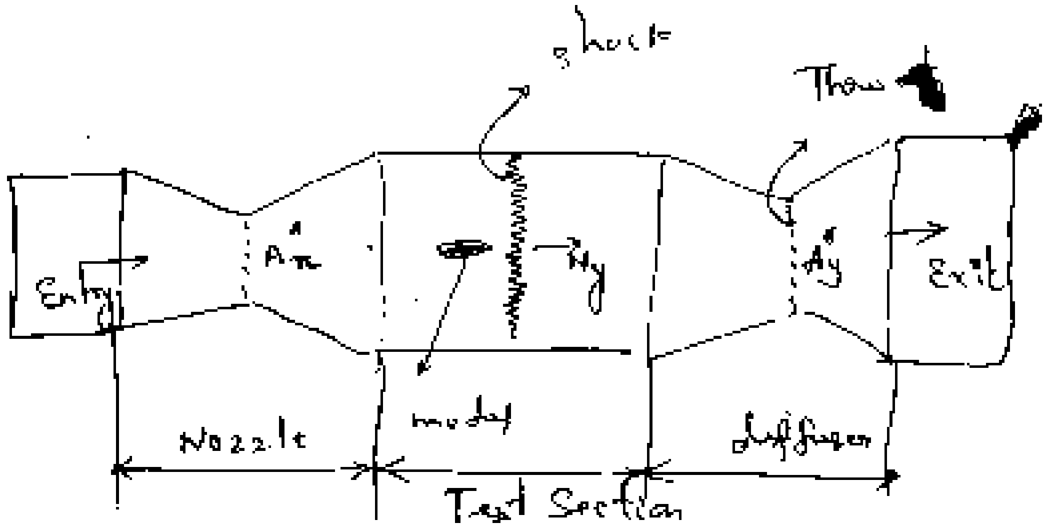
A pitot tube along with a wall tapping can be used to determine the Mach Number of a Supersonic ~~data~~ stream.

The introduction of the pitot tube produces a curved shock a little distance upstream of its mouth as shown in fig

Pitot tube measures the stagnation pressure ~~up~~ (P_{oy}) downstream of the shock wave.



Pitot tube in Supersonic flow



Supersonic wind tunnel with shocks in the test section.

Nozzle & diffuser throat areas

$$\frac{\dot{m}\sqrt{T_{on}}}{A_n^* P_{on}} = \frac{\dot{m}\sqrt{T_{oy}}}{A_y^* P_{oy}} = 0.0404.$$

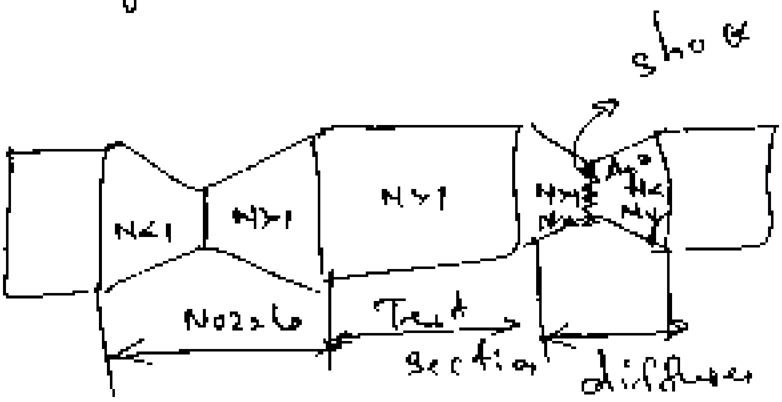
$\dot{m}$ = Constant.

$T_{on} = T_{oy} = T_o$ = Constant.

$A_n^* P_{on} = A_y^* P_{oy}$

$$\frac{A_y^*}{A_n^*} = \frac{P_{oy}}{P_{on}} > 1$$

∴ the diffuser throat area in presence of shock, always larger than the Nozzle throat area



Supersonic wind tunnel with shock at the diffuser throat

• $\frac{P_{oy}}{P_x}$ → derived under following assumption

(i) shock wave is normal at the stagnation streamline. [pitot tube is placed $\|^{el}$ to the flow]
shock wave is symmetrical on both sides

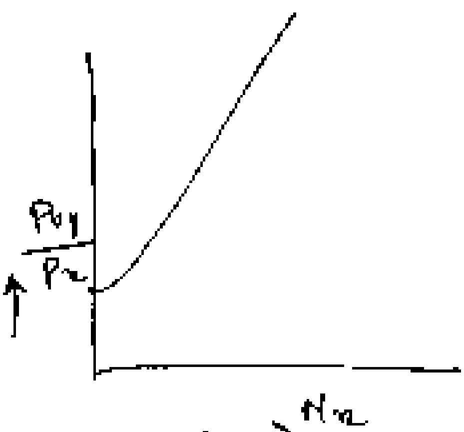
(ii) gas is brought to rest at the mouth of the tube isentropically

Now,

$$\frac{P_{oy}}{P_x} = \frac{P_{oy}}{P_y} \times \frac{P_y}{P_x} = \left(\frac{\frac{\gamma+1}{2} M_x^2}{\frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1}} \right)^{\gamma/\gamma-1} \times \left(\frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1} \right)$$

This on simplification

$$\frac{P_{oy}}{P_x} = \frac{\left(\frac{\gamma+1}{2} M_x^2 \right)^{\gamma/\gamma-1}}{\left(\frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1} \right)^{1/\gamma-1}}$$



Used only for supersonic flow

Mach numbers at pitot tube measurement

for subsonic flow

$$\frac{P_0}{P} = \left[1 + \frac{\gamma-1}{2} M^2 \right]^{\gamma/\gamma-1}$$

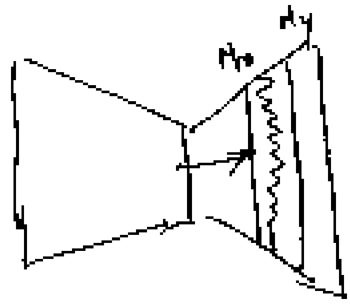
Supersonic wind tunnels

Normal shocks have application in supersonic wind tunnels where the diffusion of the supersonic flow after the test section takes place through a shock wave

This is shown in fig

A convergent divergent air nozzle has Exit to throat area ratio of 3. A Normal shock appears at the divergent section where the Existing area ratio of 2.2. find the Mach Number before and after the shock. if the inlet stagnation properties are 500 kPa, and 450 K. find the properties of air at Exit & Entropy increase across the shock.

(21)



given data

$\frac{A_2}{A_x^*} = 3$ $\frac{A_x}{A_x^*} = 2.2$

$P_0 = 500$ kPa, $T_0 = 450$ K

To find

P_2, T_2, C_2, M_2

ΔS

M_x and M_y

$\gamma = 1.4$ isentropic flow Table

@ $\frac{A_2}{A_x^*} = 2.2$

$M_x = 2.31$

$M_x < 1$ Neglected

$M_x = 0.29$

$\frac{T_x}{T_{0x}} = 0.484$, $\frac{P_x}{P_{0x}} = 0.0787$

$T_x = 217.8$ K

$P_x = 0.~~3935~~$ ~~bar~~ 39.35 kPa

Normal shock wave Table

$M_y = 0.533$ $\frac{P_{0y}}{P_{0x}} = 0.5785$

$P_{0y} = 0.5785 \times$ ~~0.5~~ 5

$P_{0y} = 2.89$ bar

$\frac{\Delta S}{R} = \ln \frac{P_{0x}}{P_{0y}}$

$\Delta S = 287 \ln \frac{5}{2.89}$

$\Delta S = 157.07$ J/kg.K

Isentropic flow Table

@ $M_y = 0.533$ $\quad \frac{A_y}{A_y^*} = 1.287$

$A_y^* = A_2^*$

$$\frac{A_2}{A_y^*} = \frac{A_2}{A_x^*} \times \frac{A_x^*}{A_x} \times \frac{A_y}{A_y^*}$$

$$= 3 \times \frac{1}{2.2} \times 1.287 = 1.755$$

$$\frac{A_2}{A_y^*} = 1.755$$

$M_2 = 0.36$ $\quad (M_2 < 1)$ Mach number should be low after shock

$$\frac{T_2}{T_{oy}} = 0.975 \qquad \frac{P_2}{P_{oy}} = 0.917$$

$T_{ox} = T_{oy}$

$T_2 = 0.975 \times 450$ $\quad P_2 = 0.917 \times 2.89$

$T_2 = 438.75 \, K$ $\quad P_2 = 2.65 \, bar$

$$M_2 = \frac{c_2}{a_2} = \frac{c_2}{\sqrt{\gamma R T_2}}$$

$$0.36 \times \sqrt{1.4 \times 287 \times 438.75} = c_2$$

$c_2 = 151.15 \, m/s$

A gas $\gamma = 1.3$ @ $P_1 = 345$ mbar, $T_1 = 350$ K, $M_1 = 1.5$, is to be isentropically expanded to 138 mbar. determine (a) deflection angle (b) final Mach Number (c) temperature of the gas

$\gamma = 1.3$
$P_1 = 345$ mbar
$T_1 = 350$ K
$M_1 = 1.5$
$P_2 = 138$ mbar

From isentropic flow Table
$\gamma = 1.3$, $M_1 = 1.5$

$$\frac{P_1}{P_{01}} = 0.284, \quad \frac{T_1}{T_{01}} = 0.748$$

$$\frac{P_2}{P_{02}} = \frac{P_2}{P_1} \times \frac{P_1}{P_{01}}$$

$$= \frac{138 \times 10^{-3}}{345 \times 10^{-3}} \times 0.284$$

$$\boxed{\frac{P_2}{P_{02}} = 0.1136}$$

$\gamma = 1.3$ isentropic flow

$M_2 = 2.08$

$$\frac{T_2}{T_{02}} = 0.606$$

$M_2 = 2.18$

$$T_2 = \frac{T_2}{T_{02}} \times \frac{T_{01}}{T_1}$$

$T_{01} = T_{02}$

$$\frac{T_2}{T_{02}} = 0.606$$

$$\boxed{T_2 = 283.56\,K}$$

Air approaches a symmetrical wedge ($\delta = 15°$) at a Mach Number of 2.0. Determine for the strong of weak waves (a) wave angle, (b) pressure ratio (c) density ratio (d) temp. ratio (e) downstream Mach Number. verify these values using gas Tables for normal shock

---

$$\tan\delta = 2\cot\alpha \frac{M_1^2 \sin^2\alpha - 1}{2 + \gamma M_1^2 + M_1^2(1 - 2\sin^2\alpha)}$$

$$\tan 15° = 2\cot\alpha \frac{2^2 \sin^2\alpha - 1}{2 + 1.4 \times (2)^2 + 2^2(1 - 2\sin^2\alpha)}$$

on right hand

$\alpha = 80°, 79, 80.3° \,\&\, 79.3°$, $0.132, 0.142, 0.128 \,\&\, 0.134$

$$\boxed{\alpha = 79.8°}$$

The angle being closer to 90° is obviously the angle of <u>strong shock wave</u>

<u>Using gas Tables</u>

$M_x = M_1 \sin \alpha$

$M_n = 2 \times \sin 79.8°$

$\boxed{M_n = 1.97}$

From Normal shock tables $\gamma = 1.4$, $M_x = 1.97$

$M_y = 0.582, \quad \frac{P_y}{P_x} = 4.35, \quad \frac{T_y}{T_x} = 1.66$

$$M_2 = \frac{M_y}{\sin(\alpha - \delta)} = \frac{0.582}{\sin(79.8° - 15°)}$$

$\boxed{M_2 = 0.644}$

$\frac{P_2}{P_1} = P_y / P_x = 4.35$

$\frac{T_2}{T_1} = T_y / T_x = 1.66$

$\frac{\rho_2}{\rho_1} = \frac{4.35}{1.66} = 2.62$

<u>weak shock wave</u> $\boxed{\alpha = 45.3}$

$M_n = M_1 \sin \alpha$

$M_n = 2 \sin 45.3$

$\boxed{M_n = 1.42}$

From Normal shock table $\gamma = 1.4$, $M_n = 1.42$

$M_y = 0.731, \quad \frac{P_y}{P_x} = 2.186, \quad \frac{T_y}{T_x} = 1.267$

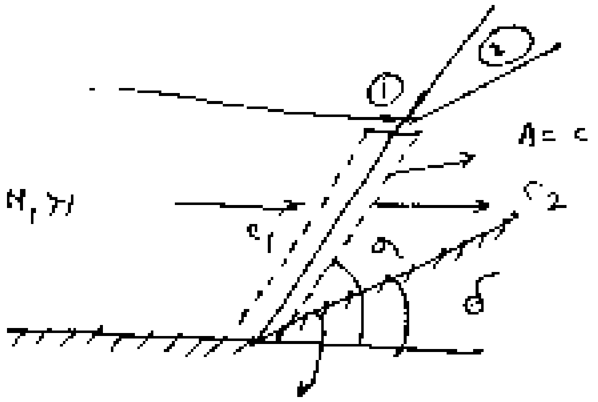
$$M_2 = \frac{M_y}{\sin(\alpha - \delta)} = \frac{0.731}{\sin 30.3} = 1.448$$

$$\frac{P_2}{P_1} = P_y/P_x = 2.186$$

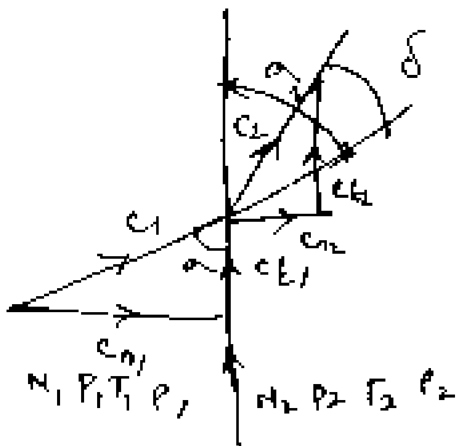
$$\frac{T_2}{T_1} = \frac{T_y}{T_x} = 1.267 \qquad \frac{\rho_2}{\rho_1} = \frac{2.186}{1.267} = 1.725$$

(10-CAQ-1)

Fundamental relations for oblique shock



flow through a oblique shock



velocity triangle upstream and downstream of an oblique shock

Continuity equation

$$\dot m = \rho_1 A C_{n_1} = \rho_2 A C_{n_2}$$

where A = constant Area ⊥ to the Normal velocity component is same on the two sides of the shock

(i.e.) $\dot m = \rho_1 C_{n_1} = \rho_2 C_{n_2}$

• Momentum Equation in the tangential direction

(25)

$$\dot{m} c_{t_1} = \dot{m} c_{t_2}$$

$$c_{t_1} = c_{t_2} = c_t$$

Momentum Equation in the normal direction

$$\dot{m} c_{n_1} - \dot{m} c_{n_2} = (P_2 - P_1) A$$

$$\rho_1 A c_{n_1}^2 - \rho_2 A c_{n_2}^2 = A (P_2 - P_1)$$

$$\rho_1 c_{n_1}^2 - \rho_2 c_{n_2}^2 = P_2 - P_1$$

Energy Equation gives

$$h_0 = h_{0_1} = h_{0_2} = \text{Constant}$$

$$h_1 + \frac{1}{2} c_1^2 = h_2 + \frac{1}{2} c_2^2$$

$$h_2 - h_1 = \frac{1}{2}(c_1^2 - c_2^2)$$

$$c_p (T_2 - T_1) = \frac{1}{2}(c_1^2 - c_2^2)$$

Velocity triangles upstream & downstream of the shock give

$$c_1^2 - c_2^2 = (c_{n_1}^2 + c_t^2) - (c_{n_2}^2 + c_t^2)$$

$$c_1^2 - c_2^2 = c_{n_1}^2 - c_{n_2}^2$$

Equation of state

$$h = c_p T = \frac{\gamma}{\gamma-1} R T = \frac{a^2}{\gamma-1} = \frac{\gamma}{\gamma-1}\frac{P}{\rho}$$

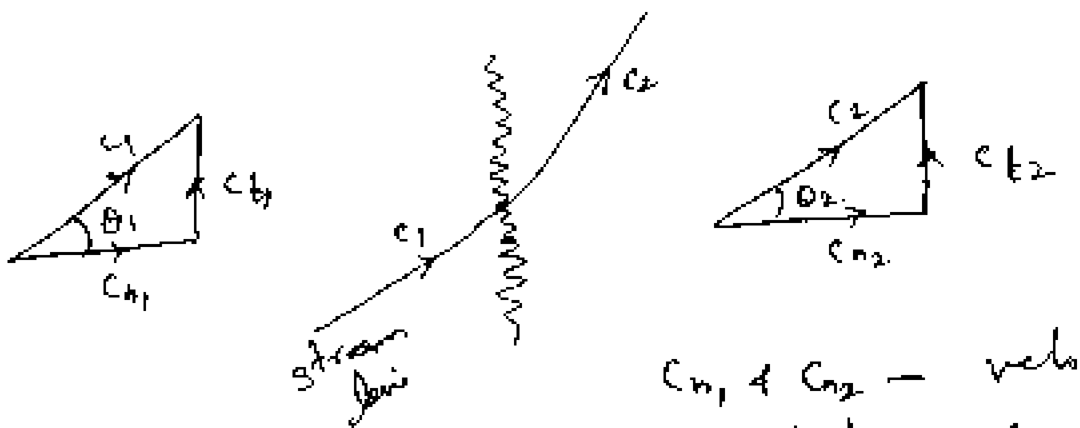
$$\frac{\gamma}{\gamma-1}\left(\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}\right) = \frac{1}{2}(c_{n_1}^2 - c_{n_2}^2)$$

$$h_0 = c_p T_0 = \frac{\gamma}{\gamma-1} R T_0 = \frac{a_0^2}{\gamma-1}$$

$$h_0 = \frac{a_0^2}{\gamma-1} = \frac{1}{2}\frac{\gamma+1}{\gamma-1} a^{*2}$$

$$h_0 = \frac{\gamma}{\gamma-1}\frac{p_0}{\rho_0} = \frac{a_0^2}{\gamma-1} = \frac{1}{2}\frac{\gamma+1}{\gamma-1}a^{*2}$$

Prandtl's Equation



C_{n_1} & C_{n_2} — velocities of upstream & downstream of a Normal shock wave

$$\tan\theta_1 = \frac{c_{t_1}}{C_{n_1}}$$

$$c_1^2 = C_{n_1}^2 + C_t^2$$
$$c_2^2 = C_{n_2}^2 + C_t^2$$

$C_{n_2} < C_{n_1}$

$C_2 < C_1$

$M_2 < 1$

$T_2 > T_1$

$$\tan\theta_2 = \frac{c_{t_2}}{C_{n_2}}$$

We know that

$$h = c_p T = \frac{\gamma}{\gamma-1} R T = \frac{a^2}{\gamma-1} = \frac{\gamma}{\gamma-1}\frac{p}{\rho} \rightarrow (1)$$

We know that

$$\frac{\gamma}{\gamma-1}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right) = \frac{1}{2}\left(C_{n_1}^2 - C_{n_2}^2\right)$$

$$C_t = c_{t_1} = c_{t_2}$$

$$= \frac{1}{2}\left(C_{n_1}^2 + c_{t_1}^2\right) - \left(C_{n_2}^2 + c_{t_2}^2\right) \rightarrow (2)$$

$$\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} + \frac{1}{2}\left(C_{n_1}^2 + c_{t_1}^2\right) = \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2} + \frac{1}{2}\left(C_{n_2}^2 + c_{t_2}^2\right) \rightarrow (3)$$

Using velocity triangle

$$\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} + \frac{1}{2}C_{n_1}^2 + c_t^2 = h_1 + \frac{1}{2}c_1^2 = h_0$$

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1}+\frac{1}{2}\left(c_{n_1}^2+c_{t_1}^2\right)=\frac{1}{2}\frac{\gamma+1}{\gamma-1}a^{*2} \quad (25)$$

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1}=\frac{1}{2}\frac{\gamma+1}{\gamma-1}a^{*2}-\frac{1}{2}\left(c_{n_1}^2+c_{t_1}^2\right)$$

$$P_1=\frac{\gamma-1}{2\gamma}\rho_1\left\{\frac{\gamma+1}{\gamma-1}a^{*2}-\left(c_{n_1}^2+c_{t_1}^2\right)\right\}$$

$$P_1=\rho_1\left\{\frac{\gamma+1}{2\gamma}a^{*2}-\frac{\gamma-1}{2\gamma}\left(c_{n_1}^2+c_{t_1}^2\right)\right\}$$

Similarly for

$$P_2=\rho_2\left\{\frac{\gamma+1}{2\gamma}a^{*2}-\frac{\gamma-1}{2\gamma}\left(c_{n_2}^2+c_{t_2}^2\right)\right\}$$

$$\rho_1 c_{n_1}^2-\rho_2 c_{n_2}^2=P_2-P_1$$

$$\rho_1 c_{n_1}^2-\rho_2 c_{n_2}^2=\rho_2\left\{\frac{\gamma+1}{2\gamma}a^{*2}-\frac{\gamma-1}{2\gamma}\left(c_{n_2}^2+c_{t_2}^2\right)\right\}-\rho_1\left\{\frac{\gamma+1}{2\gamma}a^{*2}-\frac{\gamma-1}{2\gamma}\left(c_{n_1}^2+c_{t_1}^2\right)\right\}$$

$$\boxed{c_{t_1}=c_{t_2}=c_t}$$

$$\rho_1 c_{n_1}^2-\rho_2 c_{n_2}^2=\rho_2\frac{\gamma+1}{2\gamma}a^{*2}-\frac{\gamma-1}{2\gamma}\rho_2\left(c_{n_2}^2+c_{t_2}^2\right)-\rho_1\frac{\gamma+1}{2\gamma}a^{*2}+\frac{\gamma-1}{2\gamma}\rho_1\left(c_{n_1}^2+c_{t_1}^2\right)$$

$$\rho_1 c_{n_1}^2-\rho_2 c_{n_2}^2=\rho_2\frac{\gamma+1}{2\gamma}a^{*2}-\frac{\gamma-1}{2\gamma}\rho_2 c_{n_2}^2-\frac{\gamma-1}{2\gamma}\rho_2 c_{t_2}^2-\rho_1\frac{\gamma+1}{2\gamma}a^{*2}+\frac{\gamma-1}{2\gamma}\rho_1 c_{n_1}^2+\frac{\gamma-1}{2\gamma}\rho_1 c_{t_1}^2$$

$$\rho_1 c_{n_1}^2-\frac{\gamma-1}{2\gamma}\rho_1 c_{n_1}^2-\rho_2 c_{n_2}^2+\frac{\gamma-1}{2\gamma}\rho_2 c_{n_2}^2+\frac{\gamma-1}{2\gamma}\rho_2 c_{t_2}^2-\frac{\gamma-1}{2}\rho_1 c_{t_1}^2=\rho_2\frac{\gamma+1}{2\gamma}a^{*2}-\rho_1\frac{\gamma+1}{2\gamma}a^{*2}=(\rho_2-\rho_1)\frac{\gamma+1}{2\gamma}a^{*2}$$

$$a^{*2} = \frac{\gamma-1}{\gamma+1} c_t^2 + \frac{\rho_1 C_{n_1}^2 - \rho_2 C_{n_2}^2}{\rho_2 - \rho_1}$$

$$a^{*2} - \frac{\gamma-1}{\gamma+1} c_t^2 = \frac{\rho_1 C_{n_1}^2 - \rho_2 C_{n_2}^2}{\rho_2 - \rho_1}$$

$$a^{*2} - \frac{\gamma-1}{\gamma+1} c_t^2 = \frac{C_{n_1} C_{n_2}}{\rho_2 - \rho_1}\left(\rho_1 \frac{C_{n_1}}{C_{n_2}} - \rho_2 \frac{C_{n_2}}{C_{n_1}}\right)$$

We know that

$$\frac{C_{n_1}}{C_{n_2}} = \frac{\rho_2}{\rho_1}$$

$$\boxed{a^{*2} - \frac{\gamma-1}{\gamma+1} c_t^2 = C_{n_1} C_{n_2}}$$ → Prandtl Meyer relation for oblique shock

if $c_t = 0$

C_n

Rankine - Hugoniot Equations for oblique shocks

We know that.

$$P_2 - P_1 = \rho_1 C_{n_1}^2 - \rho_2 C_{n_2}^2$$

$$P_2 - P_1 = \rho_1 C_{n_1}^2 \left(1 - \frac{\rho_2}{\rho_1}\frac{C_{n_2}^2}{C_{n_1}^2}\right)$$

$\boxed{\rho_1 C_{n_1} = \rho_2 C_{n_2}}$

$$P_2 - P_1 = \rho_1 C_{n_1}^2 \left(1 - \frac{\rho_1}{\rho_2}\right)$$

$$P_2 - P_1 = \rho_1 C_{n_1}^2 \left(\frac{\rho_2 - \rho_1}{\rho_2}\right)$$

$$P_2 - P_1 = \frac{\rho_1}{\rho_2} C_{n_1}^2 (\rho_2 - \rho_1)$$

$$C_{n_1}^2 = \frac{P_2 - P_1}{\rho_2 - \rho_1}\frac{\rho_2}{\rho_1}$$

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Similarly

$$C_{n_2}^2 = \frac{P_2 - P_1}{\rho_2 - \rho_1}\frac{\rho_1}{\rho_2}$$

We know that

$$\frac{\gamma}{\gamma-1}\left(\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}\right) = \frac{1}{2}\left(C_{n_1}^2 - C_{n_2}^2\right)$$

$$\frac{\gamma}{\gamma-1}\left(\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}\right) = \frac{1}{2}\left[\left(\frac{P_2 - P_1}{\rho_2 - \rho_1}\right)\times\left(\frac{\rho_2}{\rho_1}\right) - \left(\frac{P_2 - P_1}{\rho_2 - \rho_1}\right)\left(\frac{\rho_1}{\rho_2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{P_2 - P_1}{\rho_2 - \rho_1}\times 1\right.$$

$$= \frac{1}{2}\frac{P_2 - P_1}{\rho_2 - \rho_1}\left(\frac{\rho_2}{\rho_1} - \frac{\rho_1}{\rho_2}\right)$$

$$= \frac{1}{2}\frac{P_2 - P_1}{\rho_2 - \rho_1}\left(\frac{\rho_2^2 - \rho_1^2}{\rho_1\rho_2}\right)$$

$$= \frac{1}{2}\frac{P_2 - P_1}{\rho_2 - \rho_1}\left(\frac{\rho_2 + \rho_1}{\rho_1\rho_2}\times\frac{\rho_2 - \rho_1}{1}\right)$$

$$\frac{\gamma}{\gamma-1}\left(\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}\right) = \frac{1}{2}\frac{(P_2 - P_1)(\rho_2 + \rho_1)}{\rho_1\rho_2}$$

$$\frac{\gamma}{\gamma-1}\left(\frac{\rho_1 P_2 - P_1\rho_2}{\rho_1\rho_2}\right) = \frac{1}{2}\frac{(P_2 - P_1)(\rho_2 + P_1)}{\rho_1\rho_2}$$

$$\frac{2\gamma}{\gamma-1}\left(\rho_1 P_2 - P_1\rho_2\right) = (P_2 - P_1)(\rho_2 + \rho_1)$$

This on re-arrangement & Simplification

$$\frac{P_2}{P_1} = \frac{\frac{\gamma+1}{\gamma-1}\frac{\rho_2}{\rho_1} - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_2}{\rho_1}}.$$

again this on

$$\frac{\rho_2}{\rho_1}\left[\frac{\gamma+1}{\gamma-1} - \frac{p_2}{p_1}\right] = \frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1} - 1$$

Then on simpf

$$\frac{\rho_2}{\rho_1} = \frac{\frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1} + 1}{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}$$

Rankine-Hugoniot Equation for oblique shocks

Flow with oblique shock wave

when the direction of flow is inclined at an oblique angle to the shock wave it is known as oblique shock wave.

when the shock wave is inclined at an angle to the direction of flow, it is called oblique shock.

→ 2-dimensional plane shock wave

Flow through oblique shock

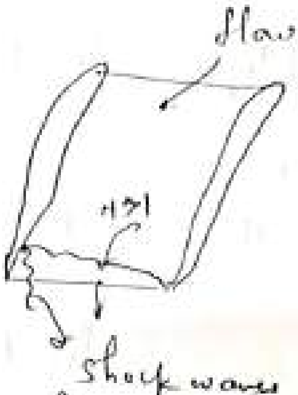
Consider the flow of gas over a wedge shaped object having an inclination (δ) to the direction of flow. A shock occurs of all in inclination of α to the direction of flow at the wedge. the wave angle (α) will be always greater than the deflection angle (δ).

when the wave angle α is Equal to 90°, then it is called normal shock.

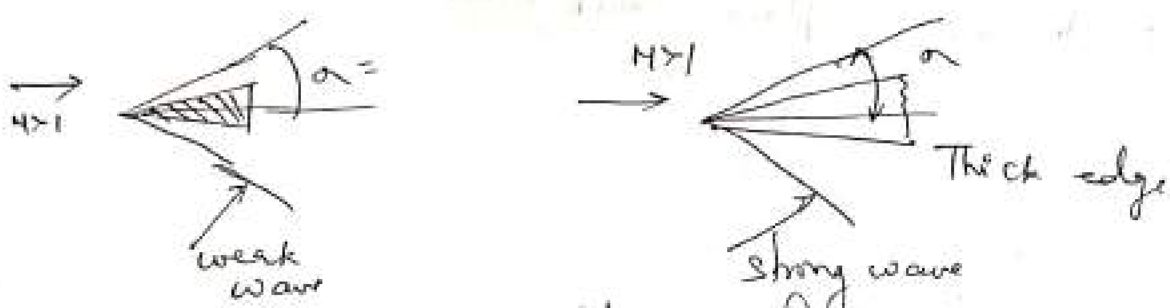
oblique shocks occur at the exit of the turbine blade passage across the supersonic flow

Assumptions

1. absence of body forces
2. the gas is perfect with constant specific heats
3. absence of External work
4. Steady adiabatic and friction less flow through a stationary oblique shock wave



Oblique shock wave in a turbine blade passage



Examples of oblique shock wave

A jet of air at a Mach Number of 2.5 is deflected inwards at the corner of a curved wall. the wave angle at the corner is 60°. Determine the deflection angle of the wall, pressure & temperature ratios & final Mach Numbers

$$\tan\delta = 2\cot\alpha \frac{M_1^2 \sin^2\alpha - 1}{2 + \gamma M_1^2 + M_1^2(1 - 2\sin^2\alpha)}$$

$$\tan\delta = 2\cot 60° \frac{(2.5)^2 \sin^2 60 - 1}{2 + 1.4(2.5)^2 + (2.5)^2(1 - 2\sin^2 60)}$$

$\tan\delta = 0.5584$

$\boxed{\delta = 29.18°}$

$M_2 = M_1 \sin\alpha = 2.5 \sin 60°$

$\boxed{M_2 = 2.165}$

From Normal shock Table $\gamma = 1.4$, $M_x = 2.165$

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$$\frac{P_y}{P_x} = \frac{P_2}{P_1} = 5.276, \quad \frac{T_y}{T_x} = \frac{T_2}{T_1} = 1.825$$

$M_y = 0.553$

$a - \delta = 60.00 - 29.18 = 30.82$

$$M_2 = \frac{M_y}{\sin(a-\delta)} = \frac{0.553}{\sin(30.82)} = \frac{0.553}{0.512}$$

$$\boxed{M_2 = 1.08}$$

A gas $\gamma = 1.3$ @ $P_1 = 345$ mbar, $T_1 = 350$ k, $M_1 = 1.5$, is to be isentropically expanded to 138 mbar. determine (a) deflection angle (b) final Mach Number (c) temperature of the gas

$\gamma = 1.3$

$P_1 = 345$ mbar

$T_1 = 350$ k

$M_1 = 1.5$

$P_2 = 138$ mbar

From isentropic flow Table

$\gamma = 1.3$, $M_1 = 1.5$

$$\frac{P_1}{P_{01}} = 0.284, \quad \frac{T_1}{T_{01}} = 0.748$$

$$\frac{P_2}{P_{02}} = \frac{P_2}{P_1} \times \frac{P_1}{P_{01}}$$

$$= \frac{138 \times 10^{-3}}{345 \times 10^{-3}} \times 0.284$$

$$\boxed{\frac{P_2}{P_{02}} = 0.1136}$$ — $M_2 = 2.08$ ($\gamma = 1.3$ isentropic flow)

$$\frac{T_2}{T_{02}} = 0.606$$

From tables for Prandl-Meyer function for $\gamma = 1.3$ at

$M_1 = 1.50 \quad \omega(M_1) = 12.693^\circ$

$M_2 = 2.08 \quad \omega(M_2) = 31.120^\circ$

deflection angle

$\delta = \omega(M_1) - \omega(M_2)$

$\delta = 12.693 - 31.120^\circ$

$\boxed{\delta = -18.427^\circ}$ clockwise

$\boxed{M_2 = 2.08}$

$T_2 = \frac{T_2}{T_{02}} \times \frac{T_{01}}{T_1}$ $\quad T_{01} = T_{02}$

$\frac{T_2}{T_{02}} = 0.606$

$\boxed{T_2 = 292.56k}$

Air approaches a symmetrical wedge ($\delta = 15^\circ$) at a Mach Number of 2.0. Determine for the strong & weak waves (a) wave angle, (b) pressure ratio (c) density ratio (d) temp. ratio (e) downstream Mach number. verify these values using gas tables for normal shock

---

$$\tan\delta = 2\cot\alpha \frac{M_1^2 \sin^2\alpha - 1}{2 + \gamma M_1^2 + M_1^2(1 - 2\sin^2\alpha)}$$

$$\tan 15^\circ = 2\cot\alpha \frac{2^2 \sin^2\alpha - 1}{2 + 1.4 \times (2)^2 + 2^2(1 - 2\sin^2\alpha)}$$

m right hand

$\alpha = 80^\circ, 79, 80.3^\circ \; \& \; 79.3^\circ$, 0.132, 0.142, 0.1284, 0.134

$\boxed{\alpha = 79.8^\circ}$

Scanned by CamScanner

The angle being closer to 90° is obviously the angle of strong shock wave

Using gas Tables

$M_x = M_1 \sin\alpha$

$M_x = 2 \times \sin 79.8^\circ$

$M_x = 1.97$

From Normal shock Table $\gamma = 1.4$, $M_x = 1.97$

$M_y = 0.582$, $\frac{P_y}{P_x} = 4.35$, $\frac{T_y}{T_x} = 1.66$

$M_2 = \frac{M_y}{\sin(\alpha - \delta)} = \frac{0.582}{\sin(79.8^\circ - 15^\circ)}$

$M_2 = 0.644$

$\frac{P_2}{P_1} = P_y/P_x = 4.35$,

$\frac{T_2}{T_1} = T_y/T_x = 1.66$

$\frac{\rho_2}{\rho_1} = \frac{4.35}{1.66} = 2.62$

Weak shock wave $\alpha = 45.3$

$M_x = M_1 \sin\alpha$

$M_x = 2 \sin 45.3$

$M_x = 1.42$

From Normal shock table $\gamma = 1.4$, $M_x = 1.42$

$M_y = 0.731$, $\frac{P_y}{P_x} = 2.186$, $\frac{T_y}{T_x} = 1.267$

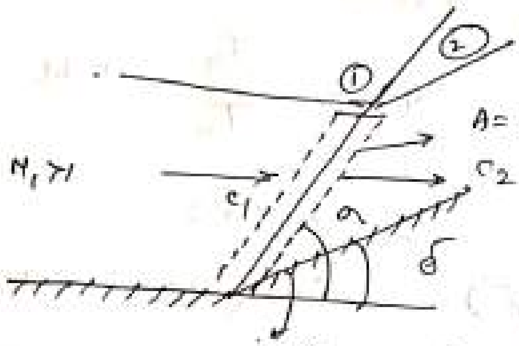
$M_2 = \frac{M_y}{\sin(\alpha - \delta)} = \frac{0.731}{\sin 30.3} = 1.448$

$$\frac{P_2}{P_1} = P_y/P_x = 2.186$$

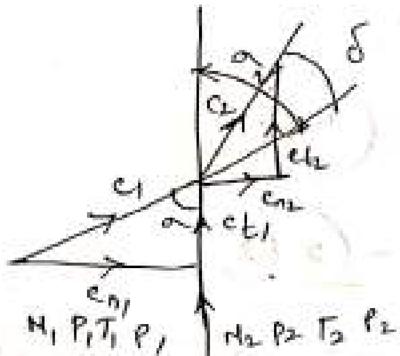
$$\frac{T_2}{T_1} = \frac{T_y}{T_x} = 1.267 \qquad \frac{\rho_2}{\rho_1} = \frac{2.186}{1.267} = 1.725$$

(10 - CAT-1)

Fundamental relations for oblique shock



flow through curved wall - oblique shock



velocity triangle upstream and downstream of an oblique shock

Continuity Equation

$$\dot{m} = \rho_1 A c_{n1} = \rho_2 A c_{n2}$$

A = constant Area ⊥ to the Normal velocity components is same on the two sides of the shock

(i.e.) $\dot{m} = \rho_1 c_{n1} = \rho_2 c_{n2}$

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Problems

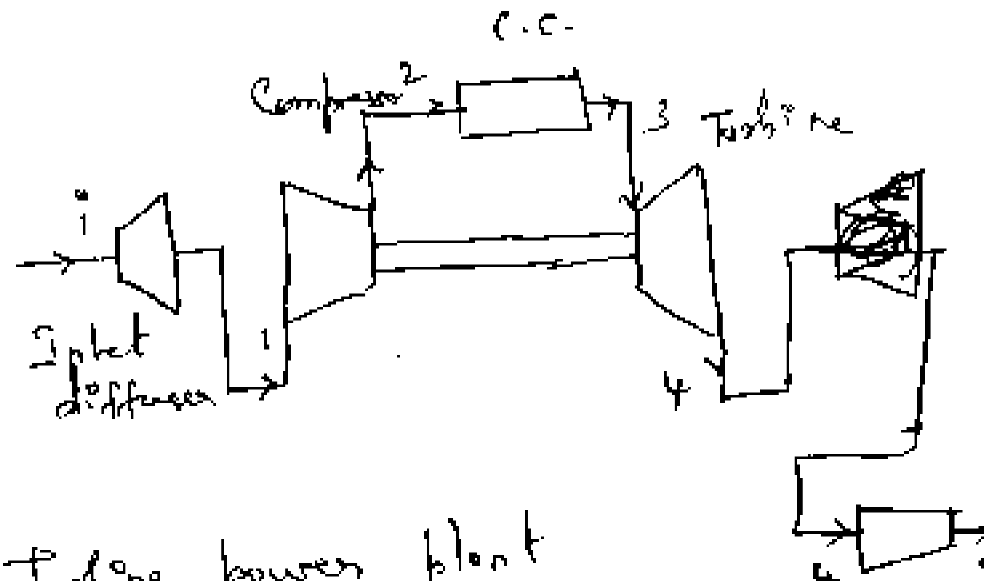
(1)

unit-IV

Theoretical relations are developed for various quantities affecting the design and performance of aircraft gas Turbine Engine and propulsion devices.

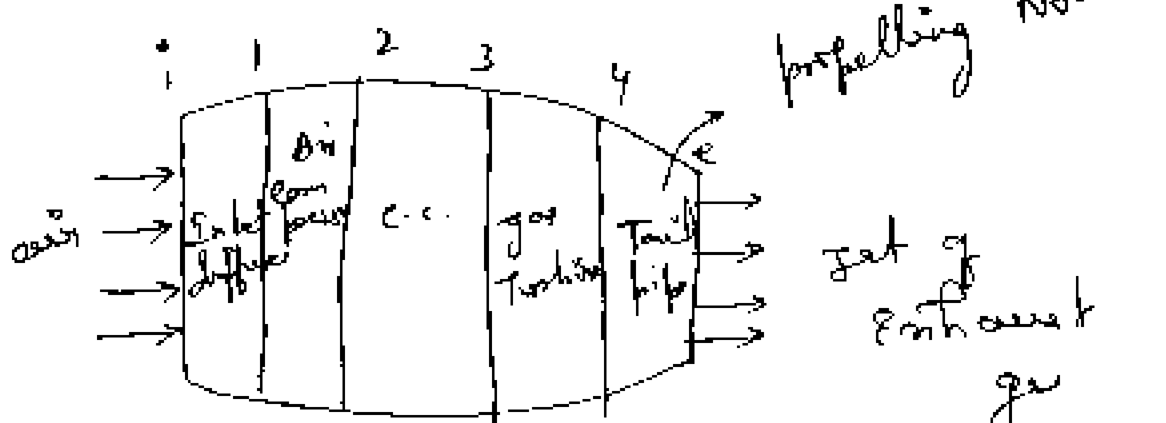
Thrust (F), Specific Thrust (F_s) or Impulse (I), aircraft to Jet speed ratio (σ), specific fuel Consumption based on Thrust (TSFC) and various Efficiency — cycle efficiency (η_j), Thermal efficiency (η_{th}), Propulsive efficiency (η_{prop}) and overall Efficiency (η_o)

Energy relations

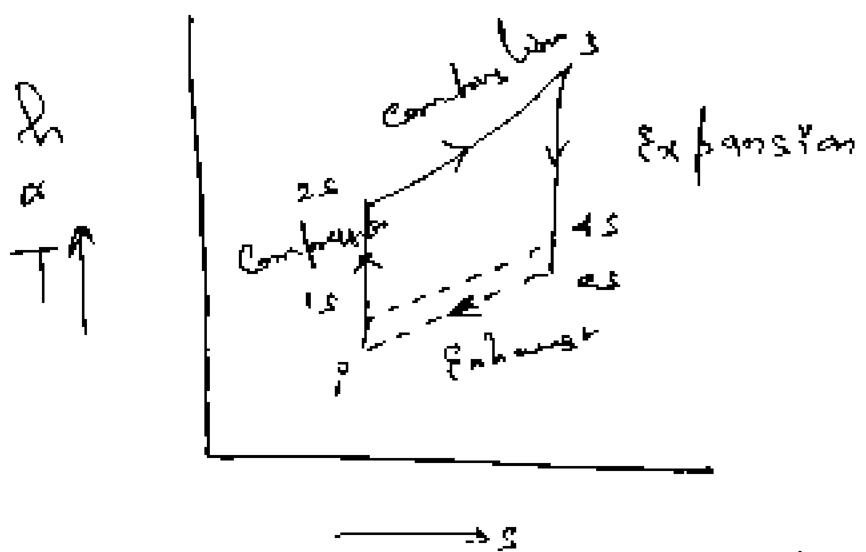


gas Turbine power plant for aircraft propulsion

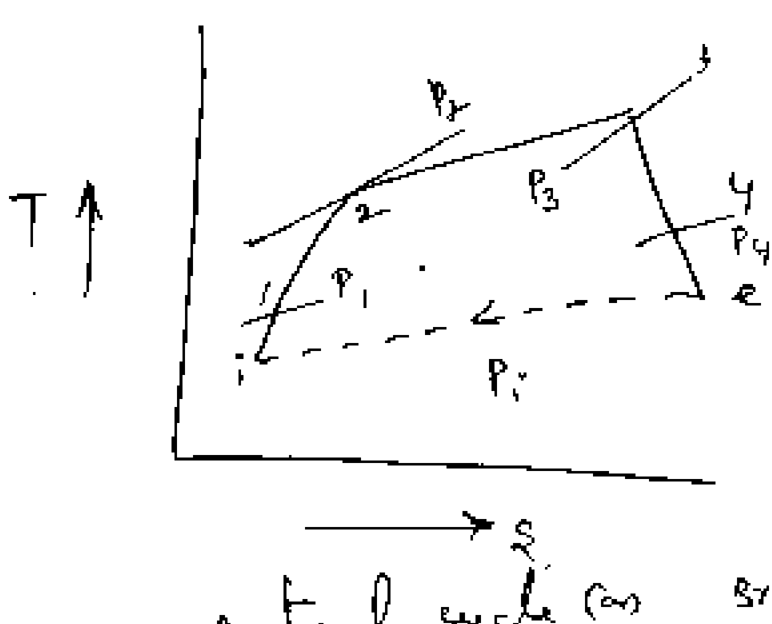
7



layout of the principal components in an aircraft gas Turbine Engine



Brayton cycle (Ideal)



Actual cycle (or) Brayton cycle.

~~Energy relations~~ (2)

Energy relations

Various processes occuring in turbojet Engines, have been depicted in fig. following relations are applicable for ideal cycle

Inlet diffuser

The state of air at the compressor is denoted by 1, instead of i

$$h_{0i} = h_{01}$$

$$h_i + \frac{1}{2} c_i^2 = h_1 + \frac{1}{2} c_1^2$$

$$T_i + \frac{c_i^2}{2c_p} = T_1 + \frac{1}{2}\frac{c_1^2}{c_p}$$

$$\frac{T_1}{T_i} = \left(\frac{P_1}{P_i}\right)^{\frac{\gamma-1}{\gamma}}, \qquad P_1/P_i = \left(\frac{T_1}{T_i}\right)^{\gamma/\gamma-1}$$

(P_1/P_i → pressure ratio)

Compressor

$$w_c = h_{02s} - h_{01} = c_p (T_{02s} - T_{01})$$

$$\text{pressure ratio} = \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1}$$

Combustion chamber

Heat supplied is given by

$$Q_s = Q_f = h_{03} - h_{02s} = c_p (T_{03} - T_{02s})$$

Turbine

$$W_T = h_{03} - h_{04s} = C_p (T_{03} - T_{04s})$$

pressure ratio $\frac{P_3}{P_4} = \left(\frac{T_3}{T_{4s}}\right)^{\gamma/\gamma-1}$

Exhaust Nozzle

$$h_{04s} = h_{0es}$$

$$h_{4s} + \frac{1}{2} C_{4s}^2 = h_{es} + \frac{1}{2} C_{es}^2$$

$$T_{4s} + \frac{C_{4s}^2}{2c_p} = T_{es} + \frac{C_{es}^2}{2c_p}$$

pressure ratio $\frac{P_4}{P_e} = \left(\frac{T_{4s}}{T_{es}}\right)^{\gamma/\gamma-1}$

Air standard Efficiency or Thermal Efficiency

$$\eta_j = \frac{W.D}{Q_s} = \frac{Q_s - Q_{rej}}{Q_s} = 1 - \frac{Q_{rej}}{Q_{sup}}$$

$$Q_s = c_p(T_3 - T_{2s})$$

$$Q_{rej} = C_p(T_{4s} - T_1)$$

$$\eta_j = 1 - \frac{c_p(T_{4s} - T_1)}{c_p(T_3 - T_{2s})} = 1 - \frac{T_{4s} - T_1}{T_3 - T_{2s}}.$$

The pressure ratio in the Compressor & Turbine are same

$$r = \frac{P_2}{P_1} = \frac{P_3}{P_4}$$

temp ratio $t = \frac{T_{2s}}{T_1} = \frac{T_3}{T_{4s}} = r^{(\gamma-1)/\gamma}$

$$\eta_j = 1 - \frac{1}{t} = 1 - \frac{1}{(r)^{\frac{\gamma-1}{\gamma}}}.$$

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(2)

Component Efficiencies

η efficiency of the inlet diffuser, compressor, combustion chambers, Turbine, exhaust (or) propelling nozzles will be separately considered.

Inlet diffuser (i-1)

Small pressure rise

$$\eta_D = \frac{\text{static pressure rise in the actual process}}{\text{static pressure rise in the isentropic process}}$$

$$\eta_D = \frac{P_1 - P_i}{P_{1ss} - P_i}$$

Considering Incompressible flow

$$P_1 - P_i = P_{01} - \frac{1}{2}\rho c_1^2 - P_{0i} + \frac{1}{2}\rho c_i^2$$

$$P_1 - P_i = \frac{1}{2}\rho\left(c_i^2 - c_1^2\right) - \left(P_{0i} - P_{01}\right)$$

→ (ΔP_0) stagnation pressure loss in diffuser.

for isentropic process $i - 1_{ss}$

$$P_{1ss} - P_i = \frac{1}{2}\rho\left(c_i^2 - c_1^2\right)$$

$$\eta_D = \frac{P_1 - P_i}{\frac{1}{2}\rho\left(c_i^2 - c_1^2\right)}$$

Large pressure rise

$$\eta_D' = \frac{\text{Enthalpy change in isentropic diffusion}}{\text{Enthalpy change in actual diffusion for the Same Exit pressure}}$$

$$\eta_D' = \frac{h_{1S} - h_i}{h_1 - h_i} = \frac{T_{1S} - T_i}{T_1 - T_i}$$

$C_{1S} > C_1$

$$\eta_D' = \frac{\frac{T_{1S}}{T_i} - 1}{\frac{T_1}{T_i} - 1} \qquad T_1 \simeq T_{01} \simeq T_{0i}$$

$$\frac{T_1}{T_i} - 1 = \frac{T_{0i}}{T_i} - 1 = \frac{\gamma - 1}{2} M_i^2$$

$$\boxed{\eta_D' = \frac{\left(\frac{P_1}{P_i}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\gamma - 1}{2} M_i^2}}$$

Compressor (23-3)

actual workdone by the Compressor on the air are

$$w_{ac} = h_{02} - h_{01} = c_p (T_{02} - T_{01})$$

work transfer in isentropic process

$$w_{sc} = h_{02s} - h_{01} = c_p (T_{02s} - T_{01})$$

$$\eta_c = \frac{w_{sc}}{w_{ac}} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

$r_{0c} = \frac{}{7}$ (4)

$$\eta_c = \frac{\left[(r_{0c})^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{02} - T_{01}}$$

Combustion chamber

$\dot{m}$ $\dot{m}_a$ $\dot{m}_f$

$\dot{m}$ $\dot{m}_a \left(1 + \frac{\dot{m}_f}{\dot{m}_a}\right) = \dot{m}_a (1 + f)$ → fuel air ratio

η_B = $\frac{\text{Increase in the Enthalpy of gases}}{\text{Energy supplied in the fuel}}$

↗ Efficiency of combustion or fuel burning

$$= \frac{(\dot{m}_a + \dot{m}_f) h_{03} - h_{02} \dot{m}_a}{\dot{m}_f \times CV} \qquad \frac{}{f \times CV}$$

Turbine

$W_{aT} = h_{03} - h_{04} = c_p (T_{03} - T_{04})$ (Turbine work in actual process)

$W_{sT} = h_{03} - h_{04s} = c_p (T_{03} - T_{04s})$ (work in isentropic process)

$$\eta_T = \frac{W_{aT}}{W_{sT}} = \frac{h_{03} - h_{04}}{h_{03} - h_{04s}}$$

$$= \frac{T_{03} - T_{04}}{T_{03} - T_{04s}}$$

stagnation pressure ratio

$P_{o4s} \simeq P_{o4}$ $\quad r_{oT} = \frac{P_1}{P_0}$

$$\eta_T = \frac{T_{o3} - T_{o4}}{T_{o3}\left[1 - \frac{1}{(r_{oT})^{\frac{\gamma-1}{\gamma}}}\right]}$$

Exhaust or propelling Nozzle

$$\eta_j = \frac{h_{o4} - h_e}{h_{o4} - h_{es}} = \frac{T_{o4} - T_e}{T_{o4} - T_{es}}$$

$$= \frac{T_{o4} - T_e}{T_{o4}\left[1 - \left(\frac{P_e}{P_{o4}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

at the velocity at the

$$\eta_j = \frac{T_4 - T_e}{T_4\left[1 - \left(\frac{P_e}{P_4}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Energy Equation gives

$$h_{o4} = h_{oe} = h_e + \frac{1}{2} c_e^2$$

$$\frac{1}{2} c_e^2 = h_{o4} - h_e = \eta_j (h_{o4} - h_{es})$$

$$c_e^2 = 2 \eta_j c_p (T_{o4} - T_{es})$$

$$c_e^2 = 2 \eta_j c_p T_{o4} \left(1 - \frac{T_{es}}{T_{o4}}\right)$$

$$= 2 \eta_j c_p T_{o4} \left[1 - \left(\frac{P_e}{P_{o4}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$r_{0c} = \frac{P_{02}}{P_{01}}$$

$$\eta_c = \frac{T_{01}\left[(r_{0c})^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{02} - T_{01}}$$

Combustion chamber

$$\dot{m} = \dot{m}_a + \dot{m}_f$$

$$\dot{m} = \dot{m}_a\left(1 + \frac{\dot{m}_f}{\dot{m}_a}\right) = \dot{m}_a(1+f)$$ → fuel air ratio

$$\eta_B = \frac{\text{Increase in the Enthalpy of gases}}{\text{Energy supplied in the fuel}}$$

Efficiency of combustion or fuel burning

$$= \frac{(\dot{m}_a + \dot{m}_f)h_{03} - h_{02}\dot{m}_a}{\dot{m}_f \times CV}$$

$$\eta_B = \frac{h_{03} - h_{02}}{f \times CV}$$

Turbine

$$W_{aT} = h_{03} - h_{04} = c_p(T_{03} - T_{04})$$ (Turbine work in actual process)

$$W_{sT} = h_{03} - h_{04s} = c_p(T_{03} - T_{04s})$$ (work in isentropic process)

$$\eta_T = \frac{W_{aT}}{W_{sT}} = \frac{h_{03} - h_{04}}{h_{03} - h_{04s}}$$

$$= \frac{T_{03} - T_{04}}{T_{03} - T_{04s}}$$

stagnation pressure ratio

$$p_{04s} \simeq p_{04} \quad r_{0T} = \frac{p_{03}}{p_{04}}$$

$$\eta_T = \frac{T_{03} - T_{04}}{T_{03}\left[1 - \frac{1}{(r_{0T})^{\frac{\gamma-1}{\gamma}}}\right]}$$

Exhaust or propelling Nozzle

$$\eta_j = \frac{h_{04} - h_e}{h_{04} - h_{es}} = \frac{T_{04} - T_e}{T_{04} - T_{es}}$$

$$= \frac{T_{04} - T_e}{T_{04}\left[1 - \left(\frac{p_e}{p_{04}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

if the velocity at the Nozzle entry is small $T_{04} \simeq T_4$

$$\eta_j = \frac{T_4 - T_e}{T_4\left[1 - \left(\frac{p_e}{p_4}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Energy Equation gives

$$h_{04} = h_{0e} = h_e + \frac{1}{2} c_e^2$$

$$\frac{1}{2} c_e^2 = h_{04} - h_e = \eta_j (h_{04} - h_{es})$$

$$c_e^2 = 2 \eta_j c_p (T_{04} - T_{es})$$

$$c_e^2 = 2 \eta_j c_p T_{04} \left(1 - \frac{T_{es}}{T_{04}}\right)$$

$$= 2 \eta_j c_p T_{04} \left[1 - \left(\frac{p_e}{p_{04}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

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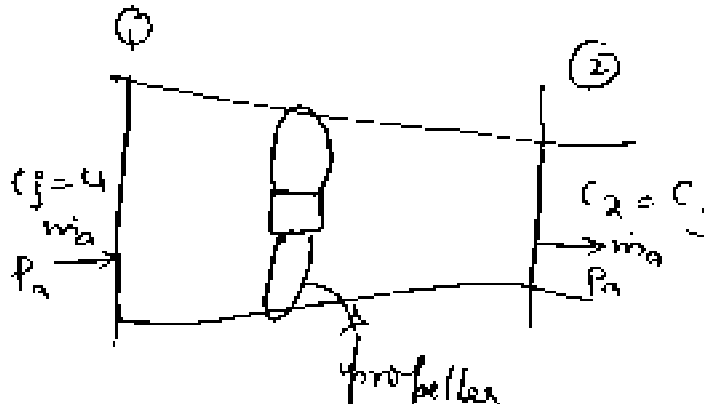
Thrust

The force which propel the aircraft at a given speed is called Thrust or propulsive force.

Propeller Thrust



flow through aircraft propeller

$$F = \dot{m}_a (C_2 - C_1)$$

$C_1 = u$ = flight speed

$C_2 = C_j$ = jet velocity or slip stream velocity

$$F = \dot{m}_a (C_j - u)$$

$$= \dot{m}_a u \left[\frac{C_j}{u} - 1 \right]$$

$$F = \dot{m}_a u \left[\frac{1}{\alpha} - 1 \right] \quad (\text{or})$$

$$F = \dot{m}_a C_j (1 - \alpha)$$

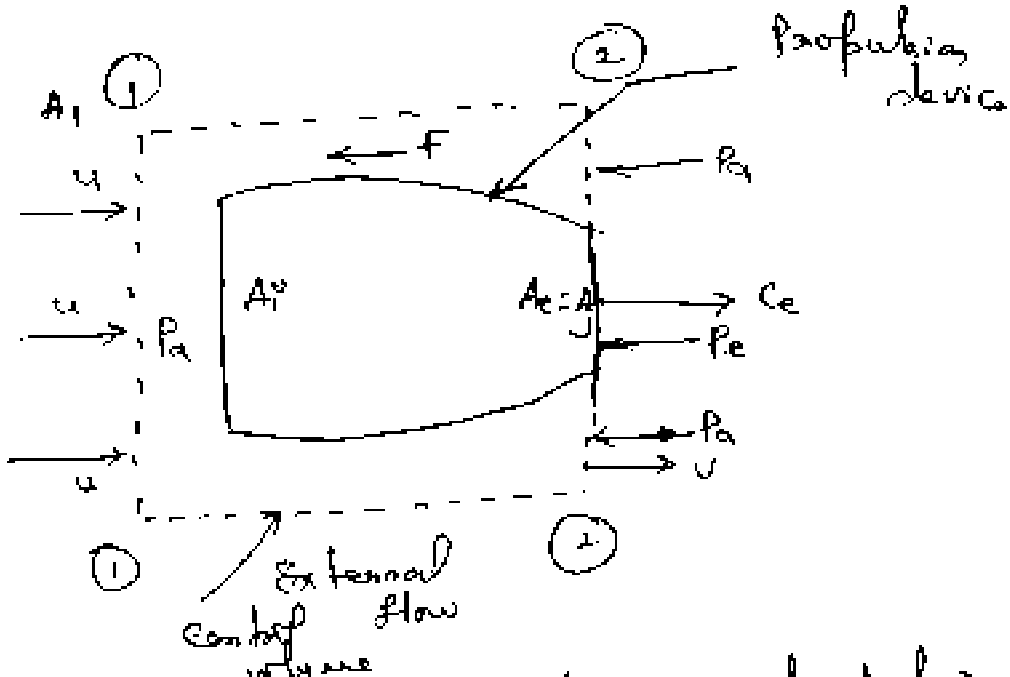
$\alpha = \frac{u}{C_j}$ = α (effective speed ratio or flight to jet velocity ratio)

$$\frac{F}{\dot{m}_a u} = \frac{1}{\alpha} - 1$$

$$\frac{F}{\dot{m}_a C_j} = 1 - \alpha$$

} non-dimensional value of Thrust

Jet Thrust



Internal & External flows for a propulsive device

For External & Internal flow for a turbojet Engine (propulsive device) occurring through an Imaginary Control surface with very large c/s. A_1 & A_2 at Entry & Exit

$P_a \rightarrow$ ambient air
u – velocity of air

} hot gases leave the Engine at pressure P and high velocity C_e

Flow rate of the Engine inlet $\dot{m}_a$ (kg/s) & Engine Exit ($A_e = A_j$) $\rightarrow$ is $(\dot{m}_a + \dot{m}_f)$ (kg/s)

Thrust on the Engine (aircraft) is exerted on account of changes in the momentum flux as well as pressure.

Net Thrust = momentum thrust + pressure thrust

$$F = F_{mom} + F_{pr}$$

for steady flow parallel to the direction of flight gives (6)

$$F_{mom} = \int_{A_e} \rho c_e^2 dA + \int_{A_2 - A_e} \rho u^2 dA - \int_{A_i} \rho u^2 dA$$

$$- \int_{A_1 - A_i} \rho u^2 dA$$

External flow does not exp. any change in the momentum flux

$$\int_{A_2 - A_e} \rho u^2 dA - \int_{A_1 - A_i} \rho u^2 dA = 0$$

$$F_{mom} = \int_{A_e} \rho c_e^2 dA - \int_{A_i} \rho u^2 dA$$

$$F_{mom} = \rho_e c_e^2 A_e - \rho_i u^2 A_i \longrightarrow (1)$$

for 1-dimensional flow at station 1 & 2

From ~~Exp.~~ Continuity Equation

$$\dot m_a = \rho_i u A_i \longrightarrow (2)$$

$$\dot m = \dot m_a + \dot m_f = \rho_e c_e A_e \longrightarrow (3)$$

$$F_{mom} = (\dot m_a + \dot m_f) c_e - \dot m_a u$$

$\dot m_f$ << compared to $\dot m_a$

$$F_{mom} = \dot m_a (c_e - u) \longrightarrow (4)$$

Pressure force on the control surface for the External flow outside the C/S (A_i is zero)

$$F_{pr} = P_e A_e + P_a (A_i - A_e) - P_a A_i$$

$F_{pr} = (P_e - P_a) A_e$

$f = m_a (C_e - u) + (P_e - P_a) A_e$

$f = m_a (C_e - u) + (P_e - P_a) A_e$ $A_e \geq A_i$

$f = \dot{m}_a (C_j - u)$

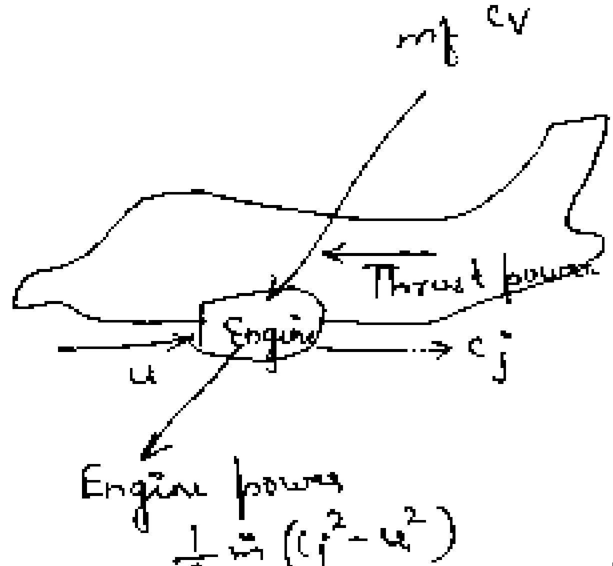
If $P_e = P_a$
$C_e = C_j$

Complete combustion.

Propulsive, Thermal and overall Efficiencies

Performance of aircraft can be judged by various Efficiencies.

Performance of Engine - rated in terms of Energy Conversion or Thermal η



Power utilization in aircraft propulsion

The power Input to the Engine or the propulsion system $\Sigma m_f \times CV$

The Engine output is the increase in the rate of Kinetic Energy of gases (i.e.) $\frac{1}{2} \dot{m} (C_j^2 - u^2)$

out come of the propulsion system (i.e.) Thrust power = $f \times u$.

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For convenience the flow rate through the Engine is taken as $\dot{m} \simeq m_a$ (kg/s)

Propulsive Efficiency

$$\eta_{prop} = \frac{\text{Propulsive power or Thrust power}}{\text{power output of the Engine}}$$

$$= \frac{\dot{m}(c_j - u) \times u}{\frac{1}{2}\dot{m}(c_j^2 - u^2)} = \frac{2(c_j - u) \times u}{(c_j + u)(c_j - u)}$$

$$= \frac{2u}{c_j + u}$$

$$= \frac{2}{\frac{c_j}{u} + 1}$$

$$\boxed{\eta_{prop} = \frac{2}{\frac{1}{a} + 1} = \frac{2a}{1+a}}$$

$$\boxed{a = \frac{c_j}{u}}$$

Thermal Efficiency

$$\eta_{ther} = \frac{\text{Power output of the Engine}}{\text{power input to the Engine through fuel}}$$

$$= \frac{\frac{1}{2}\dot{m}(c_j^2 - u^2)}{m_f \times CV}$$

Overall Efficiency

$$\eta_o = \frac{\text{propulsive power}}{\text{Power Input to the Engine through fuel}}$$

$$= \frac{\dot{m}(c_j - u) \times u}{\dot{m}_f \times CV} \rightarrow (2)$$

$$\boxed{\eta_o = \eta_{prop} \times \eta_{ther}}$$

Specific fuel Consumption

$$TSFC = \frac{\dot{m}_f}{F} \rightarrow (1)$$

fuel consumption rate per unit Thrust.

(1) in (2)

$$\boxed{\eta_o = \frac{u}{TSFC \times CV}}$$

specific Thrust

$$F_s = \frac{F}{\dot{m}}$$

Thrust produced per unit flow rate through the propulsion devices

for $TSFC = f/F_s$.

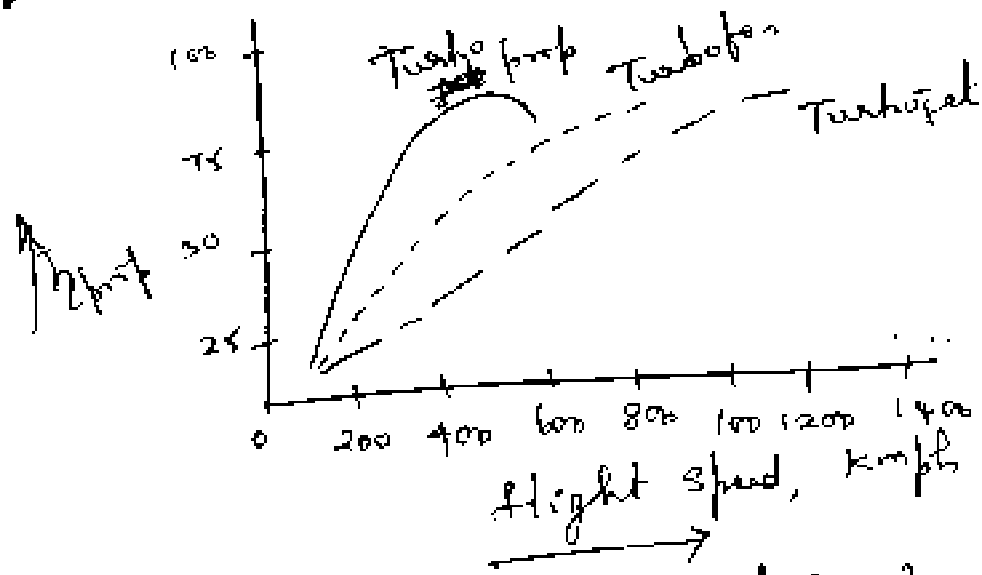
Specific Impulse

$$I_s = \frac{F}{\dot{w}} = \frac{\dot{m}(c_j - u)}{\dot{m} \times g} = \frac{c_j - u}{g} = \frac{u\left(\frac{c_j}{u} - 1\right)}{g}$$

$$= \frac{u}{g}\left[\frac{1}{\sigma} - 1\right]$$

I_s: Thrust produced per unit weight flow rate through the propulsive device

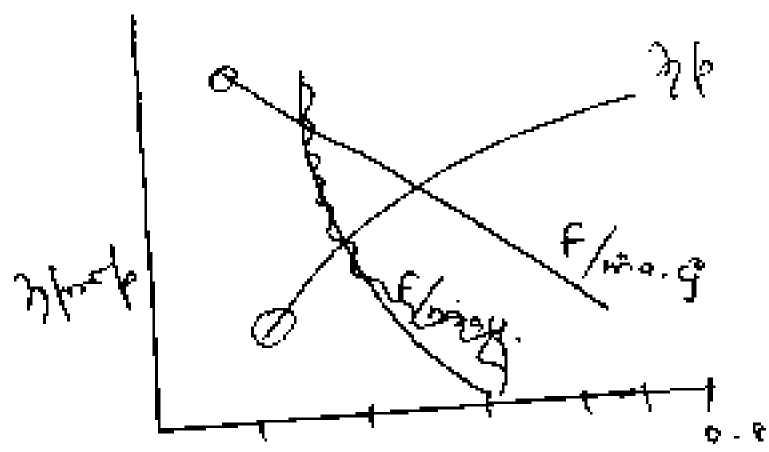
I_s (seconds) unit



Comparision of propulsive η of various propulsive devices.

Turboprop system is preferable in the lower range of speed.

performance of Turbofan & Turbojet Engines Continue to Improve with the speed.



$a = \frac{u}{c_j}$

lower the value of a, η prop are poor and Thrust is high

(1) The diameter of the propeller of an aircraft is 2.5 m, it flies at a speed of 500 kmph at an altitude of 8000 m. for a flight to jet speed ratio of 0.75. determine (a) the flow rate of air through the propeller (b) thrust produced (c) specific thrust (d) specific Impulse (e) Thrust power

---

$d = 2.5\,m$

$u = 500\ kmph = \frac{500 \times 1000}{60 \times 60} = 138.89\,m/s$

$A = \frac{\pi}{4} d^2$

$A = \frac{\pi}{4}(2.5)^2 = 4.908\,m^2.$

@ $z = 8000\,m.$

$\rho = 0.525\ kg/m^3.$

$\sigma = \frac{u}{c_j} = 0.75 = \frac{138.89}{c_j}$

$c_j = 185.18\,m/s$

velocity of air flow at the propeller disc is

$c = \frac{1}{2}(u + c_j) = \frac{1}{2}(138.89 + 185.18)$

$c = 162.035\,m/s$

Theoretical value of the flow rate is given by

$\dot{m}_a = \rho A c$

$= 0.525 \times 4.908 \times 162.035$

$\dot{m}_a = 417.516\ kg/s$

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Thrust

$F = \dot{m}_a (C_j - u)$

$= 417.516 (185.18 - 138.89)$

$F = 19.3268\ kN$

specific Thrust

$F_s = \frac{F}{\dot{m}_a} = \frac{19.3268 \times 10^3}{417.516} = 46.29\ N/(kg/s)$

specific Impulse (I_s)

$I_s = \frac{F}{\dot{w}_a} = \frac{F}{\dot{m}_a \times g} = \frac{19.3268 \times 10^3}{417.516 \times 9.81} = 4.718\ sec$

Thrust power (P) = $F \times u$

$= 19.3268 \times 10^3 \times 138.89$

$= 2684.3\ kW$

(2) An aircraft flies at 960 kmph one of its turbojet Engine takes in 40 kg/s of air & Expands the gases to the ambient pressure. the air-fuel ratio is 50 and lower calorific value is 43 MJ/kg. for maximum thrust power determine (1) jet velocity (2) Thrust (3) specific Thrust (4) Thrust power (5) η_{prop}, η_{ther} & η_o & TSFC

given data

$\dot{m}_a = 40$ kg/s

$u = \frac{960 \times 1000}{60 \times 60} = 266.7$ m/s $\qquad \frac{m_a}{m_f} = 50$

for maximum thrust power $a = \frac{1}{2} = \frac{u}{c_j}$

$0.5 = \frac{u}{c_j}$

$a = \frac{u}{c_j} = 0.5 = \frac{266.7}{c_j}$

$c_j = 533.4$ m/s

$\dot{m} = \dot{m}_a + \dot{m}_f = \dot{m}_a\left(1 + \frac{m_f}{m_a}\right) = 40\left(1 + \frac{1}{50}\right)$

$\dot{m} = 40.8$ kg/s

$F = (\dot{m}_a + \dot{m}_f)\, c_j - \dot{m}_a \times u$

$= 40.8 \times 533.4 - 40 \times 266.7$

$F = 11.094$ kN

specific Thrust $(F_s) = \frac{F}{\dot{m}_a} = \frac{11.094 \times 10^3}{40}$

$= 277.35$ N/(kg/s)

Thrust power

$P = F \times u = 11.094 \times 10^3 \times 266.7$

$P = 2958.77$ kW

$\eta_{prop} = \frac{2a}{1+a} = \frac{2 \times 0.5}{1 + 0.5} = 0.66 = 66\%$

$\eta_{ther} = \frac{\frac{1}{2}\dot{m}(c_j^2 - u^2)}{m_f \times CV} = \frac{\frac{1}{2} \times 40.8\left((533.4)^2 - (266.7)^2\right)}{0.8 \times 43 \times 10^6}$

$= 0.1265 \times 100 = 12.65\%$

$\eta_0 = \eta_{th} \times \eta_{prop}$

$= 0.1265 \times 0.666 \times 100$

$\boxed{\eta_0 = 8.42\%}$

$TSFC = \frac{\dot{m}_f}{F} = \frac{0.8 \times 3600}{11094}$

$= 0.2596\ kg/kN$

(3) Calculate the air flow through the Engine, Exit, cross-section area of the propelling Nozzle, Thrust, Thrust power, η_{prop} & η_0 for a turbojet Engine from the following data.

flight Mach Number = 0.85,

flight altitude = 12,000 m

$A_1 = 0.5 m^2$, $A/F = 60$, $\frac{m_a}{m_f} = 60$

Conditions of at the Exit of the Exhaust jet

$P_e = 477$ mbar, $T_e = 1000 K$

velocity $c_e = 660 m/s$ C.V = 43 MJ/kg

$M_1 = 0.85$ $M_1 = \frac{c_1}{a_1} = \frac{c_1}{\sqrt{\gamma R T_1}}$

@ $z = 12,000 m$,

$T_1 = 216.65 K$, $a_1 = 295.2 m/s$ $P_1 = 0.193$ bar, $\rho_1 = 0.311$

$0.85 = \frac{c_1}{\sqrt{1.4 \times 287 \times 216.65}}$

$u = c_1$ $\boxed{c_1 = 250.78 m/s}$

$\dot{m}_a = \rho_1 A_1 c_1$

$= 0.311 \times 0.5 \times 250.78$

$\boxed{\dot{m}_a = 38.99 kg/s}$

$F = m_a (c_j - u) + (P_e - P_a) A_e$

$c_j = 660\, m/s$

$F = \dot{m}_a (c_j - u) + (P_e - P_a) A_e$

$= 38.99 (660 - 250.78) + (0.477 - 0.193) \times 10^5 \times 0.355$

$= 15955.4 + 10,082$

$\frac{\dot{m}_a}{m_f} = 60$

$F = 26.037\, kN$

$\frac{38.99}{60} = m_f$

$m_f = 0.649$

Isentropic flow table

@ $M_i = 0.8$, $\gamma = 1.4$

$\frac{T_i}{T_0} = 0.87$, $\frac{P_1}{P_{01}} = 0.62$, $\frac{A_1}{A^*} = 1.041$

$P_{01} = 0.311\, bar$

$P_{01} = P_{02}$

$A_1^* = 0.4891\, m^2$

$\frac{P_e}{P_{01}} = \frac{477 \times 10^{-3}\, bar}{0.311\, bar} = 1.53$

~~$T_e = 1000\, K$~~

$\dot{m}_a = \rho_e A_e c_e$

$38.99 = \frac{P_e}{R T_e} A_e c_e$

$38.99 = \frac{477 \times 10^{-3} \times 10^5}{287 \times 1000} \times A_e \times 660$

$38.99 = 0.166 \times 660$

$A_e = 0.355\, m^2$

$\alpha = \frac{u}{c_j} = \frac{250.78}{660}$

$\alpha = 0.377$

$\eta_{prop} = \frac{2\alpha}{1+\alpha} = \frac{2 \times 0.377}{1 + 0.377}$

$\eta_{prop} = 55.18\%$

Thrust power $= F \times u = 26.037 \times 250.78 = 6529.5\, kW$

$\eta_0 = \frac{\dot{m}(c_j - u) \times u}{m_f \times CV} = \frac{(m_a + m_f)(c_j - u) \times u}{m_f \times CV}$

$= \frac{(38.99 + 0.649) \times (660 - 250.78) \times 250.78}{0.649 \times 43 \times 10^6}$

3.6×10^{-3}

$= 23.92\%$

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A turboprop Engine operates at an altitude of 3000m above mean sea level and an aircraft speed of 525 kmph, the data for the Engine is given below:

Inlet diffuser $\eta = 0.875$, velocity of air at compressor
Compressor $\eta = 0.790$, Temp. rise through the Compressor $= 230°C$.
Entry $= 90$ m/s,
properties of air $\gamma = 1.4$, $C_p = 1005$ J/kg.K.
From the above data calculate (a) pressure rise through the inlet diffuser (b) pressure ratio developed by the compressor (c) power required by the compressor per unit flow rate of air & (d) air standard η of the Engine.

given data

$Z = 3000m$, $u_i = \frac{525 \times 1000}{60 \times 60} = 145.833$ m/s

$\eta_{diff} = 0.875$

$\eta_{comp} = 0.790$ Temp rise $= 230°C$

$C_1 = 90$ m/s

@ $Z = 3000$ m

$T_i = 268.65$ K, $P_i = 0.701$ bar, $\rho_i = 0.909$ kg/m³

$a_i = 328.7$ m/s

$M_i = \frac{u_i}{a_i} = \frac{145.833}{328.7} = 0.4436$

$\frac{T_{oi}}{T_i} = 1 + \frac{\gamma - 1}{2} M_i^2 = 1 + \frac{1.4-1}{2}(0.4436)^2$

$T_{oi} = 1.039 \times 268.65 =$ $\boxed{279.44\text{ K} = T_{oi}}$

$$T_{o_1} = T_1 + \frac{c_1^2}{2c_p}$$

$$279.244 = T_1 + \frac{(90)^2}{2 \times 1005}$$

$$\boxed{T_1 = 275.194\,K}$$

For the inlet diffuser

$$\eta_D = \frac{T_{1s} - T_i}{T_1 - T_i}$$

$$T_{1s} - T_i = \eta_D (T_1 - T_i)$$

$$T_i\left(\frac{T_{1s}}{T_i} - 1\right) = \eta_D T_i \left(\frac{T_1}{T_i} - 1\right)$$

$$\frac{T_{1s}}{T_i} = 1 + \eta_D\left(\frac{T_1}{T_i} - 1\right)$$

~~$$\left(\frac{P_{1s}}{P_i}\right)^{\frac{\gamma-1}{\gamma}} = 1 + \eta_D \left(\frac{P_1}{P_i}\right)^{\frac{\gamma-1}{\gamma}} - 1$$~~

$$\left(\frac{P_{1s}}{P_i}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_1}{P_i}\right)^{\frac{\gamma-1}{\gamma}} = 1 + 0.875\left(\frac{275.194}{268.65} - 1\right)$$

$$\frac{P_1}{P_i} = 1.0766, \quad \boxed{P_1 = 0.7547\,\text{bar}}$$

(a) Pressure rise through the inlet diffuser

$$P_1 - P_i = 0.7547 - 0.7010$$
$$= 0.0537\,\text{bar}$$

(12)

Compressor η

$$\eta_{com} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

$$\eta_{com}\left(T_{02} - T_{01}\right) = T_{01}\left(\frac{T_{02s}}{T_{01}} - 1\right)$$

$$\eta_{com}\left(T_{02} - T_{01}\right) = T_{01}\left[\left(r_{0c}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\frac{0.79 \times 230}{277.244} = \left(r_{0c}\right)^{\frac{1.4-1}{1.4}} - 1$$

$$r_{0c} = \frac{P_{02}}{P_{01}} = 5.779 //$$

Power required by the Compressor is

$$= m_a C_p \left(T_{02} - T_{01}\right)$$

$$= 1 \times 1005 \; (230)$$

$$= 231.15 \text{ kW}/\text{kg/s}$$

$$\eta_j = 1 - \frac{1}{\left(r_{0c}\right)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(5.779)^{0.286}}$$

$$\boxed{\eta_j = 39.42\,\%}$$

Ramjet Engine

Ideal η

Various process occuring in ramjet Engine can also be represented by an open circuit Brayton cycle.
The cycle is considered under following assumptions

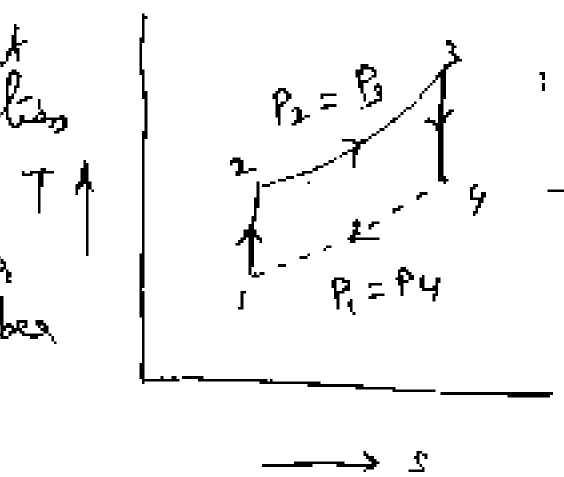
(i) steady 1-dimensional flow

(ii) isentropic Compression & Expansion $\Delta S = 0$, $\Delta P_0 = 0$

(iii) perfect gas

(iv) Constant pressure heat addition in the Combustion chamber $P_2 = P_3$

(v) very low Mach Number in the Combustion chamber



Ideal Brayton cycle for ramjet Engine.

$P_2 \simeq P_{02} \simeq P_{03}$

$T_2 \simeq T_{02} \simeq T_{01}$

$$\eta_j = 1 - \frac{1}{t}$$

$$= 1 - \frac{1}{(r_c)^{\frac{\gamma-1}{\gamma}}}$$

Temp. ratio

$$t = \frac{T_2}{T_1} = \frac{T_{02}}{T_1} = \frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2$$

$$\eta_j = 1 - \frac{1}{1 + \frac{\gamma-1}{2} M_1^2} = \frac{\frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_1^2}$$

$$\boxed{\eta_j = \frac{1}{1 + \frac{2}{\gamma-1} \frac{1}{M_1^2}} = f(M_1)}$$

(13)

$\dot{m} = m_a + m_f$

$= m_f \left[\frac{m_a}{m_f} + 1 \right] = 0.35 \left[53 + 1 \right] = 18.91 \text{ kg/s}$

$\dot{m} = \rho A c \times n$

$18.9 = 0.168 \times \frac{\pi}{4} (0.25)^2 \times c_j \times 2$

$\boxed{c_j = 1145.91 \text{ m/s}}$

absolute velocity of jet $c_a = c_j - u$

$= 1145.91 - 300$

$\boxed{c_a = 845.91 \text{ m/s}}$

Thrust or Resistance

$f = \dot{m} c_j - m_a u$

$= 18.9 \times 1145.91 - 18.55 \times 300$

$f = 16.09 \text{ kN}$

$\eta_0 = \frac{\dot{m}(c_j - u) \times u}{m_f \times CV} = \frac{18.9 (1145.91 - 300) \times 300}{0.35 \times 49 \times 10^6}$

$\boxed{\eta_0 = 27.9\%}$

$\eta_{th} = \frac{\frac{1}{2} \dot{m} (c_j^2 - u^2)}{m_f \times CV} = \frac{\frac{1}{2} 18.9 \left[(1145.91)^2 - (300)^2 \right]}{0.39 \times 49 \times 10^6}$

$\boxed{\eta_{th} = 67.3\%}$

A Turbojet Engine propels an aircraft at a Mach Number of 0.8 in level flight at an altitude of 10 km. The data for the Engine is given below

stagnation temp. at the turbine inlet = 1200 K,
stagnation temp. rise through the Compressor = 175 K,
C.V. of fuel = 43×10^6 J/kg, Compressor η = 0.75,
Combustion chamber η = 0.975, Turbine η = 0.81,
Mechanical η of the power transmission between turbine & Compressor = 0.98,
Exhaust Nozzle η = 0.97,
I_s = 25 seconds
Assuming the Same properties for air & Combustion gases calculate,
(a) fuel-air ratio, (b) Compressor pressure ratio
(c). turbine pressure ratio (d) Exhaust Nozzle pressure ratio
(d) Mach Number of Exhaust Jet

given data

$M_1 = 0.8$ $\quad Z = 10,000$ m from gas table
$\rightarrow T_i = 233.15$ K, $a_i = 299.6$ m/s

$$\frac{T_{0i}}{T_i} = 1 + \frac{\gamma-1}{2} M_1^2$$

$T_{01} = T_{0i}$

$$T_{0i} = \left(1 + \frac{1.4-1}{2}(0.8)^2\right) \times 233.15$$

$$T_{01} = 251.71\ \text{K}$$

$$T_{02} - T_{01} = 175$$

$$T_{02} = 175 + T_{01} = 175 + 251.7$$

$$T_{02} = 426.71\ \text{K}$$

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for the Combustion chamber

$$\eta_B = \frac{(\dot m_a + \dot m_f) C_p T_{o2} - m_a C_p T_{o2}}{\dot m_f \times CV} \quad \div \dot m_f$$

$$\eta_B = \frac{\left(\frac{1}{f} + 1\right) C_p T_{o2} - \frac{1}{f} T_{o2}}{CV}$$

$$0.975 = \frac{\left(\frac{1}{f} + 1\right) 1005 \times 1200 - \frac{1}{f} 426.71}{43 \times 10^6}$$

$$f = 0.01908$$

for the Compressor

$$\eta_c = \frac{T_{o1}\left[r_{o_c}^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{o2} - T_{o1}}$$

$$0.75 = \frac{251.71 \left[(r_{o_c})^{\frac{1.4-1}{1.4}} - 1\right]}{426.71 - 251.71}$$

$$r_{o_c} = 4.344$$

Compressor power required = power supplied by the turbine

$$\eta_m = \frac{\dot{m}_a C_p (T_{02} - T_{01})}{(\dot{m}_a + \dot{m}_f) C_p (T_{03} - T_{04})}$$

$$\eta_m (\dot{m}_a + \dot{m}_f) C_p (T_{03} - T_{04}) = \dot{m}_a C_p (T_{02} - T_{01})$$

$$\eta_m (1 + f)(T_{03} - T_{04}) = (T_{02} - T_{01})$$

$$0.98 (1 + 0.01908)(T_{03} - T_{04}) = 426.71 - 251.71$$

$$\boxed{T_{03} - T_{04} = 175.228\ K}$$

(d) Turbine η

$$\eta_T = \frac{T_{03} - T_{04}}{T_{03}\left[1 - \frac{1}{(r_{0T})^{\frac{\gamma-1}{\gamma}}}\right]}$$

$$\eta_T T_{03} \left[1 - \frac{1}{(r_{0T})^{\frac{\gamma-1}{\gamma}}}\right] = T_{03} - T_{04}$$

$$0.81 \times 1200 \left[1 - \frac{1}{(r_{0T})^{\frac{1.4-1}{1.4}}}\right] = 175.228$$

$$\boxed{r_{0T} = 2.005}$$

Specific Impulse

$$I_s = \frac{u}{g}\left(\frac{1}{\sigma} - 1\right)$$

$$M_i = \frac{u_i}{a_i}$$

$$u_i = \sqrt{\gamma R T_i} \times M_i$$

$$= \sqrt{1.4 \times 287 \times 233.15} \times 0.8$$

$$\boxed{u_i = 239.68\ m/s}$$

$$0.4942 = \sigma = \frac{u}{c_e} = \frac{239.68}{c_e}$$

$$\boxed{C_e = 484.93\ m/s}$$

$$\eta_B = \frac{\dot{m}_a C_p (T_{03} - T_{02})}{m_f \times CV}$$

$$0.98 = \frac{1005 (1600 - 356.54)}{f \times 40 \times 10^6}$$

$$\boxed{f = 0.03188}$$

$$P_{03} = P_{02} - 0.02 P_{02} = 0.98 P_{02}$$

$$P_{03} = 0.98 \times 1.446 = 1.417 \text{ bar}$$

Nozzle pressure ratio $r_j = \frac{P_{03}}{P_4} = \frac{1.417}{0.440} = 3.22$

$\boxed{M_{4S} = 1.41}$ from gas table

$$\frac{T_{04}}{T_{4S}} = 1 + \frac{\gamma - 1}{2} M_{4S}^2 = 1 + \frac{1.4 - 1}{2} \times (1.41)^2$$

$$= 1.3976$$

$$T_{4S} = \frac{1600}{1.3976} = 1144.82 \text{ K}$$

$$T_{04} - T_4 = \eta_j (T_{04} - T_{4S})$$

$$1600 - T_4 = 0.96 (1600 - 1144.82)$$

$$\boxed{T_4 = 1163.027 \text{ K}}$$

$$T_{04} = T_4 + \frac{C_4^2}{2 C_p}$$

$$1600 = 1163.027 + \frac{C_4^2}{2 \times 1005}$$

$$\boxed{C_4 = 937.185 \text{ m/s}}$$

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(c) Area of c/s of the diffuser inlet

$$A_1 = \frac{\pi}{4}(0.5)^2 = 0.1963\,m^2$$

$$\dot{m}_a = \rho_1 u A_1$$
$$= 0.624 \times 471.48 \times 0.1963$$

$$\boxed{\dot{m}_a = 57.752\ kg/s}$$

(d) for Negligible velocity $P_{o2} = P_{02}$

$$\eta_D = \frac{c_p(T_{02s} - T_1)}{\frac{1}{2}c_1^2} = \frac{(r_D)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\gamma-1}{2}M_1^2}$$

$$0.9 \times \frac{1.4-1}{2}(1.5)^2 = (r_D)^{\frac{1.4-1}{1.4}} - 1$$

$$\boxed{r_D = 3.2875}$$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2}M_1^2 = 1 + \frac{1.4-1}{2} \times (1.5)^2$$

$$\frac{T_{01}}{T_1} = 1.45$$

$$T_{01} = 1.45 \times 245.90$$

$$\boxed{T_{02} = T_{01} = 356.55\,K}$$

$$a_4 = \sqrt{\gamma R T_4} = \sqrt{1.4 \times 287 \times 1163.027}$$

$$\boxed{a_4 = 683.596 \, m/s}$$

$$M_4 = \frac{c_4}{a_4} = \frac{937.185}{683.596} = 0.502$$

(17)

$$\eta_{prop} = \frac{2\alpha}{1+\alpha} = \frac{2 \times 0.503}{1+0.503} \times 100 = 66.93\,\%$$

$$\frac{\dot m_a}{\dot m_f} = 43$$

$$\boxed{\dot m_f = 1.841 \, kg/s}$$

$$\dot m = \dot m_a + \dot m_f = 57.752 + 1.841$$

$$\boxed{\dot m = 59.593 \, kg/s}$$

$$F = \dot m c_4 - \dot m_a u = 59.593 \times 937.185 - 57.752 \times 471.75$$

$$\boxed{F = 28.614 \, kN}$$

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Extra problem

A Turbojet Engine flying at a speed of 960 kmph Consumes air at a rate of 54.5 kg/s

Calculate

(a) Exit velocity of jet when the change in enthalpy for the Nozzle is 200 kJ/kg and velocity coefficient 0.97

(b) A/F is 75:1, fuel rate in kg/s

(c) TSFC

(d) Thermal η, when combustion η = 93 % CV = 45,000 kJ/kg

(e) η_{prop} (f) Propulsive power (g) η_0.

given data

$u = 960$ kmph $= \frac{960 \times 1000}{60 \times 60} = 266.67$ m/s

$\dot{m}_a = 54.5$ kg/s $\Delta H = 200$ kJ/kg

$C_v = 0.97$ A/F = 75 $\eta_{cc} = 93\%$

$CV = 45 \times 10^6$ J/kg.

$\frac{m_a}{m_f} = 75$, $m_f = \frac{54.5}{75} = 0.726$ kg/s

$C_j = C_v \sqrt{2\Delta H} = 0.97 \sqrt{2 \times 200 \times 10^3}$

$C_j = 613$ m/s

$F = m_a [C_j - u] = 54.5 [613 - 266.67] = 18.901$ kN

$TSFC = \frac{m_F}{F} = \frac{0.726}{18.901 \times 10^3} \times 3600 = 0.1381$ kg/N hr

$$\text{Thermal } \eta = \frac{\text{Propulsive power}}{Q_s} = \frac{\frac{1}{2}\dot{m}\left[V_j^2 - V_a^2\right]}{m_f \times CV \times \eta_{cc}}$$

$$= \frac{18.901 \times 266.67}{0.7267 \times 0.93 \times 45{,}000 \times 10^3}$$

$= 27.4\%$

$\eta_{overall} = \eta_{prop} \times \eta_{th}$

$= 0.6048 \times 0.274 = 16.6\%$

A Turbojet plane has two jets 250 mm ϕ and the net power at the turbine is 3000 kW. The fuel consumption per kWhr is 0.42 kg with a fuel calorific value 49 MJ/kg, when flying at a speed of 300 m/s in atmospheric $\rho = 0.168\ kg/m^3$. $A/F = 53$, calculate

(i) absolute velocity of jet

(ii) Resistance of the plane

(iii) η_o , (iv) $\eta_{thermal}$

No of jets = 2

$d = 250\ mm$

$P = 3000\ kW$

$m_f = 0.42/kWhr$

$CV = 49 \times 10^6\ J/kg$

$\rho = 0.168\ kg/m^3$

$\frac{m_a}{m_f} = 53$

$$m_f = \frac{0.42 \times 3000}{3600} = 0.35\ kg/s$$

$\frac{m_a}{m_f} = 53,$

$\dot{m} = \dot{m}_a + \dot{m}_f$

$= m_f \left[\frac{m_a}{m_f} + 1 \right]$

$= 0.35 [53 + 1] = 18.9 \text{ kg/s}$

$c = u$

$\dot{m} = \rho A C_j$

$18.9 = 0.168 \times \frac{\pi}{4}(0.25)^2 \times C_j \times 2$

$C_j = 1145.91 \text{ m/s}$

Absolute velocity of jet $C_a = C_j - u$

$C_a = 845.91 \text{ m/s}$

$F = \dot{m} C_j - \dot{m}_a . u$

$= 18.9 \times 1145.91 - 18.55 \times 300$

$F = 16.09 \times 10^3 \text{ N}$

$$\eta_o = \frac{\dot{m}[C_j - u] \times u}{m_f \times CV}$$

$$= \frac{18.9 [1145.91 - 300] \times 300}{0.35 \times 49 \times 10^6} = 27.9 \%$$

$$\eta_{th} = \frac{\frac{1}{2} \dot{m} (C_j^2 - u^2)}{m_f \times CV} = \frac{\frac{1}{2} 18.9 [(1145.91)^2 - (300)^2]}{0.35 \times 49 \times 10^6} \times 100$$

UNIT- IV

JET PROPULSION

It is the propulsion of jet aircraft or other missiles by the reaction of jet coming out with a high velocity. The term jet propulsion is used where the oxygen is obtained from the surrounding atmosphere.

We know that when a fluid is to be accelerated, a force is required to produce this acceleration in the fluid. At the same time, there is an equal and opposite reaction force acting on this fluid. This opposite reaction force of the fluid on the engine is known as thrust. Hence it may be stated that the principle of jet propulsion is based on the reaction principle.

CLASSIFICATION OF JET PROPULSION:

Jet propulsion engines may be classified broadly into two groups.

(a) Air breathing engines - combustion takes place by using atmospheric air

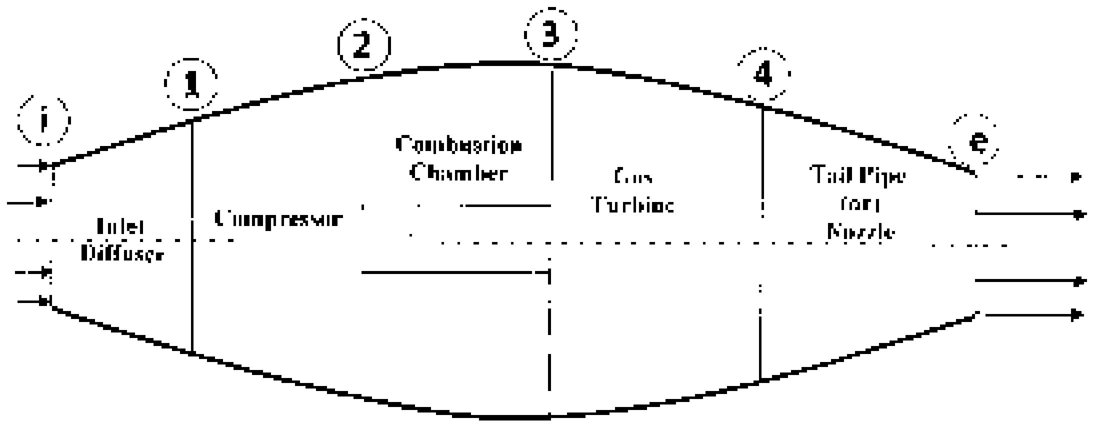
(b) Rocket engines - combustion takes place by using its own oxygen supply.

CLASSIFICATION OF AIR BREATHING ENGINES:

1. Turbojet Engine
2. Turbo Prop Engine
3. Turbo Fan Engine
4. Ramjet Engine
5. Pulse Jet Engine

1. TURBOJET ENGINE:

WORKING:



This engine consists of inlet diffuser, compressor, combustion chamber, turbine and an exhaust nozzle. The function of the diffuser is to convert the kinetic energy of the entering air into a static pressure rise. After this air enters to the compressor,(axial or centrifugal) which further compresses the air to a very high pressure and delivers it to the combustion chamber. Then fuel nozzle supplies fuel continuously and continuous combustion takes place at constant pressure. The high pressure and high temperature gases then enters the turbine, where they expand partially to provide drive power for the turbine. The turbine is directly connected to the compressor and all the power developed by the turbine is to drive the compressor and the auxiliary device. After the gases leave the turbine, they expand further in the exhaust nozzle and are ejected with a very high velocity than the flight velocity to produce a thrust for propulsion.

ADVANTAGES OF TURBOJET ENGINE:

1. Lower frontal area due to the absence of fan. Therefore the drag is less.
2. Suitable for long distance flights at higher altitudes and speeds.
3. Since this engine has a compressor, it is capable of operating under static conditions.
4. Reheat can be possible to increase the thrust.
5. Lower weight per unit thrust at design speed and altitude.
6. Since a diffuser is at the inlet, part of the compression is done by it without any work input

DISADVANTAGES OF TURBOJET ENGINE:

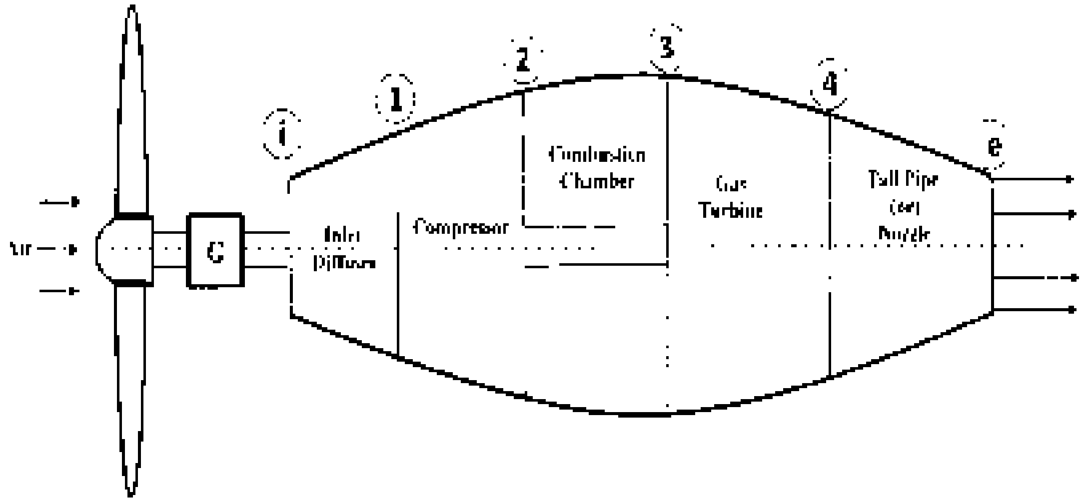
1. Propulsive efficiency and thrust are lower at lower speeds.
2. Thrust specific fuel consumption is high at low speeds and altitudes.
3. It is not economical for short distance flights.
4. Long runway is required due to slower acceleration.
5. Sudden decrease of speed is difficult to achieve.

APPLICATIONS OF TURBOJET ENGINE:

Turbo jet engines are used in military aircrafts, guided missiles and piloted aircrafts, etc.

2. TURBO PROP ENGINE

WORKING:



It is very similar to turbo jet engine, the major difference being that the turbine is designed so that it develops shaft power for driving a propeller to provide most of the propulsive thrust (90%) and only a small amount jet thrust is produced in the nozzle.

The engine consists of a diffuser, compressor, combustion chamber, turbine, exhaust nozzle, reduction gear and a propeller. The diffuse, compressor and combustion chamber functions are as same as the turbo jet engine. However, in the turbo prop engine, the turbine extracts much more power than the turbo-jet engine. because the turbine provides power for both the compressor and the propeller. When all of this energy is extracted from the high temperature gases, only little energy is left out for producing jet thrust. Thus the turbo-prop engine drives most of its propulsive thrust from the propeller and drives only a small portion (10 to 25%) from the exhaust nozzle.

Since the shaft rotation speed of gas turbine engine is very high, a reduction gear must be placed between the turbine shaft and the propeller to enable the propeller to operate efficiently.

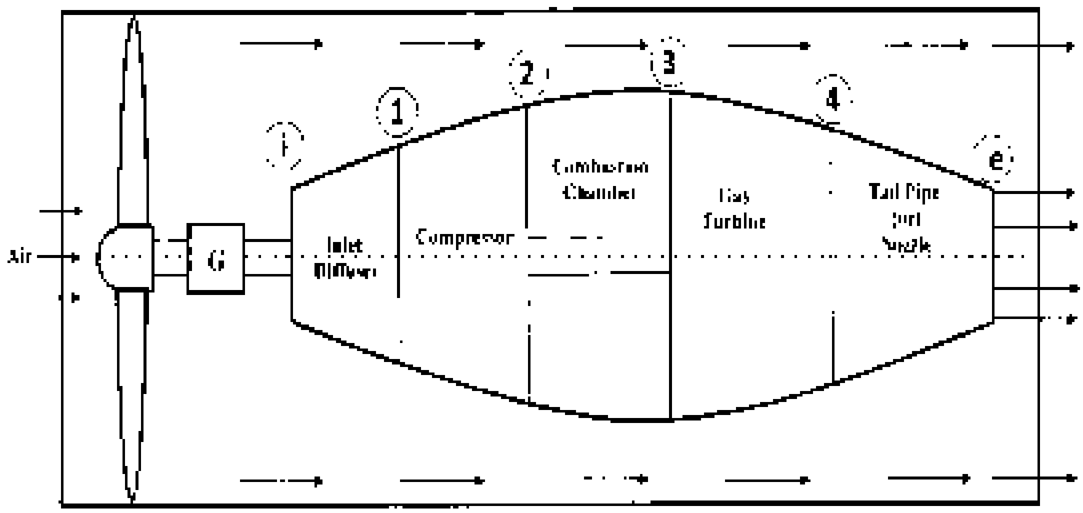
ADVANTAGES OF TURBO PROP ENGINE:

1. Propulsive efficiency is very high
2. The TSFC based on thrust is low.
3. High acceleration at lower speed enables to a shorter run way.
4. Thrust reversal is possible by varying the blade angle, this gives the advantage of decreasing the speed drastically.
5. Used for shorter distance travels.(C < 600 Kmph)

DISADVANTAGES OF TURBO PROP ENGINE:

1. Heavier propeller, compressor and turbine decreases pay load capacity.
2. A reduction gear is required to transmit the power from the turbine shaft to the propeller shaft.
3. If the speed of the engine increases above 600 Kmph, the efficiency drastically decreases.
4. The frontal area is being blocked on account of large diameter propeller which increases the co-efficient of drag.
5. Engine is heavier and more complicated

3. TURBOFAN ENGINE



The turbofan engine is a combination of the turbo prop and the turbojet engines combines the advantages of both.

ADVANTAGES OF TURBOFAN ENGINE:

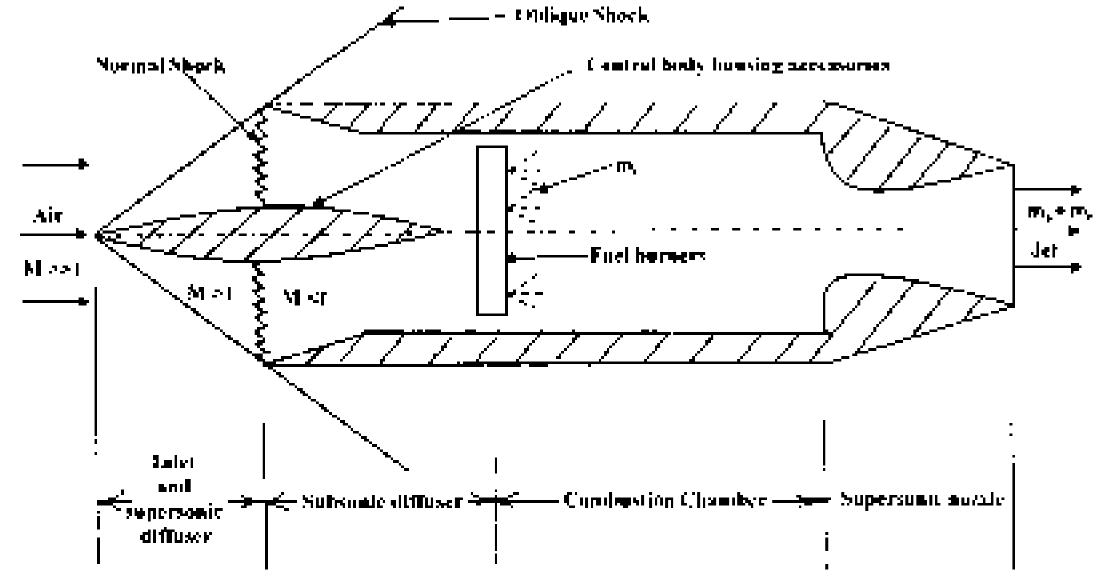
1. Thrust developed is higher than turbojet engine.
2. Weight per unit thrust is lower than turbo prop engine.
3. Less noise.
4. High take-off thrust.

DISADVANTAGES OF TURBOFAN ENGINE:

1. Increased frontal area
2. Fuel consumption is high compared to turbo prop engine.
3. Construction is complicated compared to turbojet engine
4. Lower speed limit than turbojet engine.

4. RAMJET ENGINE

WORKING:



The simplest type of air-breathing engine is the Ram jet engine. The engine consists of a supersonic diffuser, subsonic diffuser section, combustion chamber and a discharge nozzle section. The function of a supersonic and subsonic diffuser is to convert the kinetic energy of the entering air into a pressure rise, called the "**Ram pressure**".

Air from the atmosphere enters the supersonic diffuser where in its static pressure increased and the velocity of air is reduced. Then the air enters the subsonic diffuser it is compressed further. The air then flows into the combustion chamber, where the fuel burners are located and here the air is heated to a high temperature (1600°C to 2000°C) by the continuous combustion of fuel. The highly heated products of combustion are then allowed to expand in the exhaust nozzle section and are discharged from the engine with a speed greater than that of entering air. Because of the rate of increase in momentum of the working fluid flowing through the engine, a thrust 'F' is developed in the direction of flight.

The cycle pressure ratio of ram jet engine depends upon its flight velocity: the higher the flight velocity, the larger the ram pressure and consequently larger the thrust. Since the flight speed is very high, the pressure rise in the diffuser (ram pressure) is very high and this eliminates the compressor. Consequently the turbine is also eliminated, because the function of a turbine is just to run the compressor. Since the ram jet engine cannot operate under static conditions as there will be no pressure rise in the diffuser, it is not self-operating at zero flight velocity. Therefore to attain the required flight speed some kind of starting device must be required such as launching rockets.

ADVANTAGES OF RAM JET ENGINE:

1. Pay load capacity is very high due to the absence of fan, compressor and turbine.
2. Its fuel consumption decreases the flight speed and approaches reasonable values when the flight Mach number is between 2 to 4, and therefore it is suitable for propelling supersonic missiles.
3. Since the frontal area is less, the co-efficient of drag is low.
4. It increases the mechanical efficiency due to the absence of sliding and moving parts
5. High temperature and pressure can be employed

DISADVANTAGES OF RAM JET ENGINE:

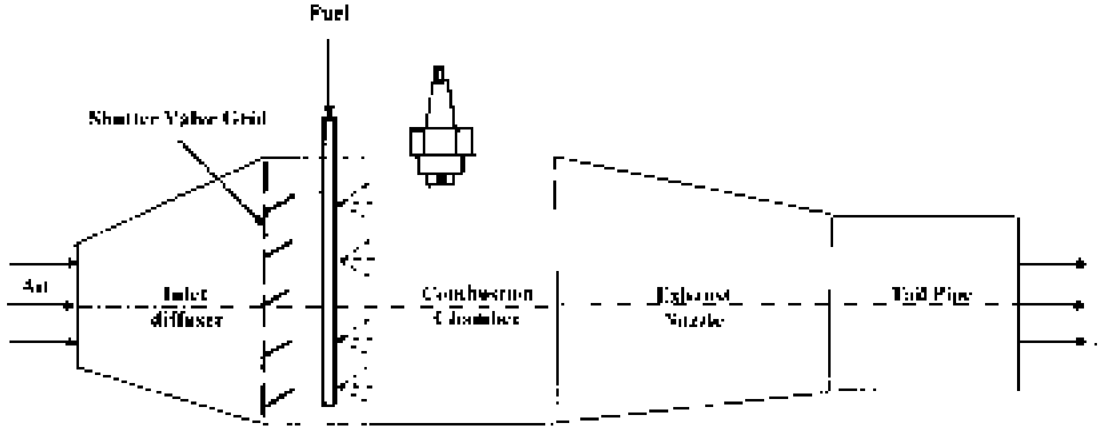
1. A starting device is required to propel ram jet up to supersonic speed.
2. Altitude limitation is there.
3. It has low thermal efficiency and high TSFC.
4. Due to high temperature of gas coming out from the nozzle, erosion occurs at the exit of the nozzle.

APPLICATIONS OF RAMJET ENGINE:

Used as guided missiles and high supersonic speed aircrafts.

5. PULSE JET OR FLYING BOMB

WORKING:



Pulse jet engine consists of an inlet diffuser, valve grid (contains springs that close on their own spring pressure), combustion chamber, spark plug and a discharge nozzle.

The function of a diffuser is to change the kinetic energy of the entering air into static pressure rise by slowing down the air velocity. When a certain pressure drop exits across the valve grid, the valves will open and allow the fresh air to enter the combustion chamber, where fuel is injected and mixed with air. Hence combustion takes place with spark ignition. There is a rapid increase in pressure, which causes the valve to close rapidly and surges the products of combustion rearward into the nozzle, where they expand and escape with higher velocity than the entrance velocity. Thus the thrust is produced at the nozzle exit.

Since firing in the combustor is intermittent and therefore intermittent thrust is produced. The pulse-jet engine is a simple, cheap for subsonic flights and well adopted to pilotless aircraft.

ADVANTAGES OF PULSE JET ENGINE:

1. It gives higher pay load capacity due to the absence of compressor, propeller and turbine.
2. It is simple in construction and cheap. It is suitable for subsonic flights.
3. Drag co-efficient is less due to smaller frontal area.
4. Due to the absence of sliding and moving parts mechanical efficiency is very high.

DISADVANTAGES OF PULSE JET ENGINE:

1. Limited flight speed and altitude.
2. Severe vibrations and high intensity of noise due to intermittent combustion.
3. Nozzle erosion occurs, due to the high temperature of gases coming out from the nozzle.

THRUST AUGMENTATION

The poor take of characteristics of a turbojet engine can be improved by augmenting the thrust. The thrust is given by

$$F = \dot{m}_a\left[C_j - u\right]$$

From the above equation the thrust can be increased either by increasing jet velocity or by increasing the mass flow rate of air. Thrust augmentation will improve

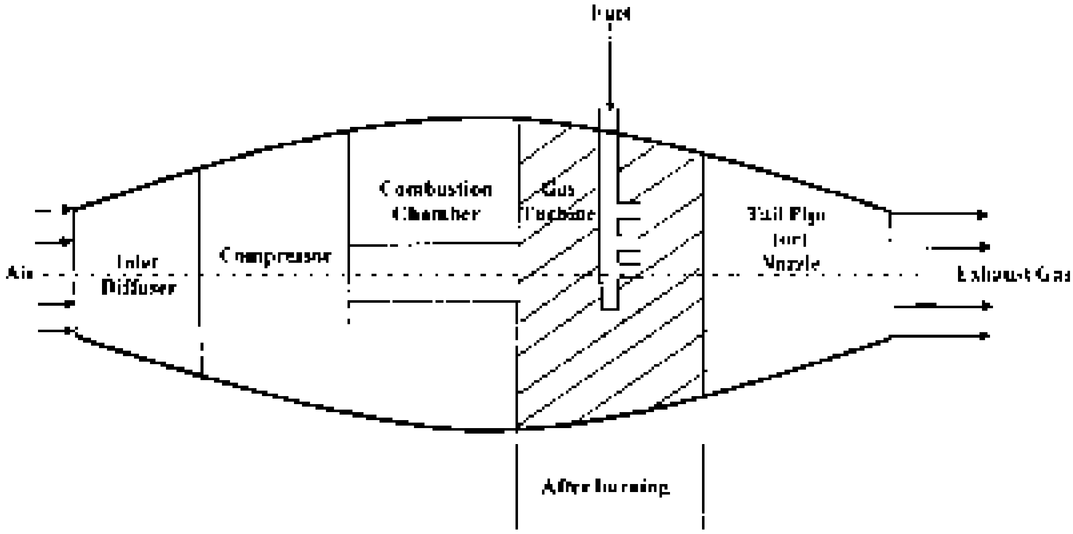
(a) Short take off distance

(b) High climb rate to very high altitude.

The methods of thrust augmentation are as follows:

(I) After burning:

Burning of additional fuel in the tail pipe between the turbine and the exhaust nozzle as shown in Fig:



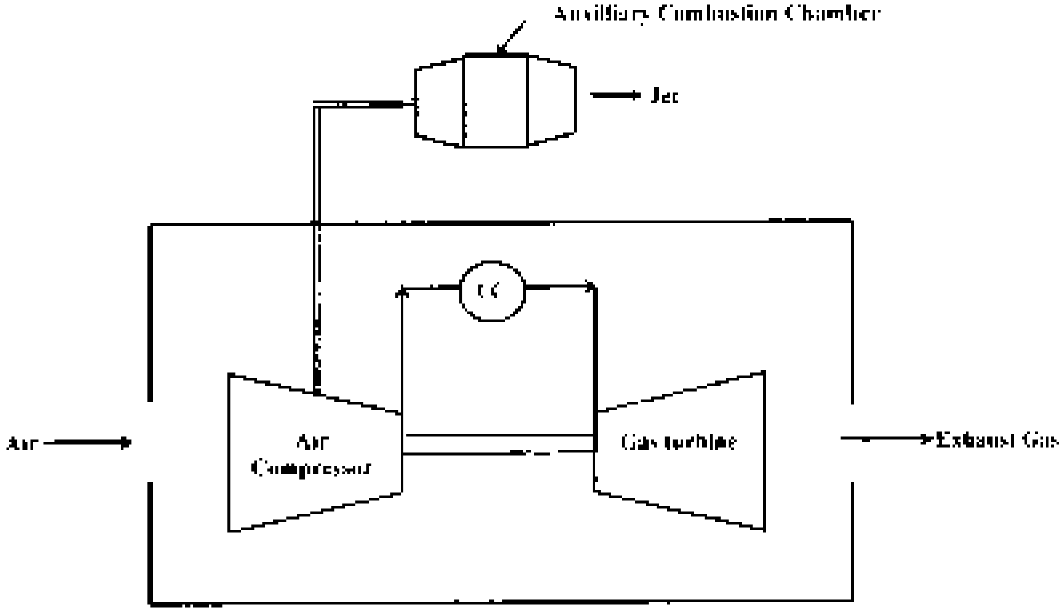
This method of thrust augmentation increases the jet velocity and it is known as after burning. The device is called after burner. A stable combustion of fuel in tail pipe would require sufficiently low velocity and to avoid excessive pressure loss, a diffuser is provided between the turbine outlet and the tail pipe burner inlet. By this method the thrust can be increased by 20 to 25% but the specific fuel consumption is increased. An aircraft will have to carry extra fuel for after burning is a disadvantage.

(b)Increasing mass flow rate of air

The thrust produced is also directly proportional to mass flow rate and therefore the density of the incoming air can be substantially increased by cooling, Water, alcohol or a mixture of water and alcohol is sprayed over the incoming air. The evaporation of liquid extracts heat from incoming air. This cooling gives an increase in both pressure ratio and the mass flow rate. This result in a thrust increase by about 20%, but that will depend upon the coolant used. And this method is used only during the take-off period.

(c) Bleed burn cycle

Fig: shows the schematic arrangement of a bleed burn cycle.



A small fraction of compressed air is bled off from the compressor and enters in an auxiliary combustion chamber. Additional fuel is injected for burning and hot gases coming out are allowed to expand through secondary nozzle to produce additional thrust. In order to compensate for mass of air extracted from the compressor, water is injected into main combustion chamber. This maintains the usual mass flow rate of hot gases through the main nozzle and this system is used to increase the thrust during the take off period.

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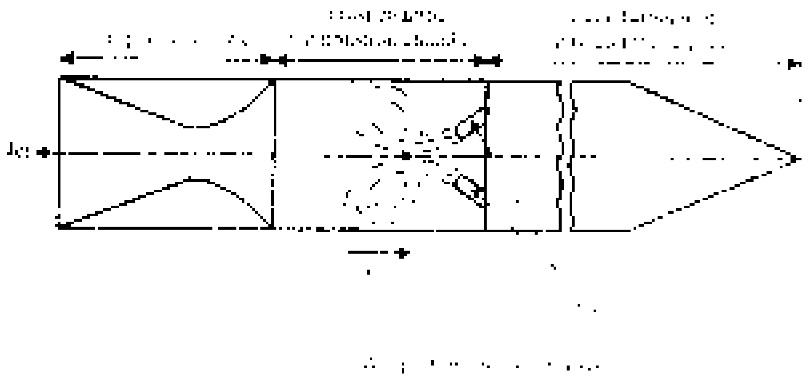
UNIT- V

ROCKET PROPULSION

1. The functioning of the air breathing propulsive device is dependent on the ambient air.
2. Air breathing propulsive devices cannot operate in vacuum.
3. Air breathing propulsive device's operation and performance depends on the altitude.
4. The thrust of air breathing engine depends on flight speed since ambient air is admitted at the front of the engine at this speed.



5. Turbo Jet and Ram Jet can operate at supersonic speeds.
6. The Ram Jet engine achieves a maximum speed of about Mach no. 4.
7. For very high speeds in the hypersonic range rockets are employed which carry their own fuel and oxidizer supply. Therefore, their operation and performance do not depend on the ambient air and altitude.
8. Rocket engines can operate in vacuum and achieve any altitude.
9. The thrust of rocket engine is independent of the flight speed.

Working principle of a Liquid Propellant Rocket:

1. A rocket engine consists of a container or containers for propellants, combustion chamber (Thrust chamber) and a propulsion or exhaust nozzle besides control and navigational equipment, payload etc.
2. The fuel and the oxidizer (known as propellants) are introduced into the thrust chamber for chemical reaction (combustion) at high pressure and temperature.
3. The products of combustion expand through the propelling nozzle to very high velocities

4. High flow rate of exhaust gases of high velocities gives very high value of thrust This is due to a large change in the momentum flux as well as the difference between the nozzle exhaust pressure and ambient pressure.
5. Both liquid and solid propellants can be used in rocket engines.
   - Larger and long range rockets use liquid propellants
   - Smaller and short range rockets us solid propellants.
6. In a hybrid type, solid fuel is used with a liquid oxidizer or vice - versa.
7. A mono propellant engine uses only one chemical (propellant) which dissociates after ignition.
   - Liquid propellant rockets - Known as ``Engines"
   - Solid propellant rockets – known as ``Motors"
8. On account of limited quantity of the propellants stored in the rockets, the exhaust jet velocities must be very high (of the order of 3000 m/s) for given operation time and targeted altitude. This requires very high pressures and temperatures of gases in the thrust chamber; their maximum values are over 200 bar and 400 K respectively.

Rockets Classification.

1. Based on Source of Energy
   - Chemical Rockets
   - Solar Rockets
   - Nuclear Rockets
   - Electrical Rockets
2. Based on Propellants
   - Liquid Propellant Rockets
   - Solid Propellant Rockets
   - Hybrid Propellant Rockets
3. Based on Application
   - Space Rockets
   - Military Rockets (missiles)
   - Weather or sound Rockets
   - Air craft propulsion, Turbojet rocket engines, Ramjet rocket engines
   - Booster Rockets
   - Sustainer Rockets
   - Retro Rockets
4. Based on Number of Stages

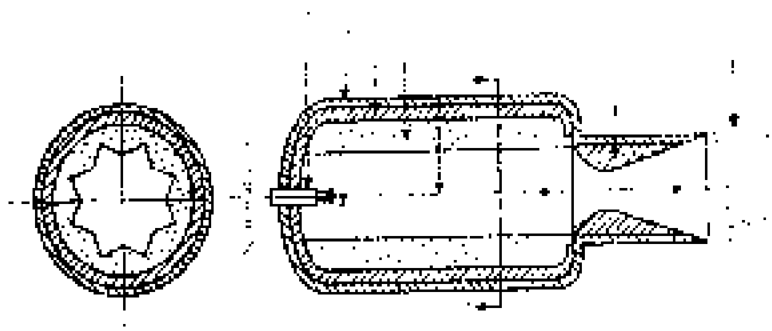
- Single Stage Rockets
- Multi Stage Rockets

5. Based on Size and Range
   - Short range small Rockets
   - Long Range large Rockets

1. **Liquid Propellant Rockets**:

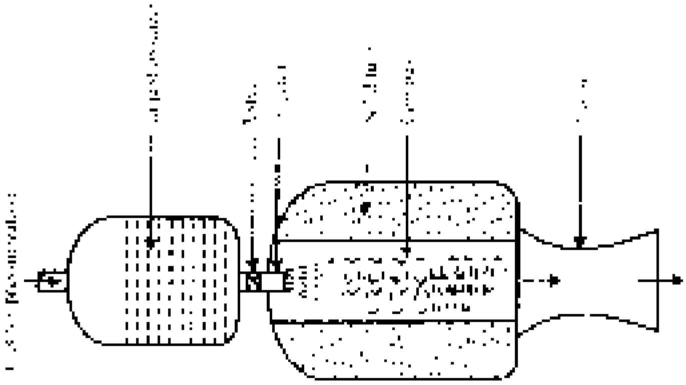
1. Rocket engines in which liquid fuels and oxidizers are used are known as liquid propellant Rocket engines.
2. The propellant is fed into the thrust chamber (combustion chamber or combustor) from their containers for combustion or chemical reaction.
3. For better mixing and efficient combustion, the fuel and the oxidizer are atomized through the injector. The feed system regulates the optimum mixture ratio for a given set of propellants.
4. Some commonly used liquid fuels are liquid hydrogen, hydrazine, alcohol etc. The oxidizers are liquid oxygen, red fuming nitric acid (RFNA), liquid fluorine, WFNA etc.
5. Most of the liquid propellants are toxic and require very high combustion temperatures. This demands special material and handling systems. One of the propellants is circulated through the outer shell of the exhaust nozzle and the inner walls of the thrust chamber before injection for combustion.
6. This circulation cools the high temperature components and also increases the temperature of the propellant, thus increasing the thermal efficiency.
7. Beside the propellant tanks, thrust chamber and exhaust nozzle which occupy a large proportion of rocket space, there are several other things that a rocket engine carries such as the propellant pumping and control system, navigational equipment, auxiliary power unit and the payload.

2. **Solid Propellant Rockets**:



1. Rockets which use solid fuels and oxidizers (solid propellants) are known as solid propellant rocket engines or motors.
2. Solid fuel (plastic or resin material) and oxidizer (nitrates, perchlorates etc.) are mixed in a single propellant grain and packed into the steel shell or case of the rocket.
3. The method of casting the fuel and the oxidizer mixture in fluid state into the rocket shell is widely employed.
4. A cylindrical star shaped cavity is provided for combustion in the centre along the axis of the shell. A variety of shapes are also employed.
5. A liner is provided between the case and the propellant to protect the case from high temperature developing inside the propellant layers.
6. The igniter is located at the top to start the combustion or chemical reaction between the fuel and the oxidizer.
7. Once the flame front is established, combustion is self - sustaining.
8. Excessive pressures which are serious problems in solid propellant rockets can be kept lower by suitably choosing the fuel and oxidizer combinations.
9. Solid propellant rockets are comparatively simpler and lighter. They are widely used in small sizes, however recent developments have made it possible to design large size solid propellant rockets also for space vehicles.

3. **Hybrid Propellant Rockets**:
    1. Hybrid rockets employ a combination of liquid and solid propellants.
    2. A hybrid rocket combines advantages of both the liquid and solid propellant rockets.

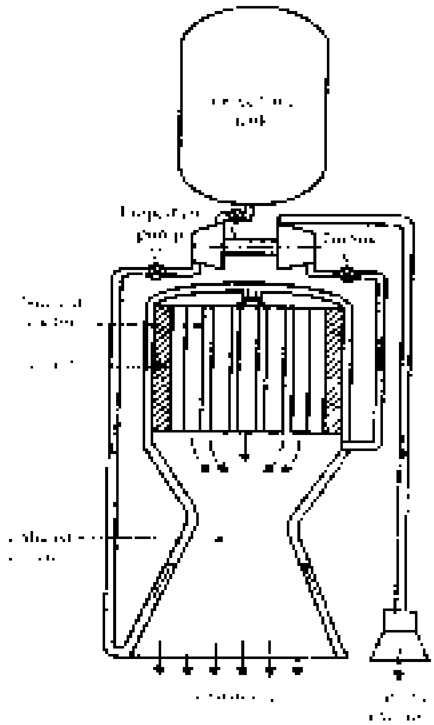


3. The liquid oxidizer is injected into the solid fuel shell where the chemical reaction between the two propellants takes place.
4. For some combinations, ignition is not required to start the reaction; merely a contact between the fuel and the oxidizer starts the required reaction. Such propellants are known as hypergolic. Such fuel - oxidizer combinations for hybrid propellant rockets are
   - Beryllium Hydride (Be-H?) - Fluorine (F_2)
   - Lithium Hydride (LiH) - Chlorine Trifluoride (ClF_3)
   - Lithium Hydride (LiH) - Nitrogen Tetroxide (N_2O_4)
   - Hydrocarbon (CH_2) - Nitrogen Tetroxide (N_2O_4)

Advantages of Hybrid Rockets:

1. Thrust control is comparatively easier because only the flow of the liquid oxidizer needs to be regulated.
2. Since the fuel and the oxidizer are kept separate, the deterioration that occurs in solid propellant rockets is absent here.
3. Hybrid rockets are lighter compared to the corresponding liquid propellant type on account of less elaborate propellant pumping equipment and higher fuel density
4. Greater choice in the selection of fuel grain configuration.
5. In case of an accident or crash, the explosion (if any) is less destructive compared to the liquid rocket engines.

4. Nuclear Rockets:

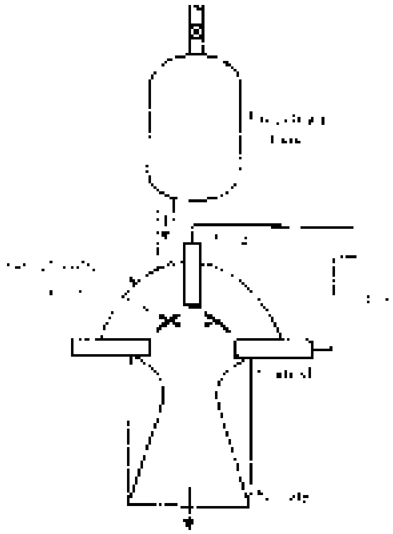


1. In a nuclear rocket, nuclear energy is used to heat the propellant (or the working fluid) to obtain high stagnation temperature in the thrust chamber).
2. Liquid hydrogen is widely used in such rockets. A nuclear reaction takes place in a reactor which is placed in place of combustor in the thrust chamber.
3. A nuclear rocket with nuclear fission of the uranium fuel consist of the conventional components such as the fuel, moderator, reflector, shield, control rods etc.
4. Liquid hydrogen is pumped from its tank to the reactor through the shell jacketing the exhaust nozzle as shown.
5. A small portion of the heated gas (propellant) is tapped out for driving the turbine which in turn drives the propellant pump. The turbine exhaust is also expanded in a small nozzle providing a fraction of the total thrust.
6. Temperature produced in thrust chamber is 2000°C.

5. <u>Solar Rockets:</u>

Electrical Rockets.

6. **Electrical Rockets:**



1. Electrical energy is used to generate thrust in the rockets by different methods.
2. In an electro – thermal rocket, the propellant is heated by an electric arc struck between the central anode and an annular housed in the thrust chamber.
3. Very high temperatures of the propellant in the thrust chamber can be obtained in this type.
4. This arrangement gives low values of mass ratio with very high specific impulse (20000 sec)

Liquid Propellants.

1. Mono Propellants

❖ Liquid propellant rocket engines are considerably simpler compared to the bipropellant engines as they require only one turbo pump (on gas pressure) feed system.

Example:

- Hydrogen Peroxide (H_2O_2)
- Hydrazine (N_2H_4)
- Nitroglycerine [CH (O No)]
- Nitromethane[CH No2]

❖ Monopropellant engines are generally small. They are more suitable for auxiliary and turbo pump power plants in rocket engine.

2. Bipropellants:

Some propellants combinations which are widely used in liquid propellant rockets engines are given below:

Oxidizer	Fuel	Mixture Ratio	Combustion Temperature (K)
Liquid Oxygen (Lox)	Gasoline	2.50	3294
	Hydrazine	0.92	3400
	UDMH	1.65	3600
	Liquid Hydrogen	4.00	3922
	Ethyl Alcohol	1.80	3422

Hydrogen Peroxide (H_2O_2)	Gasoline	-	-
	Ethanol	-	-
	Hydrazine	1.84	2817
	UDMH	4.54	2922
	Organic fuel	-	-
Nitrogen Tetraoxide (N_2O_4)	Hydrogen	5.20	2661
	UDMH	2.50	3355
	Hydrazine	1.33	3246
	Ehtanol	2.80	3242
Nitric Acid (RFNA)	Aniline	3.00	3045
	Gasoline	4.80	-
	Hydrazine	1.47	3083
	Alcohol	1.90	3045
	UDMH (50%)	1.73	2997

Oxidizers:

A. Liquid Oxygen (Lox)

- ✓ *Cryogenic Propellant (Boiling Point = 90 K)*
- ✓ Requires proper insulation to prevent evaporation
- ✓ Non - corrosive but harmful to personnel
- ✓ It can explode under impact when mixed with oil and organic substances

B. Hydrogen Peroxide

- ✓ Highly reactive liquid requiring special material for handling system

- ✓ Highly concentrated solution of hydrogen peroxide is used in rocket engines
- ✓ Harmful to the skin of the personnel and suffers from hazards of fire and explosion

C. Nitrogen Tetroxide

- ✓ Storable propellant (Boiling point = 294.4 K) used in combination with rocket fuel such as dimethyl hydrazine etc.
- ✓ It is easily ignitable

D. Nitric Acid

- ✓ RFNA - Red Fuming Nitric Acid
- ✓ WFNA - White Fuming Nitric Acid
- ✓ Nitric Acid is preferred in many applications on account of high specific gravity

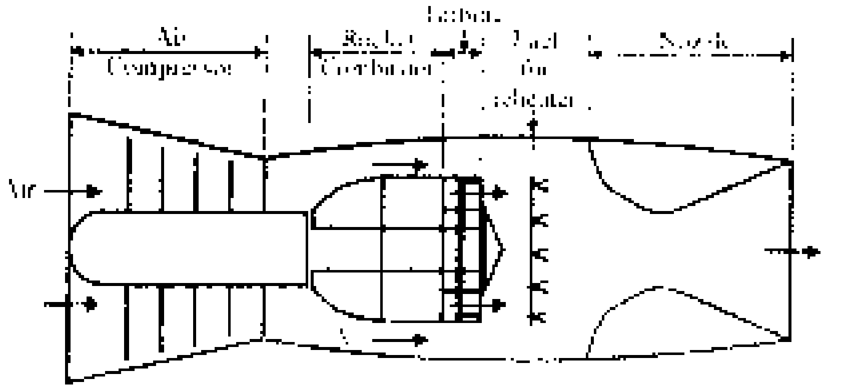
FUELS

1. Liquid Hydrogen

- ❖ Cryogenic Propellant (Boiling point = 20 K) Low specific gravity = 0.07
- ❖ Larger and well insulated storage tanks are required
- ❖ Its mixture with air (due to leakage) can cause explosion

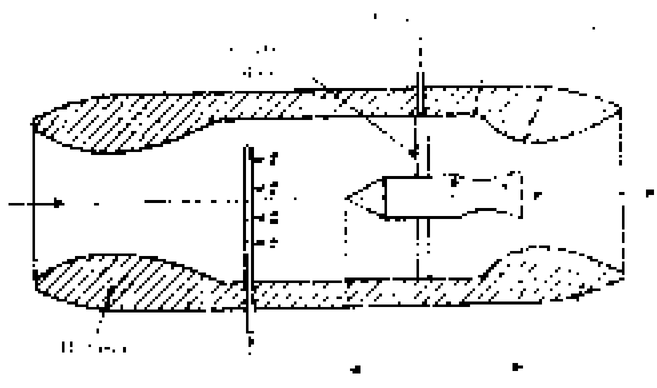
2. Ethyl Alcohol or Ethanol

- ❖ Used with several oxidizers such as Lox, H_2O_2, N_2O_4, RFNA etc.
- ❖ Gives lower combustion temperature with slightly lower performance
- ❖ Ethanol can dissolve a number of lubricants. Therefore, they are selected for ethanol engines



- "Rocket - assisted take off" planes can employ much lighter engines; this mode of take - off is frequently referred to as "RATO" or "JATO".
- Solid fuel rocket motors are generally used for this purpose on account of their smaller size and simplicity.
- It is the combination of a rocket and turbo jet engine. The engine starts with the supply of hot gases from the rocket combustor, which drives the turbine and air compressor.
- The compressor supplies combustion air directly to a reheater located upstream of the propulsion nozzle.
- Military aircrafts can take - off with the conventional turbojet and achieve high rate of climb with rocket propulsion.

Ram – Jet Rocket Engine:



- Ram jet engines are also launched at supersonic speeds by rockets.
- The operation of the rocket engine upstream of the main nozzle produces the thrust required for launching the ram jet engine.
- After achieving the desired Mach number ($M > 1$) the ram jet engine flies on its own thrust.

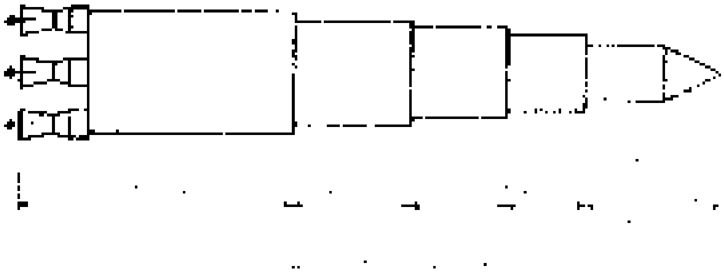
3. **Military Applications**

In addition to V-2 bombs, many types of missiles with war heads have been used. They are propelled by both solid and liquid fuel in short range and long range operations. Some important missiles are

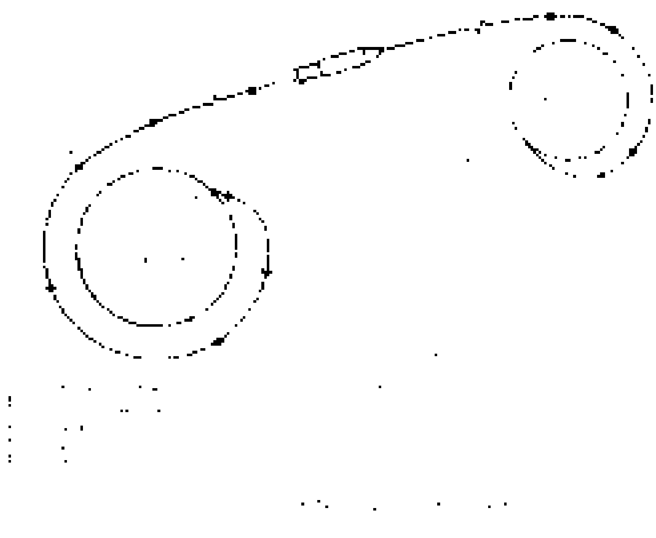
I. Surface to air missile
II. Surface to surface missile
III. Air to air missile
IV. Inter - continental Ballistic Missile
V. Intermediate Range Ballistic Missile
VI. Guided Air Missile

The inter - continental ballistic missiles can travel a maximum distance equal to about 0.25 of earth's circumference.

1. **Space Applications:**



- Both solid and liquid propellant multi - stage rockets have been used to propel space vehicles.
- All space vehicles (on satellites have pass through the atmosphere over coming forces due to drag and gravity)
- To avoid very high drag losses and prevent excessive skin temperatures space vehicles rise as a slower rate through the atmosphere



- The first stage lifts off the entire rocket - vehicles system; therefore, it is the most powerful stage often known as the "booster" or "booster stage" It can employ a single large liquid propellant rocket or a number of smaller rocket operates simultaneously in parallel as shown.

- The last stage or the "sustainer" is generally the smallest. The thrust required for propulsion along the inter planetary trajectory is much smaller. Small rockets are used on the way for various minor operations such as trajectory correction, altitude control and stage separation.
- Retro rockets are small rockets fired in the direction of motion of the space vehicle to achieve braking or deceleration
- A rocket - vehicle system may have as many as 50 to 100 big and small rockets
- Several types of space crafts are launched and propelled by rockets such as Earth satellites. Lunar satellites, inter planetary satellites, manned and unmanned satellites
- Fig 2. Shows an inter planetary flight of a rocket vehicle between the earth and a given planet. The take - off begins on earth surface with the operation of a booster rocket
- After the powered flight for some time, the vehicle coasts in earth orbit before accelerating to the inter planetary orbit.
- On account of the absence of atmospheric drag and gravity, the thrust required in the interplanetary trajectory is small and the vehicle to jet - velocity ratio is very high.
- Small rockets are fired only for trajectory correction.
- When the vehicle approaches the targeted planetary orbit it is decelerated by the action of retro - rockets. Retro - rockets are also employed for de - orbiting and landing on the planet's surface.
- Re - fuelling of rocket engines in transit at other planets can make them much lighter and simpler in design.
- Re - fuelling can be attempted at the moon surface and other distant planets for return and long journey through space; many planets have hostile environment preventing such facility. Some of them have either too high or too low ambient temperatures. Others are enveloped by a thick layer of opaque and dense gases

- Gravitational force of other planets in addition to rocket thrust can also be employed for accelerating space vehicles during long journeys. This would require comparatively smaller rocket engines.
- It is estimated that a rocket must be launched at a velocity of 16,616 m/s (54,500 fps) in the direction of earth's rotation for escaping from the solar system. By employing "Jupiter fly by" the launch velocity can be reduced considerably.

PROPELLANT FEED SYSTEM:

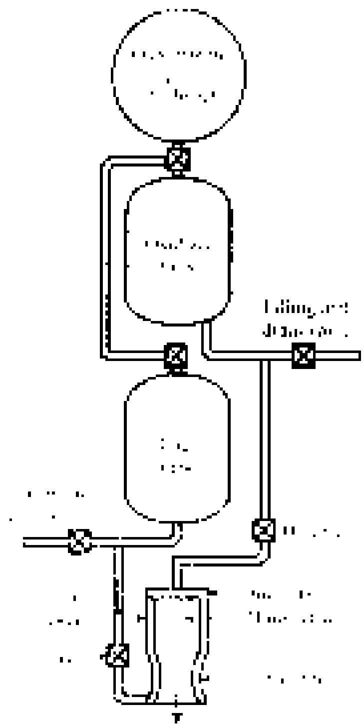
Liquid propellants are required to be injected at a pressure slightly above the combustor pressure.

Types of feed systems:

1. Gas pressure feed system - used for low thrust and short range operations
2. The pump feed system- used for large engines.

Gas Pressure Feed System:

An inert gas is separately carried at a pressure much higher than the injection pressure, this is used to exert the required pressure in the propellant tanks. The pressurizing gas is chosen on the basis of its chemical properties, density, pressure and the total weight of the gas and the tank. A gas which is ideal for one propellant may be quite unsuitable for another. Nitrogen, Helium and air have been used for pressurization. The propellants under high pressure are forced to flow into the thrust chamber through valves, feed lines and injectors. Several regulating and check valves are used for filling, draining, starting and checking the flow of propellants.



In this method no moving parts such as pump and turbines are used. Therefore, the system is considerably simpler. However, the pressurization of the propellant tanks requires them to be comparatively much heavier and introduces a weight penalty besides other problems. Therefore, this system is unsuitable for large rocket engines and long range missions.

Pressure for the injection can also be generated within the propellant tank or tanks by introducing a small quantity of a gas which reacts exothermically with the propellant: this produces the high pressure gas required to force the propellant into the combustor.

Turbo Pump Feed System:

In this system propellants from the tanks are pumped into the combustor by gas turbine driven centrifugal pumps. The turbine or turbines work on high pressure and temperature gas generated separately or tapped out from main combustor. Fig: 1 shows the general arrangement of a turbo pump system. Here both the fuel and oxidizer pumps are driven by a single turbine. In order to achieve flexibility in choosing the design and operating parameter the fuel and oxidizer pumps can be driven separately by their turbines.

upstream of the exhaust nozzle is at low Mach numbers and the equilibrium combustion pressure can be assumed to be identical with the stagnation pressure.

Linear Burning Rate

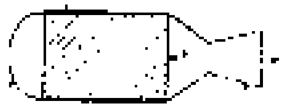
A solid propellant grain burns at its exposed surface (area = Ac). As a result of the combustion the grain surface recedes in the direction perpendicular to itself The rate at which it recedes is known as the ``Linear burning rate"® expressed in millimeters or centimeters per second. Linear burning rate mainly depends on the combustion pressure (Pc) and the propellant grain temperature (Tp) before combustion: the velocity of combustion gases and the elapsed time can be assumed to have only slight effect. The linear burning rate is given by

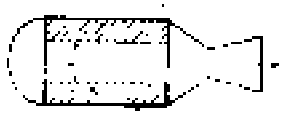
$$r = K_1 P_c^{n1}$$

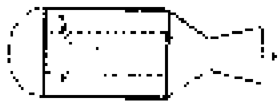
The coefficient K1 and the exponent n1 depend on the chemical composition and the pressure range; the value of the exponent n1 is approximately between 0.2 and 0.8. The coefficient K1 also depends on the propellant grain temperature. (Tp)

Restricted Burning:

Several grain configurations (shown in figure) are employed to obtain restricted burning of the solid fuel oxidizer mixture at the desired rate. The inhibiting material or restrictions prevents the propellant grain from burning in all directions. Very often the restrictions assume the form of linear between the propellant grain and the thrust chamber case or shell. The inhibiting material should not start burning along with the grain or before.







Cigarette Burning:

An older practice employs 'cigarette burning' pattern as shown in figure: here the grain is ignited at one end upstream of the nozzle from where the combustion starts and proceeds in the direction shown. The burning surface progressively recedes leftward. In this pattern of burning the entire thrist chamber is heated to a high temperature throughout the operation. This causes high temperature material and strength problems. Since the burning surface area remains constant a constant thrust profile is obtained.

Internal Burning:

It is improved method of cigarette burning. Combustion of the propellant takes place on the surface of the internal passage (of star shape cross section as shown in figure) or perforation provided along the whole length of the propellant grain. Different geometrical shapes of the internal passages offer different surface areas or propellant area ratio (Ac/A) values. Thus several thrust-time profiles can be obtained.

Neutral Burning:

A propellant grain with the configuration of a cross in the middle of the annular void space is shown in figure. Here the burning takes place only at the cylindrical surface of the grain. The gas pressure is same all around the grain. This can also give neutral burning.

In restricted burning rockets, particularly in cigarette burning pattern a larger quantity of propellants can be packed into a given volume of the combustion chamber. This offers better utilization of the available space.

Unrestricted Burning:

If the propellant grain surface is not restricted from burning and all surface except those with supports are exposed to the flame or hot gases the mode of combustion is known as "Unrestricted Burning". This requires perforations and hollow spaces running along the length of the propellant grain as shown in figure. This arrangement requires larger volume of empty spaces to permit the flow of combustion gases and provision of supports. Therefore, the quantity of propellant that can be packed into a given volume of the combustion chamber is considerably reduced; however, large values of the propellant burning surface area can be obtained by employing a number of grain configurations.

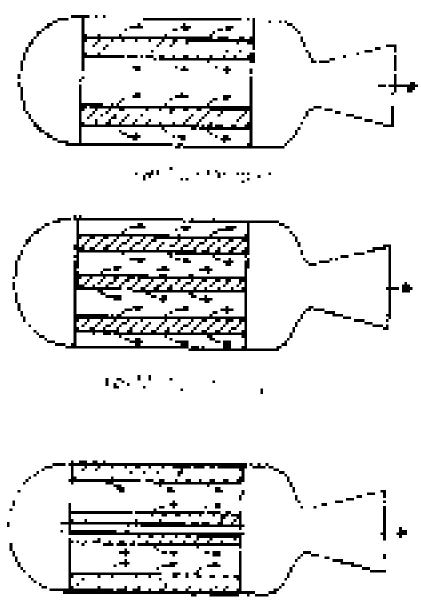


Figure (a) shows a tubular grain shape, the hollow cylindrical grain is supported within the shell on three or more sufficiently strong webs. During combustion the outer cylindrical surface area of the grain decreases while the area of the inner surface increases. Neutral burning can be obtained if the overall area of the burning surfaces remains constant.

Figure (b) shows a multiple hole grain. Burning takes place at both the outer and inner surfaces of the grain. In this configuration also burning surface area can be kept constant to obtain constant thrust

The aforementioned configurations suffer from a serious problem of differential combustion pressure. The combustion pressure and hence the burning rate on the outer and inner surfaces are different. If the pressure difference is large the grain may crack or break. In order to avoid this the pressure on different surfaces is equalized by providing holes through the grain thickness

The rod and tube type grain shown in figure (c) ever comes this problem. Burning takes place at the inner surface of the tube and outer surface of the rod. The combustion pressure in the annular space is same. Thus the burning rate is same. Since the surface area of the tube increases and that of the rod decreases during burning may desired thrust profile is obtained by choosing different sizes and the propellants for the tube and the rod.

Ref: Fundamentals of Compressible Flow with Aircraft and Rocket Propulsion

By: S.M.Yahya

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Rocket propulsion

Rocket propulsion Theory

Some of the main and basic parameters in rocket propulsion are the Thrust (F), specific Impulse (I_s) specific propellant Consumption (SPC) and Exhaust gas velocity (C_e) or effective jet velocity C_j, Area ratio and Pressure ratio across the Nozzle will affect the above quantities.

In Energy Conversion systems efficiencies are Important. The η that are Considered here are η_{prop}, η_{ther} and $\eta_{overall}$ (η_o).

High pressure and temp. gases are produced in the combustion chambers or Thrust chambers as a result of chemical reaction b/t the fuel and oxidizer.

Pressure of combustion is Very high & ambient Pressure are low gases are Expanded in C.D. Nozzle

(x') The exhaust gases in the Jet are produced by the propellants (i.e) the fuel & oxidizer therefore the mass rate of flow gases is Equal to the rate of flow of propellant

(i.e)

$$\dot{m}_p = \dot{m}_f + \dot{m}_{oxi}$$

$$\dot{m}_p = \frac{\dot{w}_p}{g} = \dot{m}_f \left[1 + \frac{\dot{m}_{ox}}{\dot{m}_f} \right]$$

The total weight of the propellants required for the duration of powered flight of the operation time t_p is given by

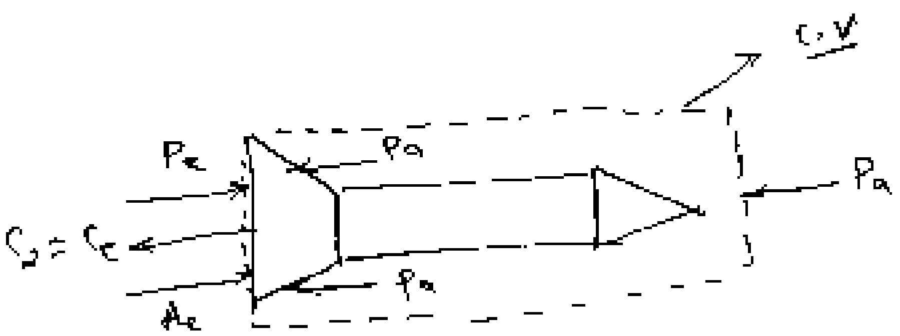
$$W_p = \int_0^{t_p} w_p \, dt$$

Thrust

The force that propels the rocket at a given velocity is known as the thrust.

This is produced due to the change in the momentum flow of the outgoing gases as well as the difference b/t the Nozzle exit pressure and the ambient pressure.

Rocket Thrust is employed to overcome the drag & gravitational forces besides providing the acceleration.



development of Thrust in a rocket.

The fig. shows the control surface enclosing the rocket Engine.

There is no inflow of air into the rocket before Expansion the initial velocity of the gases in the nozzle is zero ($C_1 = 0$), final velocity is $C_2 = C_e$.

one-dimensional steady flow at the nozzle exit the rate of change of momentum flow is

$$f_{mom} = \int_{A_e} \rho_e C_e^2 \, dA = \rho_e C_e^2 A_e \longrightarrow (1)$$

Continuity equation at the nozzle exit gives

$$\dot{m}_p = \rho_e A_e C_e \longrightarrow (2)$$

(2) in (1)

$$f_{mom} = \dot{m}_p C_e \longrightarrow (3)$$

if $P_e = P_a$ for complete combustion

if the expansion through the nozzle is not complete $P_e \neq P_a$.

Pressure forces acting on the rocket in the direction of motion is given by

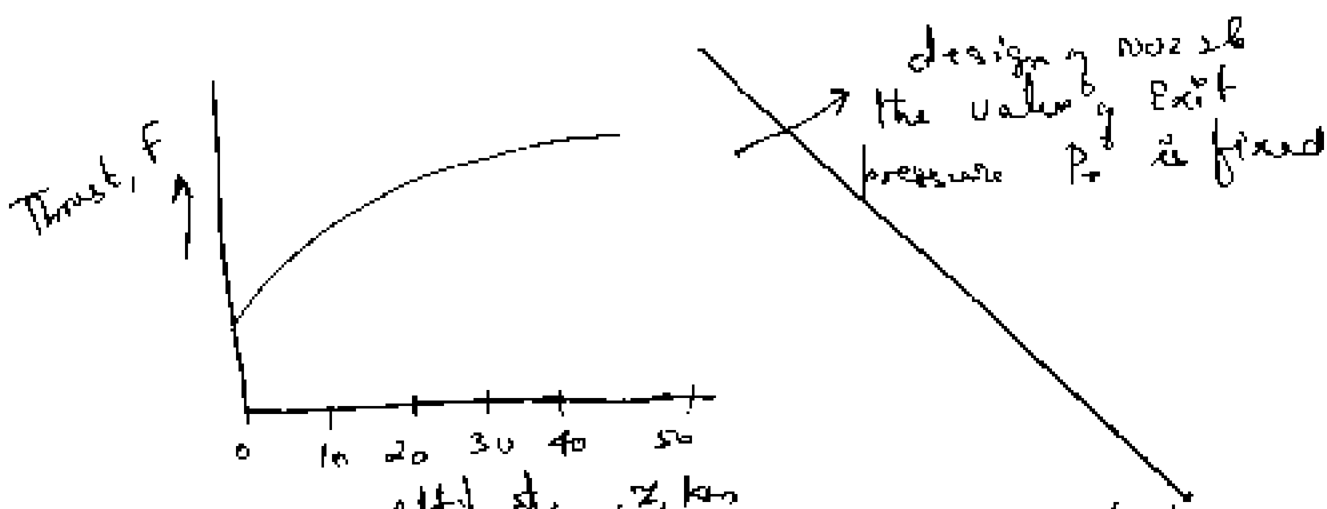
$A_e = A_a$

$$f_{pr} = P_e A_e - P_a A_e$$

$$f_{pr} = (P_e - P_a) A_e \longrightarrow (4)$$

Net Thrust

$$f = f_{mom} + f_{pr} = \dot{m}_p C_e + (P_e - P_a) A_e \longrightarrow (5)$$



Variation of Thrust with altitude

Therefore the Thrust during the flight through the atmosphere continuously increases with the altitude on account of decreases in P_a, under certain altitude to the variation is Negligible & thrust is almost constant.

$$F = \dot{m}_p C_j = \dot{m}_p C_e + (P_e - P_a) A_e$$

$$\boxed{F = \frac{\dot{w}_p}{g} \cdot C_j} \longrightarrow$$

<u>specific Impulse (I_s)</u>

$$I_s = \frac{F}{\dot{w}_p} \qquad \frac{N}{N/s} = s.$$

specific Impulse of a rocket Engine is the Thrust per unit weight flow rate of the propellant.

$$I_s = \frac{F}{\dot{w}_p}, \qquad F = \frac{\dot{w}_p}{g} \cdot C_j$$

$$\boxed{I_s = \frac{C_j}{g}}$$

$C_j \rightarrow$ depends on the Exhaust gas properties (γ, R) & Thrust chamber pressure (P_0) & Temperature (T_0)

$$I_t = \int_0^{t_p} F \cdot dt = \int_0^{t_p} I_s w_p \cdot dt$$

$$I_t = I_s \int_0^{t_p} w_p \cdot dt$$

$$\boxed{I_t = I_s W_p} \qquad I_s = \frac{F}{\frac{W_s}{t_p}}$$

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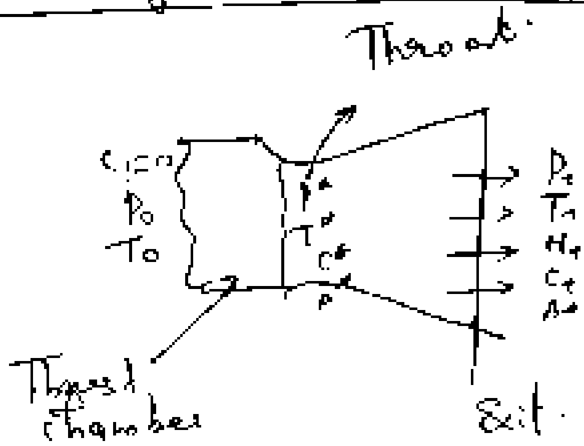
specific propellant consumption (SPC)

The weight flow rate of the propellant required to produce a thrust of one Newton is known as the SPC.

$$SPC = \frac{\dot{w}_p}{F} = \frac{1}{F/\dot{w}_p} = \frac{1}{I_s}$$

flow through Rocket Nozzles



various flow parameters in a rocket Nozzle are shown in fig.

$$\frac{T^*}{T_0} = \frac{2}{\gamma+1},$$

$$P^*/P_0 = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}$$

$$c^* = \sqrt{\gamma R T^*}$$

$$\dot{m}_p = \rho^* A^* c^* = \rho_e A_e c_e$$

$$\frac{T_0}{T_e} = 1 + \frac{\gamma-1}{2} M_e^2$$

$$\frac{P_0}{P_e} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/\gamma-1}$$

$$c_{max}^2 = 2 c_p T_0$$

weight flow coefficient (C_w) → This is the ratio of the gas or propellant flow rate & force term $P_0 A^*$.

$$C_w = \frac{\dot{w}_p}{P_0 A^*}$$

1.

Thrust coefficient (C_f)

This is the ratio of Thrust and the force term $P_0 A^*$

$$C_f = \frac{F}{P_0 A^*}$$

$$F = C_f P_0 A^*$$

$$\dot{w}_p = C_w P_0 A^* ,$$

$$I_s = \frac{F}{w_p} = \frac{C_F P_0 A^*}{C_w P_0 A^*} = \frac{C_f}{C_w}$$

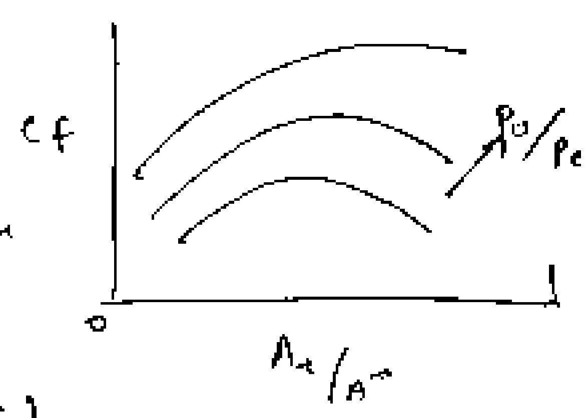
$$SPC = \frac{1}{I_s} = \frac{C_w}{C_f}.$$

Impulse to weight ratio (IWR)

$$IWR = \frac{I_t}{W_t}.$$

This is the ratio of the total Impulse of the rocket vehicle system the rocket & total weight of the

[an Increase with pressure ratio, there is a decrease with the increase in (γ), also A_e/A_0 >>]



Characteristic velocity (V^*)

$$V^* = \frac{C_j}{C_F} = \frac{\text{Effective Jet velocity}}{\text{Thrust coefficient}}$$

$$V^* = \frac{C_j}{C_f} \times \frac{g}{g} = \frac{I_s \times g}{C_F} = \frac{g}{C_F \, SPC}$$

$$\boxed{V^* = \frac{g}{C_w}}$$

Propulsive Efficiency

Propulsive η = $\dfrac{\text{Propulsion or Thrust power}}{\text{Engine output power}}$ (4)

Propulsion power = $F \times u$

$= m_p \times c_j \times u$

K.E. loss in the exhaust gas $\frac{1}{2}(c_j - u)^2$

$\therefore$ rate of Energy or power loss $= \frac{1}{2}\dot{m}_p (c_j - u)^2$

Engine o/p Power = Propulsion power + power loss in the Exhaust gas

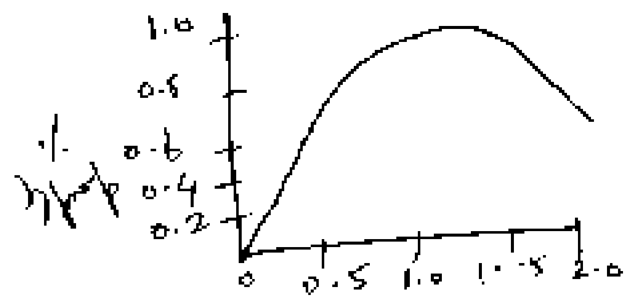
$= m_p c_j u + \frac{1}{2}\dot{m}_p (c_j - u)^2$

$= m_p c_j u + \frac{1}{2} m_p \left[c_j^2 + u^2 - 2 c_j u \right]$

$= \frac{1}{2}\dot{m}_p (c_j^2 + u^2)$

$$\eta_p = \frac{m_p c_j \times u}{\frac{1}{2}\dot{m}_p (c_j^2 + u^2)} = \frac{2 c_j \times u}{c_j^2 + u^2}$$

$$\boxed{\eta_p = \frac{2a}{1+a^2}}$$



$a = \frac{u}{c_j}$

→ operate at flight speeds much greater than jet speed.

Thermal Efficiency

$$\eta_{th} = \frac{\text{Engine o/p power}}{\text{Power Input through fuel}} = \frac{\frac{1}{2}\dot{m}_p (c_j^2 + u^2)}{m_p \times Q_R}$$

$$\boxed{\eta_{th} = \frac{c_j^2 + u^2}{2 Q_R}}$$

Over all Efficiency (η_o)

$$\eta_o = \frac{\text{Propulsion power}}{\text{Power Input through fuel}}$$

$$= \frac{F \times u}{\dot{m}_p \times Q_R} = \frac{[\dot{m}_p (c_j \times u)]\, u}{\dot{m}_p \times Q_R}$$

$$\boxed{\eta_o = \frac{c_j \times u}{Q_R}}$$

$Q_R \rightarrow$ heat of reaction per kg of products of combustion.

(F+1)

(C.V)

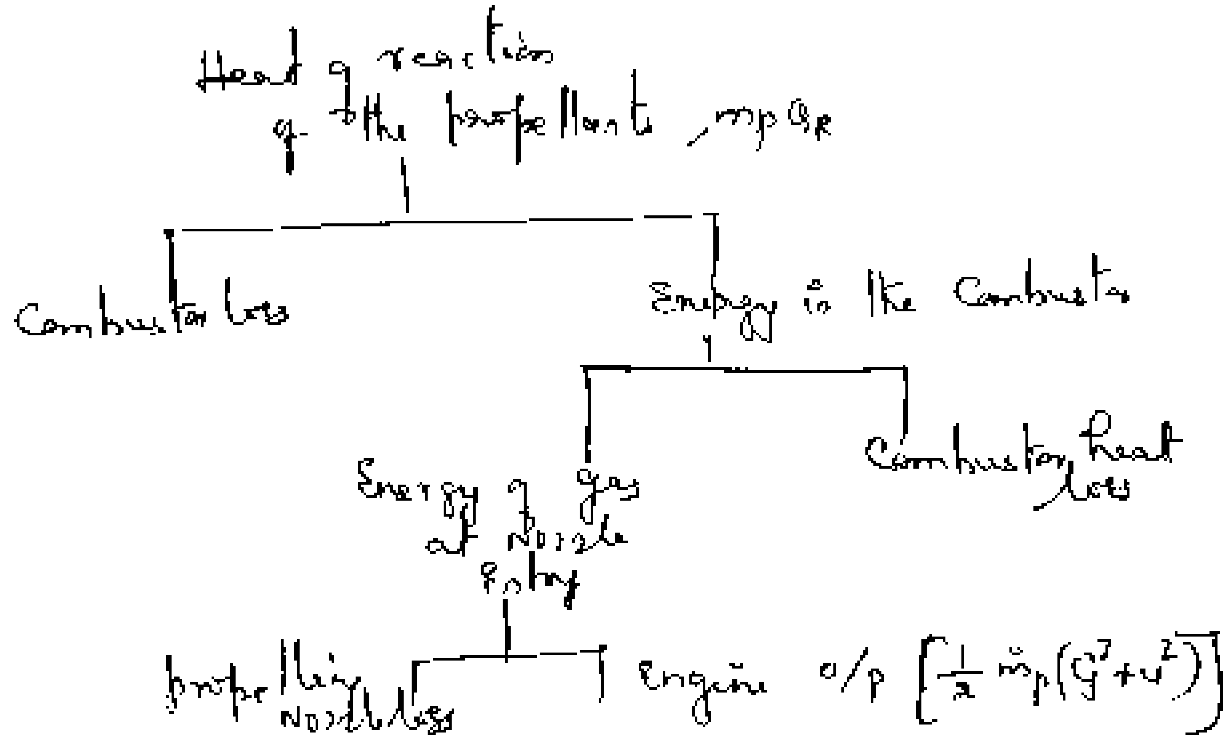
$$\eta_o = \eta_{ther} \times \eta_{prop}$$

$$= 2\left(\frac{c_j\, u}{c_j + u}\right) \times \frac{(c_j^2 + u^2)}{2\, Q_R}$$

$$\eta_o = \frac{c_j^2 + u^2}{2 \times Q_R} \times \frac{2\, c_j\, u}{(c_j^2 + u^2)} = \frac{c_j\, u}{Q_R}$$

$$\boxed{(F-1)\quad \eta_o = \eta_{ther} \times \eta_{prop}}$$

Losses



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Engine o/p — propulsive power
— Energy loss in the Exhaust gas $\frac{1}{2}\dot{m}_p(c_j - u)^2$

1) A rocket has the following data,
Propellant flow rate = 5 kg/s ($\gamma = 1.6$)
Nozzle exit diameter = 10 cm.
Nozzle exit pressure = 1.02 bar.
Ambient pressure = 1.013 bar
Thrust chamber pressure (P_0) = 20 bar
Thrust = 7 kN

(a) Determine the effective jet velocity, actual jet velocity, I_s and spc

(b) Recalculate the values of thrust & specific Impulse for an altitude where the ambient pressure is 10 mbar.

Case (a)

$\dot{m}_p = 0.5$ kg/s, $d_e = 0.10$ m, $P_e = 1.02$ bar, $P_a = 1.013$ bar
$F = 7\times10^3$ N, $P_0 = 20$ bar

c_j, c_e, spc, I_s

$$I_s = \frac{F}{\dot{w}_p}$$

$$\dot{w}_p = \frac{\dot{w}_p}{g} = \frac{0.5}{9.81}$$

$$\frac{\dot{w}_p}{g} = \dot{m}_p$$

$\dot{w}_p = 0.05 \times 9.81$

$\dot{w}_p = 49.01$ N/s

$I_s = \frac{F}{\dot{w}_p} = \frac{7000}{49.05} = 142.71$ seconds

$F = \dot{m}_p \times C_j$

$7000/5 = C_j = 1400\ m/s$

$A_e = A_j = \frac{\pi}{4} d_e^2 = \frac{\pi}{4}(0.10)^2 = 7.85 \times 10^{-3}\ m^2$

$F = \dot{m}_p C_e + (P_e - P_a) A_e$

$7000 = 5 \times C_e + (1.02 - 1.013) \times 10^5 \times 7.85 \times 10^{-3}$

$C_e = 1398.9\ m/s$

$SPC = \frac{1}{I_s} = \frac{1}{142.71} = 0.007\ s^{-1}$

Case b

$F' = \dot{m}_p C_e + (P_e - P_a') A_e$

$= 5 \times 1398.9 + (1.02 - 0.01) \times 10^5 \times 78.54 \times 10^{-4}$

$F' = 7.787\ kN$

$I_s' = \frac{7.787 \times 10^3}{49.05} = 158.77$ Seconds

Same as problem 1, $\gamma = 1.3$, determine the following quantities for the exit pressure of 1.02 bar, H_2, A_e/A^*, Thrust & propellant weight flow coeff & v^*

$\frac{P_e}{P_o} = \frac{1.02}{20} = 0.051$

6

From isentropic flow gas Table

$$\frac{P_e}{P_0} = 0.051, \quad M_e = 2.56, \quad \frac{A_e}{A^*} = 3.154$$

$$A_e = \frac{\pi}{4}(0.10)^2 = 78.54 \times 10^{-4}\ m^2$$

$$\frac{78.54 \times 10^{-4}}{A^*} = 3.154$$

$$\boxed{A^* = 24.90\ cm^2}$$

$$C_F = \frac{F}{P_0 A^*} = \frac{7 \times 1000}{20 \times 10^5 \times 24.90 \times 10^{-4}}$$

$$\boxed{C_F = 1.405}$$

$$C_\omega = \frac{\dot{\omega}_p}{P_0 \times A^*} = \frac{49.05}{20 \times 10^5 \times 24.90 \times 10^{-4}}$$

$$\boxed{C_\omega = 0.00985\ s^{-1}}$$

$$v^* = \frac{c_j}{c_F} = \frac{1400}{1.405} = \boxed{996.44\ m/s = v^*}$$

(3) A rocket flies at 10,080 kmph with an effective exhaust jet velocity of 1400 m/s & the propellant flow rate of 5 kg/s. If the heat of reaction of the propellants is 6500 kJ/kg of the propellant mixture determine (i) η_{prop} & propulsive power (ii) Engine output & η_{ther} (iii) η_0.

$$c_j = 1400\ m/s \qquad u = \frac{10,080 \times 1000}{60 \times 60} = 2800\ m/s$$

$$\sigma = \frac{u}{c_j} = \frac{2800}{1400} = 2.0$$

$$\eta_p = \frac{2a}{1+a^2} = \frac{2\times 2}{1+2^2} = 0.80 \times 100$$

80%

Propulsion

Thrust $f = \dot{m}_p \times C_j$

$= 5\times 1400 = 7000\ N$

propulsive power $= f\times u$

$= 7000 \times 2800 = 19.6\ MW$

Engine output $= \frac{1}{2}\dot{m}_p (C_j^2 + u^2)$

Eoy $= \frac{1}{2}\times 5 \;(1.4^2 + 2.8^2)$

$= 24.5 \times 10^6\ W$

$$\eta_{thermal} = \frac{\text{Engine o/p power}}{\dot{m}_p \times Q_R} = \frac{24.5\times 10^6}{5\times 6500\times 10^3}\times 100$$

$= 75.38\%$

$\eta_o = \eta_{ther} \times \eta_{prop}$ $= 0.7538 \times 0.80$

$= 60.3\%$

---

(4) A rocket Engine has the following data.
Thrust coefficient $= 1.2$, propellant flow rate $= 20\ N/s$
combustion chamber pressure $= 15$ bar, Exhaust
Nozzle throat diameter $= 5$ cm, from the above
data compute the values of Thrust, I_s, C_j & $\dot{V}^*$

$C_F = 12$, $\dot{\omega}_p = 20\ N/s$ / f, I_s, C_j & $\dot{V}^*$

$P_0 = 15$ bar $\quad d_t = 5\ cm = 5\times 10^{-2}\ m$

$A^* = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 5\times 10^{-2} = 1.96\times 10^{-3}\ m^2$

$$\boxed{A^* = 1.96\times 10^{-3}\ m^2}$$

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$$C_f = \frac{F}{P_0 A^*}$$

$$1.2 = \frac{F}{15\times10^5 \times 1.96\times10^{-3}}$$

$$\boxed{F = 3543.3 N}$$

$$I_{sp} = \frac{F}{W_p} = \frac{3543.3}{20} = 176.71 \text{ Sec}$$

$$F = \dot{m}_p \times C_j$$

$$3543.3 = \frac{\dot{W}_p}{g} \times C_j = \frac{20}{9.81} \times C_j$$

$$\boxed{C_j = 1733.57 \text{ m/s}}$$

$$V^* = \frac{C_j}{C_f} = \frac{1733.57}{1.2} = 1444.6 \text{ m/s}$$

(5) The data for a rocket Engine is given below, combustion chamber pressure = 40bar, combustion chamber temperature = 3500K, oxidizer flow rate = 40kg/s, mixture ratio = 5, properties of the Exhaust gases $\gamma = 1.3$, if the expansion in the rocket nozzle takes place to the ambient pressure 600 N/m².

Given data

$P_0 = 40$ bar, $T_0 = 3500$ K, $\dot{m}_{ox} = 40$ kg/s

$\frac{\dot{m}_o}{\dot{m}_f} = 5$,

$P_a = 600$ N/m²

$\gamma = 1.3$

$R = 287$ J/kg.K.

A^*, F, C_F, V^*, C_e 17

for complete expansion in Nozzle $P_e = P_a$.

$P_e = P_a = 600\ N/m^2$.

$$\frac{P_e}{P_0} = \frac{600}{40 \times 10^5} = 1.5 \times 10^{-4}$$

isentropic flow Table ($\gamma = 1.3$)

$$\frac{P_e}{P_0} \approx 10 \times 10^{-5}, \quad \frac{T_e}{T_0} = 0.119, \quad \frac{A_e}{A^*} = 285.3$$

$M_e = 7$..

$T_e = 0.119 \times 3500$

$$\boxed{T_e = 416.5\ K}$$

$$\frac{A_e}{A^*} = 285.3$$

From given data

$$\frac{\dot m_{oxi}}{\dot m_f} = 5, \quad \frac{40}{5} = \dot m_f = 8\ kg/s$$

$$\dot m_p = \dot m_o + \dot m_f = 40 + 8 = 48\ kg/s$$

$$M_e = \frac{C_e}{a_e} = \frac{C_e}{\sqrt{\gamma R T_e}}$$

$$7 = \frac{C_e}{\sqrt{1.3 \times 287 \times 416.5}}$$

$$\boxed{C_e = 2759.4\ m/s}$$

$$\dot m_p = \rho_e A_e C_e$$

$$48 = \frac{P_e}{R T_e} \times A_e \times C_e = \frac{600}{287 \times 416.5} \times A_e \times 2759.4$$

$$\boxed{A_e = 3.46\ m^2}$$

$$\frac{A_e}{A^*} = ~~285~~\ 285.3, \quad \boxed{A^* = 0.0121\ m^2}$$

Thrust $f = \dot{m}_p C_e$

$F = 48 \times 2759.4$

$f = 132451.2\ N$

$C_e = C_j$

$P_e = P_a$

$C_f = \frac{f}{P_0 A^*} = \frac{132451.2}{40 \times 10^5 \times 0.0121}$

$C_F = 2.73$

$v^* = \frac{C_j}{C_f} = \frac{C_e}{C_f} = \frac{2759.4}{2.73}$

$v^* = 1010.76\ m/s$

(b) A rocket Engine has following data effective jet velocity = 1250 m/s, flight to jet speed ratio = 0.80, oxidizer flow rate = 3.5 kg/s fuel flow rate = 1 kg/s, heat of reaction per kg of the exhaust gases = 2500 KJ/kg. determine (i) Thrust (ii) specific Impulse (iii) η_{prop} (iv) η_{ther} (v) η_o.

given data

$C_j = 1250\ m/s$, $\sigma = 0.80$, $\dot{m}_{oxi} = 3.5\ kg/s$

$\dot{m}_{fuel} = 1\ kg/s$

$Q.R = 2500 \times 10^3\ J/kg$

$\dot{m}_p = \dot{m}_{oxi} + \dot{m}_{fuel} = 3.5 + 1 = 4.5\ kg/s$

$\sigma = \frac{u}{C_j} = 0.8 = \frac{u}{1250}$

$u = 1000\ m/s$

Thrust $f = \dot{m}_p \times C_j$

$= 4.5 \times 1250$

$f = 5625 N$

$I_s = \frac{f}{\dot{\omega}_p} = \frac{5625}{\dot{m}_p \times g} = \frac{5625}{4.5 \times 9.81}$

$I_s = 127.42 sec$

$\eta_{prop} = \frac{2a}{1+a^2} = \frac{2 \times 0.8}{1+(0.8)^2} = 97\%$

$\eta_{ther} = \frac{C_j^2 + u^2}{2 \times C.V} = \frac{1250^2 + 1000^2}{2 \times 2500 \times 10^3}$

$\eta_{th} = 51.25\%$

$\eta_o = \eta_p \times \eta_{ther} = 0.97 \times 0.5125$

$= 50\%$

(9) A rocket Nozzle has a throat area of $20 cm^2$ and combustion chamber pressure of 25bar. If the specific impulse is 130 sec & weight flow rate 45 N/s determine (a) Thrust coefficient (b) propellant weight flow coefficient (c) SPC, (d) V^*

$A^* = 20 cm^2 = 20 \times 10^{-4} m^2$ $P_o = 25 bar$ $I_s = 130$, $\dot{\omega}_p = 45 N/s$

specific impulse $I_s = \frac{F}{\dot{\omega}_p}$

$130 = \frac{f}{45}$

$f = 5850 N$

$C_f = \frac{F}{P_o A^*} = \frac{5850}{25 \times 10^5 \times 20 \times 10^{-4}}$

$C_F = 1.17$

$C_w = \frac{\dot{\omega}_p}{P_o \times A^*} = \frac{45}{25 \times 10^5 \times 20 \times 10^{-4}}$

$C_w = 9 \times 10^{-3}$

(9)

Sri Venkateswara College of Engineering
Continuous Assessment Test

ROLL NUMBER	
NAME	
YEAR & BRANCH	
SUBJECT	

$SPC = \frac{\omega_p}{F} = \frac{45}{5850} = 7.69 \times 10^{-3}$

$F = m_p \times C_j$

$5850 = \frac{45}{9.81} \times C_j$

$C_j = 1275.3 m/s$

$C \quad V^* = \frac{C_j}{C_f}$

$= \frac{1275.3}{1.17}$

$V^* = 1090 m/s$

(8) ~~Calculate the thrust, C_j, I_s for a rocket operating at ~~z=~~ 20 km with the following data.~~

propellant flow rate = 1.0 kg/s.

Thrust chamber pressure = 27.5 bar

" " Temp = 2400 K

$F =$

$C_j, \quad I_s$

$\frac{A_e}{A^*} = 10.2$, $\gamma = 1.3$, $R = 355$ J/kg.K for exhaust gases

$z = 20 \times 10^3 m \longrightarrow T_1 = 216.65$, $a_1 = 295.2$, $P_1 = 0.054 P$

$\rho_1 = 0.08 \rho$

$\dot{m}_p = 1.0$

$P_0 = 27.5$ bar, $T_0 = 2400$ k

@ $\frac{A_e}{A^*} = 10.2$ → (NYI), $M_e = 3.60$, $\frac{T_e}{T_0} = 0.339$

$\frac{P_e}{P_0} = 0.0093$

$\frac{T_e}{T_0} = 0.339$

$T_e = 813.6 K$

$\frac{P_e}{P_0} = 0.0093$

$P_e = 0.255575$ bar

(P.S no. 26)

$\dot{m}_p = \rho_e A_e C_e$

$m_p = 1.0 \, kg/s$

$1.0 = \frac{P_e}{R T_e} \times A_e \times C_e$ (struck through)

$M_e = \frac{C_e}{a_e}$

$M_e = \frac{C_e}{\sqrt{\gamma R T_e}}$

$3.6 = \frac{C_e}{\sqrt{1.3 \times 355 \times 813.6}}$

$C_e = 2205.9 \, m/s$

$F = m_p \times C_e$

$= 1 \times 2205.9$

$F = 2.205 \, kN$

$I_s = \frac{F}{\omega_p} = \frac{2.205 \times 10^3}{m_p \times g} = \frac{2.205 \times 10^3}{1 \times 9.81} = 224.85 \, sec$